Exam 2A Solution

1. A baseball is thrown vertically upward and feels no air resistance. As it is rising

Solution: Possible answers:
A) both its momentum and its mechanical energy are conserved - incorrect. The momentum of the ball itself is not conserved, since its velocity decreases as it rises.

B) its kinetic energy is conserved, but its momentum is not conserved - incorrect. Kinetic energy is also not conserved, since the ball slows down as it rises.

C) its gravitational potential energy is not conserved, buts its momentum is conserved - incorrect. Gravitational potential energy clearly increases as the height of the ball increases.

D) its momentum is not conserved, but its mechanical energy is conserved - correct. The momentum of the ball itself is not conserved because the ball slows down as it rises. Mechanical energy, the sum of kinetic and gravitational potential energies, is conserved in the absence of air resistance.

E) both its momentum and its kinetic energy are conserved - incorrect. See rationale for A and B.

2. A 2.00 -kg object traveling east at 20.0 m/s collides with a 3.00 -kg object traveling west at 10.0 m/s. After the collision, the 2.00 -kg object has a velocity 5.00 m/s to the west. How much kinetic energy was lost during the collision?

Solution: We don't know the final velocity of the second object after the collision, we'll need to find it. For that, we can use conservation of momentum. Once we have all the velocities, the calculation is straightforward. Let $m_1 = 2.0\,\mathrm{kg}$, $m_2 = 3.0\,\mathrm{kg}$. With east being +x, then $v_{1i} = 20.0\,\mathrm{m/s}$, $v_{2i} = -10.0\,\mathrm{m/s}$, $v_{1f} = -5.00\,\mathrm{m/s}$. First, conservation of momentum.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \tag{1}$$

$$v_{2f} = \frac{m_1 v_{1i} + m_2 v_{2i} - m_1 v_{1f}}{m_2} \approx 6.67\,\mathrm{m/s} \tag{2}$$

The change in kinetic energy is:

$$\Delta K = K_f - K_i = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 - \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} m_2 v_{2i}^2 \approx -458\,\mathrm{J} \tag{3}$$

$$\tag{4}$$

Since the question asked how much energy was *lost*, the answer is 458 J, E.

3. Consider what happens when you jump up in the air. Which of the following is the most accurate statement?

Solution: Possible answers:

A) Since the ground is stationary, it cannot exert the upward force necessary to propel you into the air. Instead, it is the internal forces of your muscles acting on your body itself that propels your body into the air. Incorrect - Newton's third law plainly tells us that you exert a force on the ground, and the ground exerts a force back on you. The ground is also not really 'stationary .' Stationary is a relative term, so stationary relative to what?

B) When you jump up the earth exerts a force F1 on you and you exert a force F2 on the earth. You go up because F1>F2. Incorrect - Newton's third law tells us that |F1|=|F2|.

C) It is the upward force exerted by the ground that pushes you up, but this force cannot exceed your weight. Incorrect - if this force could not exceed your weight, how would you leave the ground?

D) When you push down on the earth with a force greater than your weight, the earth will push back with the same magnitude force and thus propel you into the air. Correct: the earth exerts a force on you equal and opposite to the force you exert on it. If you push down with a force greater than your weight, the earth pushes back with a force of identical magnitude. Since the force pushing you exceeds your weight, you have a net acceleration upward.

E) You are able to spring up because the earth exerts a force upward on you that is greater than the downward force you exert on the earth. Incorrect - Newton's third law tells us that |F1|=|F2|.

4. A force on a particle depends on position such that $F(x) = (3.0\,\mathrm{N/m^2})x^2 + (6.00\,\mathrm{N/m})x$ for a particle constrained to move along the x-axis. What work is done by this force on a particle that moves from $x = 0.00\,\mathrm{m}$ to $x = 2.00\,\mathrm{m}$?

