

## Week 3 Homework

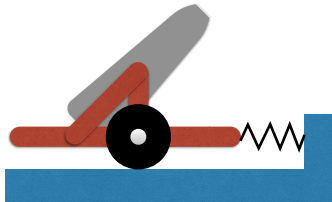
### Instructions:

1. There are problems assigned for each day of class.
2. The following class, you will turn in the one problem the instructor requests.
3. Only the chosen problem for the day is graded.
4. Please follow the homework template provided.
5. You may collaborate, but everyone must turn in their own work.

### Problems for 9 June (due 10 June)

1. On a frictionless table, a mass  $m$  moving at speed  $v$  collides with another mass  $m$  initially at rest. The masses stick together. How much energy is converted to heat?
2. A very light ping pong ball bounces elastically head-on off a very heavy bowling ball that is initially at rest. What is the fraction of the ping pong ball's initial kinetic energy that is transferred to the bowling ball?
3. A large howitzer is rigidly attached to a carriage, which can move along horizontal rails but is connected to a sturdy wall by a large spring, initially unstretched and with force constant  $k = 1.90 \times 10^4 \text{ N/m}$ , as shown below. The cannon fires a 200 kg projectile at a velocity of 125 m/s directed  $45.0^\circ$  above the horizontal.

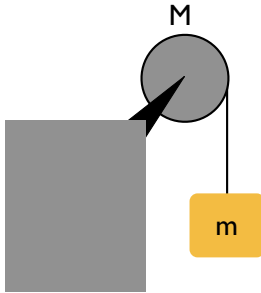
If the mass of the cannon and its carriage is 4780 kg, find the maximum extension of the spring.



### Problems for 10 June (due 11 June)

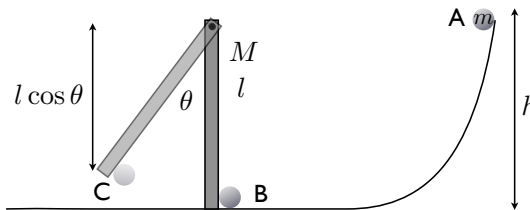
4. A uniformly dense rope of length  $b$  and mass per unit length  $\lambda$  is coiled on a smooth table. One end is lifted by hand with constant velocity  $v_0$ . Find the force of the rope on the hand when the rope is a distance  $a$  above the table ( $b > a$ ).

5. A uniform disk with mass  $M = 2.5 \text{ kg}$  and radius  $R = 20 \text{ cm}$  is mounted on a fixed horizontal axle, as shown below. A block of mass  $m = 1.2 \text{ kg}$  hangs from a massless cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord. *Note: the moment of inertia of a disk about its center of mass is  $I = \frac{1}{2}MR^2$ .*

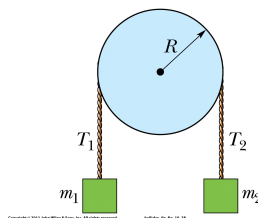


**Problems for 11 June (due 12 June)**

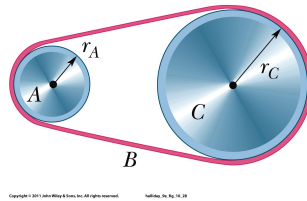
6. In the figure below, a small block of mass  $m$  slides down a frictionless surface through height  $h$  and then sticks to a uniform rod of mass  $M$  and length  $L$ . The rod pivots about point  $O$  through angle  $\theta$  before momentarily stopping. Find  $\theta$ .



7. In the figure below, block 1 has mass  $m_1$ , block 2 has mass  $m_2$  (with  $m_2 > m_1$ ), and the pulley (a solid disc), which is mounted on a horizontal axle with negligible friction, has radius  $R$  and mass  $M$ . When released from rest, block 2 falls a distance  $d$  in  $t$  seconds without the cord slipping on the pulley. (a) What are the magnitude of the accelerations of the blocks? (b) What is  $T_1$ ? (c) What is  $T_2$ ? (d) What is the pulley's angular acceleration? The moment of inertia of a solid disc is  $I = \frac{1}{2}MR^2$ .



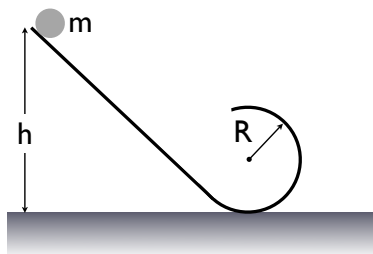
8. A flywheel rotating freely on a shaft is suddenly coupled by means of a drive belt to a second flywheel sitting on a parallel shaft (see figure below). The initial angular velocity of the first flywheel is  $\omega$ , that of the second is zero. The flywheels are uniform discs of masses  $M_a$  and  $M_c$  with radii  $R_a$  and  $R_c$  respectively. The moment of inertia of a solid disc is  $I = \frac{1}{2}MR^2$ . The drive belt is massless and the shafts are frictionless. (a) Calculate the final angular velocity of each flywheel. (b) Calculate the kinetic energy lost during the coupling process. *Hint: if the belt does not slip, the linear speeds of the two rims must be equal.*



9. A solid sphere, a solid cylinder, and a thin-walled pipe, all of mass  $m$ , roll smoothly along identical loop-the-loop tracks when released from rest along the straight section (see figure below). The circular loop has radius  $R$ , and the sphere, cylinder, and pipe have radius  $r \ll R$  (i.e., the size of the objects may be neglected when compared to the other distances involved). If  $h = 2.8R$ , which of the objects will make it to the top of the loop? Justify your answer with an explicit calculation. The moments of inertia for the objects are listed below.

$$I = \begin{cases} \frac{2}{5}mr^2 & \text{sphere} \\ \frac{1}{2}mr^2 & \text{cylinder} \\ mr^2 & \text{pipe} \end{cases} \quad (1)$$

*Hint: consider a single object with  $I = kmr^2$  to solve the general problem, and evaluate these three special cases only at the end.*



10. The rotational inertia (moment of inertia) of a collapsing spinning star drops to  $\frac{1}{3}$  its initial value. What is the ratio of the new rotational kinetic energy to the initial rotational kinetic energy?

**Problems for 12 June (due 15 June)**

**11.** The fastest possible rate of rotation of a planet is that for which the gravitational force on material at the equator just barely provides the centripetal force needed for the rotation. Show that the corresponding shortest period of rotation is

$$T = \sqrt{\frac{3\pi}{G\rho}}$$

where  $\rho$  is the uniform density (mass per unit volume) of the spherical planet. The volume of a sphere is  $\frac{4}{3}\pi r^3$ , where  $r$  is the radius of the sphere.

**12.** The period of the earth's rotation about the sun is 365.256 days. It would be more convenient to have a period of exactly 365 days. How should the mean distance from the sun be changed to correct this anomaly?

**13.** The space shuttle releases a 470 kg satellite while in an orbit 280 km above the surface of the earth. A rocket engine on the satellite boosts it to a geosynchronous orbit. How much energy is required for the orbit boost? (Note: the earth's radius is 6378 km, its mass is  $5.98 \times 10^{24}$  kg, and  $G = 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2 \text{kg}^{-2}$ . Hint: "geosynchronous" means the satellite's period  $T$  is 24 hrs.)

**14.** Calculate the mass of the Sun given that the Earth's distance from the Sun is  $1.496 \times 10^{11}$  m. (Hint: you already know the period of the Earth's orbit.)