Formula sheet

$$g = 9.81\,\mathrm{m/s^2} \quad G = 6.67\times10^{-11}\,\mathrm{N\,m^2/kg^2}$$
$$N_A = 6.022\times10^{23}\,\text{things/mol}$$
$$k_B = 1.38065\times10^{-23}\,\mathrm{J\cdot K^{-1}} = 8.6173\times10^{-5}\,\mathrm{eV\cdot K^{-1}}$$
$$\text{sphere} \quad V=\frac{4}{3}\pi r^3 \quad A = 4\pi r^2$$
$$ax^2+bx^2+c=0 \Longrightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$\sin\alpha\pm\sin\beta = 2\sin\frac{1}{2}(\alpha\pm\beta)\cos\frac{1}{2}(\alpha\mp\beta)$$
$$\cos\alpha\pm\cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$$
$$c^2 = a^2+b^2-2ab\cos\theta_{ab}$$
$$\frac{d}{dx}\sin ax = a\cos ax \qquad \frac{d}{dx}\cos ax = -a\sin ax$$
$$\int\cos ax\,\mathrm{d}x = \frac{1}{a}\sin ax \qquad \int\sin ax\,\mathrm{d}x = -\frac{1}{a}\cos ax$$
$$\sin\theta\approx\theta \qquad \cos\theta\approx 1-\frac{1}{2}\theta^2 \qquad \text{small }\theta$$

$$\vec v = \lim_{\Delta t\to0}\frac{\Delta\vec r}{\Delta t}\equiv\frac{d\vec r}{dt}$$
$$a_x = \lim_{\Delta t\to0}\frac{\Delta v_x}{\Delta t}\equiv\frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$
$$\Delta v_x = \int_{t_i}^{t_f} a_x(t)\,dt \qquad \Delta x = \int_{t_i}^{t_f} v_x(t)\,dt$$
$$x(t) = x_i + v_{x,i}t + \frac{1}{2}a_x t^2$$
$$v_x(t) = v_{x,i} + a_x t$$
$$v_{x,f}^2 = v_{x,i}^2 + 2a_x\Delta x$$
$$a_{x,\mathrm{ramp}} = g\sin\theta$$

$$\Delta U^G = mg\Delta x \qquad \frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}$$
$$E_{\mathrm{mech}} = K+U \quad K=\frac{1}{2}mv^2$$
$$\Delta E = \Delta K+\Delta U = 0 \quad \text{non-dissipative, closed}$$

$$\Delta E = W \qquad P = \frac{dE}{dt}$$
$$\Delta U_{\mathrm{spring}} = \frac{1}{2}k(x-x_o)^2$$
$$P = F_{\mathrm{ext,x}}\,v_x \quad \text{one dimension}$$
$$W = \left(\sum\vec F\right)\Delta x_F \quad \text{constant force 1D}$$
$$W = \int_{x_i}^{x_f} F_x(x)\,dx \quad \text{nondiss. force, 1D}$$

$$\Delta\vec p = \vec 0 \quad \text{isolated system}$$
$$\vec p_f = \vec p_i \quad \text{isolated system}$$
$$\vec p \equiv m\vec v$$
$$m_u = -\frac{\Delta v_{s,x}}{\Delta v_{u,x}}m_s$$
$$\vec J = \Delta\vec p$$
$$v_{1f} = \left(\frac{m_1-m_2}{m_1+m_2}\right)v_{i1} + \left(\frac{2m_2}{m_1+m_2}\right)v_{2i} \quad \text{1D elastic}$$
$$v_{2f} = \left(\frac{2m_1}{m_1+m_2}\right)v_{1i} + \left(\frac{m_2-m_1}{m_1+m_2}\right)v_{2i} \quad \text{1D elastic}$$
$$\Delta E = 0 \quad \text{isolated system}$$
$$K = \frac{1}{2}mv^2$$
$$\vec v_{12} = \vec v_2 - \vec v_1 \quad \text{relative velocity}$$

$$\vec a = \frac{\sum\vec F}{m} \qquad a_{\vec{cm}} = \frac{\sum\vec F_{\mathrm{ext}}}{m} \qquad \sum\vec F\equiv\frac{d\vec p}{dt}$$
$$\vec J = \left(\sum\vec F\right)\Delta t \quad \text{constant force}$$
$$\vec J = \int_{t_i}^{t_f}\sum\vec F(t)\,dt \quad \text{time-varying force}$$
$$\vec F_{12} = -\vec F_{21} \qquad F_{\mathrm{grav}} = -mg \qquad F_{\mathrm{spring}} = -k\Delta x$$

Rotation: we use radians

$$s = \theta r \quad \leftarrow \text{arclength}$$
$$\omega = \frac{d\theta}{dt} = \frac{v_t}{r} \qquad \alpha = \frac{d\omega}{dt}$$
$$a_t = \alpha r \quad \text{tangential} \qquad a_r = -\frac{v^2}{r} = -\omega^2 r \quad \text{radial}$$
$$v_t = r\omega \qquad v_r = 0$$

$$(F_{12}^s)_{\max} = \mu_s F_{12}^n$$
$$F_{12}^k = \mu_k F_{12}^n$$
$$W = \vec F\cdot\Delta\vec r_F \quad \text{const non-diss force}$$
$$W = \int_{\vec r_i}^{\vec r_f}\vec F(\vec r)\cdot d\vec r \quad \text{variable nondiss force}$$

