Name ____________________ Date ____________

25Sept06
LeClair
FormA

PH 105-2 Exam I SOLUTION

The bits in red are my solutions. The ratios in blue are the number of people out of the total that got that question right. Additional notes in blue refer to the source of the question.

Class average was 80.5%, with a standard deviation of 16%. Based on this, letter grades are: A=(14,15), B=(12,13), C=11, D=(9,10), F<9. There were 15 As, 22 Bs, 7 Cs, 9 Ds, and 5 Fs.

1. Which of the following *could* be the proper equation for the energy of an object moving at relativistic velocities (*i.e.*, close to the speed of light)? Note that energy E has units of kg·m^2/s^2, the speed of light c is in m/s, mass m is in kg, and momentum p is in kg·m/s. 43/57

- ◯ $E = pc + m^2c^2$
- ◯ $E^2 = p^2c + (mc^2)^2$
- ◯ $E = p^2c^2 + (mc^2)^2$
- ◯ $E^2 = p^2c^2 + (mc^2)^2$

The basic solution here, unless you know which equation is right to being with, is to test each possible answer *via* dimensional analysis.

$$E = \left[\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}\right] \stackrel{?}{=} pc + m^2c^2 = \left[\frac{\text{kg}\cdot\text{m}}{\text{s}}\right]\left[\frac{\text{m}}{\text{s}}\right] + \left[\text{kg}^2\right]\left[\frac{\text{m}^2}{\text{s}^2}\right] = \left[\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}\right] + \left[\frac{\text{kg}^2\cdot\text{m}^2}{\text{s}^2}\right]$$

Since the units of the two terms on the right side of the equation are not the same, this is already not a valid choice (even though one of them does have units of energy

$$E^2 = \left[\frac{\text{kg}^2\cdot\text{m}^4}{\text{s}^4}\right] \stackrel{?}{=} p^2c + \left(mc^2\right)^2 = \left[\frac{\text{kg}^2\cdot\text{m}^2}{\text{s}^2}\right]\left[\frac{\text{m}}{\text{s}}\right] + \left[\text{kg}^2\right]\left[\frac{\text{m}^4}{\text{s}^4}\right] = \left[\frac{\text{kg}^2\cdot\text{m}^3}{\text{s}^3}\right] + \left[\frac{\text{kg}^2\cdot\text{m}^4}{\text{s}^4}\right]$$

Again, the two terms on the right hand side have different units, so this is not a valid equation, even though the second term by itself has the proper units.

$$E = \left[\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}\right] \stackrel{?}{=} p^2c^2 + (mc^2)^2 = \left[\frac{\text{kg}^2\cdot\text{m}^2}{\text{s}^2}\right]\left[\frac{\text{m}^2}{\text{s}^2}\right] + \left[\text{kg}^2\right]\left[\frac{\text{m}^4}{\text{s}^4}\right] = \left[\frac{\text{kg}^2\cdot\text{m}^4}{\text{s}^4}\right] + \left[\frac{\text{kg}^2\cdot\text{m}^4}{\text{s}^4}\right]$$

Although both terms have the same units this time ... they are both the wrong units.

$$E^2 = \left[\frac{\text{kg}^2\cdot\text{m}^4}{\text{s}^4}\right] \stackrel{?}{=} p^2c^2 + \left(mc^2\right)^2 = \left[\frac{\text{kg}^2\cdot\text{m}^2}{\text{s}^2}\right]\left[\frac{\text{m}^2}{\text{s}^2}\right] + \left[\text{kg}^2\right]\left[\frac{\text{m}^4}{\text{s}^4}\right] = \left[\frac{\text{kg}^2\cdot\text{m}^4}{\text{s}^4}\right] + \left[\frac{\text{kg}^2\cdot\text{m}^4}{\text{s}^4}\right]$$

Finally, this time both terms on the right hand side have the same units, and they are the *correct* units. This is it (and it is actually the equation for the energy of a body traveling at relativistic speeds).

