Name & CWID ______________________________

PH 253 Final Exam

Instructions

1. Solve six of the problems below. All problems have equal weight.
2. Clearly mark your which problems you have chosen.
3. Do your work on separate sheets. Staple them to this exam paper when you are finished.
4. You are allowed 2 sheet of standard 8.5x11 in paper and a calculator.

1. A particle of mass m is confined to a one-dimensional box of width L, that is, the potential energy of the particle is infinite everywhere except in the interval $0<x<L$, where its potential energy is zero. The particle is in its ground state. What is the probability that a measurement of the particle's position will yield a result in the left quarter of the box? The wavefunction for a particle in a 1D box may be written

$$\psi(x) = A \sin Bnx \tag{1}$$

where A and B are constants you will need to find, and n is an integer. *Hint: normalize and apply boundary conditions.*

2. The wave function for the ground state of hydrogen ($n=1$) is

$$\psi_1 = \frac{1}{\sqrt{\pi a_o^3}} e^{-r/a_o} \tag{2}$$

where a_o is the Bohr radius.

(a) What is the *most probable* value of r for the ground state?
(b) What is the total probability of finding the electron at a distance greater than this radius?

3. Schrödinger's equation for a simple harmonic oscillator reads

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{1}{2}m\omega^2x^2\psi = E\psi \tag{3}$$

The ground state wave function has the form

$$\psi_o = ae^{-\alpha^2x^2} \tag{4}$$

Determine the value of the constant α and the energy of the state.

4. By considering the visible spectrum of hydrogen and He^+, show how you could determine spectroscopically if a sample of hydrogen was contaminated with helium. (Hint: look for differences in the visible emission lines, $\lambda \approx 390 \sim 750$ nm. A difference of 10 nm is easily measured.)

Name & CWID ____

5. A meterstick makes an angle of $30°$ with respect to the x'-axis of O'. What must be the value of v if the meterstick makes an angle of $45°$ with respect to the x-axis of O?

6. A Σ^0 particle at rest decays to a Λ^0 particle and a photon. Determine the energy of the released photon, given that the Σ^0 has rest energy $m_\Sigma c^2 = 1192\,\text{MeV}$ and the Λ^0 has rest energy $m_{\Lambda^0} c^2 = 1116\,\text{MeV}$.

7. A plane, $300\,\text{MHz}$ electromagnetic wave is incident normally on a surface of area $50\,\text{cm}^2$. If the intensity of the wave is $9 \times 10^{-5}\,\text{W/m}^2$,
(a) Determine the rate at which photons strike the surface.
(b) Determine the force on the surface if it is perfectly reflecting.

8. A $0.3\,\text{MeV}$ X-ray photon makes a "head on" collision with an electron initially at rest. Using conservation of energy and momentum, find the recoil velocity of the electron. Check your result with the Compton formula.

9. **(a)** How many different photons can be emitted by hydrogen atoms that undergo transitions from the ground state from the $n = 4$ state? **(b)** Enumerate their energies, in electron volts.

10. Neglecting spin, in a strong external magnetic field of $5\,\text{T}$, determine the lines resulting from the $2p \to 1s$ transition ($\lambda_o = 121.0\,\text{nm}$) in hydrogen. Provide a sketch of the energy levels and their m_l values.

11. **(a)** For a free relativistic quantum particle moving with speed v, the total energy is $E = hf = \hbar\omega = \sqrt{p^2c^2 + m^2c^4}$ and the momentum is $p = h/\lambda = \hbar k = \gamma m v$. For the quantum wave representing the particle, the group speed is $v_g = d\omega/dk$. Prove that the group speed of the wave is the same as the speed of the particle.

(b) It is convenient to describe the motion of an electron (or a hole) in a band by giving it an *effective mass* m^*, defined by

$$\frac{1}{m^*} \equiv \frac{1}{\hbar^2}\frac{d^2E}{dk^2} \tag{5}$$

where $k = 2\pi/\lambda$ is the wave number. For a free electron ($p = \hbar k$), show that $m^* = m$.

12. An interstellar space probe is moving at a constant speed relative to earth of $0.76c$ toward a distant planet. Its radioisotope generators have enough energy to keep its data transmitter active continuously for 15 years, as measured in their own reference frame.

(a) How long do the generators last as measured from earth?
(b) How far is the probe from earth when the generators fail, as measured from earth?
(c) How far is the probe from earth when the generators fail, *as measured by its built-in trip odometer*?

