UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 125 / LeClair Spring 2009

Random sample problems

1. The position of a particle in meters can be described by $x = 10t - 2.5t^2$, where t is in seconds. What is the position of the particle when it changes direction?

When it changes direction, its velocity is zero. Find the time at which that is true, evaluate the position there.

$$v = \frac{dx}{dt} = 10 - 5t = 0 \qquad \Longrightarrow \qquad t = 2\,\text{s}$$
$$x(2\,\text{s}) = 10\,\text{m}$$

2. For the particle in the question above, what is its velocity when it returns to its original $t = 0$ position?

The $t = 0$ position is $x = 0$. We need to find another time for which $x = 0$ and evaluate the velocity there.

$$x = 10t - 2.5t^2 = t(10 - 2.5t) = 0 \qquad \Longrightarrow \qquad t = \{0, 4\,\text{s}\}$$
$$v(4\,\text{s}) = -10\,\text{m/s}$$

3. A projectile is launched with an initial velocity of $\vec{\mathbf{v}} = (17\,\hat{\imath} + 3.0\,\hat{\jmath})$ m/s. What is the angle of launch?

We are given the launch velocity in component form, $\vec{\mathbf{v}} = v_x\,\hat{\imath} + v_y\,\hat{\jmath}$. The angle is simply

$$\tan\theta = \frac{v_y}{v_x} = \frac{3}{17} \approx 10^\circ$$

4. How far does the projectile above travel in the $\hat{\imath}$ direction, assuming that gravity acts in the $-\hat{\jmath}$ direction? (*I.e.,* what is the range of the particle?)

We know the x and y components of the velocity, and the y component of the acceleration. Let the launch position be our origin.

$$x(t) = x_i + v_{ix}t + \frac{1}{2}a_xt^2 = 17t$$
$$y(t) = y_i + v_{iy}t + \frac{1}{2}a_yt^2 = 3t - \frac{1}{2}gt^2$$

We need the time when the projectile reaches the ground ($y = 0$), which we can then use to find the horizontal distance.

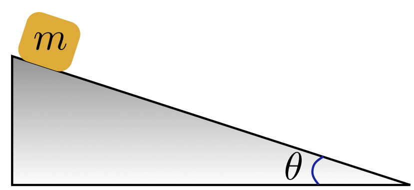
$$y(t) = 3t - \frac{1}{2}gt^2 = t(3 - \frac{1}{2}gt) = 0 \qquad \Longrightarrow \qquad t = \{0, 6/g\}$$
$$x(6/g) = 102/g \approx 10\,\mathrm{m}$$

5. A car is traveling at $20.0\,\mathrm{m/s}$ around a circular curve of radius $15.0\,\mathrm{m}$. Calculate the centripetal acceleration of the car in $\mathrm{m/s^2}$.

Centripetal acceleration is just v^2/r:

$$a_c = \frac{v^2}{r} = \frac{\left[20.0\,\mathrm{m/s^2}\right]^2}{15.0\,\mathrm{m}} \approx 26.7\,\mathrm{m/s^2}$$

6. A block slides down a frictionless plane having an inclination of $\theta = 16.9°$ as shown in the figure below. The block starts from rest at the top, and the length of the incline is $2.10\,\mathrm{m}$. Find the speed of the block at the bottom of the incline.

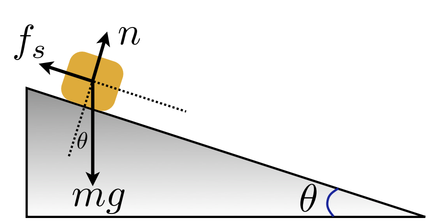


The component of gravitational acceleration parallel to the ramp plane is $g \sin\theta$. Starting from rest, and going a distance $d = 2.10\,\mathrm{m}$,

$$v_f^2 = v_i^2 + 2ad = 0 + 2gd\sin\theta$$
$$v_f = \sqrt{2gd\sin\theta} \approx 3.46\,\mathrm{m/s}$$

7. Consider the block on the incline above, but now in the presence of friction. What is the minimum coefficient of static friction μ_s such that the block *does not slide down the incline?*

Make use of the free-body diagram below.