2. A car is traveling at a constant velocity of 18 m/s and passes a police cruiser. Exactly 2 seconds after passing, the cruiser begins pursuit, with a constant acceleration of 2.5 m/s^2. How long does it take for the cruiser to overtake the car (from the moment the cop car starts)? 51/57 From the practice exam.

- ◯ 16.2 sec
- ◯ 13.2 sec
- ◯ 24.3 sec
- ◯ 0.22 sec

Basically, we just want to write down the equations describing the position as a function of time for both the car and the cop, and set them equal to each other. We start with our basic equation for 1-D motion and fill in the terms we know: $x(t) = x_i + v_{ix}t + \frac{1}{2}a_x t^2$

The car is just traveling at constant velocity, with no acceleration. Call its starting position $x_i = 0$, and we can write:

$$x_{\text{car}} = 18t$$

The cop starts two seconds later, we have to be careful to use $t - 2$ wherever time appears. The initial velocity for the cop is zero, as is the initial position. We only need to account for the acceleration of 2.5 m/s^2.

$$x_{\text{cop}} = \frac{1}{2}\left(2.5\right)\left(t-2\right)^2 = 1.25t^2 - 5t + 5$$

At the moment when the cop overtakes the car, their positions are equal:

$$x_{\text{car}} = x_{\text{cop}} = 18t = 1.25t^2 - 5t + 5 \quad \Rightarrow \quad 1.25t^2 - 23t + 5 = 0$$

Now we just solve the quadratic equation, which gives us roots of $t = [0.22, 18.2]$. What we were looking for was the time *relative to when the cop started*, which means we want $t-2$. The negative root is not physical (it corresponds to a time before the cop started), so **16.2 s** is the answer.

Incidentally, we could have set $t = 0$ at the moment the cop started, and not have to worry about subtracting 2 seconds off of the final result. This would give us the following:

$$x_{\text{car}} = 18\left(t-2\right) = x_{\text{cop}} = \frac{1}{2}\left(2.5\right)t^2 \quad \Rightarrow \quad 1.25t^2 - 18t - 36 = 0$$

You can verify yourself that the positive root of this quadratic equation is 16.2 sec, the time we were looking for.

3. The position of a particle can be described by the equation $x(t) = 2.5 + 1.7t - 3.9t^2$. When the particle returns to its $t = 0$ position some time later, what is its velocity? 33/57 Similar question on a quiz

- ◯ 1.7 m/s
- ◯ -16.2 m/s
- ◯ 2.5 m/s
- ◯ -1.7 m/s

We want to find the velocity of the particle at some time t when it has the same position as it did at $t = 0$. So, first we need to find the time t at which the position $x(t)$ is the same as $x(0)$. One can see readily that $x(0) = 2.5$. Therefore:

$$x(t) = x(0) = 2.5 = 2.5 + 1.7t - 3.9t^2 \quad \Rightarrow \quad 1.7t - 3.9t^2 = 0 = t(1.7 - 3.9t) \quad \Rightarrow \quad t = 0.436\,\text{s}$$

Now that we know *when* the particle returns to its $t = 0$ position, we can calculate the velocity at that time.

$$v(t) = \frac{\mathrm{d}}{\mathrm{d}t}x(t) = 1.7 - 7.8t \quad \Rightarrow \quad v(0.436) = 1.7 - 7.8(0.436) = -1.7$$

A slightly faster way? The equation given is just that for projectile motion. We know that projectile motion is totally symmetric - the velocity at some position x is the same whether the particle is on its way up or on its way down. That is, the velocity at $x(0)$ is the same in magnitude whether the particle is going up or down, it just differs in direction. So finding $-v(0)$ works just as well.