Constants:

$$N_A = 6.022\times10^{23}\text{ things/mol}$$
$$k_e \equiv 1/4\pi\epsilon_o = 8.98755\times10^{9}\,\text{N}\cdot\text{m}^2\cdot\text{C}^{-2}$$
$$\epsilon_o = 8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2$$
$$\mu_o \equiv 4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}$$
$$e = 1.60218\times10^{-19}\,\text{C}$$
$$h = 6.6261\times10^{-34}\,\text{J}\cdot\text{s} = 4.1357\times10^{-15}\,\text{eV}\cdot\text{s}$$
$$\hbar = \frac{h}{2\pi} \qquad hc = 1239.84\,\text{eV}\cdot\text{nm}$$
$$k_B = 1.38065\times10^{-23}\,\text{J}\cdot\text{K}^{-1} = 8.6173\times10^{-5}\,\text{eV}\cdot\text{K}^{-1}$$
$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 2.99792\times10^{8}\,\text{m/s}$$
$$m_e = 9.10938\times10^{-31}\,\text{kg} \qquad m_e c^2 = 510.998\,\text{keV}$$
$$m_p = 1.67262\times10^{-27}\,\text{kg} \qquad m_p c^2 = 938.272\,\text{MeV}$$
$$m_n = 1.67493\times10^{-27}\,\text{kg} \qquad m_n c^2 = 939.565\,\text{MeV}$$

Schrödinger

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi + V(x)\Psi$$
$$E\psi = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V(x)\psi$$
$$\int_{-\infty}^{\infty}|\psi(x)|^2\,dx = 1 \qquad P(\text{in }[x, x+dx]) = |\psi(x)|^2 \qquad \text{1D}$$
$$\int_{0}^{\infty}|\psi(r)|^2\,4\pi r^2\,dr = 1 \qquad P(\text{in }[r, r+dr]) = 4\pi r^2|\psi(r)|^2 \qquad \text{3D}$$
$$\langle x^n\rangle = \int_{-\infty}^{\infty}x^n P(x)\,dx \qquad \text{1D} \qquad \langle r^n\rangle = \int_0^{\infty}r^n P(r)\,dr \qquad \text{3D}$$
$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$$

Basic Equations:

$$\vec{F}_{net} = m\vec{a}\quad\text{Newton's Second Law}$$
$$\vec{F}_{centr} = -\frac{mv^2}{r}\,\hat{r}\quad\text{Centripetal}$$
$$\vec{F}_{12} = k_e\frac{q_1q_2}{r_{12}^2}\,\hat{r}_{12} = q_2\vec{E}_1 \qquad \vec{r}_{12} = \vec{r}_1 - \vec{r}_2$$
$$\vec{E}_1 = \vec{F}_{12}/q_2 = k_e\frac{q_1}{r_{12}^2}\,\hat{r}_{12}$$
$$\vec{F}_B = q\vec{v}\times\vec{B}$$
$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Oscillators

$$E = \left(n + \frac{1}{2}\right)hf$$
$$E = \frac{1}{2}kA^2 = \frac{1}{2}\omega^2 mA^2 = 2\pi^2 mf^2A^2$$
$$\omega = 2\pi f = \sqrt{k/m}$$

Approximations, $x \ll 1$

$$(1+x)^n \approx 1 + nx + \frac{1}{2}n(n+1)x^2 \qquad \tan x \approx x + \frac{1}{3}x^3$$
$$e^x \approx 1 + x + \frac{1}{2}x \qquad \sin x \approx x - \frac{1}{6}x^3 \qquad \cos x \approx 1 - \frac{1}{2}x^2$$

Misc Quantum

$$E = hf \qquad p = h/\lambda = E/c \qquad \lambda f = c \qquad \text{photons}$$
$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1-\cos\theta)$$
$$\lambda = \frac{h}{|\vec{p}|} = \frac{h}{\gamma mv} \approx \frac{h}{mv}$$
$$\Delta x\Delta p \geqslant \frac{h}{4\pi} \qquad \Delta E\Delta t \geqslant \frac{h}{4\pi}$$
$$eV_{\text{stopping}} = KE_{\text{electron}} = hf - \varphi = hf - W$$

Bohr

$$E_n = -13.6\,\text{eV}/n^2 \qquad \text{Hydrogen}$$
$$E_n = -13.6\,\text{eV}\left(Z^2/n^2\right) \qquad Z\text{ protons, 1 }e^-$$
$$\Delta E = -13.6\,\text{eV}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = hf$$
$$L = mvr = n\hbar$$
$$v^2 = \frac{n^2\hbar^2}{m_e^2r^2} = \frac{k_e e^2}{m_e r}$$

Quantum Numbers

$$l = 0, 1, 2, \ldots, (n-1) \qquad L^2 = l(l+1)\hbar^2$$
$$m_l = -l, (-l+1), \ldots, l \qquad L_z = m_l\hbar$$
$$m_s = -\pm\frac{1}{2} \qquad S_z = m_s\hbar \qquad S^2 = s(s+1)\hbar^2$$
$$\text{dipole transitions: } \Delta l = \pm1, \Delta m_l = 0, \pm1, \Delta m_s = 0$$
$$\mu_{sz} = \pm\mu_B$$
$$\vec{\mu}_s = 2\vec{S}\,\mu_B$$
$$E_\mu = -\vec{\mu}\cdot\vec{B}$$
$$J^2 = j(j+1)\hbar^2 \qquad j = l\pm\frac{1}{2}$$
$$J_z = m_j\hbar \qquad m_j = -j, (-j+1), \ldots, j$$

