Exam 2 solutions (form A)

If you had exam version B, this is how the questions were scrambled.

Version A	Version B
1	3
2	6
3	12
4	14
5	8
6	10
7	7
8	11
9	2
10	5
11	9
12	1
13	13
14	4
15	15
16	16
17	19
18	17
19	20
20	18

1. B. Momentum is always conserved. The only interactions of interest are between the masses and the spring, and spring forces are conservative. That means the sum of kinetic and potential energy - the mechanical energy - is also conserved. Neither kinetic nor potential energy is separately conserved in this case - some of the block's initial kinetic energy will end up as elastic potential energy in the spring.

2. D. The momentum of the *baseball* is not conserved since it slows down as it rises. By the same logic, kinetic energy is not conserved. Gravitational potential energy is also not conserved since the ball's height is changing. Mechanical energy is conserved, since the only interaction considered is earth's gravity acting on the ball.

3. C. The car and truck form an interaction pair, and Newton's third law says their forces are equal and opposite.

4. D. Constant velocity means zero acceleration, which means the *net* force is zero.

5. C. At the highest point, the velocity is instantaneously zero. The acceleration, however, is constant and directed downward, as is the force of gravity on the ball.

6. A. Letting the object fall can only cause the rope to slacken and reduce the tension. To prove this, draw a free body diagram of m_2: there is tension acting upward, m_2g downward, and a net downward acceleration. That gives:

$$T - m_2g = -m_2a \qquad \Longrightarrow \qquad T = m_2g - m_2a$$

This shows that for any non-zero acceleration, the tension is always reduced. If the rope were detached from m_1, the object would be in free-fall, and the acceleration would be $a = -g$. In that case $T \to 0$ as you would expect.

7. C. The force F acting to cause the braking does some work W, which must be equal to the initial kinetic energy once the object stops. If the stopping distance is Δx, that is

$$W = F\Delta x = K = \frac{1}{2}mv^2$$

If you calculate the kinetic energy, it turns out to be the same in both cases. That means the work is the same for both, and given that both have the same braking force, they will take the same distance to stop.

8. B. The maximum height will be dictated by the initial kinetic energy, since $\Delta U = \Delta K$ in the absence of air resistance. We know $\Delta U = mgh$, so that tells us that the final height is just proportional to the initial kinetic energy. If ball B has twice the speed, it has four times the kinetic energy, and will reach a maximum height four times larger than ball A.

9. B. Kinetic energy is proportional to mass. Both objects will have the same *speed* reaching the ground, since they experience the same acceleration, but the heavier one will have more kinetic energy.

10. D. The force for an ideal spring is a straight line graph, which is graph C. Less stiff implies that there is less force for a given displacement as the total displacement increases. In other words, the curve should change slower than linear as x increases, which is what happens in graph D. Graph B would be the case where the spring gets stiffer as x increases, since the force per unit displacement is getting bigger as x increases. Graph A represents a constant force, which is clearly not something a spring does.

11. E. The initial energy of the spring gets converted to kinetic energy, which then gets converted to gravitational potential energy. That means the initial kinetic energy of the mass is equal to the initial spring energy, and also equal to the gravitational potential energy at the highest point. That tells us

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

Given the same x, the heavier mass will have a lower velocity leaving the spring for the equality to hold, but the two boxes will have the same kinetic energy at least. The maximum height is governed by

$$mgh = \frac{1}{2}kx^2$$

Since the right hand side is the same for both, this tells us that twice the mass means half the height for the equality to hold. This leaves option E as the only reasonable one.

12. C. Constant acceleration along y of $-g$, zero acceleration along x. That means constant velocity along x, linearly changing velocity along y.

13. C. The vertical component of the velocity dictates how long it takes to hit the surface. For Len's case, we have $v_y = v_{\text{len}} \sin 30 = \frac{1}{2}v_{\text{len}}$. Since Len's velocity is twice Jan's, that means they both have the same vertical velocity. The vertical motion is thus exactly the same for both rocks, and they hit at the same time.

14. C. The acceleration is caused only by the horizontal component of the tension, $T \cos\theta = ma$. The tension is then $T = ma/\cos\theta$. For any θ such that $0 < \theta < 90$, the tension is always greater than ma. Put simply, because only the horizontal component provides acceleration, and the vertical component is essentially wasted, the overall tension force is bigger than it needs to be.

15. A. Remember speed is $v = \sqrt{v_x^2 + v_y^2}$. Tom has $v_x = 0$, and Alice has $v_x \neq 0$. Both have the same v_y at all times since neither has an initial vertical velocity. Since Alice has a nonzero v_x but the same v_y as Tom, her speed is always larger. The difference does *not* stay at 25 m/s since both are accelerating along y.

