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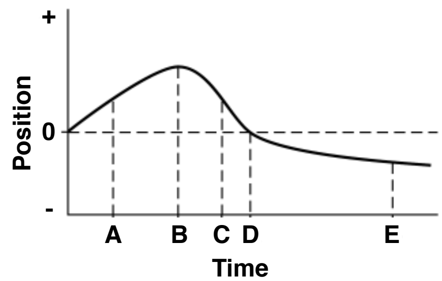
PH105 Exam 1 Solution

1. The graph in the figure shows the position of an object as a function of time. The letters A-E represent particular moments of time. At which moment shown (A, B, etc.) is the **speed** of the object the greatest?



Solution: Speed is the absolute value of the slope of the $x-t$ curve, so the region with the the largest slope (in magnitude, positive or negative) has the largest speed. That is region C

2. Two objects have masses m and $5m$, respectively. They both are placed side by side on a frictionless inclined plane and allowed to slide down from rest.

A) It takes the heavier object 5 times longer to reach the bottom of the incline than the lighter object.
B) The two objects reach the bottom of the incline at the same time.
C) It takes the lighter object 10 times longer to reach the bottom of the incline than the heavier object.
D) It takes the lighter object 5 times longer to reach the bottom of the incline than the heavier object.
E) It takes the heavier object 10 times longer to reach the bottom of the incline than the lighter object.

Solution: The time it takes depends only on the length of the ramp and acceleration, and is independent of mass.

3. A child on a sled starts from rest at the top of a 15° slope. If the trip to the bottom takes 15.2 s how long is the slope? Assume that frictional forces may be neglected.

A) 147 m **B) 293 m** C) 586 m D) 1130 m

Solution: We know acceleration and time. Given a ramp of angle θ, the acceleration is $+g\sin\theta$, with the positive direction being down the ramp. With zero initial velocity, we can write the displacement $\Delta x(t)$ as

$$\Delta x = x_f - x_i = v_i t + \frac{1}{2}at^2 = \frac{1}{2}\left(g\sin\theta\right)t^2 = 293\,\text{m} \tag{1}$$

4. As part of an exercise program, a woman walks south at a speed of 2.00 m/s for 60.0 minutes. She then turns around and walks north a distance 3000 m in 25.0 minutes. What is the woman's average velocity during her entire motion?

A) 0.824 m/s south B) 1.93 m/s south C) 2.00 m/s south D) 1.79 m/s south E) 800 m/s south

Solution: Average velocity is the *net* distance covered (displacement) divided by the elapsed time. At 2 m/s for 60 min, the woman covers 7200 m south, and then backtracks 3000 m north. That gives a total displacement of $7200\,\mathrm{m} - 3000\,\mathrm{m} = 4200\,\mathrm{m}$. The net elapsed time is 85 min, or 5100 s. The average velocity is then

$$|\vec{v}| = \frac{\Delta x}{\Delta t} = \frac{4200\,\mathrm{m}}{5100\,\mathrm{s}} \approx 0.824\,\mathrm{m/s} \tag{2}$$

5. Referring to the previous question, what is the woman's average speed during her entire motion?

A) 0.824 m/s B) 1.93 m/s **C) 2.00 m/s** D) 1.79 m/s E) 800 m/s

Solution: In this case, the average *speed* is the total distance covered (the entire path) divided by time. The total distance covered is $7200\,\mathrm{m} + 3000\,\mathrm{m} = 10200\,\mathrm{m}$, and the average speed is then

$$|\vec{v}| = \frac{10200\,\mathrm{m}}{5100\,\mathrm{s}} \approx 2.00\,\mathrm{m/s} \tag{3}$$

6. A car accelerates from $10.0\,\mathrm{m/s}$ to $30.0\,\mathrm{m/s}$ at a rate of $3.00\,\mathrm{m/s^2}$. How far does the car travel while accelerating?