Along the direction perpendicular to the ramp surface, we have the normal force and part of the block's weight:

$$\sum F_n = n - mg\cos\theta = 0 \qquad \Longrightarrow n = mg\cos\theta$$

The maximum static friction force scales with the normal force, $f_s = \mu_s n$, and counteracts the gravitational force on the ramp parallel to the ramp. The net acceleration should be zero just before motion occurs.

$$\sum F_t = mg\sin\theta - f_s = mg\sin\theta - \mu_s mg\cos\theta = 0 \qquad \mu_s = \tan\theta \approx 0.30$$

8. A 5 kg object has an initial velocity of $7\,\hat{\imath}$ m/s on a surface with no friction. A force acts on the object, and 6 seconds later its velocity is $2\,\hat{\imath} + 12\hat{\jmath}$ m/s. What was the magnitude of the force?

We can find the components of acceleration by noting the change in velocity components:

$$\begin{aligned}
a_x &= \frac{\Delta v_x}{\Delta t} = \frac{-5\,\mathrm{m/s}}{6\,\mathrm{s}} = -\frac{5}{6}\,\mathrm{m/s^2} \\
a_y &= \frac{\Delta v_y}{\Delta t} = \frac{12\,\mathrm{m/s}}{6\,\mathrm{s}} = 2\,\mathrm{m/s^2} \\
|\vec{\mathbf{a}}| &= \sqrt{a_x^2 + a_y^2} \approx 2.2\,\mathrm{m/s^2} \\
|\vec{\mathbf{F}}| &= m|\vec{\mathbf{a}}| \approx 11\,\mathrm{N}
\end{aligned}$$

9. A box begins moving across a floor with an initial velocity of 7 m/s. It slides 10 m before coming to rest. What is the coefficient of kinetic friction μ_k?

The normal force on the box is just its weight mg, so the frictional force opposing the motion is $f_k = \mu_k mg$. Since this is the only force acting along the direction of motion, it must equal the box's mass times acceleration:

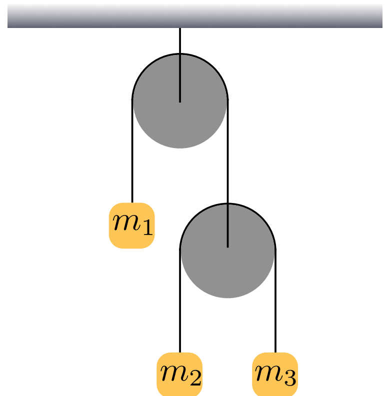
$$\sum F_{\text{horz}} = f_k = \mu_k mg = ma \qquad \Longrightarrow \qquad \mu_k g$$

We can relate the initial velocity and acceleration to the distance traveled:

$$\begin{aligned}
v_f^2 &= v_i^2 + 2a\Delta x = 0 + 2\mu_k g\Delta x \\
\mu_k &= \frac{v_f^2}{2g\Delta x} \approx 0.25
\end{aligned}$$

10. Consider the double pulley system below with $m_1 = 5$ kg, and $m_2 = m_3 = 2.5$ kg. You may neglect the mass of the pulleys and strings as well as friction. What is the acceleration of m_1?

First, look at the bottom pulley. Since $m_2 = m_3$, there is zero net force on either mass, and both should be stationary (even the amount of rope on each side is the same, though we assumed it massless). Since m_2 and m_3 are stationary, and the pulley they are attached to is massless, we can treat everything on



the right side of the upper pulley as one single mass of $m_2 + m_3 = 5\,\text{kg}$. This is precisely the same as m_1, so both sides of the upper pulley also carry the same weight. Assuming massless pulleys and strings, everything is balanced and there is **no acceleration.**

11. A plane travels horizontally at a constant speed of $40\,\text{m/s}$, $100\,\text{m}$ above the ground. It drops a package out of its hold. How far does the dropped package travel horizontally before hitting the ground?

