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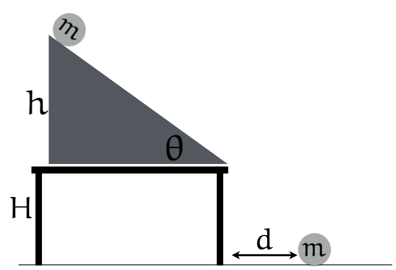
PH105 Final Exam

Instructions

1. Solve 6 of 8 problems.
2. All problems have equal weight. Show your work for full credit.
3. You are allowed 2 sheets of an 8.5 x 11 in piece of paper with notes and a calculator.

1. A ramp of height h and angle θ sits at the edge of a table of height H. A solid sphere rolls down a ramp. At the bottom of the ramp, the ball is launched and lands a distance d from the base of the table. Find d. For concreteness, let $h = 0.5\,\text{m}$, $H = 1\,\text{m}$, and $\theta = 45°$.



2. A mass $m_1 = 1.0\,\text{kg}$ travels over a frictionless surface at $5.0\,\text{m/s}$ and strikes a mass $m_2 = 2.0\,\text{kg}$. After the collision, mass m_2 slides over a region with kinetic friction $\mu_k = 0.5$. How far does m_2 slide before stopping?

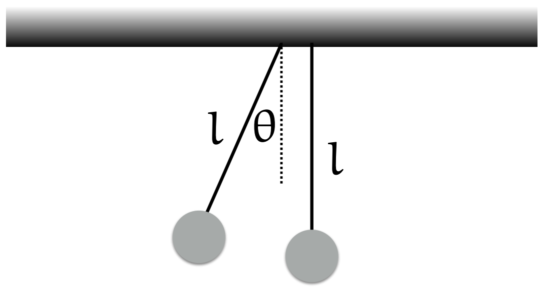
3. When a car is braking, the friction between the brake drums and brake shoes converts kinetic energy into heat. If a 2000 kg car brakes from 25 m/s to 0 m/s, **(a)** how much heat is generated in the brakes? **(b)** If each of the four brake drums has a mass of 9.0 kg and a specific heat of 450 J/kg·K, how much does the temperature of each one of the brake drums rise? (Assume all heat accumulates in the brake drums.)

4. A string of mass per unit length μ_1 is tied to a second string of mass per unit length μ_2. A harmonic wave of speed v_1 traveling along the first string reaches the junction and enters the second string. What will be the speed v_2 of this wave in the second string? *Hint: your answer should be in terms of v_1, μ_1, and μ_2. Figure out what is the same for both strings.*

5. A supertanker has a submerged depth of 30 m when in seawater (density $\rho = 1025\,\text{kg/m}^3$). What will be the submerged depth of the tanker be when it enters a river estuary with freshwater (density $\rho = 1000\,\text{kg/m}^3$)? Assume the sides of the ship are vertical.

6. A hoop of mass M and radius R rolls down a sloping ramp that makes an angle of 30.0° with the ground. The surface of the ramp has a coefficient of static friction $\mu_s = 0.589$. What is the acceleration of the hoop if it rolls without slipping? *Hint: it is not $g \sin 30.0°$.*

7. Two identical small steel balls are suspended from strings of length l so they touch when hanging straight down, in their equilibrium position. If we pull one of the balls back until its string makes an angle θ with the vertical and let go, it will collide elastically with the other ball. How high will the other ball rise?



8. High-altitude mountain climbers do not eat snow, but always melt it first with a stove. To see why, calculate the energy absorbed from a climber's body under the following conditions. The specific heat of ice is 2100 J/kg·K, the latent heat of fusion is 333 kJ/kg, the specific heat of water is 4186 J/kg·K. **(a)** Calculate the energy absorbed from a climber's body if she eats 0.95 kg of −15°C snow which her body warms to a temperature of 37°C. **(b)** Calculate the energy absorbed from a climber's body if she melts 0.95 kg of −15°C snow using a stove and drinks the resulting 0.95 kg of water at 2°C, which her body has to warm to 37°C.