A) 133 m B) 399 m C) 80.0 m D) 226 m

Solution: We know the initial and final velocities and the acceleration, and we don't know time. We want the displacement. The simplest solution is to use $v_f^2 = v_i^2 + 2a\Delta x$. Solving for Δx and substituting in the given numbers,

$$\Delta x = \frac{v_f^2 - v_i^2}{2a} \approx 133\,\mathrm{m} \tag{4}$$

7. Two objects are dropped from a bridge, an interval of 1.0 s apart, and experience no appreciable air resistance. As time progresses, the DIFFERENCE in their speeds

A) decreases at first, but then stays constant.
B) increases.
C) remains constant.
D) increases at first, but then stays constant.
E) decreases.

Solution: Both objects experience the same acceleration, meaning their speed changes by the same amount per second, and thus their difference in speed doesn't change (once both objects are moving anyway). Mathematically, we can write down velocity versus time for each, noting that one object has a 1 second head start and both start with an initial velocity of zero:

$$v_1(t) = g(t+1) \tag{5}$$
$$v_2(t) = gt \tag{6}$$
$$v_1(t) - v_2(t) = g(t+1) - gt = (1\,\mathrm{s})g \tag{7}$$

The difference in speeds is just the head start (1 second) times the gravitational acceleration, which is nothing more than the velocity the first object picked up during its head start.

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8. A car is 200 m from a stop sign and traveling toward the sign at 40.0 m/s. At this time, the driver suddenly realizes that she must stop the car. If it takes 0.200 s for the driver to apply the brakes, what must be the magnitude of the constant acceleration of the car after the brakes are applied so that the car will come to rest at the stop sign?

A) 4.17 m/s^2 B) 2.89 m/s^2 C) 3.89 m/s^2 D) 3.42 m/s^2 E) 2.08 m/s^2

Solution: During the reaction time of $t_r = 0.200\,\text{s}$, the car is going to travel an additional distance of $x_r = v_i t_r \approx 8\,\text{m}$, where v_i is the car's initial velocity of 40.0 m/s. That means that instead of having $x_s = 200\,\text{m}$ to stop, the driver has only $x_s - x_r \approx 192\,\text{m}$ to do so. We know the acceleration a, the initial speed, and the final speed (zero), and want a displacement. We can use $v_f^2 = v_i^2 + 2a\Delta x$ to relate everything,

9. The position of an object is given by $x = at^3 - bt^2 + ct$, where $a = 4.1\,\text{m/s}^3$, $b = 2.2\,\text{m/s}^2$, $c = 1.7\,\text{m/s}$, and x and t are in SI units. What is the instantaneous acceleration of the object when $t = 0.7\,\text{s}$?

A) 4.6 m/s^2 **B) 13 m/s^2** C) -13 m/s^2 D) 2.9 m/s^2

Solution: Acceleration obtained from the position function $x(t)$ by differentiation, $a(t) = d^2x/dt^2$. Do that. Evaluate at the time of interest.

$$x(t) = at^3 - bt^2 + ct \tag{8}$$
$$\frac{dx(t)}{dt} = v(t) = 3at^2 - 2bt + c \tag{9}$$
$$\frac{d^2x(t)}{dt^2} = \frac{dv(t)}{dt} = a(t) = 6at - 2b \tag{10}$$
$$a(0.7\,\text{s}) \approx 13\,\text{m/s}^2 \tag{11}$$

10. The position of an object as a function of time is given by $x = bt^2 - ct$, where $b = 2.0\,\text{m/s}^2$ and $c = 6.7\,\text{m/s}$, and x and t are in SI units. What is the instantaneous velocity of the object when $t = 2.2\,\text{s}$?

