

# Formula sheet

## Math & numbers:

$$g = 9.81 \text{ m/s}^2$$

$$ax^2 + bx^2 + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} \quad \text{magnitude}$$

$$\theta = \tan^{-1} \left[ \frac{F_y}{F_x} \right] \quad \text{direction}$$

## Rotation: we use radians

$$s = \theta r \quad \leftarrow \text{arclength}$$

$$\omega = \frac{d\theta}{dt} = \frac{v}{r} \quad \alpha = \frac{d\omega}{dt}$$

$$a_t = \alpha r \quad \text{tangential} \quad a_r = \frac{v^2}{r} = \omega^2 r \quad \text{radial}$$

$$I = \sum_i m_i r_i^2 \implies \int r^2 dm = kmr^2$$

$$I_z = I_{com} + md^2 \quad \text{axis } z \text{ parallel, dist } d$$

$$\tau_{net} = \sum \vec{r} \times \vec{F} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$$

$$\vec{\tau} = \vec{r} \times \vec{F} \quad |\vec{\tau}| = rF \sin \theta_{r,F}$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

$$K = \frac{1}{2} I \omega^2 = L^2 / 2I$$

$$\Delta K = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2 = W = \int \tau d\theta$$

$$P = \frac{dW}{dt} = \tau \omega$$

## 1-D motion:

$$v(t) = \frac{d}{dt} x(t)$$

$$a(t) = \frac{d}{dt} v(t) = \frac{d^2}{dt^2} x(t)$$

const. acc.

$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x \Delta x$$

$$v_f = v_i + at$$

## 2-D motion:

$$\vec{r} = x(t) \hat{i} + y(t) \hat{j}$$

$$x(t) = x_i + v_{ix} t + \frac{1}{2} a_x t^2$$

$$y(t) = y_i + v_{iy} t + \frac{1}{2} a_y t^2$$

$$v_x(t) = \frac{dx}{dt} = v_{xi} + a_x t$$

$$v_y(t) = \frac{dy}{dt} = v_{yi} + a_y t$$

$$a_c = \frac{v^2}{r} \quad \text{circular}$$

## Force:

$$\Sigma \vec{F} = \vec{F}_{net} = m\vec{a}$$

$$\Sigma F_x = ma_x$$

$$\Sigma F_y = ma_y$$

$$F_{grav} = mg = \text{weight}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$f_s \leq \mu_s n$$

$$f_{s,max} = \mu_s n$$

$$f_k = \mu_k n$$

$$|\vec{F}_{drag}| = -\frac{1}{2} C \rho A v^2$$

$$\vec{F}_c = -\frac{mv^2}{r} \hat{r} \quad \text{circular}$$

## Work-Energy-Potential:

$$K = \frac{1}{2} mv^2 = \frac{p^2}{2m}$$

$$\Delta K = K_f - K_i = W$$

$$W = \int F(x) dx = -\Delta U$$

$$U_g(y) = mgy$$

$$U_s(x) = \frac{1}{2} kx^2$$

$$F = -\frac{dU(x)}{dx}$$

$$K_i + U_i = K_f + U_f + W_{ext} = K_f + U_f + \int F_{ext} dx$$

## Momentum, etc.:

$$x_{com} = \frac{1}{M_{tot}} \sum_{i=1}^n m_i x_i = \frac{m_1 x_1 + m_2 x_2 + \dots + m_n x_n}{m_1 + m_2 + \dots + m_n}$$

$$v_{com} = \frac{1}{M_{tot}} \sum_{i=1}^n m_i v_i = \frac{m_1 v_1 + m_2 v_2 + \dots + m_n v_n}{m_1 + m_2 + \dots + m_n}$$

$$F_{net} = M_{tot} a_{com} = \frac{dp}{dt}$$

$$p_{tot} = M_{tot} v_{com}$$

$$\Delta p = p_f - p_i = J = \int_{t_i}^{t_f} F(t) dt = F_{avg} \Delta t$$

$$\Delta p = 0 \quad \text{closed}$$

$$\sum_i m_i v_i = \sum_f m_f v_f$$

**Isolated systems:**  $\vec{p}$ ,  $E = K + PE$ ,  $L$  are all conserved.

**Static equilibrium:**  $\Sigma F = 0$  and  $\Sigma \tau = 0$  about any axis.

**Elastic collision:** KE and  $p$  are both conserved.

**Inelastic collision:** only  $p$  is conserved, not KE.

Derived unit	Symbol	equivalent to
newton	N	kg·m/s <sup>2</sup>
joule	J	kg·m <sup>2</sup> /s <sup>2</sup> = N·m
watt	W	J/s = m <sup>2</sup> ·kg/s <sup>3</sup>