## PH105 Final Exam

## Instructions

1. Answer all questions.
2. Record your responses on a scantron sheet.
3. On your scantron sheet, be sure to bubble in your full name and CWID.
4. A ball with a weight of 1.5 N is thrown at an angle of $30^{\circ}$ above the horizontal with an initial speed of $12 \mathrm{~m} / \mathrm{s}$. If air resistance is negligible, at its highest point, the net force on the ball is:
a) $9.8 \mathrm{~N}, 30^{\circ}$ below horizontal
b) zero
c) $9.8 \mathrm{~N}, \mathrm{up}$
d) 9.8 N , down
e) 1.5 N , down
5. A car is able to accelerate linearly with time, with $a(t)=\left(2.00 \mathrm{~m} / \mathrm{s}^{3}\right) t$ to a good approximation. If the car starts at rest, how far does it travel in 10.0 s ?
a) 200 m
b) 666 m
c) 100 m
d) 333 m
e) 1000 m
6. A 4.0 kg mass sitting on a horizontal, frictionless $\mathrm{x} / \mathrm{y}$ plane, is subjected to two horizontal forces: an 4.0 N force in the x -direction and a 7.0 N force in the y -direction. What is the magnitude of the acceleration of this mass?
a) $0.75 \mathrm{~m} / \mathrm{s}^{2}$
b) $1.0 \mathrm{~m} / \mathrm{s}^{2}$
c) $1.4 \mathrm{~m} / \mathrm{s}^{2}$
d) $2.0 \mathrm{~m} / \mathrm{s}^{2}$
e) $2.8 \mathrm{~m} / \mathrm{s}^{2}$
7. A 0.50 kg block is pushed by a horizontal force against a vertical wall. The coefficient of static friction, $\mu_{s}$, between the block and the wall is 0.90 . Which minimum horizontal force do you have to apply in order to prevent the block from sliding down the wall?
a) 4.9 N
b) 5.44 N
c) 4.41 N
d) 10.34 N
e) None of these

8. A ball of mass 500 g is dropped from a height of 9.2 m . If it is found to reach the ground with a final speed of $8.5 \mathrm{~m} / \mathrm{s}$, calculate the work done by the air resistance that it must have experienced on its descent.
a) 18 J
b) 45 J
c) 0 J
d) 27 J
e) $27,000 \mathrm{~J}$
9. A 4.0 kg object is traveling at $3.0 \mathrm{~m} / \mathrm{s}$ on a horizontal frictionless table. It strikes an object with a mass of 8.0 kg head-on. The heavier object is initially at rest. During the collision the two objects stick together. Their final speed is:
a) $1.0 \mathrm{~m} / \mathrm{s}$
b) $1.5 \mathrm{~m} / \mathrm{s}$
c) $9.0 \mathrm{~m} / \mathrm{s}$
d) $1.7 \mathrm{~m} / \mathrm{s}$
e) $3.0 \mathrm{~m} / \mathrm{s}$
10. A satellite is in circular orbit around a spherical asteroid, at an altitude of 100 km above the surface, moving at a uniform speed of $80 \mathrm{~m} / \mathrm{s}$. If the asteroid has a radius of 321 km , what is the mass of the asteroid?
a) $4.04 \times 10^{19} \mathrm{~kg}$
b) $3.08 \times 10^{19} \mathrm{~kg}$
c) $9.60 \times 10^{18} \mathrm{~kg}$
d) $4.04 \times 10^{16} \mathrm{~kg}$
e) $9.60 \times 10^{15} \mathrm{~kg}$
11. A rotating gas cloud with initial rotational (moment of) inertia $I_{0}$ and initial angular speed $\omega_{0}$ collapses under the influence of internal forces until its rotational (moment of) inertia is $I_{0} / 3$. What is its final angular speed?
a) $\omega_{0} / 3$
b) $3 \omega_{0}$
c) $\omega_{0} / \sqrt{3}$
d) $\sqrt{3} \omega_{0}$
e) $\omega_{0}$
12. A harmonic wave travels in a wire with amplitude 6.00 mm , wavelength 120 cm , and period of $1.05 \cdot 10^{-2} \mathrm{~s}$. What is the speed with which the wave travels?
a) $3.60 \mathrm{~m} / \mathrm{s}$
b) $114 \mathrm{~m} / \mathrm{s}$
c) $720 \mathrm{~m} / \mathrm{s}$
d) $11500 \mathrm{~m} / \mathrm{s}$
e) $72000 \mathrm{~m} / \mathrm{s}$
13. Two identical blocks of ice float in water oriented as shown. Which of the following statements is true?
a) Block A displaces a greater volume of water since the pressure acts on a smaller area at the bottom
b) Block B displaces a greater volume of water since the pressure is less at the bottom
c) The two blocks displace equal volumes of water since they have same weight and volume
d) Block A displaces a greater volume of water since its bottom is submerged lower in the water
e) Block B displaces a greater volume of water since the pressure acts on a greater area at the bottom