Calculus of possible utility:

$$\int\frac{1}{x}\,dx = \ln x + c$$
$$\int u\,dv = uv - \int v\,du$$
$$\int\sin ax\,dx = -\frac{1}{a}\cos ax + C$$
$$\int\cos ax\,dx = \frac{1}{a}\sin ax + C$$
$$\frac{d}{dx}\tan x = \sec^2 x = \frac{1}{\cos^2 x}$$
$$\int_0^{\infty}x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$
$$\int_0^{\infty}x^2e^{-ax^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$
$$\int_{-\infty}^{\infty}x^3e^{-ax^2}\,dx = \int_{-\infty}^{\infty}xe^{-ax^2}\,dx = 0$$
$$\int_0^{\infty}x^4e^{-ax^2}\,dx = \frac{3}{8}\sqrt{\frac{\pi}{a^5}}$$

E & M

$$\vec{F}_{12} = k_e\frac{q_1q_2}{r_{12}^2}\,\hat{r}_{12} = q_2\vec{E}_1 \qquad \vec{r}_{12} = \vec{r}_1 - \vec{r}_2$$
$$\vec{E}_1 = \vec{F}_{12}/q_2 = k_e\frac{q_1}{r_{12}^2}\,\hat{r}_{12}$$
$$\vec{F}_B = q\vec{v}\times\vec{B}$$

Blackbody

$$E_{tot} = \sigma T^4 \qquad \sigma = 5.672 \times 10^{-8}\,\mathrm{W \cdot m^{-2} \cdot K^{-4}}$$

$$T\lambda_{max} = 0.29 \times 10^{-2}\,\mathrm{m \cdot K} \qquad \text{Wien}$$

$$E_{quantum} = hf$$

$$E_{oscillator} = hf/\left(e^{hf/k_B T} - 1\right)$$

$$I(\lambda, T) = \frac{(\text{const})}{\lambda^5}\left[e^{\frac{hc}{\lambda k_b T}} - 1\right]^{-1}$$

$$I(f, t) = (\text{const})\, f^3 \left[e^{\frac{hf}{k_b T}} - 1\right]^{-1}$$

Relativity

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t'_{moving} = \gamma \Delta t_{stationary} = \gamma \Delta t_p$$

$$L'_{moving} = \frac{L_{stationary}}{\gamma} = \frac{L_p}{\gamma}$$

$$x' = \gamma\,(x - vt)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$v_{obj} = \frac{v + v'_{obj}}{1 + \frac{v v'_{obj}}{c^2}} \qquad v'_{obj} = \frac{v_{obj} - v}{1 - \frac{v v_{obj}}{c^2}}$$

$$KE = (\gamma - 1)mc^2 = \sqrt{m^2c^4 + c^2p^2} - mc^2$$

$$E_{rest} = mc^2$$

$$p = \gamma m v$$

$$E^2 = p^2c^2 + m^2c^4 = \left(\gamma mc^2\right)^2$$

Vectors:

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} \quad \text{magnitude} \qquad \theta = \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{direction}$$

$$\hat{r} = \vec{r}/|\vec{r}| \quad \text{construct any unit vector}$$

$$\text{let} \quad \vec{a} = a_x\,\hat{x} + a_y\,\hat{y} + a_z\,\hat{z} \quad \text{and} \quad \vec{b} = b_x\,\hat{x} + b_y\,\hat{y} + b_z\,\hat{z}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = \sum_{i=1}^{n} a_i b_i = |\vec{a}||\vec{b}|\cos\theta$$

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y)\,\hat{x} + (a_z b_x - a_x b_z)\,\hat{y} + (a_x b_y - a_y b_x)\,\hat{z}$$

Units

$$1\,\mathrm{T \cdot m/A} = 1\,\mathrm{N/A^2}$$

$$1\,\mathrm{T \cdot m^2} = 1\,\mathrm{V \cdot s}$$

$$1\,\mathrm{T} = 1\,\mathrm{kg/A \cdot s^2}$$

$$1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$$

$$1\,\mathrm{J} = 1\,\mathrm{N \cdot m} = 1\,\mathrm{kg \cdot m^2/s^2}$$

$$1\,\mathrm{N} = 1\,\mathrm{kg \cdot m/s^2}$$

$$1\,\mathrm{W} = 1\,\mathrm{J/s} = 1\,\mathrm{kg \cdot m^2/s^3}$$

$$1\,\mathrm{F} = 1\,\mathrm{C/V} \qquad 1\,\mathrm{C} = 1\,\mathrm{A/s}$$

$$1\,\mathrm{N/C} = 1\,\mathrm{V/m}$$