Let the horizontal direction be x, with $+x$ in the direction of the plane's travel, and the vertical direction be y, with $+y$ being up. Let $y = 0$ be the ground level, and $x = 0$ be the plane's horizontal position at the moment the package is dropped.

The package has an initial velocity equal to the plane's, so $v_x = 40\,\text{m/s}$. It has no initial velocity in the y direction, but has an acceleration $a_y = -g$. The motion in the y direction is just free fall, so we can easily find the time it takes the package to fall. The motion in the x direction is characterized by constant velocity, so once we have the time it takes to reach the ground, we can find the horizontal distance easily.

$$y(t) = y_i + v_{iy}t + \frac{1}{2}a_y t^2 = 100 - \frac{1}{2}gt^2 = 0 \qquad \Longrightarrow \qquad t \approx 4.52\,\text{s}$$

$$x(t) = x_i + v_{ix}t + \frac{1}{2}a_x t^2 = 40t \approx 181\,\text{m}$$

12. A particle's velocity is given by:

$$v(t) = \frac{mg}{b}\left(1 - e^{-\frac{b}{m}t}\right)$$

where b is a constant (units of kg/sec), and the particles mass is m. Find the acceleration at $t = 0$.

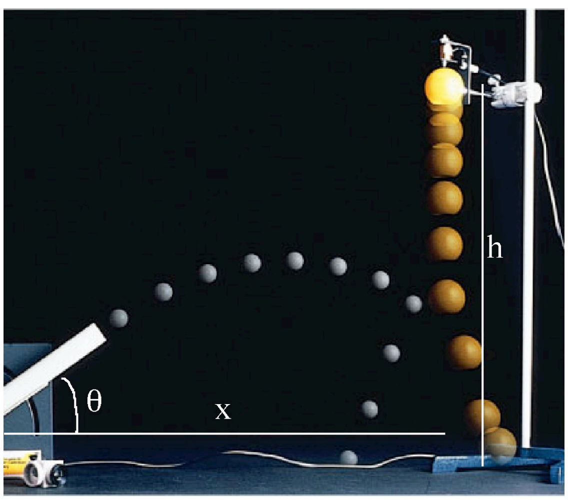
This one is just math.

$$a(t) = \frac{dv}{dt} = \frac{mg}{b}\left[0 - \left(\frac{-b}{m}\right)e^{-\frac{b}{m}t}\right] = ge^{-\frac{b}{m}t}$$
$$a(0) = g$$

13. A projectile is launched at an initial angle of $75°$ over level ground, and lands at a distance d away. Neglecting air resistance, at what other launch angle ($< 90°$) would the projectile have landed at the same distance d?

At this point, just recall that the range equation is a function of $\sin 2\theta$, which means it is symmetric about $\theta = 45°$. If $75°$ works, then so does $45 - (75 - 45) = 15°$

14. A projectile is launched at an initial angle of θ over level ground, and at the same time, a second projectile is dropped from a height h a distance x away from the launch site. Neglecting air resistance, what is the required height h such that the two projectiles collide? Your answer should be in terms of x and θ.



The projectile has a lateral velocity of $v_x = v\cos\theta$, meaning it will cover the horizontal distance in $t = x/\cos\theta$. Its y coordinate at that time is easily found:

$$y_p(t) = v\sin\theta t - \frac{1}{2}gt^2 = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$

If dropped from heigh th exactly when the first projectile is fired, after t seconds the second projectile has a y coordinate

$$y_{p2}(t) = h - \frac{1}{2}gt^2 = h - \frac{gx^2}{2v^2\cos^2\theta}$$

If we desire the two projectiles to hit, it is clear from inspection of the previous two equations that we require

$$h = x \tan\theta$$

Basically: just point the first projectile directly at the second if they are launched at the same time.