14. An ideal monatomic gas expands at a constant pressure of 10 kPa from 10 liters to 30 liters. If absorbs 500 J of heat during this process, what is its change in internal energy?
a) 200 J
b) 300 J
c) -200 kJ
d) 500 J
e) 700 J
15. If the fastest you can safely drive is $65 \mathrm{mi} / \mathrm{h}$, what is the longest time you can stop for dinner if you must travel 541 mi in 9.6 h total? If the fastest you can safely drive is $65 \mathrm{mi} / \mathrm{h}$, what is the longest time you can stop for dinner if you must travel 541 mi in 9.6 h total?
a) 1.0 h
b) 1.3 h
c) 1.6 h
d) You can't stop at all.
16. A car accelerates from $10.0 \mathrm{~m} / \mathrm{s}$ to $30.0 \mathrm{~m} / \mathrm{s}$ at a rate of $3.00 \mathrm{~m} / \mathrm{s}^{2}$. How far does the car travel while accelerating?
a) 80.0 m
b) 399 m
c) 226 m
d) 133 m
17. A $480-\mathrm{kg}$ car moving at $14.4 \mathrm{~m} / \mathrm{s}$ hits from behind a $570-\mathrm{kg}$ car moving at $13.3 \mathrm{~m} / \mathrm{s}$ in the same direction. If the new speed of the heavier car is $14.0 \mathrm{~m} / \mathrm{s}$, what is the speed of the lighter car after the collision, assuming that any unbalanced forces on the system are negligibly small?
a) $5.24 \mathrm{~m} / \mathrm{s}$
b) $19.9 \mathrm{~m} / \mathrm{s}$
c) $13.6 \mathrm{~m} / \mathrm{s}$
d) $10.5 \mathrm{~m} / \mathrm{s}$
18. Jacques and George meet in the middle of a lake while paddling in their canoes. They come to a complete stop and talk for a while. When they are ready to leave, Jacques pushes George's canoe with a force $\vec{F}$ to separate the two canoes. What is correct to say about the final momentum and kinetic energy of the system if we can neglect any resistance due to the water?
a) The final momentum is zero and the final kinetic energy is zero.
b) The final momentum is in the direction of F and the final kinetic energy is positive.
c) The final momentum is in the direction opposite of $F$ but the final kinetic energy is zero.
d) The final momentum is in the direction of $F$ but the final kinetic energy is zero.
e) The final momentum is zero but the final kinetic energy is positive.
19. A $2000-\mathrm{kg}$ truck is sitting at rest (in neutral) when it is rear-ended by a $1000-\mathrm{kg}$ car going $26 \mathrm{~m} / \mathrm{s}$. After the collision, the two vehicles stick together. What is the final velocity of the combined mass after the collision?
a) $2.9 \mathrm{~m} / \mathrm{s}$
b) $15 \mathrm{~m} / \mathrm{s}$
c) $13 \mathrm{~m} / \mathrm{s}$
d) $8.7 \mathrm{~m} / \mathrm{s}$
e) $26 \mathrm{~m} / \mathrm{s}$
20. A $1.00-\mathrm{kg}$ particle that moves with potential energy given by $U(x)=(-2.00 \mathrm{~J} \cdot \mathrm{~m}) / x+\left(4.00 \mathrm{~J} \cdot \mathrm{~m}^{2}\right) / x^{2}$. Suppose the particle is moving with a speed of $3.00 \mathrm{~m} / \mathrm{s}$ when it is located at $\mathrm{x}=1.00 \mathrm{~m}$. What is the speed of the object when it is located at $\mathrm{x}=5.00 \mathrm{~m}$ ?
a) $4.68 \mathrm{~m} / \mathrm{s}$
b) $3.67 \mathrm{~m} / \mathrm{s}$
c) $3.00 \mathrm{~m} / \mathrm{s}$
d) $2.13 \mathrm{~m} / \mathrm{s}$
21. An object attached to an ideal massless spring is pulled across a frictionless surface. If the spring constant is $45 \mathrm{~N} / \mathrm{m}$ and the spring is stretched by 0.88 m when the object is accelerating at $2.0 \mathrm{~m} / \mathrm{s}^{2}$, what is the mass of the object?
a) 22 kg
b) 20 kg
c) 17 kg
d) 26 kg
22. At the end of a delivery ramp, a skid pad exerts a constant force on a package so that the package comes to rest in a distance $d$. When the pad is replaced by one that requires a distance of 7 d to stop the same package, what happens to the length of time it takes for the package to stop?
a) decreases by a factor of 7
b) increases by a factor of 7
c) increases by a factor of $\sqrt{7}$
d) decreases by a factor of $\sqrt{7}$
23. What is the maximum distance we can shoot a dart, from ground level, provided our toy dart gun gives a maximum initial velocity of $2.78 \mathrm{~m} / \mathrm{s}$ and air resistance is negligible?
a) 0.789 m
b) 0.394 m
c) 1.39 m
d) 1.58 m
24. An open door of inertia $m_{d}$ and width $l_{d}$ is at rest when it is struck by a thrown ball of clay of inertia $m_{b}$ that is moving at speed $v_{b}$ at the instant it strikes the door (see figure below). The impact location is a distance $d=\frac{2}{3} l_{d}$ from the rotation axis through the hinges. The clay ball strikes perpendicular to the door face and sticks after it hits. What is the angular velocity of the system about the hinges after the impact? Do not ignore the inertia $m_{b}$.
a) $\omega=\frac{4 m_{b} v_{b}}{\left(m_{b}+2 m_{d}\right) l_{d}}$
b) $\omega=\frac{6 m_{b} v_{b}}{\left(4 m_{b}+3 m_{d}\right) l_{d}}$
c) $\omega=\frac{3 m_{b} v_{b}}{\left(2 m_{b}+m_{d}\right) l_{d}}$
d) $\omega=\frac{12 m_{b} v_{b}}{\left(8 m_{b}+3 m_{d}\right) l_{d}}$