$\downarrow$ launched from origin, level ground

$$y(x) = (\tan\theta_o)\,x - \frac{gx^2}{2v_o^2\cos^2\theta_o}$$
$$\text{max height } = H = \frac{v_i^2\sin^2\theta_i}{2g}$$
$$\text{Range } = R = \frac{v_i^2\sin 2\theta_i}{g}$$
$$(F_{12}^2)_{\max} = \mu_s F_{12}^n \qquad F_{12}^k = \mu_k F_{12}^n$$

fluids:

$$P = F/A \qquad P(d) = P_{\text{surface}} + \rho g d$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \quad F_1 x_1 = F_2 x_2 \qquad \text{hydraulics}$$

$$B = \text{buoyant force} = \text{weight of water displaced} = \rho_f V_{\text{displ}} g$$

$$P = P_{\text{gauge}} + P_{\text{atm}} \qquad \rho = M/V$$

$$I = \sum_i m_i r_i^2 \Rightarrow \int r^2\, dm = kmr^2 \qquad I = mr^2 \quad \text{point particle}$$

$$I_z = I_{com} + md^2 \quad \text{axis } z \text{ parallel, dist } d$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

$$K = \frac{1}{2} I\omega^2 = L^2/2I$$

$$\Delta K = \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2 = W = \int \tau\, d\theta$$

$$P = \frac{dW}{dt} = \tau\omega$$

$$\tau = rF\sin\theta_{rF} = r_\perp F = rF_\perp$$

$$\tau_{net} = \sum \vec{\tau} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$$

$$K_{\text{tot}} = K_{cm} + K_{rot} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I\omega^2$$

Oscillations:

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \quad \omega = \frac{2\pi}{T} = 2\pi f$$

$$x(t) = A\sin(\omega t + \varphi_i)$$

$$v(t) = \frac{dx}{dt} = \omega A\cos(\omega t + \varphi_i)$$

$$a(t) = \frac{d^2x}{dt^2} = -\omega^2 A\sin(\omega t + \varphi_i)$$

$$\varphi(t) = \omega t + \varphi_i$$

$$a = -\omega^2 x \qquad \frac{d^2x}{dt^2} = -\omega^2 x \qquad \frac{d^2\theta}{dt^2} = -\omega^2\theta$$

$$E = \frac{1}{2} m\omega^2 A^2$$

$$\omega = \sqrt{k/m} \quad \text{spring.}$$

$$T = \begin{cases} 2\pi\sqrt{L/g} & \text{simple pendulum} \\ 2\pi\sqrt{I/mgl_{cm}} & \text{physical pendulum} \end{cases}$$

Gravitation

$$F_{12}^G = G\frac{m_1 m_2}{r_{12}^2} \quad G = 6.67 \times 10^{-11}\,\text{N}\,\text{m}^2/\text{kg}^2$$

$$U^G(r) = -G\frac{m_1 m_2}{r_{12}^2}$$

$$E_{\text{mech}} = \frac{1}{2} mv^2 - G\frac{m_1 m_2}{r_{12}^2} \begin{cases} < 0 & \text{bound; ellipse} \\ = 0 & \text{parabola} \\ > 0 & \text{hyperbola} \end{cases}$$

Waves:

$$y = f(x - ct) \quad \text{along +x} \qquad y = f(x + ct) \quad \text{along -x}$$

$$k = \frac{2\pi}{\lambda} \quad \lambda = cT \quad \omega = \frac{2\pi}{T} \quad c = \lambda f$$

$$y(x,t) = f(x,t) A\sin(kx - \omega t + \varphi_i)$$

$$y(x,t) = 2A\sin kx\cos\omega t \quad \text{standing wave}$$

$$\text{nodes at } x = 0, \pm\frac{\lambda}{2}, \pm\lambda, \pm\frac{3\lambda}{2}$$

$$v = \sqrt{T/\mu} \quad \mu = M_{\text{string}}/L_{\text{string}} \quad T = \text{tension} \qquad \text{strings}$$

$$P_{\text{av}} = \frac{1}{2}\mu\lambda A^2\omega^2/T = \frac{1}{2}\mu A^2\omega^2 c$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

$$f_n = \frac{nv}{\lambda} = \frac{nv}{2L} \quad \lambda_n = \frac{2L}{n} \quad n = 1, 2, 3\ldots \qquad \text{strings \& open-open pipe}$$

$$f_n = \frac{nv}{\lambda} = \frac{nv}{4L} \quad \lambda_n = \frac{4L}{n} \quad n = 1, 3, 5\ldots \qquad \text{closed-open pipe}$$

thermal stuff:

$$PV = Nk_BT = nRT$$

$$T(K) = T(^\circ C) + 273.15^\circ$$

$$Q = mc\Delta t \quad c = \text{specific heat} \qquad \text{no phase chg}$$

$$Q = \pm mL \qquad \text{phase chg}$$

Isolated systems: $\vec{p}$, $E = K + PE$, L are all conserved.
Static equilibrium: $\sum F = 0$ and $\sum \tau = 0$ about any axis.
Elastic collision: KE and p are both conserved.
Inelastic collision: only p is conserved, not KE.

Derived unit	**Symbol**	**equivalent to**
newton	N	kg·m/s^2
joule	J	kg·m^2/s^2 = N·m
watt	W	J/s=m^2·kg/s^3

Power	**Prefix**	**Abbreviation**
10^{-6}	micro	µ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M

Process	**const**	W	Q	ΔE_{th}	ideal gas E law
isochoric	V	0	$NC_V\Delta T$		$\Delta E_{\text{th}} = Q$
isobaric	P	$-Nk_B\Delta T$	$NC_P\Delta T$	$NC_V\Delta T$	$\Delta E_{\text{th}} = W + Q$
isothermal	T	$-Nk_BT\ln\left(\frac{V_f}{V_i}\right)$		0	$Q = -W$