A) 2.1 m/s B) 1.7 m/s C) 2.3 m/s D) 2.7 m/s

Solution: Just like the previous question, we find $v(t)$ by differentiating with respect to time.

$$x(t) = bt^2 - ct \tag{12}$$
$$v(t) = \frac{dx(t)}{dt} = 2bt \tag{13}$$
$$v(2.2\,\text{s}) \approx 2.1\,\text{m/s} \tag{14}$$

11. If the fastest you can safely drive is 65 km/h, what is the longest time you can stop for dinner if you must travel 541 km in 9.6 h total?

A) 1.0 h **B) 1.3 h** C) 1.4 h D) You can't stop at all.

Solution: At a constant velocity of 65 km/h, to drive 541 km will take you

$$t = \frac{\Delta x}{v} = \frac{541\,\text{km}}{65\,\text{km/h}} \approx 8.3\,\text{hr} \tag{15}$$

Given that you have 9.6 hr to do it, you have 1.3 hr to spare.

12. A 1000.0 kg car is moving at 15 km/h. If a 2000.0 kg truck has 18 times the kinetic energy of the car, how fast is the truck moving?

A) 63 km/h B) 54 km/h **C) 45 km/h** D) 36 km/h

Solution: Let the car's mass and velocity be m_c and v_c, respectively, with the truck's mass and velocity as m_t and v_t. We can write down the kinetic energy of each, enforce the condition that the truck's energy is 18 times greater, and then solve for the truck's velocity.

$$K_c = \frac{1}{2} m_c v_c^2 \tag{16}$$

$$K_t = \frac{1}{2} m_t v_t^2 \tag{17}$$

$$18 K_c = K_t \tag{18}$$

$$18 \frac{1}{2} m_c v_c^2 = \frac{1}{2} m_t v_t^2 \tag{19}$$

$$|v_t| = v_c \sqrt{\frac{18 m_c}{m_t}} \approx 45\,\text{km/h} \tag{20}$$

13. A ball is thrown directly upward and experiences no air resistance. Which one of the following statements about its motion is correct?

A) The acceleration of the ball is downward while it is traveling up and upward while it is traveling down.
B) The acceleration of the ball is upward while it is traveling up and downward while it is traveling down.
C) The acceleration is downward during the entire time the ball is in the air.
D) The acceleration of the ball is downward while it is traveling up and downward while it is traveling down but is zero at the highest point when the ball stops.

Solution: We've been over this a lot already: an object in free fall (thrown or not) experiences a constant downward acceleration of g. It is the *velocity* that changes direction.

14. A test rocket is fired straight up from rest with a net acceleration of $20.0\,\text{m/s}^2$. After 4.00 s the motor turns off, but the rocket continues to coast upward with no appreciable air resistance. What maximum elevation does the rocket reach?

A) 487 m B) 327 m C) 160 m D) 408 m E) 320 m

Solution: There are two parts to this one. First, the rocket accelerates upward. Then it doesn't, and is under the influence of gravity alone. That means we have two calculations: how hight did it go while accelerating upward, and then how high does it go before reaching its maximum height? For the second part, we'll need to find the velocity at the end of the phase of upward acceleration.

Let the upward direction be $+x$. During the first phase, we have positive acceleration a for a given time t_1.

The distance covered, starting from zero velocity, is then

$$\Delta x_1 = \frac{1}{2} a t_1^2 \tag{21}$$

At that point, its velocity is

$$v_1 = a t_1 \tag{22}$$

For the second phase of motion, free fall, we have an object with acceleration $-g$ and initial velocity v_1. At the top of the rocket's motion, we know the velocity will be $v_f = 0$. Since we have no information about how long this takes, we can find the distance Δx_2 covered for this phase from the known velocities and acceleration:[i]

$$v_f^2 = 0 = v_1^2 - 2g\Delta x_2 \tag{23}$$

$$\Delta x_2 = \frac{v_1^2}{2g} \tag{24}$$

The total height the rocket reaches is the sum of these two displacements:

$$\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2 = \frac{1}{2} a t_1^2 + \frac{v_1^2}{2g} \approx 486\,\text{m} \tag{25}$$