25. A uniform solid cylinder of radius $R$ and a thin uniform spherical shell of radius $R$ both roll without slipping. If both objects have the same mass and the same kinetic energy, what is the ratio of the linear speed of the cylinder to the linear speed of the spherical shell?
a) $\frac{4}{3}$
b) $\sqrt{\frac{4}{3}}$
c) $\frac{4}{\sqrt{3}}$
d) $\frac{\sqrt{3}}{2}$
e) $\frac{\sqrt{10}}{3}$
26. A 0.25 kg ideal harmonic oscillator has a total mechanical energy of 4.0 J . If the oscillation amplitude is 20.0 cm , what is the oscillation frequency?
a) 3.2 Hz
b) 2.3 Hz
c) 1.4 Hz
d) 4.5 Hz
27. A heavy stone of mass m is hung from the ceiling by a thin $8.25-\mathrm{g}$ wire that is 65.0 cm long. When you gently pluck the upper end of the wire, a pulse travels down the wire and returns 7.84 ms later, having reflected off the lower end. The stone is heavy enough to prevent the lower end of the wire from moving. What is the mass m of the stone?
a) 23.1 kg
b) 349 kg
c) 35.6 kg
d) 8.90 kg
e) 227 kg
28. A $12,000-\mathrm{N}$ car is raised using a hydraulic lift, which consists of a U-tube with arms of unequal areas, filled with incompressible oil and capped at both ends with tight-fitting pistons. The wider arm of the U-tube has a radius of 18.0 cm and the narrower arm has a radius of 5.00 cm . The car rests on the piston on the wider arm of the U-tube. The pistons are initially at the same level. What is the initial force that must be applied to the smaller piston in order to start lifting the car? (For purposes of this problem, you can neglect the weight of the pistons.)
a) 3330 N
b) 2900 N
c) 926 N
d) 1.20 kN
e) 727 N

