PH105 Spr 17
Exam 3A

1. which version

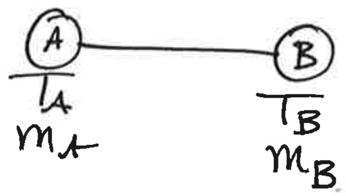
2. centr. force is the result of other forces, not a force itself
$\underline{\underline{B}}$ tension & gravity

3. $T = \dfrac{2\pi r}{v} \qquad a = \dfrac{v^2}{R} = \dfrac{(2\pi r/T)^2}{r} \propto \dfrac{1}{T^2}$

for $a \rightarrow \dfrac{a}{2}$ need $T \rightarrow T\sqrt{2}$ $\underline{\underline{B}}$

4. $K_r = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v^2}{R^2}\right) = \frac{1}{5}mv^2 < K_t = \frac{1}{2}mv^2$ $\underline{\underline{B}}$

5.



$a_A = 8.5\, a_B \qquad T_B = 2T_A$

$a_A = \dfrac{v_A^2}{R_A} = 8.5\,\dfrac{v_B^2}{R_B} \qquad T_B = \dfrac{2\pi R_B}{v_B} = 2 \cdot \dfrac{2\pi R_A}{v_A}$

$\Downarrow \qquad \Downarrow$

$\dfrac{R_A}{R_B} = \left(\dfrac{v_a}{v_b}\right)^2 \cdot \dfrac{1}{8.5} \qquad \dfrac{v_A}{v_B} = 2\,\dfrac{R_A}{R_B}$

$\dfrac{R_A}{R_B} = \left(2\,\dfrac{R_A}{R_B}\right)^2 \cdot \dfrac{1}{8.5}$

$\dfrac{R_A}{R_B} = \dfrac{8.5}{4} \sim 2.1$ $\underline{A}$

6. $K_s = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega_s^2$

$K_c = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_c^2$ (note length of cyl. irrelevant)

$K_s = K_c \Rightarrow \frac{2}{5}\omega_s^2 = \frac{1}{2}\omega_c^2 \Rightarrow \omega_c = \sqrt{\frac{4}{5}}\,\omega_s = \dfrac{2\omega_s}{\sqrt{5}}$ $\underline{\underline{C}}$

7. $\Delta t_a = 10\,s \quad \omega_f = 25\,s^{-1} \quad \Delta t_{run} = 37\,s \quad \alpha = 1.5\,s^{-2}$

$0 = \omega_f - \alpha t \Rightarrow t = \dfrac{\omega_f}{\alpha} \sim 17\,s$ $\underline{\underline{C}}$

8. larger I needs more energy for rotation ⇒ less for translation ⇒ slower.

sphere, disc, hoop since $I_s < I_d < I_h$

D

9. torque balance is $r_b m_b g = r_g m_g g$

if both r_b and r_g halve, balance still satisfied D

10. $\sum \tau = rF = I\alpha$

$$I = \frac{rF}{\alpha} = \frac{(0.8\,m)(5N)}{2s^{-1}} \simeq 2\ kg\cdot m^2 \quad \underline{\underline{B}}$$

11.

I Δx

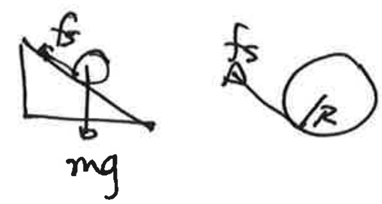
$$\Delta x = \tfrac{1}{2}at^2 \Rightarrow a = \frac{2\Delta x}{t^2}$$

$$\sum \tau = RF = I\alpha = I\frac{a}{R} = \frac{2\Delta x}{Rt^2} I$$

$$\Rightarrow I = \frac{R^2 F t^2}{2\Delta x} \simeq 0.2\ kg\cdot m^2 \quad \underline{\underline{D}}$$

12. $\sum F = -f_s + mg\sin\theta = ma$

$\sum \tau = f_s R = I\alpha = I\frac{a}{R}$



combine, $a = \dfrac{g\sin\theta}{1 + I/mR^2}$ then $\Delta x = \frac{1}{2}at^2$ or $t = \sqrt{\dfrac{2\Delta x}{a}}$

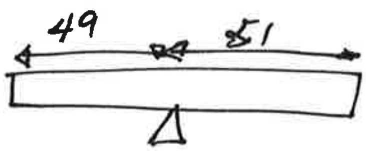
$$\Rightarrow t = \sqrt{\frac{2\Delta x(1 + I/mR^2)}{g\sin\theta}} \simeq 1.59s \quad \underline{\underline{E}}$$

13. $$\frac{K_{cyl}}{K_{sh}} = \frac{\frac{1}{2}mv_{cyl}^2(1+C_{cyl})}{\frac{1}{2}mv_{sh}^2(1+C_{sh})} = 1 \Rightarrow \frac{v_{cyl}}{v_{sh}} = \sqrt{\frac{1+C_{sh}}{1+C_{cyl}}} = \sqrt{\frac{1+\frac{2}{3}}{1+\frac{1}{2}}} = \frac{\sqrt{10}}{3}$$

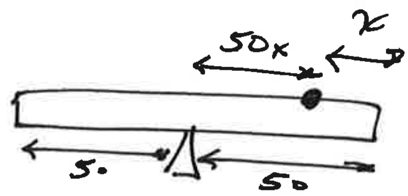
$C_{cyl} = \frac{1}{2}$, $C_{sh} = \frac{2}{3}$

$\underline{\underline{C}}$

14.