4. A rubber ball was thrown at a brick wall with an initial velocity of 10 m/s, and rebounded with a velocity of 8.5 m/s. The rebound was found to take 3.5×10^{-3} sec. What was the acceleration experienced by the ball during the rebound? 50/57 on the homework

- ◯ -9.8 m/s^2
- ◯ -5300 m/s^2
- ◯ 480 m/s^2
- ◯ 2600 m/s^2

All we need to know for this one is the definition of average acceleration. Straight off of the homework too.

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{-8.5 - 10}{3.5 \times 10^{-3}} = -5286 \approx -5300$$

5. A projectile will be launched with an initial velocity of 750 m/s, and needs to hit a target 23 km away. What should the launch angle be? You can ignore air resistance. 46/57 these are realistic numbers, neglecting air resistance

- ◯ 12°

○ 24°
○ 7°
○ 20°

The easiest way to solve this one is to use the formula for the range of a projectile, which we derived in class (and which was given on the formula sheet). Of course it is only slightly more work just to write down $x(t)$ and $y(t)$ explicitly and solve ... Note the distance is given in *km*.

$$R = \frac{v_i^2 \sin 2\theta}{g} = \frac{750^2 \sin 2\theta}{g} = 23000 \quad \Rightarrow \quad \sin 2\theta = \frac{23000g}{750^2} \quad \Rightarrow \quad \theta = 11.8^\circ \approx 12^\circ$$

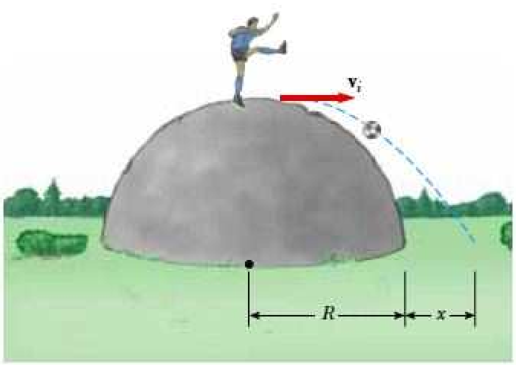
6. Joe foolishly fires his .270 Winchester, which has a muzzle velocity of 957 m/s using a 130 grain load, into the air at a 17° angle. Ignoring air resistance, how far away from Joe will the bullet land? 50/57 these are realistic numbers, neglecting air resistance

○ 2.7×10^4 m
○ 5.2×10^4 m
○ 1.1×10^4 m
○ 9.8×10^3 m

Once again, using the range equation is most straightforward

$$R = \frac{v_i^2 \sin 2\theta}{g} = \frac{957^2 \sin 34^\circ}{g} = 5.2 \times 10^4$$

7. A person standing at the top of a hemispherical rock of radius R kicks a ball (initially at rest on the top of the rock) to give it horizontal velocity $\vec{\mathbf{v}}_\mathbf{i}$ as shown below. What must be its minimum initial speed if the ball is never to hit the rock after it is kicked? 33/57 Chapter 4, number 62. On a graduate qualifier a few years ago



○ $|\vec{\mathbf{v}}_\mathbf{i}| > gR$
○ $|\vec{\mathbf{v}}_\mathbf{i}| > \sqrt{gR}$
○ $|\vec{\mathbf{v}}_\mathbf{i}| > \sqrt{2gR}$
○ $|\vec{\mathbf{v}}_\mathbf{i}| > R^2/g$

The Harder (but more general) Way: solve the problem as stated, without looking at the possible answers. We know that the ball being kicked off of the rock will follow projectile motion, which describes a parabolic curve $y(x)$. The rock, being a hemisphere, can be described in the x-y plane by a circle, $y_{\text{rock}}^2 + x_{\text{rock}}^2 = R^2$. What we want to know, then, is for what minimum value of

v_i does the parabola describing the ball's motion *not* intersect the quarter circle?