## Formula sheet

$$
\begin{aligned}
g & =9.81 \mathrm{~m} / \mathrm{s}^{2} \quad G=6.67 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2} \\
N_{A} & =6.022 \times 10^{23} \text { things } / \mathrm{mol} \quad 1 \mathrm{~L}=10^{-3} \mathrm{~m}^{3} \\
k_{B} & =1.38065 \times 10^{-23} \mathrm{~J} \cdot \mathrm{~K}^{-1}=8.6173 \times 10^{-5} \mathrm{eV} \cdot \mathrm{~K}^{-1}
\end{aligned}
$$

$$
\text { sphere } \quad V=\frac{4}{3} \pi r^{3} \quad A=4 \pi r^{2}
$$

$$
a x^{2}+b x^{2}+c=0 \Longrightarrow x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

$$
\sin \alpha \pm \sin \beta=2 \sin \frac{1}{2}(\alpha \pm \beta) \cos \frac{1}{2}(\alpha \mp \beta)
$$

$$
\cos \alpha \pm \cos \beta=2 \cos \frac{1}{2}(\alpha+\beta) \cos \frac{1}{2}(\alpha-\beta)
$$

$$
c^{2}=a^{2}+b^{2}-2 a b \cos \theta_{a b}
$$

$$
\frac{d}{d x} \sin a x=a \cos a x \quad \frac{d}{d x} \cos a x=-a \sin a x
$$

$$
\int \cos a x \mathrm{dx}=\frac{1}{a} \sin a x \quad \int \sin a x \mathrm{dx}=-\frac{1}{a} \cos a x
$$

$$
\sin \theta \approx \theta \quad \cos \theta \approx 1-\frac{1}{2} \theta^{2} \quad \text { small } \theta
$$

$$
\overrightarrow{\mathbf{v}}=\lim _{\Delta t \rightarrow 0} \frac{\Delta \overrightarrow{\mathbf{r}}}{\Delta t} \equiv \frac{d \overrightarrow{\mathbf{r}}}{d t}
$$

$$
a_{x}=\lim _{\Delta t \rightarrow 0} \frac{\Delta v_{x}}{\Delta t} \equiv \frac{d v_{x}}{d t}=\frac{d}{d t}\left(\frac{d x}{d t}\right)=\frac{d^{2} x}{d t^{2}}
$$

$$
\Delta v_{x}=\int_{t_{i}}^{t_{f}} a_{x}(t) d t \quad \Delta x=\int_{t_{i}}^{t_{f}} v_{x}(t) d t
$$

$$
x(t)=x_{i}+v_{x, i} t+\frac{1}{2} a_{x} t^{2}
$$

$$
v_{x}(t)=v_{x, i}+a_{x} t
$$

$$
v_{x, f}^{2}=v_{x, i}^{2}+2 a_{x} \Delta x
$$

$$
a_{x, \mathrm{ramp}}=g \sin \theta
$$

$$
\begin{aligned}
\Delta U^{G} & =m g \Delta x \quad \frac{a_{1 x}}{a_{2 x}}=-\frac{m_{2}}{m_{1}} \\
E_{\text {mech }} & =K+U \quad K=\frac{1}{2} m v^{2} \\
\Delta E & =\Delta K+\Delta U=0 \quad \text { non-dissipative, closed }
\end{aligned}
$$

$$
\begin{aligned}
\Delta E & =W \quad P=\frac{d E}{d t} \\
\Delta U_{\text {spring }} & =\frac{1}{2} k\left(x-x_{o}\right)^{2} \\
P & =F_{\text {ext }, \mathrm{x}} v_{x} \quad \text { one dimension } \\
W & =\left(\sum \overrightarrow{\mathbf{F}}\right) \Delta x_{F} \quad \text { constant force 1D } \\
W & =\int_{x_{i}}^{x_{f}} F_{x}(x) d x \quad \text { nondiss. force, 1D }
\end{aligned}
$$

