

Constants:

$$\begin{aligned}
 N_A &= 6.022 \times 10^{23} \text{ things/mol} \\
 k_e &\equiv 1/4\pi\epsilon_0 = 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} \\
 \epsilon_0 &= 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \\
 \mu_0 &\equiv 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \\
 e &= 1.60218 \times 10^{-19} \text{ C} \\
 \hbar &= 6.6261 \times 10^{-34} \text{ J} \cdot \text{s} = 4.1357 \times 10^{-15} \text{ eV} \cdot \text{s} \\
 \hbar &= \frac{h}{2\pi} \quad hc = 1239.84 \text{ eV} \cdot \text{nm} \\
 k_B &= 1.38065 \times 10^{-23} \text{ J} \cdot \text{K}^{-1} = 8.6173 \times 10^{-5} \text{ eV} \cdot \text{K}^{-1} \\
 c &= \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.99792 \times 10^8 \text{ m/s} \\
 m_e &= 9.10938 \times 10^{-31} \text{ kg} \quad m_e c^2 = 510.998 \text{ keV} \\
 m_p &= 1.67262 \times 10^{-27} \text{ kg} \quad m_p c^2 = 938.272 \text{ MeV} \\
 m_{n_0} &= 1.67493 \times 10^{-27} \text{ kg} \quad m_n c^2 = 939.565 \text{ MeV}
 \end{aligned}$$

Schrödinger

$$\begin{aligned}
 i\hbar \frac{\partial \Psi}{\partial t} &= -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \Psi + V(x)\Psi \\
 E\Psi &= -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \Psi + V(x)\Psi \\
 \int_{-\infty}^{\infty} |\psi(x)|^2 dx &= 1 \quad P(\text{in } [x, x+dx]) = |\psi(x)|^2 \quad 1D \\
 \int_{-\infty}^{\infty} |\psi(r)|^2 4\pi r^2 dr &= 1 \quad P(\text{in } [r, r+dr]) = 4\pi r^2 |\psi(r)|^2 \quad 3D \\
 \langle x^n \rangle &= \int_{-\infty}^{\infty} x^n P(x) dx \quad 1D \quad \langle r \rangle = \int_{-\infty}^{\infty} r^n P(r) dr \quad 3D \\
 \Delta x &= \sqrt{\langle x^2 \rangle - \langle x \rangle^2}
 \end{aligned}$$

Basic Equations:

$$\begin{aligned}
 \vec{F}_{\text{net}} &= m\vec{a} \quad \text{Newton's Second Law} \\
 \vec{F}_{\text{centr}} &= -\frac{mv^2}{r} \hat{r} \quad \text{Centripetal} \\
 \vec{F}_{12} &= k_e \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} = q_2 \vec{E}_1 \quad \vec{r}_{12} = \vec{r}_1 - \vec{r}_2 \\
 \vec{E}_1 &= \vec{F}_{12}/q_2 = k_e \frac{q_1}{r_{12}^2} \hat{r}_{12} \\
 \vec{F}_B &= q\vec{v} \times \vec{B} \\
 0 &= ax^2 + bx^2 + c \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
 \end{aligned}$$

Oscillators

$$\begin{aligned}
 E &= \left(n + \frac{1}{2}\right) \hbar f \\
 E &= \frac{1}{2} k A^2 = \frac{1}{2} \omega^2 m A^2 = 2\pi^2 m f^2 A^2 \\
 \omega &= 2\pi f = \sqrt{k/m}
 \end{aligned}$$

Approximations, $x \ll 1$

$$\begin{aligned}
 (1+x)^n &\approx 1 + nx + \frac{1}{2} n(n+1)x^2 \quad \tan x \approx x + \frac{1}{3}x^3 \\
 e^x &\approx 1 + x + \frac{1}{2}x^2 \quad \sin x \approx x - \frac{1}{6}x^3 \quad \cos x \approx 1 - \frac{1}{2}x^2
 \end{aligned}$$

Misc Quantum

$$\begin{aligned}
 E &= hf \quad p = h/\lambda = E/c \quad \lambda f = c \quad \text{photons} \\
 \lambda_f - \lambda_i &= \frac{h}{m_e c} (1 - \cos \theta) \\
 \lambda &= \frac{h}{|\vec{p}|} = \frac{h}{\gamma m v} \approx \frac{h}{m v} \\
 \Delta x \Delta p &\geq \frac{\hbar}{4\pi} \quad \Delta E \Delta t \geq \frac{\hbar}{4\pi} \\
 eV_{\text{stopping}} &= KE_{\text{electron}} = hf - \phi = hf - W
 \end{aligned}$$

Bohr

$$\begin{aligned}
 E_n &= -13.6 \text{ eV}/n^2 \\
 E_i - E_f &= -13.6 \text{ eV} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = hf \\
 L = mvr &= n\hbar \\
 v^2 &= \frac{n^2 \hbar^2}{m_e^2 r^2} = \frac{k_e e^2}{m_e r}
 \end{aligned}$$

Quantum Numbers

$$\begin{aligned}
 l &= 0, 1, 2, \dots, (n-1) \quad L^2 = l(l+1)\hbar^2 \\
 m_l &= -l, (-l+1), \dots, l \quad L_z = m_l \hbar \\
 m_s &= \pm \frac{1}{2} \quad S_z = m_s \hbar \quad S^2 = s(s+1)\hbar^2 \\
 \text{dipole transitions: } \Delta l &= \pm 1, \Delta m_l = 0, \pm 1, \Delta m_s = 0 \\
 \mu_{sz} &= \pm \mu_B \\
 \vec{\mu}_s &= 2\vec{S} \mu_B \\
 E_{\mu} &= -\vec{\mu} \cdot \vec{B} \\
 J^2 &= j(j+1)\hbar^2 \quad j = l \pm \frac{1}{2} \\
 J_z &= m_j \hbar \quad m_j = -j, (-j+1), \dots, j
 \end{aligned}$$

Calculus of possible utility:

$$\begin{aligned}
 \int \frac{1}{x} dx &= \ln x + c \\
 \int u dv &= uv - \int v du \\
 \int \sin ax dx &= -\frac{1}{a} \cos ax + C \\
 \int \cos ax dx &= \frac{1}{a} \sin ax + C \\
 \frac{d}{dx} \tan x &= \sec^2 x = \frac{1}{\cos^2 x} \\
 \int_0^{\infty} x^n e^{-ax} dx &= \frac{n!}{a^{n+1}} \\
 \int_0^{\infty} x^2 e^{-ax^2} dx &= \frac{1}{4} \sqrt{\frac{\pi}{a^3}} \\
 \int_{-\infty}^{\infty} x^3 e^{-ax^2} dx &= \int_{-\infty}^{\infty} x e^{-ax^2} dx = 0 \\
 \int_0^{\infty} x^4 e^{-ax^2} dx &= \frac{3}{8} \sqrt{\frac{\pi}{a^5}}
 \end{aligned}$$