Formula sheet

$$g = 9.81\,\mathrm{m/s^2} \quad G = 6.67\times10^{-11}\,\mathrm{N\,m^2/kg^2}$$

$$N_A = 6.022\times10^{23}\ \text{things/mol}$$

$$k_B = 1.38065\times10^{-23}\ \mathrm{J\cdot K^{-1}} = 8.6173\times10^{-5}\ \mathrm{eV\cdot K^{-1}}$$

$$\text{sphere} \quad V = \frac{4}{3}\pi r^3 \quad A = 4\pi r^2$$

$$ax^2 + bx^2 + c = 0 \Longrightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha\pm\beta)\cos\frac{1}{2}(\alpha\mp\beta)$$

$$\cos\alpha \pm \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$$

$$c^2 = a^2 + b^2 - 2ab\cos\theta_{ab}$$

$$\frac{d}{dx}\sin ax = a\cos ax \qquad \frac{d}{dx}\cos ax = -a\sin ax$$

$$\int \cos ax\,\mathrm{dx} = \frac{1}{a}\sin ax \qquad \int \sin ax\,\mathrm{dx} = -\frac{1}{a}\cos ax$$

$$\sin\theta\approx\theta \qquad \cos\theta \approx 1-\frac{1}{2}\theta^2 \qquad \text{small } \theta$$

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$$d \equiv |x_1 - x_2|$$

$$b \equiv |\vec{b}| = |b_x| \quad \text{one dimension}$$

$$\vec{r} = x\,\hat{\boldsymbol{\imath}} \quad \text{one dimension}$$

$$\vec{b} = b_x\,\hat{\boldsymbol{\imath}} \quad \text{one dimension}$$

$$\text{speed} = v = |\vec{v}|$$

$$\vec{v} = \lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t} \equiv \frac{d\vec{r}}{dt}$$

$$a_x = \lim_{\Delta t\to0}\frac{\Delta v_x}{\Delta t} \equiv \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

$$\Delta v_x = \int_{t_i}^{t_f} a_x(t)\,dt \qquad \Delta x = \int_{t_i}^{t_f} v_x(t)\,dt$$

$$x(t) = x_i + v_{x,i}t + \frac{1}{2}a_xt^2$$

$$v_x(t) = v_{x,i} + a_xt$$

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x\Delta x$$

$$a_{x,\mathrm{ramp}} = g\sin\theta$$

$$\Delta U^G = mg\Delta x$$

$$\frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}$$

$$E_{\mathrm{mech}} = K + U \quad K = \frac{1}{2}mv^2$$

$$\Delta E = \Delta K + \Delta U = 0 \quad \text{non-dissipative, closed}$$

$$\Delta\vec{p} = \vec{0} \quad \text{isolated system}$$

$$\vec{p}_f = \vec{p}_i \quad \text{isolated system}$$

$$\vec{p} \equiv m\vec{v}$$

$$m_u = -\frac{\Delta v_{s,x}}{\Delta v_{u,x}}m_s$$

$$\vec{J} = \Delta\vec{p}$$

$$v_{1f} = \left(\frac{m_1-m_2}{m_1+m_2}\right)v_{i1} + \left(\frac{2m_2}{m_1+m_2}\right)v_{2i} \quad \text{1D elastic}$$

$$v_{2f} = \left(\frac{2m_1}{m_1+m_2}\right)v_{1i} + \left(\frac{m_2-m_1}{m_1+m_2}\right)v_{2i} \quad \text{1D elastic}$$

$$\Delta E = 0 \quad \text{isolated system}$$

$$K = \frac{1}{2}mv^2$$

$$\vec{v}_{12} = \vec{v}_2 - \vec{v}_1 \quad \text{relative velocity}$$

$$v_{12} = |\vec{v}_2 - \vec{v}_1| \quad \text{relative speed}$$

$$\vec{a} = \frac{\sum\vec{F}}{m} \qquad a_{\vec{cm}} = \frac{\sum\vec{F}_{\mathrm{ext}}}{m} \qquad \sum\vec{F} \equiv \frac{d\vec{p}}{dt}$$

$$\vec{J} = \left(\sum\vec{F}\right)\Delta t \quad \text{constant force}$$

$$\vec{J} = \int_{t_i}^{t_f}\sum\vec{F}(t)\,dt \quad \text{time-varying force}$$

$$\vec{F}_{12} = -\vec{F}_{21} \qquad F_{\mathrm{grav}} = -mg \qquad F_{\mathrm{spring}} = -k\Delta x$$

$$\Delta E = W$$

$$\Delta U_{\mathrm{spring}} = \frac{1}{2}k\left(x-x_o\right)^2$$

$$P = \frac{dE}{dt}$$

$$P = F_{\mathrm{ext,x}}\,v_x \quad \text{one dimension}$$

$$W = \left(\sum\vec{F}\right)\Delta x_F \quad \text{constant force 1D}$$

$$W = \int_{x_i}^{x_f}F_x(x)\,dx \quad \text{nondiss. force, 1D}$$

Rotation: we use radians

$$s = \theta r \quad \leftarrow \text{arclength}$$

$$\omega = \frac{d\theta}{dt} = \frac{v_t}{r} \qquad \alpha = \frac{d\omega}{dt}$$

$$a_t = \alpha r \quad \text{tangential} \qquad a_r = -\frac{v^2}{r} = -\omega^2 r \quad \text{radial}$$

$$v_t = r\omega \qquad v_r = 0$$

$$(F_{12}^s)_{\max} = \mu_s F_{12}^n$$
$$F_{12}^k = \mu_k F_{12}^n$$
$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\,\hat{\imath} + A_y\,\hat{\jmath}$$
$$\vec{A}\cdot\vec{B} = AB\cos\phi = A_xB_x + A_yB_y$$
$$W = \vec{F}\cdot\Delta\vec{r}_F \quad \text{const non-diss force}$$
$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}(\vec{r})\cdot d\vec{r} \quad \text{variable nondiss force}$$