15. How much energy is needed to change the speed of a 1600 kg sport utility vehicle from 15.0 m/s to 40.0 m/s?

A) 10.0 kJ B) 40.0 kJ **C) 1.10 MJ** D) 20.0 kJ E) 0.960 MJ

Solution: The only thing changing is the vehicle's speed, so it must be the case that the energy required is the difference in kinetic energy between initial and final states. That is simple enough:

$$E = \Delta K = K_f - K_i = \frac{1}{2} m v_f^2 + \frac{1}{2} m v_i^2 = \frac{1}{2} m (v_f^2 - v_i^2) \approx 1.1 \times 10^6\,\text{J} = 1.1\,\text{MJ} \tag{26}$$

16. A 2.3 kg object traveling at 6.1 m/s collides head-on with a 3.5 kg object traveling in the opposite direction at 4.8 m/s. If the collision is **perfectly elastic**, what is the final speed of the 2.3 kg object?

A) 4.3 m/s B) 3.8 m/s C) 6.6 m/s D) 0.48 m/s **E) 7.1 m/s**

Solution: Let $m_1 = 2.3\,\text{kg}$ and $m_2 = 3.5\,\text{kg}$. The initial velocities, following the same notion convention, would be $v_{1i} = 6.1\,\text{m/s}$ and $v_{2i} = -4.8\,\text{m/s}$. Take care with the second one: you're told it is moving in the opposite direction, so *its velocity must be negative.* Now, given that this is an elastic collision, we have already derived the necessary equations to find the final velocity of the first object. We just need to plug in the given numbers.

[i] We could also say $v(t) = v_1 - gt$, find the time at which $v = 0$, and put that time in the $x(t)$ formula to get the maximum height. The result is the same, but with one more step.

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{i1} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i} \tag{27}$$

$$= \left(\frac{2.3 - 3.5}{2.3 + 3.5}\right) (6.1\,\mathrm{m/s}) + \left(\frac{2 \cdot 3.5}{2.3 + 3.5}\right) (-4.8\,\mathrm{m/s}) \approx -7.1\,\mathrm{m/s} \tag{28}$$

Since the problem asks for speed, them minus sign is irrelevant.

17. A shell explodes into two pieces, one piece 25 times heavier than the other. If any gas from the explosion has negligible mass, then

A) the momentum change of the lighter piece is 25 times as great as the momentum change of the heavier piece.
B) the momentum change of the heavier piece is 25 times as great as the momentum change of the lighter piece.
C) the kinetic energy change of the lighter piece is 25 times as great as the kinetic energy change of the heavier piece.
D) the momentum change of the lighter piece is exactly the same as the momentum change of the heavier piece.
E) the kinetic energy change of the heavier piece is 25 times as great as the kinetic energy change of the lighter piece.

Solution: Conservation of momentum: if we consider the shell as our system, and imagine we are at rest with respect to it, it starts out with zero momentum. Afterwards, the two fragments must move in opposite directions with the same momentum for momentum to be conserved.

18. In a perfectly ELASTIC collision between two perfectly rigid objects

A) the kinetic energy of the system is conserved, but the momentum of the system is not conserved.
B) the momentum of each object is conserved.
C) both the momentum and the kinetic energy of the system are conserved.
D) the momentum of the system is conserved but the kinetic energy of the system is not conserved.
E) the kinetic energy of each object is conserved.

Solution: Definitional.

19. In an INELASTIC collision between two objects

A) the momentum of each object is conserved.
B) the kinetic energy of the system is conserved, but the momentum of the system is not conserved.
C) the momentum of the system is conserved but the kinetic energy of the system is not conserved.
D) the kinetic energy of each object is conserved.
E) both the momentum and the kinetic energy of the system are conserved.

Solution: Definitional.

20. Two ice skaters push off against one another starting from a stationary position. The 45.0 kg skater acquires a speed of 0.375 m/s. What speed does the 60.0 kg skater acquire? Assume that any other unbalanced forces during the collision are negligible.