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Formula sheet

$$g = 9.81\,\mathrm{m/s^2} \quad G = 6.67\times10^{-11}\,\mathrm{N\,m^2/kg^2}$$

$$N_A = 6.022\times10^{23}\,\text{things/mol} \qquad 1\,\mathrm{L} = 10^{-3}\,\mathrm{m^3}$$

$$k_B = 1.38065\times10^{-23}\,\mathrm{J\cdot K^{-1}}$$

$$R = 8.31446\,\mathrm{J\cdot mol^{?1}\cdot K^{?1}}$$

$$\text{sphere} \quad V = \frac{4}{3}\pi r^3 \quad A = 4\pi r^2$$

$$ax^2 + bx^2 + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\vec{\mathbf{v}} = \lim_{\Delta t\to 0}\frac{\Delta\vec{\mathbf{r}}}{\Delta t} \equiv \frac{d\vec{\mathbf{r}}}{dt}$$

$$a_x = \lim_{\Delta t\to 0}\frac{\Delta v_x}{\Delta t} \equiv \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

$$\Delta v_x = \int_{t_i}^{t_f} a_x(t)\,dt \qquad \Delta x = \int_{t_i}^{t_f} v_x(t)\,dt$$

$$x(t) = x_i + v_{x,i}t + \frac{1}{2}a_x t^2$$

$$v_x(t) = v_{x,i} + a_x t$$

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x\Delta x$$

$$a_{x,\mathrm{ramp}} = g\sin\theta$$

$$\Delta U^G = mg\Delta x \qquad \frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}$$

$$E_{\mathrm{mech}} = K + U \quad K = \frac{1}{2}mv^2$$

$$\Delta E = \Delta K + \Delta U = 0 \quad \text{non-dissipative, closed}$$

$$\Delta E = W \qquad P = \frac{dE}{dt}$$

$$\Delta U_{\mathrm{spring}} = \frac{1}{2}k\,(x - x_o)^2$$

$$P = F_{\mathrm{ext,x}}\,v_x \quad \text{one dimension}$$

$$W = \left(\sum\vec{\mathbf{F}}\right)\Delta x_F \quad \text{constant force 1D}$$

$$W = \int_{x_i}^{x_f} F_x(x)\,dx \quad \text{nondiss. force, 1D}$$

Derived unit	Symbol	equivalent to
newton	N	kg·m/s^2
joule	J	kg·m^2/s^2 = N·m
watt	W	J/s=m^2·kg/s^3

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{0}} \quad \text{isolated system}$$

$$\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_i \quad \text{isolated system}$$

$$\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}$$

$$m_u = -\frac{\Delta v_{s,x}}{\Delta v_{u,x}}m_s$$

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{i1} + \left(\frac{2m_2}{m_1+m_2}\right)v_{2i} \quad \text{1D elastic}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1+m_2}\right)v_{2i} \quad \text{1D elastic}$$

$$\vec{\mathbf{a}} = \frac{\sum\vec{\mathbf{F}}}{m} \qquad \vec{\mathbf{a_{cm}}} = \frac{\sum\vec{\mathbf{F}}_{\mathrm{ext}}}{m} \qquad \sum\vec{\mathbf{F}} \equiv \frac{d\vec{\mathbf{p}}}{dt}$$

$$\vec{\mathbf{J}} = \left(\sum\vec{\mathbf{F}}\right)\Delta t \quad \text{constant force}$$

$$\vec{\mathbf{J}} = \int_{t_i}^{t_f}\sum\vec{\mathbf{F}}(t)\,dt \quad \text{time-varying force}$$

$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21} \qquad F_{\mathrm{grav}} = -mg \qquad F_{\mathrm{spring}} = -k\Delta x$$

Rotation: we use radians

$$s = \theta r \quad \leftarrow \text{arclength}$$

$$\omega = \frac{d\theta}{dt} = \frac{v_t}{r} \qquad \alpha = \frac{d\omega}{dt}$$

$$a_t = \alpha r \quad \text{tangential} \qquad a_r = -\frac{v^2}{r} = -\omega^2 r \quad \text{radial}$$

$$v_t = r\omega \qquad v_r = 0 \qquad \omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

$$(F_{12}^s)_{\max} = \mu_s F_{12}^n$$

$$F_{12}^k = \mu_k F_{12}^n$$

$$W = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}_F \quad \text{const non-diss force}$$

$$W = \int_{\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}_f}\vec{\mathbf{F}}(\vec{\mathbf{r}})\cdot d\vec{\mathbf{r}} \quad \text{variable nondiss force}$$