$\downarrow$ launched from origin, level ground

$$y(x) = (\tan\theta_o)\,x - \frac{gx^2}{2v_o^2\cos^2\theta_o}$$
$$\text{max height } = H = \frac{v_i^2\sin^2\theta_i}{2g}$$
$$\text{Range } = R = \frac{v_i^2\sin 2\theta_i}{g}$$
$$(F_{12}^2)_{\max} = \mu_s F_{12}^n \qquad F_{12}^k = \mu_k F_{12}^n$$

fluids:

$$P = F/A$$
$$P(d) = P_{\text{surface}} + \rho g d$$
$$\rho = M/V$$
$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \quad F_1x_1 = F_2x_2 \qquad \text{hydraulics}$$
$$B = \text{buoyant force} = \text{weight of water displaced} = \rho_f V_{\text{displ}} g$$
$$P = P_{\text{gauge}} + P_{\text{atm}}$$

thermal stuff:

$$PV = Nk_BT = nRT$$
$$T(K) = T(^\circ C) + 273.15^\circ$$
$$Q = mc\Delta t \quad c = \text{specific heat} \qquad \text{no phase chg}$$
$$Q = \pm mL \qquad \text{phase chg}$$

$$I = \sum_i m_i r_i^2 \Rightarrow \int r^2\,dm = kmr^2 \qquad I = mr^2 \quad \text{point particle}$$
$$I_z = I_{com} + md^2 \quad \text{axis } z \text{ parallel, dist } d$$
$$\vec{L} = \vec{r}\times\vec{p} = I\vec{\omega}$$
$$K = \frac{1}{2}I\omega^2 = L^2/2I$$
$$\Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W = \int \tau\,d\theta$$
$$P = \frac{dW}{dt} = \tau\omega$$
$$\tau = rF\sin\theta_{rF} = r_\perp F = rF_\perp$$
$$\tau_{net} = \sum\vec{\tau} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$$
$$K_{\text{tot}} = K_{cm} + K_{rot} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$$

Oscillations:

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \quad \omega = \frac{2\pi}{T} = 2\pi f$$
$$x(t) = A\sin(\omega t + \varphi_i)$$
$$v(t) = \frac{dx}{dt} = \omega A\cos(\omega t + \varphi_i)$$
$$a(t) = \frac{d^2x}{dt^2} = -\omega^2 A\sin(\omega t + \varphi_i)$$
$$\varphi(t) = \omega t + \varphi_i$$
$$a = -\omega^2 x \qquad \frac{d^2x}{dt^2} = -\omega^2 x \qquad \frac{d^2\theta}{dt^2} = -\omega^2\theta$$
$$E = \frac{1}{2}m\omega^2A^2$$
$$\omega = \sqrt{k/m} \quad \text{spring.}$$
$$T = \begin{cases} 2\pi\sqrt{L/g} & \text{simple pendulum} \\ 2\pi\sqrt{I/mgl_{cm}} & \text{physical pendulum}\end{cases}$$

Waves:

$$y = f(x-ct) \quad \text{along +x} \qquad y = f(x+ct) \quad \text{along -x}$$
$$k = \frac{2\pi}{\lambda} \quad \lambda = cT \quad \omega = \frac{2\pi}{T} \quad c = \lambda f$$
$$y(x,t) = f(x,t)A\sin(kx - \omega t + \varphi_i)$$
$$y(x,t) = 2A\sin kx\cos\omega t \quad \text{standing wave}$$
$$\text{nodes at } x = 0, \pm\frac{\lambda}{2}, \pm\lambda, \pm\frac{3\lambda}{2}$$
$$v = \sqrt{T/\mu} \quad \mu = M_{\text{string}}/L_{\text{string}} \quad T = \text{tension} \qquad \text{strings}$$
$$P_{\text{av}} = \frac{1}{2}\mu\lambda A^2\omega^2/T = \frac{1}{2}\mu A^2\omega^2 c$$
$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$
$$f_n = \frac{nv}{\lambda} = \frac{nv}{2L} \quad \lambda_n = \frac{2L}{n} \quad n = 1,2,3\ldots \qquad \text{strings \& open-open pipe}$$
$$f_n = \frac{nv}{\lambda} = \frac{nv}{4L} \quad \lambda_n = \frac{4L}{n} \quad n = 1,3,5\ldots \qquad \text{closed-open pipe}$$

Isolated systems: $\vec{p}$, $E = K + PE$, L are all conserved.
Static equilibrium: $\sum F = 0$ and $\sum\tau = 0$ about any axis.
Elastic collision: KE and p are both conserved.
Inelastic collision: only p is conserved, not KE.

Derived unit	**Symbol**	**equivalent to**
newton	N	$kg\cdot m/s^2$
joule	J	$kg\cdot m^2/s^2 = N\cdot m$
watt	W	$J/s = m^2\cdot kg/s^3$

Power	**Prefix**	**Abbreviation**
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	µ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T