Solution: If the force in question is not constant, the work is given by

$$W = \int_{x_i}^{x_f} F(x)\,dx = \int_0^2 3x^2 + 6x\,dx = x^3 + 3x^2\Big|_0^2 = 20\,\mathrm{J} \tag{5}$$

5. It requires 49 J of work to stretch an ideal very light spring from a length of 1.4 m to a length of 2.9 m. What is the value of the spring constant of this spring?

Solution: The work required would be the net work required to go to $x_2 = 2.9\,\mathrm{m}$, minus the work already expended to get to $x_1 = 1.4\,\mathrm{m}$

$$W = W_{x_2} - W_{x_1} = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 = \frac{1}{2}k\left(x_2^2 - x_1^2\right) \tag{6}$$

$$k = \frac{2W}{x_2^2 - x_1^2} \approx 15\,\mathrm{N/m} \tag{7}$$

6. On a smooth horizontal floor, an object slides into a spring which is attached to another mass that is initially stationary. When the spring is most compressed, both objects are moving at the same speed. Ignoring friction, what is conserved during this interaction?

Solution: The collision is elastic due to the spring. Both momentum and mechanical energy are conserved since there are no dissipative forces present.

7. You slam on the brakes of your car in a panic, and skid a certain distance on a straight, level road. If you had been traveling twice as fast, what distance would the car have skidded, under identical conditions?

Solution: The car will come to a stop when its initial kinetic energy converted to thermal energy via the work done by friction. The work done by the friction force F_s over a displacement x is just $F_s x$ since F_s is constant. If the car of mass m has initial velocity v, then

$$\frac{1}{2}mv^2 = F_s x \tag{8}$$

$$x = \frac{mv^2}{2F_s} \tag{9}$$

Since the stopping distance $x \propto v^2$, if the speed is doubled the car skids four times farther.

8. A 60.0 kg person drops from rest a distance of 1.20 m to a platform of negligible mass supported by an ideal stiff spring of negligible mass. The platform drops 6.00 cm before the person comes to rest. What is the spring constant of the spring?

Solution: Conservation of energy. The person's initial gravitational potential energy will end up as stored potential energy in the spring. If the person falls a distance h and the spring compresses a distance y, the net vertical distance is $h + y$. For a spring of constant k,

$$mg(h+y) = \frac{1}{2}ky^2 \tag{10}$$

$$k = \frac{2mg(h+y)}{y^2} \approx 4.12 \times 10^5\,\mathrm{N/m} \tag{11}$$

9. A crane lifts a 425 kg steel beam vertically a distance of 117 m. How much work does the crane do on the beam if the beam accelerates upward at $1.8\,\mathrm{m/s^2}$? Neglect frictional forces.

Solution: A free body diagram for the crane should give you $F - mg = ma$. That means the force F is given by $F = m(g + a)$. The work done by that force, since it is constant, is $W = F\Delta y$, where Δy is the vertical distance covered. With the numbers given, $W \approx 5.8 \times 10^5\,\mathrm{J}$.

10. Two objects, each of weightW, hang vertically by spring scales as shown in the figure. The pulleys and the strings attached to the objects have negligible weight, and there is no appreciable friction in the pulleys. The reading in each scale is

Solution: **D** Ask yourself this: would the tension change one of the weights W was removed and the end of the rope tied to the ground? It would not, and as a result the tension in the rope must be W whether we have a weight on each end or a weight on only one end with the other end tied off.

11. A 7.0 kg object is acted on by two forces. One of the forces is 10.0 N acting toward the east. Which of the following forces is the other force if the acceleration of the object is $1.0\,\mathrm{m/s^2}$.

Solution: The sum of the forces must give mass times the net acceleration, $F_{\mathrm{net}} = ma$. This tells us $F_{\mathrm{net}} = 7\,\mathrm{N}$, with the sign indicating the direction must be east. If one force is 10 N east, the other must be 3.0 N west for this to work out.