Measure heights relative to the ground and call the ball's starting position $x = 0$. The x position of the ball at any time t is just $x = v_i t$. The y position of the ball at any time t can be described by:

$$y_{\text{ball}} = y_i + v_{iy}t + \frac{1}{2}a_y t^2 = R + 0 - \frac{1}{2}gt^2 = R - \frac{gx^2}{2v_i^2}$$

For the last step, we made use of $t = x/v_i$. Note that this is just the usual trajectory for projectile motion $y(x)$ with the starting height $y_i = R$ added in. In order for the ball not to hit the rock, the parabola above must everywhere lie above the circle describing the rock. In other words, $y_{\text{ball}} > y_{\text{rock}}$ at every x. We can write this as an inequality:

$$y_{\text{ball}}^2 + x_{\text{ball}}^2 > R^2$$

Now plug in what we know ...

$$\begin{aligned}
\left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 &> R^2 \\
R^2 - \frac{gx^2R}{v_i^2} + \frac{g^2x^4}{4v_i^4} + x^2 &> R^2 \\
\frac{g^2x^4}{4v_i^4} + x^2 &> \frac{gx^2R}{v_i^2} \\
\frac{g^2x^2}{4v_i^4} + 1 &> \frac{gR}{v_i^2}
\end{aligned}$$

Now we note that if this inequality is true at $x = 0$, it is true for all $x > 0$ (physically, this means that if the initial trajectory has a high enough radius of curvature, it clears the whole rock). This leads us to:

$$v_i > \sqrt{gR}$$

The Easier Way: rely on eliminating the answers that are not possible, and brute-forcing the remaining ones. We first notice that only the second and third choices even have units of velocity, the first and fourth choices can be ruled out immediately. This give you a 50-50 chance right off the bat. For the remaining choices, put the possible v_i into the trajectory equation for projectile motion $y(x)$ with $\theta = 0$, being sure to add in the starting height of $y(0) = R$. First we'll try $v_i = \sqrt{gR}$

$$y_{\text{ball}}(x) = R - \frac{g}{2v_i^2}x^2 = R - \frac{g}{2gR}x^2 = R - \frac{1}{2R}x^2$$

Now square both sides ...

$$y_{\text{ball}}^2 = \left(R - \frac{x^2}{2R}\right)^2 = R^2 - x^2 + \frac{x^4}{4R^2} \quad \Rightarrow \quad x^2 + y_{\text{ball}}^2 = R^2 + \frac{x^4}{4R^2}$$

Since the rock can be described by $x^2 + y^2 = R^2$, this trajectory is clearly always above the rock (by $\frac{x^4}{4R^2}$) . The question asked for the *minimal* initial velocity, so the remaining possible answer, $v_i = \sqrt{2gR}$, need not be considered.

8. After being struck by a hockey stick, a hockey puck slides across the ice with an initial velocity of 7.0 m/s. If the coefficient of kinetic friction μ_k between the ice and the puck is 0.15, what is the velocity of the puck when it reaches the goal 10 m down the ice? 49/57 from the homework, done in class

- ◯ 4.4 m/s
- ◯ it does not reach the goal, μ_k is too big
- ◯ 2.2 m/s
- ◯ 19.6 m/s

First draw a free-body diagram for the hockey puck, let us call the x axis parallel to the ice and the y axis normal to it. In the y direction, we have only the normal force n and the puck's weight $-mg$.

$$\Sigma F_y = n - mg = ma_y = 0 \Rightarrow n = mg.$$

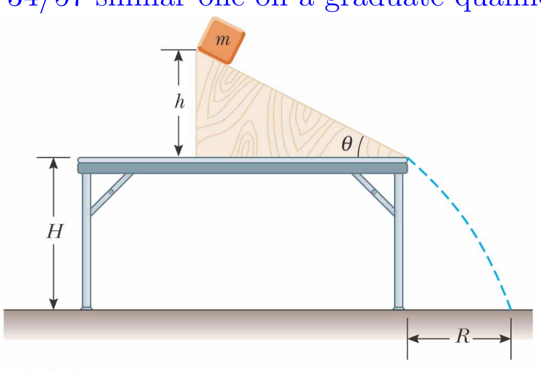
In the x direction, we have only kinetic friction.