A) 0.000 m/s B) 0.500 m/s **C) 0.281 m/s** D) 0.750 m/s E) 0.375 m/s

Solution: All we need is conservation of momentum for this one. The two skaters start at rest, with zero net momentum. That means their momenta after push off must be equal in magnitude and opposite in sign. Let the lighter skater be 1 and the heavier skater 2. Then conservation of momentum gives

$$0 = m_1 v_{1f} + m_2 v_{2f} \tag{29}$$
$$v_{2f} = -\frac{m_1}{m_2} v_{1f} \approx 0.281\,\mathrm{m/s} \tag{30}$$

21. A 480 kg car moving at 14.4 m/s hits from behind a 570 kg car moving at 13.3 m/s in the same direction. If the new speed of the heavier car is 14.0 m/s, what is the speed of the lighter car after the collision, assuming that any unbalanced forces on the system are negligibly small? It is not known whether the collision was elastic or not.

A) 10.5 m/s **B) 13.6 m/s** C) 19.9 m/s D) 5.24 m/s

Solution: Unlike question 16, we are told this is not an elastic collision. Given that, the only thing we can really rely on is conservation of momentum. If we assign the $+x$ direction to be the original direction of motion, all given velocities are positive. Let the lighter car be 1, and the heavier car 2.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \tag{31}$$
$$m_1 v_{1f} = m_1 v_{1i} + m_2 v_{2i} - m_2 v_{2f} \tag{32}$$
$$v_{1f} = \frac{m_1 v_1 i + m_2 v_{2i} - m_2 v_{2f}}{m_1} \approx 13.6\,\mathrm{m/s} \tag{33}$$

22. You are standing on a skateboard, initially at rest. A friend throws a very heavy ball towards you. You can either catch the object or deflect the object back towards your friend (such that it moves away from you with the same speed as it was originally thrown). What should you do in order to MINIMIZE your speed on the skateboard?

A) Deflect the ball.
B) Catch the ball.
C) Your final speed on the skateboard will be the same regardless whether you catch or deflect the ball.

Solution: Your change in speed will be your change in momentum divided by your mass. Your change in momentum will be equal to the ball's change in momentum. The less the ball changes its momentum, the less you do, and the smaller your speed.

Let's say you catch the ball. Its momentum goes from mv to 0, for a change of mv. How about if you deflect it back at the same speed? Momentum is a vector, so direction matters. It comes in with momentum mv, but leaves with momentum $-mv$ (since it is going in the opposite direction), for a change of $mv-(-mv)=2mv$. You're better off catching the ball.

23. Two objects of the same mass move along the same line in opposite directions. The first mass is moving with speed v. The objects collide, stick together, and move with speed $0.100v$ in the direction of the velocity of the first mass before the collision. What was the speed of the second mass before the collision?

A) $0.00v$ **B)** $0.800v$ C) $1.20v$ D) $0.900v$ E) $10.0v$

Solution: Clearly this is a totally inelastic collision, since the objects stick together. That means conservation of momentum. Let the initial velocity of the first mass be the positive direction. Initially, one object of mass m moves with speed v and the other, also of mass m, with $-v_2$. After the collision, the combined mass $2m$ moves at $0.100v$. Conservation of momentum gives

$$p_i = mv + m(-v_2) = mv - mv_2 = p_f = 2m(0.100v) = 0.200mv \tag{34}$$
$$0.800mv = mv_2 \tag{35}$$
$$v_2 = 0.800v \tag{36}$$

24. On the earth, when an astronaut throws a 0.250 kg stone vertically upward, it returns to his hand a time T later. On planet X he finds that, under the same circumstances, the stone returns to his hand in $2T$. In both cases, he throws the stone with the same initial velocity and it feels negligible air resistance. The acceleration due to gravity on planet X (in terms of g) is