12. Two weights are connected by a massless wire and pulled upward with a constant speed of 1.50 m/s by a vertical pullP. The tension in the wire is T (see figure in exam). Which one of the following relationships between T and P must be true?

Solution: Construct at a free body diagram for the upper rope, noting that there is no net acceleration.. The force P pulls up, and a net weight of 100 N (both boxes together) pulls down. That tells us $P = 100\,\mathrm{N}$, so answer A is clearly out. Now construct a free body diagram for the upper box. A force P pulls it up, while a tension T and its weight of 25 N pull down. That tells us $P - T - 25 = ma = 0$, or $P = T + 25\,\mathrm{N}$.

13. Is it possible for a system to have negative potential energy?

Solution: Yes, since the choice of the zero of potential energy is arbitrary. None of the other conditions are correct - kinetic energy and potential energy are in general not equal (since the zero of potential energy is arbitrary anyway this wouldn't make sense). Similarly, the kinetic energy is always positive, so condition D makes no sense. Finally, since the zero point for potential energy is arbitrary, the same must be true of total energy, so restricting it to positive numbers doesn't make sense.

14. An 1100 kg car traveling at 27.0 m/s starts to slow down and comes to a complete stop in 578 m. What is the magnitude of the average braking force acting on the car?

Solution: The work done by the breaking force must equal the change in the car's kinetic energy.

If the brake force F is constant, the work done is Fx where x is the stopping distance. The change in kinetic energy for the car of mass m traveling at velocity v initially and coming to a stop is $\frac{1}{2}mv^2$. Thus,

$$\frac{1}{2}mv^2 = Fx \tag{12}$$

$$F = \frac{mv^2}{2x} \approx 690\,\text{N} \tag{13}$$

15. A spring stretches by 21.0 cm when a 135 N object is attached. What is the weight of a fish that would stretch the spring by 31.0 cm?

Solution: If the spring of constant k stretches by a distance x_1 when a weight W_1 is applied, then it must be true that $kx_1 = W_1$, or $k = W_1/x_1$. For the fish, it is then true that $kx_f = W_f$, or

$$W_f = kx_f = \frac{W_1}{x_1}x_f \approx 199\,\text{N} \tag{14}$$

16. On a horizontal frictionless floor, a worker of weight 0.900 kN pushes horizontally with a force of 0.200 kN on a box weighing 1.80 kN. As a result of this push, which statement could be true?

Solution: From Newton's third law, we know the force of the worker on the box must be equal and opposite the force of the box on the worker. That means:

$$F_w = -F_b = m_w a_w = m_b a_b \tag{15}$$

Give that the box has twice the inertia of the worker, it must have half the acceleration. That leaves A as the only answer. Numerically, we can calculate $a_w = F_w/m_w = 2.17\,\text{m/s}^2$ (noting that m_w is the given weight divided by g) and $a_b = F_b/a_b = -1.08\,\text{m/s}^2$.

17. A 60.0 kg person rides in an elevator while standing on a scale. The scale reads 400 N. The acceleration of the elevator is closest to

Solution: The net acceleration experienced is $F/m \approx 6.67\,\text{m/s}^2$. That tells us that the acceleration must be downward. A free body diagram has the normal force upward (this is what the scale measures), the person's weight downward, and the acceleration of the elevator downward (it must be downward if the net acceleration is less than g). This gives you $F_N - mg = -ma$, or $F_n = m(g-a)$. The net acceleration, F_n/m, is $g - a$. Thus, $g - a = 6.67\,\text{m/s}^2$, or $a = 3.13\,\text{m/s}^2$ in the downward direction.

18. If electricity costs 6.00¢/kWh (kilowatt-hour), how much would it cost you to run a 120 W 18) stereo system 4.0 hours per day for 4.0 weeks?