$$\Sigma F_x = -f_k = -\mu_k n = -\mu_k mg = ma_x \quad \Rightarrow \quad a_x = -\mu_k g$$

Given the acceleration, we can readily find the final velocity:

$$v_f^2 = v_i^2 + 2a_x \Delta x = v_i^2 - 2\mu_k g \Delta x = 7^2 - 2\,(0.15)\,(9.8)\,(10) \quad \Rightarrow \quad v_f = 4.42\,\text{m/s} \approx 4.4\,\text{m/s}$$

9. Consider the figure below. Let $h = 1$ m, $\theta = 37°$, $H = 2$ m, and $m = 1.1$ kg. There is a coefficient of kinetic friction $\mu_k = 0.2$ between the mass and the inclined plane, and the mass m starts out at the very top of the incline with a velocity of 0.1 m/s. Ignoring air resistance, what is R? 34/57 similar one on a graduate qualifier last year



- ◯ 4.0 m
- ◯ 0.5 m
- ◯ 2.6 m
- ◯ 1.4 m

This is a problem that really has to be worked in stages. First focus on the block and incline, and draw the free body diagram for those two. Let the x axis point down the incline, and the y axis point up normal to the incline. In the y direction, we have the normal force, and part of the weight of the block:

$$\Sigma F_y = n - mg\cos\theta = ma_y = 0 \qquad \Rightarrow \qquad n = mg\cos\theta$$

In the x direction, we have the other component of the weight, and opposing that we have friction:

$$\Sigma F_x = mg\sin\theta - f_k = mg\sin\theta - \mu_k n = mg\sin\theta - \mu_k mg\cos\theta = ma_x$$
$$\Rightarrow \qquad a_x = g\left(\sin\theta - \mu_k\cos\theta\right)$$

Given the acceleration a_x, the initial velocity $v_i = 0.1\,\mathrm{m/s}$, and the length of the ramp being $h/\sin\theta$, we can readily calculate the speed at the bottom of the ramp, which we'll call v_f:

$$v_f^2 = v_i^2 + 2a_x\Delta x = v_i^2 + 2g\left(\sin\theta - \mu_k\cos\theta\right)\left(\frac{h}{\sin\theta}\right) \qquad \Rightarrow \qquad v_f \approx 3.79\,\frac{\mathrm{m}}{\mathrm{s}}$$

Next, to find the position R we can just use the usual equations for projectile motion. In this case, the projectile starts off with a velocity of v_f, with $y_i = H$, and an angle of $-\theta$. We can easily write down the equations for $y(t)$ and $x(t)$, noting that $\cos(-\theta) = \cos\theta$ and $\sin(-\theta) = -\sin\theta$:

$$\begin{aligned} x(t) &= \left(v_f\cos\theta\right)t \\ y(t) &= H - \left(v_f\sin\theta\right)t - \frac{1}{2}gt^2 \end{aligned}$$

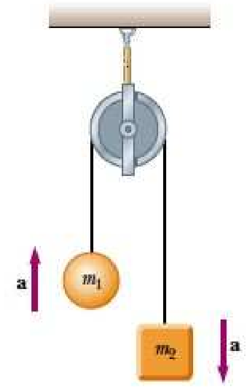
At the point where the block strikes the floor, $y = 0$. Setting $y(t) = 0$ and using the quadratic equation, the block must hit the floor at:

$$t_{\text{hit}} = \frac{1}{g}\left(-v_f\sin\theta \pm \sqrt{v_f^2\sin^2\theta + 2gH}\right)$$

Only the positive root is physical, which gives $t_{\text{hit}} = 0.449\,\text{sec}$. Finally, the quantity R must be

$$x(t_{\text{hit}}) = \left(v_f\cos\theta\right)t_{\text{hit}} = 1.35 \approx 1.4\,\mathrm{m}$$