15. A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. The puck always remains on the ice and slides 115 m before coming to rest. Determine the coefficient of kinetic friction between the puck and the ice.

The normal force on the puck is just its weight mg. Thus, the frictional force retarding motion is $\mu_k mg$, and the puck's acceleration is $-\mu_k g$ (defining the direction of $\vec{\mathbf{v}}$ be positive). We can relate the initial velocity, final velocity, acceleration, and stopping distance:

$$\begin{aligned} v_f^2 &= 0 = v_i^2 + 2a\Delta x = -2\mu_k g \Delta x \\ \mu_k &= \frac{v_i^2}{2g\Delta x} \approx 0.18 \end{aligned}$$

16. A ball starts from rest and accelerates at $0.500\,\mathrm{m/s^2}$ while moving down an inclined plane which is 9.00 m long. What is the speed of the ball at the bottom of the plane?

We can relate initial and final velocities to distance traveled and acceleration:

$$\begin{aligned} v_f^2 - v_i^2 = v_f^2 &= 2a\Delta x \\ v_f &= \sqrt{2a\Delta x} = 3\,\mathrm{m/s} \end{aligned}$$

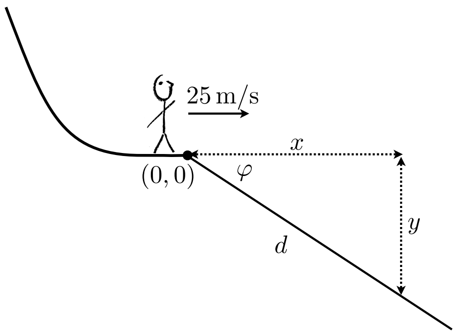
17. A ski jumper leaves the ski track moving in the horizontal direction with a speed of 25.0 m/s. The landing incline below him falls off with a slope of 35°. Where does he land on the incline (x and y coordinates)?

The crude drawing below helps us set up the situation. Let the jumper's position at the end of the ramp be our origin, with $+y$ running vertically and $+x$ running to the right.

We can readily write down the parametric equations of motion, given the initial velocity purely in the x direction $v_{ix} = 25\,\mathrm{m/s}$ and the vertical acceleration $a_y = -g$.

$$\begin{aligned} x(t) &= v_{ix} t = 25t \\ y(t) &= \frac{1}{2} a_y t^2 = -\frac{1}{2} g t^2 \end{aligned}$$

We need to find the time t when the jumper hits the ramp. From the geometry of the ramp, we can write the jumper's landing point (x_f, y_f) in terms of the lateral distance down the ramp d and the ramp's angle. Combinging this with the parametric equations above,

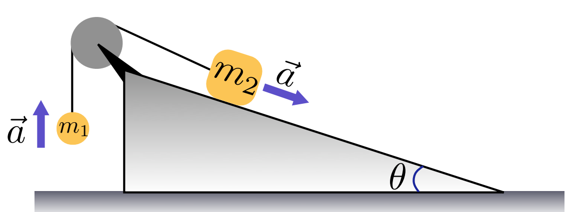


$$x_f = d\cos\varphi = v_{ix}t \qquad \Longrightarrow \qquad t = \frac{d\cos\varphi}{v_{ix}}$$
$$y_f = -d\sin\varphi = -\frac{1}{2}gt^2 = -\frac{gd^2\cos^2\varphi}{2v_{ix}^2} \qquad \Longrightarrow \qquad d = \frac{2v_{ix}^2\sin\varphi}{g\cos^2\varphi} \approx 109\,\mathrm{m}$$

Given the distance down the slope, we can find the (x, y) position of the landing easily:

$$(x_f, y_f) = (d\cos\varphi, -d\sin\varphi) \approx (89.3\,\mathrm{m}, -62.5\,\mathrm{m})$$

18. A ball of mass m_1 and a block of mass m_2 are connected by a lightweight cord that passes over a frictionless pulley of negligible mass, as shown below. The block lies on a frictionless incline of angle θ. Find the magnitude of the acceleration of the two objects.