Solution: This one is just unit conversions:

$$\frac{6\,¢}{\text{kW}\cdot\text{h}}\cdot\frac{1\,\text{kW}}{1000\,\text{W}}\cdot 120\,\text{W}\cdot\frac{4.0\text{h}}{\text{day}}\cdot\frac{7\,\text{days}}{1\,\text{week}}\cdot 4.0\,\text{weeks}\approx 81\,¢ \tag{16}$$

19. Alice and Tom dive from an overhang into the lake below. Tom simply drops straight down from the edge, but Alice takes a running start and jumps with an initial horizontal velocity of 25 m/s. Neither person experiences any significant air resistance. Just as they reach the lake below

Solution: D, the speed of alice is larger than the speed of Tom. Remember speed is $v=\sqrt{v_x^2+v_y^2}$. Tom has $v_{x,i}=0$, and Alice has $v_{x,i}\neq 0$. Both have the same v_y at all times since neither has an initial vertical velocity. Since Alice has a nonzero v_x but the same v_y as Tom, her speed is always larger. The difference does *not* stay at 25 m/s since both are accelerating along y.

(Also, as clarified during the exam, "splashdown speed" means the vertical component of the velocity in this context.)

20. A boy throws a rock with an initial velocity of 2.15 m/s at 30.0° above the horizontal. If air resistance is negligible, how long does it take for the rock to reach the maximum height of its trajectory?

Solution: All that matters is the vertical component of the velocity, $v_{i,y}=v_i\sin\theta$. Once you know that, it is a 1D motion problem just like we've done in the past. We know the maximum height will be when $v_y=0$.

$$v_{i.y} = v_i\sin\theta \tag{17}$$

$$v_y(t) = v_{i,y} - gt = 0 \tag{18}$$

$$t = \frac{v_{i,y}}{g} = \frac{v_i\sin\theta}{g} \approx 0.110\,\text{s} \tag{19}$$

21. A catapult is tested by Roman legionnaires. They tabulate the results in a papyrus and 2000 years later the archaeological team reads (distances translated into modern units): Range = 0.20 km; angle of launch = $\pi/4$; landing height = launch height. What is the initial velocity of launch of the boulders if air resistance is negligible?

Solution: We know that the launch is over level ground, and for a given angle we know the range. The two easy approaches are the $y(x)$ trajectory equation, or the range equation. In the former case we know $y(0)=0$ and $y(\text{range})=0$. That plus the given angle is enough to find the velocity. The other, slightly easier, way is to use the range equation. Let the range be R, the initial velocity v_i, and the launch angle θ. Then we have

$$R = \frac{v_i^2 \sin 2\theta}{g} \tag{20}$$

$$v_i = \sqrt{\frac{gR}{\sin 2\theta}} \approx 44\,\mathrm{m/s} \tag{21}$$

22. A rock is thrown at a window that is located 18.0 m above the ground. The rock is thrown at an angle of 40.0° above horizontal. The rock is thrown from a height of 2.00 m above the ground with a speed of 30.0 m/s and experiences no appreciable air resistance. If the rock strikes the window on its upward trajectory, from what horizontal distance from the window was it released?

Solution: We know where the ball starts and where it lands. Given the launch angle, those two points are enough to determine the entire trajectory, including the horizontal distance. The difference in height between the rock's launch and the window is $h = (18.0 - 2.0)\,\mathrm{m}$. Let the origin be the point at which the rock is launched, with $+x$ downrange and $+y$ upward. Let the launch angle be θ, let the initial velocity v_i, and let the horizontal distance to the window be d. The trajectory of the rock is then

$$y(x) = x \tan\theta - \frac{gx^2}{2v_i^2 \cos^2\theta} \tag{22}$$

Now we know that $y(d) = h$:

$$h = d \tan\theta - \frac{gd^2}{2v_i^2 \cos^2\theta} \tag{23}$$

$$0 = \frac{g}{2v_i^2 \cos^2\theta} d^2 - (\tan\theta) d + h \tag{24}$$

This is a quadratic equation in d, which will give us two distances. Since we want the rock to strike on the upward portion of the trajectory, we want the smaller root. Using the quadratic equation,