PH105-004
Fall 2015
Exam 2 (v.A)

16. potential energy known – conservative force (non-dissipative)

$\Rightarrow$ consv. of energy applies $K + U = \text{const}$

$K_i = K(x = 1\text{m}) = \frac{1}{2} m v_i^2$ $\qquad K_f = K(x = 5\text{m}) = \frac{1}{2} m v_f^2$

$U_i = U(x = 1\text{m}) = \frac{-2}{x_i} + \frac{4}{x_i^2}$ $\qquad U_f = U(x = 5\text{m}) = \frac{-2}{x_f} + \frac{4}{x_f^2}$

$K_i + U_i = K_f + U_f$

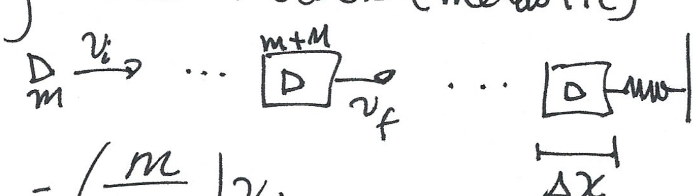
$K_f = K_i + U_i - U_f = \frac{1}{2} m v_f^2 = \frac{1}{2} m v_i^2 - \frac{2}{x_i} + \frac{4}{x_i^2} + \frac{2}{x_f} - \frac{4}{x_f^2}$

with $x_i = 1\text{m}$, $x_f = 5\text{m}$, $v_i = 3\text{m/s}$, $m = 1\text{kg}$

$$\boxed{v_f \simeq 3.67 \frac{\text{m}}{\text{s}}}$$

---

17. – Consv. $\vec{p}$ to handle "collision" of bullet & block (inelastic)
– then consv E, $KE \to U^s$

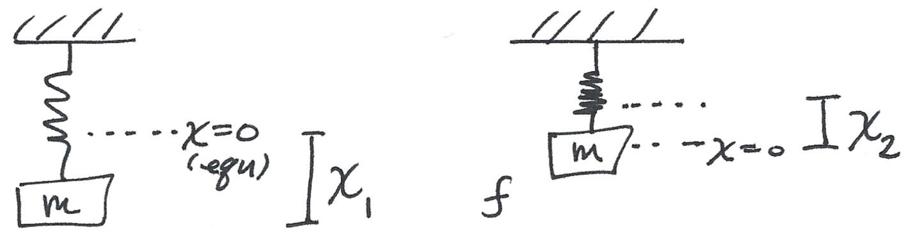


$$p_i = \underset{\text{bullet}}{m} v_i = p_f = (m + \underset{\text{block}}{M}) v_f \Rightarrow v_f = \left(\frac{m}{m+M}\right) v_i$$

$$\underbrace{\frac{1}{2}(m+M) v_f^2}_{K_i \text{ after coll.}} = \frac{1}{2}(m+M) \frac{m^2}{(m+M)^2} v_i^2 = \underbrace{\frac{1}{2} k \Delta x^2}_{U^s \text{ after}}$$

$$\Rightarrow v_i = \sqrt{k(m+M)} \frac{\Delta x}{m} \simeq 1100 \text{ m/s}$$

18.



$|x_2| = 1\,cm = 0.01\,m$

$|x_1| = 2\,cm = 0.02\,m$

$m = 4\,kg$

$k = 1\,N/cm = 100\,\frac{N}{m}$

• either consv E or work spring → KE
(same eqns either way)

$$K_i + U_i = K_f + U_f$$

$K_i = 0$ $\qquad$ $K_f = \frac{1}{2} m v_f^2$

$U_i = U_i^S + U_i^G = \frac{1}{2} k x_1^2 - mg x_1$ $\qquad$ $U_f = U_f^S + U_f^G = \frac{1}{2} k x_2^2 + mg x_2$

(call x=0 our zero for U^S)

$$\frac{1}{2} k x_1^2 - mg x_1 = \frac{1}{2} m v_f^2 + \frac{1}{2} k x_2^2 + mg x_2$$

$$\frac{1}{2} m v_f^2 = \frac{1}{2} k (x_1^2 - x_2^2) - mg(x_1 + x_2)$$

$$v_f^2 = \frac{k}{m} (x_1^2 - x_2^2) - 2g(x_1 + x_2)$$

! with given numbers, we find $v_f^2 = -0.58$. Clearly this is impossible. The mass should have been 4g not 4kg. Partial credit was generous as a result.

(if you ignore the gravitational potential it works out, but that's silly.)

19. $a = \frac{\Delta v}{\Delta t} = \frac{v_i}{\Delta t}$ $\quad F = ma = \frac{v_i m}{\Delta t} \simeq 3.97\,N$

– or –

$$F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{m v_i}{\Delta t}$$

20. $R = 111\,m$, $h = 72.3\,m$

$v_y = v_0 \sin\theta - gt = 0$ at max height $\Rightarrow t_{max} = \frac{v_0 \sin\theta}{g}$

$$y_{max} = h = v_0 \sin\theta \, t_{max} - \frac{1}{2} g t_{max}^2 = v_0 \sin\theta \left(\frac{v_0 \sin\theta}{g}\right) - \frac{1}{2} g \left(\frac{v_0 \sin\theta}{g}\right)^2$$

$$\Rightarrow h = \frac{v_0^2 \sin^2\theta}{2g}$$

at range, twice this time

$$x(2t_{max}) = R = v_0 \cos\theta (2t_{max}) = \frac{2 v_0^2 \sin\theta \cos\theta}{g}$$

divide: $\frac{h}{R} = \frac{v_0^2 \sin^2\theta}{2g} \cdot \frac{g}{2 v_0^2 \sin\theta \cos\theta} = \tan\theta \cdot \frac{1}{4}$

$$\text{or } \theta = \tan^{-1}\left(\frac{4h}{R}\right) \simeq 69^\circ$$