A) $g/\sqrt{2}$ **B)** $g/2$ C) $2g$ D) $g/4$ E) $g\sqrt{2}$

Solution: We know that the time it takes to reach maximum height is the same time it takes to fall back down. That means we really only need to figure out how long it takes to reach maximum height and double it. We will reach the top when the velocity is zero, and we can find out how long this takes from our $v(t)$ equation. Given an initial velocity v_i on earth, with acceleration $-g$, .

$$v(t) = v_i - gt = 0 \qquad \Longrightarrow \qquad t = \frac{v_i}{g} \tag{37}$$

That's how long it takes to go up. It takes just as long to go down, so the total time is $t_{\text{net}} = T = 2v_i/g$. Now we see that the time is *inversely proportional to* g. That means given the same initial velocity, if it takes double the time the acceleration must be half as much. To see this mathematically, write down the equations for both cases: a time $2T$ with acceleration a and a time T with acceleration g.

$$2T = \frac{v_i}{a} \tag{38}$$
$$T = \frac{v_i}{g} \tag{39}$$

Now divide those two:

$$\frac{2T}{T} = \frac{v_i/a}{v_i/g} \tag{40}$$
$$2 = \frac{g}{a} \tag{41}$$
$$a = \frac{1}{2}g \tag{42}$$

25. To determine the height of a flagpole, Abby throws a ball straight up and times it. She sees that the ball goes by the top of the pole after 0.50 s and then reaches the top of the pole again after a total elapsed time of 4.1 s. How high is the pole above the point where the ball was launched? (You can ignore air resistance.)

A) 16 m B) 13 m C) 18 m D) 26 m **E) 10 m**

Solution: We know that the ball reaches the exact same height at two different times. There are many ways to proceed from there. One is to just write down the $x(t)$ equation at both times and set them equal to one another. Let the ground level be $x=0$, with $+x$ upward. That makes the acceleration $-g$ and the initial velocity $+v_o$. That gives us

$$x(t) = v_o t - \frac{1}{2}gt^2 \tag{43}$$

If the ball reaches the top of the flagpole of height h at times t_1 and t_2, then $x(t_1)=x(t_2)=h$.

$$h = v_o t_1 - \frac{1}{2}gt_1^2 = v_o t_2 - \frac{1}{2}gt_2^2 \tag{44}$$

$$v_o t_1 - v_o t_2 = v_o\left(t_1 - t_2\right) = \frac{1}{2}gt_1^2 - \frac{1}{2}gt_2^2 = \frac{1}{2}g\left(t_1^2 - t_2^2\right) = \frac{1}{2}g\left(t_1 - t_2\right)\left(t_1 + t_2\right) \tag{45}$$

$$v_o\left(t_1 - t_2\right) = \frac{1}{2}g\left(t_1 - t_2\right)\left(t_1 + t_2\right) \tag{46}$$

$$v_o = \frac{1}{2}g\left(t_1 + t_2\right) \tag{47}$$

Now we know the initial velocity, and we can plug that back into the $x(t)$ equation to find the height.[ii] Incidentally, we could have found the velocity another way. If the velocity is v when passing the top of the flagpole on the way up, it is $-v$ passing it on the way down. From our $v(t)$ equation, this means

$$v = v_o - gt_1 = -(v_o - gt_2) \tag{48}$$

$$2v_o = gt_1 + gt_2 \tag{49}$$

$$v_o = \frac{1}{2}g\left(t_1 + t_2\right) \tag{50}$$

In either case, given an expression for v_o, our $x(t)$ becomes

$$x(t_1) = h = \frac{1}{2}g\left(t_1 + t_2\right)t_1 - \frac{1}{2}gt_1^2 = \frac{1}{2}g\left(t_1^2 + t_1t_2 - t_1^2\right) = \frac{1}{2}gt_1t_2 \approx 10\,\mathrm{m} \tag{51}$$

[ii]The analysis above works only because we know $t_1 \neq t_2$, so we don't divide by zero getting to the fourth line.