$$
\begin{aligned}
\Delta \overrightarrow{\mathbf{p}} & =\overrightarrow{\mathbf{0}} \quad \text { isolated system } \\
\overrightarrow{\mathbf{p}}_{f} & =\overrightarrow{\mathbf{p}}_{i} \quad \text { isolated system } \\
\overrightarrow{\mathbf{p}} & \equiv m \overrightarrow{\mathbf{v}} \\
m_{u} & =-\frac{\Delta v_{s, x}}{\Delta v_{u, x}} m_{s} \\
\overrightarrow{\mathbf{J}} & =\Delta \overrightarrow{\mathbf{p}} \\
v_{1 f} & =\left(\frac{m_{1}-m_{2}}{m_{1}+m_{2}}\right) v_{i 1}+\left(\frac{2 m_{2}}{m_{1}+m_{2}}\right) v_{2 i} \quad \text { 1D elastic } \\
v_{2 f} & =\left(\frac{2 m_{1}}{m_{1}+m_{2}}\right) v_{1 i}+\left(\frac{m_{2}-m_{1}}{m_{1}+m_{2}}\right) v_{2 i} \quad \text { 1D elastic } \\
\Delta E & =0 \quad \text { isolated system } \\
K & =\frac{1}{2} m v^{2} \\
\vec{v}_{12} & =\overrightarrow{\mathbf{v}}_{2}-\overrightarrow{\mathbf{v}}_{1} \quad \text { relative velocity }
\end{aligned}
$$

$$
\begin{aligned}
\overrightarrow{\mathbf{a}} & =\frac{\sum \overrightarrow{\mathbf{F}}}{m} \quad \mathbf{a}_{\mathbf{c m}}=\frac{\sum \overrightarrow{\mathbf{F}}_{\mathrm{ext}}}{m} \quad \sum \overrightarrow{\mathbf{F}} \equiv \frac{d \overrightarrow{\mathbf{p}}}{d t} \\
\overrightarrow{\mathbf{J}} & =\left(\sum \overrightarrow{\mathbf{F}}\right) \Delta t \quad \text { constant force } \\
\overrightarrow{\mathbf{J}} & =\int_{t_{i}}^{t_{f}} \sum \overrightarrow{\mathbf{F}}(t) d t \quad \text { time-varying force } \\
\overrightarrow{\mathbf{F}}_{12} & =-\overrightarrow{\mathbf{F}}_{21} \quad F_{\mathrm{grav}}=-m g \quad F_{\text {spring }}=-k \Delta x
\end{aligned}
$$

## Rotation: we use radians

$$
s=\theta r \quad \leftarrow \text { arclength }
$$

$$
\omega=\frac{d \theta}{d t}=\frac{v_{t}}{r} \quad \alpha=\frac{d \omega}{d t}
$$

$$
a_{t}=\alpha r \quad \text { tangential } \quad a_{r}=-\frac{v^{2}}{r}=-\omega^{2} r \quad \text { radial }
$$

$$
v_{t}=r \omega \quad v_{r}=0 \quad \omega_{f}^{2}=\omega_{i}^{2}+2 \alpha \Delta \theta
$$

$$
\begin{aligned}
\left(F_{12}^{s}\right)_{\max } & =\mu_{s} F_{12}^{n} \\
F_{12}^{k} & =\mu_{k} F_{12}^{n} \\
W & =\overrightarrow{\mathbf{F}} \cdot \Delta \overrightarrow{\mathbf{r}}_{F} \quad \text { const non-diss force } \\
W & =\int_{\overrightarrow{\mathbf{r}}_{i}}^{\overrightarrow{\mathbf{r}}_{f}} \overrightarrow{\mathbf{F}}(\overrightarrow{\mathbf{r}}) \cdot d \overrightarrow{\mathbf{r}} \quad \text { variable nondiss force }
\end{aligned}
$$

$\downarrow$ launched from origin, level ground

$$
y(x)=\left(\tan \theta_{o}\right) x-\frac{g x^{2}}{2 v_{o}^{2} \cos ^{2} \theta_{o}}
$$

$$
\text { max height }=H=\frac{v_{i}^{2} \sin ^{2} \theta_{i}}{2 g}
$$

$$
\text { Range }=R=\frac{v_{i}^{2} \sin 2 \theta_{i}}{g}
$$

$$
\left(F_{12}^{2}\right)_{\max }=\mu_{s} F_{12}^{n} \quad F_{12}^{k}=\mu_{k} F_{12}^{n}
$$