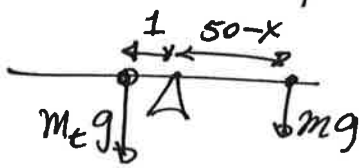


A) balances



B) still balances

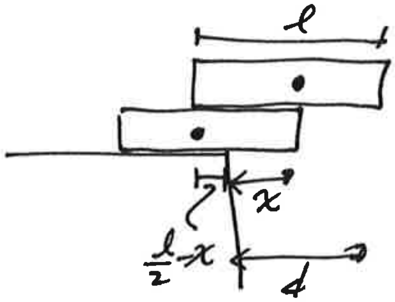
B) can treat whole rod as having its center of mass 1cm to the left of the pivot!



$$\sum_i \tau = m_t g \cdot 1 - mg(50-x) = 0$$

$m_t = -50m + mx$ $\quad$ $x = \dfrac{-m_t + 50m}{m} \approx \underline{\underline{34\,cm}}$

15.



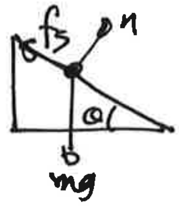
• about table corner, upper block center on edge of lower gives max possible overhang

$$\sum \tau = mg\left(\frac{l}{2}-x\right) - mgx = 0$$

$$\Rightarrow \quad \frac{l}{2} - x = x \quad \Rightarrow \quad x = \frac{l}{4} = 3\,\text{cm}$$

total is $d = x + \frac{l}{2} = 3 + 6\,\text{cm} = \underline{9\,\text{cm}}$ B

16. Slides first.



usual analysis.

$$\sum F_x = mg\sin\theta - f_s = mg\sin\theta - \mu mg\cos\theta = 0$$

$\Rightarrow \theta = \tan^{-1}\mu_k \simeq 24°$ slips for any larger θ

clearly doesn't tip until 45° $\underline{\underline{E}}$

17. from previous problems, $a = \frac{g\sin\theta}{1+c} \simeq 3.07\,\text{m/s}^2$ $\underline{\underline{C}}$

18. $T = 2\pi\sqrt{\frac{m}{k}}$ does not depend on A. $\underline{\underline{A}}$

19. $T = 2\pi\sqrt{\frac{m}{k}}$ double m, $T \to T\sqrt{2}$ $\underline{\underline{E}}$

20. $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow k = \frac{4\pi^2 m}{T^2} \simeq 24\,N$ $\underline{\underline{A}}$

note you're given $mg = 2.450\,N$, not m!

21. $F_{tot} = -k_1\Delta x - k_2\Delta x = ma$ $\quad a = -\left(\frac{k_1+k_2}{m}\right)\Delta x$ $\quad \Rightarrow \omega = \sqrt{\frac{k_1+k_2}{m}}$

$\simeq 25\,s^{-1}$ $\underline{\underline{C}}$

22. $x_i = 0.08\,m \quad \omega = \sqrt{\frac{k}{m}}$

$x(t) = x_i \cos(\omega t + \varphi_i)$ since $x(0) = x_i$, $\varphi_i = 0$

$\Rightarrow x(t) = x_i \cos(\omega t)$ $\quad v(t) = \frac{dx}{dt} = -\omega x_i \sin(\omega t)$

$v(0.4s) \simeq 0.82 \frac{m}{s}$ $\underline{\underline{E}}$

23. collision: $m v_i = (m+m) v_f \Rightarrow v_f = \frac{1}{2} v_i$

$K_i = \frac{1}{2}(2m) v_f^2 = \frac{1}{4} m v_i^2$ but $K_i = U_{max} = \frac{1}{2} k A^2$

$\Rightarrow \frac{1}{2} k A^2 = \frac{1}{4} m v_i^2 \Rightarrow A = \sqrt{\frac{1}{2} \frac{m v_i^2}{k}} \simeq \underline{0.3 m}$

$T = 2\pi \sqrt{\frac{(2m)}{k}} \simeq \underline{1.26 s}$ $\quad \underline{\underline{C}}$

24. $E = \frac{1}{2} m \omega^2 A^2$ $\quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi A}\sqrt{\frac{2E}{m}} \simeq 4.5\, s^{-1} = 4.5\,Hz$ $\underline{\underline{B}}$

$\Rightarrow \omega = \sqrt{\frac{2E}{m}} \frac{1}{A}$

25. $T = 2\pi\sqrt{l/g}$ independent of mass!

doubling l, $T \rightarrow T\sqrt{2}$ $\quad \underline{\underline{C}}$

26. $E = \frac{1}{2} k A^2$ mass irrelevant $\underline{\underline{E}}$