10. Consider the so-called "Atwood's machine" below. What is the acceleration of the two masses, if one ignores friction and the mass of the pulley and rope? 47/57 worked in class, basically the subject of a lab



- ◯ $|\vec{\mathbf{a}}| = \left(\frac{m_2+m_1}{m_2-m_1}\right) g$
- ◯ $|\vec{\mathbf{a}}| = (m_2 - m_1)\, g$
- ◯ $|\vec{\mathbf{a}}| = \left(\frac{m_2-m_1}{m_1+m_2}\right) g$
- ◯ $|\vec{\mathbf{a}}| = \left(\frac{m_2-m_1}{m_2}\right) g$

Pick out each mass separately, and draw the free body diagram. For each mass, there is only its weight and the tension in the string, which we'll call T. We note that since a is the same for both masses, this means the string does not go slack and T is the same on both sides of the pulley. First, we write the force balance for m_1, calling up the $+y$ direction.

$$\Sigma F = T - m_1 g = m_1 a$$

For m_1 the acceleration is positive, since it moves m_1 in the $+y$ direction. Now we write the force balance for m_2, noting that the acceleration is *negative* for m_2!

$$\Sigma F = T - m_2 g = -m_2 a$$

Now, subtract the second equation from the first, being very careful with signs:

$$-m_1 g + m_2 g = m_1 a + m_2 a \qquad \Rightarrow \qquad a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

11. A block with mass 5 kg has an initial velocity of $\vec{\mathbf{v}}_{\mathbf{i}} = (7.0\,\hat{\mathbf{i}} + 3.1\,\hat{\mathbf{j}})$ m/s on a surface with no friction. A force acts on the block, and 17 seconds later its velocity is $\vec{\mathbf{v}}_{\mathbf{f}} = (2.0\,\hat{\mathbf{i}} + 15\,\hat{\mathbf{j}})$ m/s. What was the magnitude of the force? 54/57 similar one on the practice exam

- ◯ 3.8 N
- ◯ 5.5 N
- ◯ 14 N
- ◯ 4.4 N

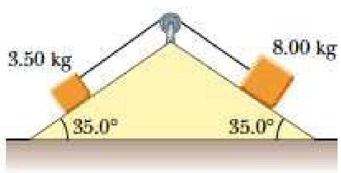
We know what the initial and final velocities are, and the time elapsed. Given these, we can calculate an acceleration that must be present to have caused the change in velocity. From the acceleration, we can get force *via* $\Sigma F = ma$. First, the acceleration by component:

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{2.0 - 7.0}{17} = \frac{-5}{17}$$
$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{15 - 3.1}{17} = \frac{-11.9}{17}$$

The force components follow directly, $F_x = ma_x$, $F_y = ma_y$, and from the components we get the magnitude of the force itself:

$$|F| = \sqrt{F_x^2 + F_y^2} = m\sqrt{a_x^2 + a_y^2} = 3.8\,\mathrm{N}$$

12. Two blocks of mass 3.50 kg and 8.00 kg are connected by a massless string that passes over a frictionless pulley, as shown in the figure below. The inclines are frictionless. Find the tension in the string. 29/57 Chapter 5, number 68



◯ 110 N
◯ 27 N
◯ 12 N
◯ 64 N

The first most crucial thing to realize is that *both blocks are moving.* Think about it for a moment. There is no friction, and one mass is much heavier than the other ... so the 8 kg mass has to slide down the ramp. Call the 3.50 kg mass m_1 and the 8.00 kg mass m_2. Let the tension in the string be T, and presume the string does not slack so T and $|a|$ are the same for both masses. Take the x axis in each case along the incline direction, with $+x$ to the right, and the y axis in each case normal to the incline. For each mass this gives a force balance:

$$\begin{aligned} T - m_1 g \sin 35^\circ &= m_1 a \\ -T + m_2 g \sin 35^\circ &= m_2 a \end{aligned}$$

