NAME & ID ____

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Quiz 1: Relativity

Instructions:

1. Answer all questions below. They have equal weight.
2. Express your answer with the appropriate units and significant digits
3. Show your work for full credit.

1. Neutrons have an average lifetime of 15 minutes when at rest in the laboratory. What is the average lifetime (measured in the lab) of a neutron moving at a speed of 0.5c?

The neutron's lifetime is always the same in its own reference frame, a constant 15 minutes which we will call the 'proper' time interval Δt_p. In the laboratory, we are in motion *relative to the neutron*, and hence we measure a dilated (longer) time interval $\Delta t'$. For each speed, then, we just need to calculate the dilated time interval, and that is the observed lifetime in the laboratory frame. The dilated interval measured in the lab is then:

$$\Delta t' = \gamma \Delta t_p = \gamma \left(15\,\text{min}\right)$$

Basically we just need to calculate γ for the given speed v.

$$\Delta t' = \gamma \left(15\,\text{min}\right) = \frac{1}{\sqrt{1-\left(0.5c\right)^2/c^2}} \left(15\,\text{min}\right) = \frac{15\,\text{min}}{\sqrt{1-0.5^2}} = 1.15 \left(15\,\text{min}\right) = 17.3\,\text{min}$$

2. If a duck has a time dilation factor of 10, what is its speed?

What does this mean? If the duck has a time dilation factor of 10, that means it is moving so fast relative to an external observer that it feels a passage of time which is 10 times slower than the observer. The situation is then that we have an observer on the ground, measuring proper time, who sees a duck fly by at some velocity v. The 'time dilation factor' is just the ratio of the time that passes according to the duck compared to that measured by the observer.

First, we must assume that the duck carries some sort of accurate timepiece. Second, if the duck's time is dilated, the observer on the ground must measure the 'proper' time. We know how to relate these times: the duck is moving, the observer is watching it go by at some velocity v, so

$$\Delta t'_{\text{duck}} = \gamma \Delta t_{\text{observer}}$$

The dilation factor is then:

$$[\text{dilation factor}] = \frac{\Delta t'_{\text{duck}}}{\Delta t_{\text{observer}}} = \gamma = 10$$

We now just need the definition of γ, and we can solve for the velocity of the clock v

$$\begin{aligned}
\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} &= 10 \\
\sqrt{1 - \frac{v^2}{c^2}} &= \frac{1}{10} \\
1 - \frac{v^2}{c^2} &= \frac{1}{10^2} = \frac{1}{100} \\
\frac{v^2}{c^2} &= 1 - \frac{1}{100} \\
v^2 &= c^2 \left(1 - \frac{1}{100}\right) \\
v &= \pm\, c\sqrt{1 - \frac{1}{100}} \\
v &\approx \pm\, 0.995c
\end{aligned}$$

One minor point: remember that when we take a square root, we have a positive and a negative answer, hence the $\pm$. Physically, this represents the fact that we can't tell from the information given whether the duck is coming or going - the answer is the same no matter what direction the duck is moving relative to the observer, it is only important that it *is* moving.

3. A bassist taps the lowest E on his bass at 140 beats per minute during one portion of a song. What tempo would an observer on a ship moving *toward* the bassist at $0.70c$ hear?

What we are really interested in is the time interval between the taps. That time interval will be dilated (longer) for the moving observer, and hence the taps will sound farther apart (the tapping will be slower).

The 'proper time' interval Δt_p is that measured by the bassist, which is $1/140$ min/beat (so that there are 140 beats/min).[i] The time interval *between taps* measured by the moving observer $\Delta t'$ is longer by a factor gamma:

$$\Delta t' = \gamma \Delta t_p = \frac{1}{\sqrt{1 - \frac{0.7^2 c^2}{c^2}}} \cdot \left(\frac{1\,\text{min}}{140\,\text{beats}}\right) = \frac{1}{\sqrt{1 - 0.7^2}} \cdot \left(\frac{1\,\text{min}}{140\,\text{beats}}\right) \approx \frac{0.1\,\text{min}}{\text{beat}}$$

The beats per minute heard by the moving observer is just $1/\Delta t'$:

[i] The time interval is just the inverse of the *rate* of tapping in beats per unit time.

$$[\text{beats per minute, heard by observer}] = \frac{1}{\Delta t'} = \frac{1}{0.1} = 100$$

So, the bass line seems to be moving about 40% slower.