First, make a free body diagram for m_1. We have only weight and tension in the string, and we postulate that acceleration is upward. Let the upward direction be positive. Then it is clear

$$T - m_1g = m_1a \qquad \Longrightarrow \qquad T = m_1(a + g)$$

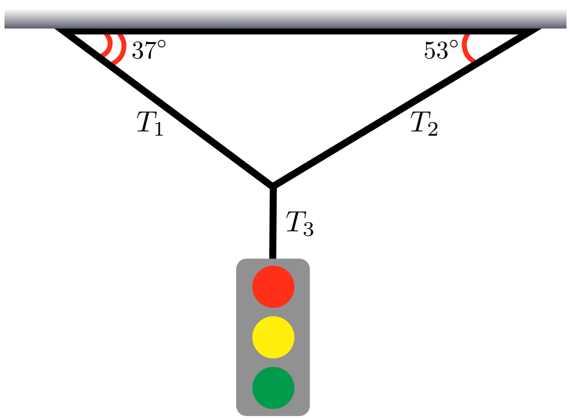
where T is the tension in the string. For m_2 on the ramp, let the x axis point down the ramp, and the y axis vertically normal to it. Along the ramp direction, a balance of forces reads:

$$\sum F_t = m_2g\sin\theta - T = m_2a \qquad \Longrightarrow \qquad a = g\sin\theta - \frac{T}{m_2}$$

Of course, the tension is the same everywhere in the string, and the acceleration is the same for both masses. Solving the first equation for T and substituting into the second:

$$
\begin{aligned}
a &= g\sin\theta - \frac{T}{m_2} = g\sin\theta - \frac{m_1}{m_2}(a+g) \\
a\left(\frac{m_1}{m_2}+1\right) &= g\sin\theta - \frac{m_1}{m_2}g \\
a &= g\left[\frac{\sin\theta - \frac{m_1}{m_2}}{\frac{m_1}{m_2}+1}\right] = g\left[\frac{m_2\sin\theta - m_1}{m_1+m_2}\right]
\end{aligned}
$$

19. Consider the traffic light hanging from 3 cables below. If the traffic light weighs 200 N, find the tension in cable 1, T_1.



We need a free body diagram for the light itself, and for the point at which all three cables meet. For the traffic light itself, we simply have $mg = T_3$, where mg is the traffic light's weight of 200 N. At the point where all three cables meet,

$$
\begin{aligned}
\sum F_{\text{horz}} &= T_2\cos 53° - T_1\cos 37° = 0 \\
\sum F_{\text{vert}} &= T_1\sin 37° + T_2\sin 53° - T_3 = 0
\end{aligned}
$$

Solve the first equation for T_2 in terms of T_1, and substitute into the second:

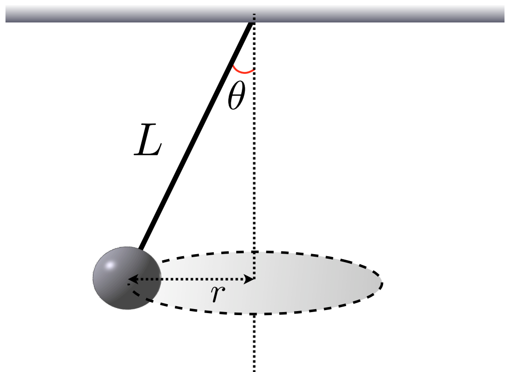
$$
\begin{aligned}
T_1\sin 37° + T_2\sin 53° - T_3 &= T_1\sin 37° + \left(T_1\frac{\cos 37°}{\cos 53°}\sin 53°\right) - T_3 = 0 \\
T_1 &= \frac{T_3}{\sin 37° + \cos 37°\tan 53°} = \frac{mg}{\sin 37° + \cos 37°\tan 53°} \approx 120\,\text{N}
\end{aligned}
$$

20. A block is given an initial velocity of 5.00 m/s up a frictionless 20° incline. How far up the incline does the block slide before coming to rest?

