Name Date

PH 102 Quiz 7: Have you done your homework yet?

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$$E = hf = \frac{hc}{\lambda}$$
$$e\Delta V = KE_{\text{max}} = hf - \phi$$
$$\lambda_{\text{out}} - \lambda_{\text{in}} = \frac{h}{m_e c}(1-\cos\theta)$$
$$h = \lambda|\vec{\mathbf{p}}|$$

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$
$$h = 6.624\times 10^{-34}\,\text{J}\cdot\text{s}$$
$$e = 1.602\times 10^{-19}\,\text{C}$$
$$c = 3.00\times 10^{8}\,\text{m/s}$$
$$m_e = 9.11\times 10^{-31}\,\text{kg}$$

1. An FM radio transmitter has a power output of 130 kW and operates at a frequency of 98.3 MHz. How many photons per second does the transmitter emit?

- ⊗ 2×10^{30}
- ○ 5×10^{-29}
- ○ 1×10^{15}
- ○ 7×10^{18}

The power output $\mathcal{P}$ is the total energy per unit time, or the energy *per photon* times the number of photons per unit time:

$$\mathcal{P} = \frac{E_{\text{tot}}}{\Delta t} = \frac{(\#\text{ photons})\,E_{\text{photon}}}{\Delta t} \tag{1}$$

The energy per photon is given above, $E=hf$. Combining those two:

$$\mathcal{P} = \frac{(\#\text{ photons})\,E_{\text{photon}}}{\Delta t} = \frac{(\#\text{ photons})\,hf}{\Delta t} \tag{2}$$

Solve for the number of photons per unit time, and plug in the numbers:

$$\frac{(\#\text{ photons})}{\Delta t} = \frac{\mathcal{P}}{hf} = \frac{130\times 10^{6}\,\text{W}}{[6.624\times 10^{-34}\,\text{J}\cdot\text{s}]\times[98.3\times 10^{6}\,\text{s}^{-1}]} \approx 2\times 10^{30}\,\text{photons/s} \tag{3}$$

2. Light of wavelength 220 nm falls on a carbon surface, and electrons with 0.64 eV kinetic energy are emitted. What is the work function of carbon?

- ○ 4 eV
- ○ 3 eV
- ⊗ 5 eV
- ○ 0.2 eV

This is the photoelectric effect. The work function ϕ can be related to the maximum kinetic energy of ejected electrons and the energy of the incident light:

$$KE_{\text{max}} = hf - \phi = E_{\text{photon}} - \phi \tag{4}$$

The work function is just the amount of energy required to liberate an electron from the metal surface into the vacuum. Thus, the incident light energy per photon, hf, first has to overcome the work function ϕ, and what ever energy is left over is given to the ejected electron as kinetic energy.

First, find we need the energy of the incident photons, hf. We will want this in eV, not J, so get that out of the way ...

$$E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{\left[6.624 \times 10^{-34}\,\text{J}\cdot\text{s}\right]\left[3.00 \times 10^{8}\,\text{m/s}\right]}{220 \times 10^{-9}\,\text{m}} \times \frac{1\,\text{eV}}{1.602 \times 10^{-19}\,\text{J}} \approx 5.64\,\text{eV} \tag{5}$$

$$\phi = hf - KE = E_{\text{photon}} - KE = 5.64 - 0.64\,\text{eV} \approx 5\,\text{eV} \tag{6}$$

Incidentally: these problems are much easier if you remember (or write down) that the product hc is about 1240 eV·nm, so one has $E_{\text{photon}}(\text{eV}) = 1240/\lambda(nm)$. This already takes into account dividing hc by e to get units of eV.

3. What is the minimum accelerating voltage required to produce a photon with $\lambda = 1$ mm?

- ⊗ 1.2 mV
- ◯ 1.2 V
- ◯ 1.2 kV
- ◯ 0.12 V

The word "accelerating" is a decoy here, of sorts. What we are really asking is what is the minimum potential energy, given by an accelerating voltage, that could be used to produce a photon of the desired wavelength? The idea is that all of the potential energy from the voltage applied to a single charge e is used for creating a photon.

Now, remember the formula from the last problem, putting the photon energy in eV, and note that 1 mm is 10^6 nm:

$$\begin{aligned} PE &= hf = \frac{hc}{\lambda} \\ eV &= \frac{hc}{\lambda} \\ V &= \frac{hc}{e\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{\lambda}\,\text{eV} = \frac{1.240 \times 10^{3}}{10^{6}} = 1.24 \times 10^{-3}\,\text{eV} = 1.24\,\text{meV} \end{aligned}$$

Since we are talking about the energy given to a single charge, we can just write 1.24 mV in place of 1.24 meV.