$$d = \frac{\tan\theta \pm \sqrt{\tan^2\theta - 4 \cdot \frac{g}{2v_i^2 \cos^2\theta} \cdot h}}{2 \cdot \frac{g}{2v_i^2 \cos^2\theta}} \tag{25}$$

$$= \left(\tan\theta \pm \sqrt{\tan^2\theta - \frac{2gh}{2v_i^2 \cos^2\theta}}\right) \cdot \frac{v_i^2 \cos^2\theta}{g} \tag{26}$$

Further simplification is not really useful. From this you find $d \in \{27.3, 63, 0\}\,\mathrm{m}$, with $d = 27.3\,\mathrm{m}$ corresponding to the upward portion of the trajectory.

23. A rescue plane flying horizontally at 72.6 m/s spots a survivor in the ocean 182 m directly below and releases an emergency kit with a parachute. Because of the shape of the parachute, it experiences insignificant horizontal air resistance. If the kit descends with a constant vertical acceleration of $5.82\,\mathrm{m/s^2}$, how far away from the survivor will it hit the waves?

Solution: We can use $x(t)$ and $y(t)$ most easily here. First, find out how long it takes to fall $h = 182\,\mathrm{m}$, then figure out how far the kit travels horizontally in that time.

The vertical initial speed is zero, and we know $a_y = 5.82\,\mathrm{m/s^2}$. With $+y$ upward and the origin at the ocean's surface, we know how to write down $y(t)$. We want to find out when $y = 0$

$$y(t) = y_i + v_{iy}t + \frac{1}{2}a_y t^2 = h - \frac{1}{2}a_y t^2 = 0 \tag{27}$$

$$t = \sqrt{\frac{2h}{a_y}} \tag{28}$$

Next, the kit will have an initial and constant velocity along x: that of the plane it was traveling in, $v_x = 72.6\,\mathrm{m/s}$. The displacement in time t, given no horizontal acceleration, is just

$$\Delta x = v_x t = v_x\sqrt{\frac{2h}{a_y}} \approx 574\,\mathrm{m} \tag{29}$$

24. A 4.00 kg block rests between the floor and a 3.00 kg block as shown in the figure. The 3.00 kg block is tied to a wall by a horizontal rope. If the coefficient of static friction is 0.800 between each pair of surfaces in contact, what horizontal force F must be applied to the 4.00 kg block to make it move?

Solution: Let the force of friction between the blocks be $F_{f,2}$ and the force of friction between the lower block and the floor be $F_{f,1}$. The upper mass is m_1 and the lower mass m_2. On the lower mass, we have $F_{f,1}$ and $F_{f,2}$ pulling to the left, and force F pulling to the right. The force balance is then

$$F - F_{f,1} - F_{f,2} = 0 \tag{30}$$

The force of friction between the two blocks is determined by the normal force on m_1 ($m_1 g$) and the coefficient of friction μ, while the force of friction between the lower block and the floor is determined by the normal force on m_2 (the weight of both blocks, $(m_1 + m_2)g$) and μ:

$$F_{f,1} = \mu m_1 g \tag{31}$$

$$F_{f,2} = \mu(m_1 + m_2)g \tag{32}$$

Putting it all together,

$$0 = F - \mu m_1 g - \mu(m_1 + m_2)g \tag{33}$$

$$F = \mu(2m_1 + m_2)g \approx 78.4\,\mathrm{N} \tag{34}$$

25. A 6.0 kg box slides down an inclined plane that makes an angle of 39° with the horizontal. If the coefficient of kinetic friction is 0.19, at what rate does the box accelerate down the slope?

Solution: We've established a few different ways at this point that along the ramp $a_x = g \sin\theta \approx 6.2\,\mathrm{m/s^2}$.