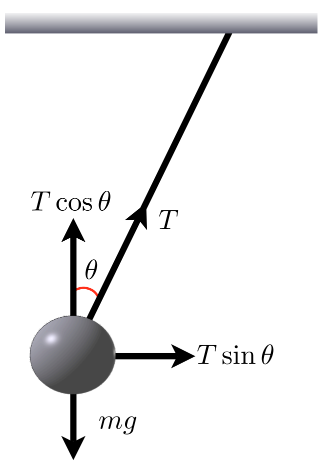
The acceleration down the ramp plane is $-g\sin 20°$. If it is to travel a distance d up the ramp given an initial velocity v_i,

$$v_f^2 - v_i^2 = 2ad = -2gd\sin 20°$$
$$d = \frac{v_i^2}{2g\sin 20°} \approx 3.72\,\mathrm{m}$$

21. Consider the conical pendulum below. Find an expression for the *linear* velocity v of the mass m as it orbits, in terms of L, g, and θ. (Hint: note that one can relate r, L, and θ in a simple geometric manner.)



See the free body diagram below.



Along the vertical direction, we have

$$T\cos\theta = mg$$

Along the radial direction, all forces must sum to give the centripetal force, the requisite constraint when an object follows a circular path:

$$\sum F_r = T\sin\theta = ma_c = \frac{mv^2}{r}$$

Divide the two equations we have to eliminate tension and mass:

$$\begin{aligned}\tan\theta &= \frac{v^2}{rg}\\ v &= \sqrt{rg\tan\theta}\end{aligned}$$

Noting that $r = L\sin\theta$ we can eliminate radius in favor of the more easily-measured pendulum length:

$$v = \sqrt{Lg\sin\theta\tan\theta}$$

22. A rubber ball was thrown at a brick wall with an initial speed of 10 m/s, and rebounded in the opposite direction with a speed of -8.5 m/s. The rebound was found to take 3.5×10^{-3} sec. What was the acceleration experienced by the ball during the rebound?

We only need the definition of acceleration, and to recognize that the velocity changes sign upon rebound.

$$a = \frac{\Delta v}{\Delta t} = \frac{-8.5\,\mathrm{m/s} - 10\,\mathrm{m/s}}{3.5\times10^{-3}\,\mathrm{s}} \approx -5300\,\mathrm{m/s^2}$$

23. A projectile will be launched with an initial velocity of 750 m/s, and needs to hit a target 23 km away. What should the launch angle be? You can ignore air resistance.

We have derived the projectile range equation (over level ground) many times, and you should be able to do this on your own now. Let the origin be the launch position. The range is then

$$R = \frac{v_i^2\sin 2\theta_i}{g}$$

Given v_i and R, we want to find θ_i.

$$\theta_i = \frac{1}{2}\sin^{-1}\left[\frac{gR}{v_i^2}\right] \approx 11.8^\circ$$

24. Joe foolishly fires his .270 Winchester, which has a muzzle velocity of 957 m/s using a 130 grain load, into the air at a 17° angle. Ignoring air resistance, how far away from Joe will the bullet land?

Same as the last problem ... except that now we wish to find R.