Add these two equations together:

$$(m_2 - m_1)\, g \sin 35^\circ = (m_1 + m_2)\, a \qquad \Rightarrow \qquad a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g \sin 35^\circ = 0.22g$$

Plug that value for a into either equation above, and you should get $T = 27\,\mathrm{N}$

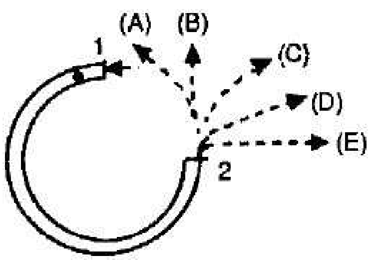
13. A 1500 kg car moving on a flat, horizontal road negotiates a curve. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is $\mu_s = 0.500$, find the maximum speed the car can have and still make the turn successfully. (*Hint: what is balancing the force of friction?*) 31/57

◯ 13.1 m/s
◯ 27.4 m/s
◯ 18.5 m/s
◯ 6.5 m/s

In this case the frictional force must be balanced by the centripetal force (or, more accurately, the frictional force must have the form of centripetal force to maintain circular motion). In this case the friction force is *static*, since we don't want any movement along the radial direction. Since the curve is flat and horizontal, the normal force on the car is just the weight of the car, mg. This gives:

$$f_s = \mu_s n = \mu_s mg = F_{\text{centr}} = \frac{mv^2}{r} \quad \Rightarrow \quad v = \sqrt{\mu_s g r} = 13.1\,\text{m/s}$$

14. The diagram below depicts a semicurcular channel that has been securely attached, in a **horizontal plane**, to the table top. A ball enters at channel "1", and exits at "2". Which of the path representations would most nearly correspond to the path of the ball as it exits the channel at "2" and rolls across the table top? 50/57



- ◯ A
- ◯ B
- ◯ C
- ◯ D
- ◯ E

Newton's 1st law states that in the absence of force, a body continues moving in a straight line at constant velocity. Already, this means that only B and E could possibly be correct - once the ball leaves the tube, there are no forces acting on it, so it must move in a straight line.

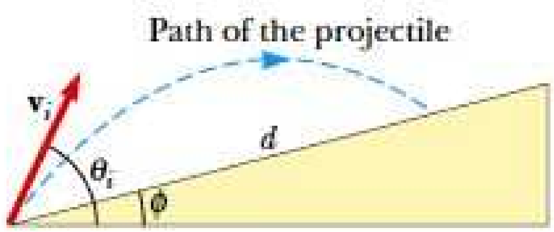
So, basically after exiting the tube, the ball will continue moving in the straight line corresponding to its instantaneous velocity at the moment it reached the end of the tube. For perfect circular motion, we know that the magnitude of the velocity is constant and its direction is tangent to the circle. This corresponds to path **B**, which is the correct answer.

15. Two metal balls are the same size, but one weights twice as much as the other. The balls are dropped from the top of a two story building at the same instant of time. The time it takes the balls to reach the ground below will be: 54/57

- ◯ about half as long for the heavier ball
- ◯ about half as long for the lighter ball
- ◯ about the same for both balls ←
- ◯ considerably less for the heavier ball, but not necessarily half as long
- ◯ considerably less for the lighter ball, but not necessarily half as long

From our knowledge of projectile motion, we know that a dropped object falls a distance $\frac{1}{2}gt^2$ in a time t, *independent of its mass*. The heavier and lighter objects will take essentially the same amount of time to fall, the only possible small difference arising from air resistance. Since we are talking about metal balls, presumably spheres, and this difference is entirely negligible anyway (if their mass is a factor of two different, their radius would only be a factor of $\sqrt[3]{2} \approx 1.26$ different).

BONUS 1 (+1pt): A projectile is fired up an incline (incline angle φ) with an initial speed v_i at an angle θ_i with respect to the horizontal ($\theta_i > \varphi$), as shown in the figure below. How far up the incline does the projectile land? One person got this, two got half credit.