## fluids:

$$
\begin{aligned}
P & =F / A \quad P(d)=P_{\text {surface }}+\rho g d \\
\frac{F_{1}}{A_{1}} & =\frac{F_{2}}{A_{2}} \quad F_{1} x_{1}=F_{2} x_{2} \quad \text { hydraulics } \\
B & =\text { buoyant force }=\text { weight of water displaced }=\rho_{f} V_{\text {displ }} g \\
P & =P_{\text {gauge }}+P_{\mathrm{atm}} \quad \rho=M / V
\end{aligned}
$$

$$
\begin{aligned}
I & =\sum_{i} m_{i} r_{i}^{2} \Rightarrow \int r^{2} d m=k m r^{2} \quad I=m r^{2} \quad \text { point particle } \\
I_{z} & =I_{c o m}+m d^{2} \quad \text { axis } z \text { parallel, dist } d \\
\overrightarrow{\mathbf{L}} & =\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{p}}=I \overrightarrow{\boldsymbol{\omega}} \\
K & =\frac{1}{2} I \omega^{2}=L^{2} / 2 I \\
\Delta K & =\frac{1}{2} I \omega_{f}^{2}-\frac{1}{2} I \omega_{i}^{2}=W=\int \tau d \theta \\
P & =\frac{d W}{d t}=\tau \omega \\
\tau & =r F \sin \theta_{r F}=r_{\perp} F=r F_{\perp} \\
\tau_{n e t} & =\sum \vec{\tau}=I \overrightarrow{\boldsymbol{\alpha}}=\frac{d \overrightarrow{\mathbf{L}}}{d t} \\
K_{\mathrm{tot}} & =K_{c m}+K_{r o t}=\frac{1}{2} m v_{c m}^{2}+\frac{1}{2} I \omega^{2}
\end{aligned}
$$

## Oscillations:

$$
\left.\begin{array}{rl}
T & =\frac{1}{f}=\frac{2 \pi}{\omega} \quad \omega=\frac{2 \pi}{T}=2 \pi f \quad k=\frac{2 \pi}{\lambda} \\
x(t) & =A \sin \left(\omega t+\varphi_{i}\right) \\
v(t) & =\frac{d x}{d t}=\omega A \cos \left(\omega t+\varphi_{i}\right) \\
a(t) & =\frac{d^{2} x}{d t^{2}}=-\omega^{2} A \sin \left(\omega t+\varphi_{i}\right) \\
\varphi(t) & =\omega t+\varphi_{i} \\
a & =-\omega^{2} x=\frac{d^{2} x}{d t^{2}} \quad \frac{d^{2} \theta}{d t^{2}}=-\omega^{2} \theta \\
E & =\frac{1}{2} m \omega^{2} A^{2} \quad F_{\text {spring }}=-k x
\end{array}\right] \begin{array}{lll}
\omega & =\sqrt{k / m} \quad \text { spring. } \\
T & = \begin{cases}2 \pi \sqrt{L / g} & \text { simple pendulum } \\
2 \pi \sqrt{I / m g l_{c m}} & \text { physical pendulum }\end{cases}
\end{array}
$$

## Gravitation

$$
\begin{aligned}
F_{12}^{G} & =G \frac{m_{1} m_{2}}{r_{12}^{2}} \quad G=6.67 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{kg}^{2} \\
U^{G}(r) & =-G \frac{m_{1} m_{2}}{r_{12}} \\
E_{\text {mech }} & =\frac{1}{2} m v^{2}-G \frac{m_{1} m_{2}}{r_{12}} \begin{cases}<0 & \text { bound; ellipse } \\
=0 & \text { parabola } \\
>0 & \text { hyperbola }\end{cases}
\end{aligned}
$$

