Formula sheet

$$g = 9.81\,\mathrm{m/s^2}$$

$$\text{sphere} \quad V = \frac{4}{3}\pi r^3 \quad A = 4\pi r^2$$

$$ax^2 + bx^2 + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$c^2 = a^2 + b^2 - 2ab\cos\theta_{ab}$$

$$\sin\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{small } \theta$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \equiv \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

$$\Delta v_x = \int_{t_i}^{t_f} a_x(t)\,dt \qquad \Delta x = \int_{t_i}^{t_f} v_x(t)\,dt$$

$$x(t) = x_i + v_{x,i}t + \frac{1}{2}a_x t^2$$

$$v_x(t) = v_{x,i} + a_x t$$

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x\Delta x$$

$$a_{x,\text{ramp}} = g\sin\theta$$

$$\Delta U^G = mg\Delta x$$

$$\frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}$$

$$E_{\text{mech}} = K + U \quad K = \frac{1}{2}mv^2$$

$$\Delta E = \Delta K + \Delta U = 0 \quad \text{non-dissipative, closed}$$

$$\Delta E = W$$

$$\Delta U_{\text{spring}} = \frac{1}{2}k\left(x - x_o\right)^2$$

$$P = \frac{dE}{dt}$$

$$P = F_{\text{ext,x}}\,v_x \quad \text{one dimension}$$

Rotation: we use radians

$$s = \theta r \quad \leftarrow \text{arclength}$$

$$\omega = \frac{d\theta}{dt} = \frac{v_t}{r} \qquad \alpha = \frac{d\omega}{dt}$$

$$a_t = \alpha r \quad \text{tangential}$$

$$a_r = -\frac{v^2}{r} = -\omega^2 r \quad \text{radial}$$

$$v_t = r\omega \qquad v_r = 0$$

$$\Delta\Theta = \omega_i t + \frac{1}{2}\alpha t^2 \quad \text{const } \alpha$$

$$\omega_f = \omega_i + \alpha t \quad \text{const } \alpha$$

$$\Delta x = r\theta \quad v = r\omega \quad a = r\alpha \quad \text{no slipping}$$

$$I = \sum_i m_i r_i^2 \Rightarrow \int r^2\,dm = cmr^2 \qquad I = mr^2 \quad \text{point particle}$$

$$I_z = I_{com} + md^2 \quad \text{axis } z \text{ parallel, dist } d$$

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = I\vec{\boldsymbol{\omega}} \qquad L = I\omega = mvr$$

$$K = \frac{1}{2}I\omega^2 = L^2/2I$$

$$\Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W = \int \tau\,d\theta$$

$$P = \frac{dW}{dt} = \tau\omega$$

$$\tau = rF\sin\theta_{rF} = r_\perp F = rF_\perp$$

$$\tau_{net} = \sum \vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}} = \frac{d\vec{\mathbf{L}}}{dt}$$

$$K_{\text{tot}} = K_{cm} + K_{rot} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$$

Oscillations:

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \quad \omega = \frac{2\pi}{T} = 2\pi f$$

$$x(t) = A\sin\left(\omega t + \varphi_i\right) = A\sin\varphi(t)$$

$$v(t) = \frac{dx}{dt} = \omega A\cos\left(\omega t + \varphi_i\right)$$

$$a(t) = \frac{d^2x}{dt^2} = -\omega^2 A\sin\left(\omega t + \varphi_i\right) = -\omega^2 x(t)$$

$$\varphi(t) = \omega t + \varphi_i$$

$$a = -\omega^2 x = \frac{d^2x}{dt^2} \qquad \frac{d^2\theta}{dt^2} = -\omega^2\theta$$

$$E = \frac{1}{2}m\omega^2 A^2$$

$$\omega = \sqrt{k/m} \quad \text{spring.}$$

$$T = \begin{cases} 2\pi\sqrt{m/k} & \text{spring} \\ 2\pi\sqrt{L/g} & \text{simple pendulum} \\ 2\pi\sqrt{I/mgl_{cm}} & \text{physical pendulum} \end{cases}$$

Derived unit	**Symbol**	**equivalent to**
newton	N	$\mathrm{kg{\cdot}m/s^2}$
joule	J	$\mathrm{kg{\cdot}m^2/s^2} = \mathrm{N{\cdot}m}$
watt	W	$\mathrm{J/s} = \mathrm{m^2{\cdot}kg/s^3}$

Moments of inertia of things of mass M

Object	axis	**dimension**	**I**
solid sphere	central axis	radius R	$\frac{2}{5}MR^2$
hollow sphere	central axis	radius R	$\frac{2}{3}MR^2$
solid disc/cylinder	central axis	radius R	$\frac{1}{2}MR^2$
hoop	central axis	radius R	MR^2
point particle	pivot point	distance R to pivot	MR^2
rod	center	length L	$\frac{1}{12}ML^2$
rod	end	length L	$\frac{1}{3}ML^2$
solid regular octahedron	through vertices	side a	$\frac{1}{10}ma^2$