This one is far harder and more annoying than it looks. First, here is one form of the answer, depending on what trigonometric identities one prefers ...

$$d = \frac{2v_i^2 \cos\theta_i \sin\left(\theta_i - \varphi\right)}{g\cos^2\varphi} \tag{1}$$

Basically, what you want to do is this. Find the general equation for the trajectory of the projectile $y_{\text{proj}}(x)$. Find the equations for a point on the ramp, *viz.*, $y_{\text{ramp}} = d\sin\varphi$ and $x_{\text{ramp}} = d\cos\varphi$. When the projectile hits the ramp, $y_{\text{proj}}(x) = y_{\text{ramp}}$

You can either choose the x and y axis to be along the ground and straight up, or along the ramp. Both ways will work, and the math is not that much simpler either way. We will choose the former, with x along the level ground and y straight up. First, we worry about the projectile. We will write down the parametric equations $x_{\text{proj}}(t)$ and $y_{\text{proj}}(t)$ and solve for $y_{\text{proj}}(x)$.

$$\begin{aligned}
x_{\text{proj}}(t) &= \left(v_i\cos\theta\right)t \\
\Rightarrow \quad t &= \frac{x}{v_i\cos\theta} \\
y_{\text{proj}}(t) &= \left(v_i\sin\theta\right)t - \frac{1}{2}gt^2 \\
y_{\text{proj}}(x) &= \left(v_i\sin\theta\right)\left(\frac{x}{v_i\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{v_i\cos\theta}\right)^2 = x\tan\theta - \frac{gx^2}{2v_i^2\cos^2\theta}
\end{aligned}$$

The result for $y_{\text{proj}}(x)$ was given on the formula sheet as well. When the projectile hits the ramp, its x coordinate must be $d\cos\varphi$, which we plug into the equation for $y_{\text{proj}}(x)$ above, which must also equal $d\sin\varphi$.

$$\begin{aligned}
y_{\text{proj}}(x) &= x\tan\theta - \frac{gx^2}{2v_i^2\cos^2\theta} = y_{\text{ramp}} = d\sin\varphi \\
d\sin\varphi &= \left(d\cos\varphi\right)\tan\theta - \frac{g\left(d\cos\varphi\right)}{2v_i^2\cos^2\theta}
\end{aligned}$$

Solving this for d ...

$$
\begin{aligned}
d &= \frac{2v_i^2 \cos\theta \left[\sin\theta \cos\varphi - \sin\varphi \cos\theta\right]}{g \cos^2\varphi} \\
d &= \frac{2v_i^2 \cos\theta \sin\left(\theta - \varphi\right)}{g \cos^2\varphi}
\end{aligned}
$$

For the last step, we made use of the identity for $\sin(x-y)$. One can further find what angle gives the maximum distance up the ramp by finding $\frac{\mathrm{d}d}{\mathrm{d}\theta}$ and setting it equal to zero. The optimal angle and maximum distance are then:

$$
\begin{aligned}
\theta_{\text{max}} &= 45° + \frac{\varphi}{2} \\
d_{\text{max}} &= \frac{v_i^2 \left(1 - \sin\varphi\right)}{g \cos^2\varphi}
\end{aligned}
$$

BONUS 2 (+1pt): You push an object, initially at rest, across a frictionless floor with a constant force for a time interval t, resulting in a final speed of v for the object. You repeat the experiment, but with a force that is twice as large. What time interval is now required to reach the same final speed v? 47/57 Quick quiz 5.4 in Chapter 5

- ◯ $4t$
- ◯ $2t$
- ◯ t
- ◯ $t/2$
- ◯ $t/4$

A given force F gives you an acceleration $a = F/m$. The velocity resulting from this force is $v = at = (F/m)t$. If we double the force F, then the same final speed is reached by $t/2$, since everything is linear. Twice the force, half the time to the same velocity.