Waves:

$$
\begin{aligned}
y= & f(x-c t) \quad \text { along }+\mathrm{x} \quad y=f(x+c t) \quad \text { along -x } \\
k= & \frac{2 \pi}{\lambda} \quad \lambda=c T \quad \omega=\frac{2 \pi}{T} \quad c=\lambda f \\
y(x, t)= & f(x, t)=A \sin \left(k x-\omega t+\varphi_{i}\right) \\
y(x, t)= & 2 A \sin k x \cos \omega t \quad \text { standing wave } \\
& \text { nodes at } x=0, \pm \frac{\lambda}{2}, \pm \lambda, \pm \frac{3 \lambda}{2} \\
v= & \sqrt{T / \mu} \quad \mu=M_{\text {string }} / L_{\text {string }} \quad T=\text { tension } \quad \text { strings } \\
P_{\mathrm{av}}= & \frac{1}{2} \mu \lambda A^{2} \omega^{2} / T=\frac{1}{2} \mu A^{2} \omega^{2} c \\
\frac{\partial^{2} f}{\partial x^{2}}= & \frac{1}{c^{2}} \frac{\partial^{2} f}{\partial t^{2}} \\
f_{n}= & \frac{n v}{\lambda}=\frac{n v}{2 L} \quad \lambda_{n}=\frac{2 L}{n} \quad n=1,2,3 \ldots \\
f_{n}= & \frac{n v}{\lambda}=\frac{n v}{4 L} \quad \lambda_{n}=\frac{4 L}{n} \quad n=1,3,5 \ldots \quad \text { strings \& open-open pipe }
\end{aligned}
$$

## thermal stuff:

$$
\begin{aligned}
P V & =N k_{B} T=n R T \\
W & =P \Delta V \\
T(K) & =T\left({ }^{\circ} C\right)+273.15^{\circ} \\
Q & =m c \Delta t \quad c=\text { specific heat } \quad \text { no phase chg } \\
Q & = \pm m L \quad \text { phase chg }
\end{aligned}
$$

Isolated systems: $\overrightarrow{\mathbf{p}}, E=K+P E, L$ are all conserved.
Static equilibrium: $\sum F=0$ and $\sum \tau=0$ about any axis.
Elastic collision: KE and $p$ are both conserved.
Inelastic collision: only $p$ is conserved, not KE.

| Derived unit | Symbol | equivalent to |
| :--- | :---: | :---: |
| newton | N | $\mathrm{kg} \cdot \mathrm{m} / \mathrm{s}^{2}$ |
| joule | J | $\mathrm{kg} \cdot \mathrm{m}^{2} / \mathrm{s}^{2}=\mathrm{N} \cdot \mathrm{m}$ |
| watt | W | $\mathrm{J} / \mathrm{s}=\mathrm{m}^{2} \cdot \mathrm{~kg} / \mathrm{s}^{3}$ |


| Power | Prefix | Abbreviation |
| :--- | :--- | :---: |
| $10^{-6}$ | micro | $\mu$ |
| $10^{-3}$ | milli | m |
| $10^{-2}$ | centi | c |
| $10^{3}$ | kilo | k |
| $10^{6}$ | mega | M |

Moments of inertia of things of mass $M$

| Object | axis | dimension | I |
| :--- | :---: | :---: | :---: |
| solid sphere | central axis | radius $R$ | $\frac{2}{5} M R^{2}$ |
| hollow sphere | central axis | radius $R$ | $\frac{2}{3} M R^{2}$ |
| solid disc/cylinder | central axis | radius $R$ | $\frac{1}{2} M R^{2}$ |
| hoop | central axis | radius $R$ | $M R^{2}$ |
| point particle | pivot point | distance $R$ to pivot | $M R^{2}$ |
| rod | center | length $L$ | $\frac{1}{12} M L^{2}$ |
| rod | end | length $L$ | $\frac{1}{3} M L^{2}$ |

Thermodynamic processes

| Process | const | $W$ | $Q$ | $\Delta E_{\mathrm{th}}$ | ideal gas $E$ law |
| :--- | :--- | :--- | :--- | :--- | :--- |
| isochoric | $V$ | 0 | $N C_{V} \Delta T$ |  | $\Delta E_{\text {th }}=Q$ |
| isobaric | $P$ | $-N k_{B} \Delta T$ | $N C_{P} \Delta T$ | $N C_{V} \Delta T$ | $\Delta E_{\text {th }}=W+Q$ |
| isothermal | $T$ | $-N k_{B} T \ln \left(\frac{V_{f}}{V_{i}}\right)$ |  | 0 | $Q=-W$ |