↓ launched from origin, level ground

$$y(x) = (\tan\theta_o)\,x - \frac{gx^2}{2v_o^2\cos^2\theta_o}$$

$$\text{max height } = H = \frac{v_i^2\sin^2\theta_i}{2g}$$

$$\text{Range } = R = \frac{v_i^2\sin 2\theta_i}{g}$$

$$(F_{12}^2)_{\max} = \mu_s F_{12}^n \qquad F_{12}^k = \mu_k F_{12}^n$$

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fluids:

$$P = F/A \qquad P(d) = P_{\text{surface}} + \rho g d$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \quad F_1 x_1 = F_2 x_2 \qquad \text{hydraulics}$$

$$B = \text{buoyant force} = \text{weight of water displaced} = \rho_f V_{\text{displ}} g$$

$$P = P_{\text{gauge}} + P_{\text{atm}} \qquad \rho = M/V$$

$$I = \sum_i m_i r_i^2 \Rightarrow \int r^2\, dm = kmr^2$$

$$I_z = I_{com} + md^2 \quad \text{axis } z \text{ parallel, dist } d$$

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = I\vec{\boldsymbol{\omega}}$$

$$K = \frac{1}{2}I\omega^2 = L^2/2I$$

$$\Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W = \int \tau\, d\theta$$

$$P = \frac{dW}{dt} = \tau\omega$$

$$\tau = rF\sin\theta_{rF} = r_\perp F = rF_\perp$$

$$\tau_{net} = \sum \vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}} = \frac{d\vec{\mathbf{L}}}{dt}$$

$$K_{\text{tot}} = K_{cm} + K_{rot} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$$

Moments of inertia of things of mass M

Object	axis	**dimension**	**I**
solid sphere	central	radius R	$\frac{2}{5}MR^2$
hollow sphere	central	radius R	$\frac{2}{3}MR^2$
solid disc	central	radius R	$\frac{1}{2}MR^2$
hoop	central	radius R	MR^2
point particle	pivot	distance R to pivot	MR^2
rod	center	length L	$\frac{1}{12}ML^2$
rod	end	length L	$\frac{1}{3}ML^2$

Oscillations:

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \quad \omega = \frac{2\pi}{T} = 2\pi f \quad k = \frac{2\pi}{\lambda}$$

$$x(t) = A\sin(\omega t + \varphi_i)$$

$$v(t) = \frac{dx}{dt} = \omega A\cos(\omega t + \varphi_i)$$

$$a(t) = \frac{d^2x}{dt^2} = -\omega^2 A\sin(\omega t + \varphi_i)$$

$$\varphi(t) = \omega t + \varphi_i$$

$$a = -\omega^2 x = \frac{d^2x}{dt^2} \qquad \frac{d^2\theta}{dt^2} = -\omega^2\theta$$

$$E = \frac{1}{2}m\omega^2 A^2 \quad F_{\text{spring}} = -kx$$

$$\omega = \sqrt{k/m} \quad \text{spring.}$$

$$T = \begin{cases} 2\pi\sqrt{L/g} & \text{simple pendulum} \\ 2\pi\sqrt{I/mgl_{cm}} & \text{physical pendulum} \end{cases}$$

Waves:

$$y = f(x - ct) \quad \text{along } +\text{x} \qquad y = f(x + ct) \quad \text{along -x}$$

$$k = \frac{2\pi}{\lambda} \quad \lambda = cT \quad \omega = \frac{2\pi}{T} \quad c = \lambda f$$

$$y(x,t) = f(x,t) = A\sin(kx - \omega t + \varphi_i)$$

$$y(x,t) = 2A\sin kx\cos\omega t \quad \text{standing wave}$$

$$\text{nodes at } x = 0, \pm\frac{\lambda}{2}, \pm\lambda, \pm\frac{3\lambda}{2}$$

$$v = \sqrt{T/\mu} \quad \mu = M_{\text{string}}/L_{\text{string}} \quad T = \text{tension}$$

$$P_{\text{av}} = \frac{1}{2}\mu\lambda A^2\omega^2/T = \frac{1}{2}\mu A^2\omega^2 c$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

$$f_n = \frac{nv}{\lambda} = \frac{nv}{2L} \quad \lambda_n = \frac{2L}{n} \quad n = 1,2,3\ldots \quad \text{strings \& open-open pipe}$$

$$f_n = \frac{nv}{\lambda} = \frac{nv}{4L} \quad \lambda_n = \frac{4L}{n} \quad n = 1,3,5\ldots \quad \text{closed-open pipe}$$

thermal stuff:

$$PV = Nk_BT = nRT$$

$$W = P\Delta V$$

$$T(K) = T(^\circ C) + 273.15^\circ$$

$$Q = mc\Delta t \quad c = \text{specific heat} \qquad \text{no phase chg}$$

$$Q = \pm mL \qquad \text{phase chg}$$

Isolated systems: $\vec{\mathbf{p}}$, $E = K + PE$, L are all conserved.
Static equilibrium: $\sum F = 0$ and $\sum \tau = 0$ about any axis.
Elastic collision: KE and p are both conserved.
Inelastic collision: only p is conserved, not KE.

Power	**Prefix**	**Abbreviation**
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M