Of course, you can just plug in the constants and numbers in the usual way, and not use the formula from problem 2, it will work out the same way.

4. X-rays with an energy of 320 keV undergo Compton scattering, and are deflected by 42°. What is the energy of the scattered X-ray?

- ◯ 302 keV
- ◯ 161 keV
- ⊗ 275 keV
- ◯ 381 keV

This one has a couple of steps. First, Compton scattering involves a photon coming in, and scattering off of an electron. Afterward, the photon has a reduced energy, and the electron has gained energy. So right off, we know the scattered X-ray had better have a lower energy than the incident X-ray. This rules out the last choice already.

Now, remember the formula for Compton scattering. This gives you the *change* in wavelength between the incident and scattered light:

$$\begin{aligned} \lambda_{\text{out}} - \lambda_{\text{in}} &= \frac{h}{m_e c}(1 - \cos\theta) \\ \lambda_{\text{out}} &= \lambda_{\text{in}} + \frac{h}{m_e c}(1 - \cos\theta) \end{aligned}$$

We can find λ_{in} from the known energy of the incident X-ray:

$$\lambda_{\text{in}} = \frac{hc}{E_{\text{in}}} = \frac{1240}{320 \times 10^3}\,\text{nm} \approx 3.875 \times 10^{-12}\,\text{m} \tag{7}$$

Here we again used the modified formula from problem 2. Now we can calculate the scattered X-ray's wavelength:

$$\begin{aligned}\lambda_{\text{out}} &= \lambda_{\text{in}} + \frac{h}{m_e c}\left(1 - \cos\theta\right) = 3.875 \times 10^{-12}\,\text{m} + \frac{6.624 \times 10^{-34}\,\text{J}\cdot\text{s}}{[9.11 \times 10^{-31}\,\text{kg}]\,[3.00 \times 10^{8}\,\text{m}]}\left(1 - \cos 42^\circ\right) \\ &\approx 3.875 \times 10^{-12} + 6.23 \times 10^{-13}\,\text{m} \approx 4.5 \times 10^{-12}\,\text{m}\end{aligned}$$

Finally, given the scattered X-ray's wavelength, we can find its energy.

$$E_{\text{out}} = \frac{1240\,\text{eV}\cdot\text{nm}}{4.5 \times 10^{-3}\,\text{nm}} \approx 2.75 \times 10^5\,\text{eV} = 275\,\text{keV} \tag{8}$$

With Compton scattering, the wavelength change is small ... be sure to carry 4-5 digits precision in all stages of the calculation (which I did *not* show here) to avoid rounding errors.

5. A molecule is known to exist in an unstable higher energy configuration for $\Delta t = 10\,\text{nsec}$, after which it relaxes to its lower energy stable state by emitting a photon. What uncertainty in the frequency of the emitted photon is implied?

- ◯ 6 kHz
- ◯ 7 GHz
- ⊗ 8 MHz
- ◯ 9 Hz

Energy-time uncertainty tells us that there is a limit on the accuracy with which the energy of a system can be measured over a given time interval Δt. The lifetime of the excited state gives us a specific time interval over which the molecule is in an excited state, and hence, in a specific energy state. This implies an uncertainty in the energy of the excited state ΔE.

$$\Delta E \Delta t \geq \frac{h}{4\pi} \tag{9}$$

When the molecule decays into the lower, stable state, a photon is emitted. The photon has to have an energy equal to the difference in energies between the excited and lower energy states of the molecule, ΔE. This is just conservation of energy. Since the emitted photon's frequency is simply related to its frequency, $E_{\text{photon}} = hf$, we can write:

$$\begin{aligned}\Delta E \Delta t = \Delta\left(hf\right)\Delta t = h\Delta f \Delta t &\geq \frac{h}{4\pi} \\ \not{h}\Delta f \Delta t &\geq \frac{\not{h}}{4\pi} \\ \Delta f &\geq \frac{1}{4\pi\Delta t} = \frac{1}{4\pi \times 10^{-9}\,\text{s}} \approx 8 \times 10^6\,\text{Hz} = 8\,\text{MHz}\end{aligned}$$