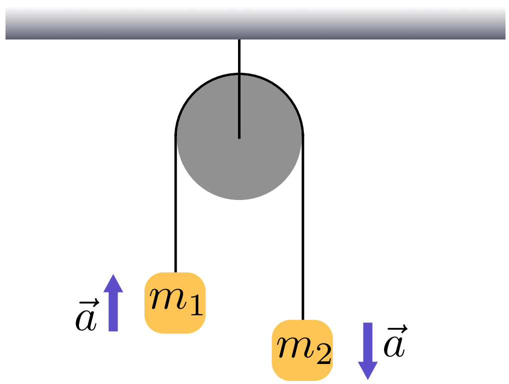
$$R = \frac{v_i^2\sin 2\theta_i}{g} = 5.2\times10^4\,\mathrm{m}$$

25. After being struck by a hockey stick, a hockey puck slides across the ice with an initial velocity of 7.0 m/s. If the coefficient of kinetic friction μ_k between the ice and the puck is 0.15, what is the velocity of the puck when it reaches the goal 10 m down the ice?

The normal force is just the weight of the puck itself, which means the friction force is $f_k = \mu_k mg$. Since this is the only force acting along the direction of motion, it follows that $a = -\mu_k g$. Given the initial velocity and sliding distance,

$$\begin{aligned} v_f^2 &= v_i^2 + 2a\Delta x \\ v_f &= \sqrt{v_i^2 - 2\mu_k g \Delta x} \approx 4.4\,\text{m/s} \end{aligned}$$

26. Consider the so-called "Atwood's machine" below. What is the acceleration of the two masses, if one ignores friction and the mass of the pulley and rope?



With the direction of acceleration given, the force balances on m_1 and m_2 read

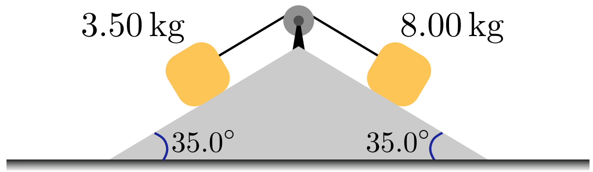
$$\begin{aligned} m_1 a &= T - m_1 g \\ -m_2 a &= T - m_2 g \end{aligned}$$

Subtract both equations and solve for a

$$(m_1 + m_2)\, a = (m_2 - m_1)\, g \qquad \Longrightarrow \qquad a = \left[\frac{m_2 - m_1}{m_2 + m_1}\right] g$$

27. Two blocks of mass 3.50 kg and 8.00 kg are connected by a massless string that passes over a frictionless pulley, as shown in the figure below. The inclines are frictionless. Find the tension in the string. (*Hint: do you expect the blocks to be moving?*)

Given the symmetry of the situation, the blocks should be moving to the right since the 8 kg mass is heavier. The acceleration for both blocks should be the same, since they are connected by our



ideal string. Let the direction of motion for both blocks define positive acceleration. With a bit of geometry,

$$
\begin{aligned}
m_1 a &= T - m_1 g \sin 35° \\
m_2 a &= m_2 g \sin 35° - T
\end{aligned}
$$

Add the two equations together and solve for a.

$$
\begin{aligned}
(m_2 + m_1)\, a &= m_2 g \sin 35° - m_1 g 35° \\
a &= \left[\frac{m_2 - m_1}{m_2 + m_1}\right] g \sin 35° \approx 2.20\,\mathrm{m/s^2}
\end{aligned}
$$

28. A 1500 kg car moving on a flat, horizontal road negotiates a curve. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is $\mu_s = 0.500$, find the maximum speed the car can have and still make the turn successfully. (*Hint: what is balancing the force of friction?*)

In this case the frictional force between the car's tires and the road must provide the centripetal acceleration to maintain a circular path. The normal force acting on the car is simply its weight, so the maximum static frictional force is $f_s = \mu_s mg$. This is the only force acting in the radial direction, so it must give the centripetal acceleration.

$$
f_s = \mu_s mg = \frac{mv^2}{r} \qquad \Longrightarrow \qquad v = \sqrt{\mu_s g r} \approx 13.1\,\mathrm{m/s}
$$

Note that the car's mass has nothing to with the maximum speed – the mass-dependence of the frictional force is precisely cancelled by the mass-dependence of centripetal force.