University of Alabama
Department of Physics and Astronomy

PH 105 LeClair | Summer 2012

Problem Set 10 Solutions

1. An object of mass m is dropped from a height h above the surface of a planet of mass M and radius R. Assume the planet has no atmosphere so that friction can be ignored. Further assume the planet has no life that may be harmed by subsequent portions of this problem.

What is the speed of the mass just before it strikes the surface of the planet? Do *not* assume that h is small compared with R.

Solution: Conservation of energy, and no more. The only tricky point is to remember that since we must use general gravitation (since we may not assume $h \ll R$), we must take into account the potential energy at the planet's surface too. For general gravitation, potential energy is by convention only zero for an infinite separation of two objects.

$$E_i = U_i = -\frac{GMm}{R+h} \tag{1}$$

$$E_f = K_f + U_f = \frac{1}{2}mv^2 - \frac{GMm}{R} \tag{2}$$

$$\Longrightarrow \qquad v = \sqrt{2GM}\sqrt{\frac{1}{R} - \frac{1}{R+h}} \tag{3}$$

2. A satellite is in a circular Earth orbit of radius r. The area A enclosed by the orbit depends on r^2 because $A = \pi r^2$. Determine how the following properties depend on r: **(a)** period, **(b)** kinetic energy, **(c)** angular momentum, and **(d)** speed.

Solution: Period is given by Kepler's law:

$$T^2 \propto r^3 \qquad \Longrightarrow \qquad T \propto r^{3/2} \tag{4}$$

For a circular orbit, we can show that the kinetic energy is exactly $-\frac{1}{2}$ times the potential. Briefly, for a circular orbit the gravitational force must provide the centripetal force to maintain the circular path. Thus,

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \tag{5}$$
$$\frac{r}{2}\frac{GMm}{r^2} = \frac{r}{2}\frac{mv^2}{r} \tag{6}$$
$$\frac{GMm}{r} = \frac{1}{mv^2} = K = -\frac{1}{2}U \tag{7}$$

Thus, we can see $K \propto 1/r$. Since $K \propto^2$, it also follows immediately that $v \propto 1/\sqrt{r}$. Since angular momentum is, in magnitude, $L{=}mvr$, it must be the case that $L \propto vr \propto (1/\sqrt{r})(r)$, or $L \propto \sqrt{r}$.

3. The density of water is $1000\,\mathrm{kg/m^3}$, while that of ice is $916.7\,\mathrm{kg/m^3}$. If a block of ice is placed in water, what volume fraction of the ice is below the surface?

Solution: Let the block of ice under consideration have an area A exposed to the water, with total thickness in the vertical direction be d, with a thickness x below the water. We will assume that the ice is uniform in the lateral directions, having the same thickness everywhere. There are two forces acting on the block of ice: its weight pulling own, and the buoyant force of the displaced water pushing it upward. The former is easy to find given the density of ice ρ_i and the volume of the block Ad:

$$W = mg = \rho_i A d g \tag{8}$$

The buoyant force is due to the water displaced, and it is only the ice below the surface of volume Ax that displaces any water. The buoyant force is the weight of the displaced water, and given a density ρ_w of water:

$$B = \rho_i A x g \tag{9}$$

In equilibrium, the two forces must be equal:

$$W = \rho_i A d g = B = \rho_i A x g \tag{10}$$
$$\implies \quad \frac{x}{d} = \text{fraction below water} = \frac{\rho_i}{\rho_w} \approx 0.917 \tag{11}$$

Thus, the old adage that nine tenths of an iceberg is underwater is basically correct.

4. Superman attempts to drink water through a very long straw. With his great strength, he achieves maximum possible suction. The walls of the straw do not collapse. Find the maximum height through which he can lift the water.

Solution: Let's say Superman reduces the pressure inside the straw to some value P_o compared to the outside atmosphere of pressure P_a. The net force on a column of water of cross-sectional area A would then be $(P_a - P_o)\,A$. If this force is to lift the column of liquid through a height h, it must be equal to the weight of a column of said liquid of area A and height h.

$$\text{net suction force} = (P_a - P_o)\,A \tag{12}$$

$$\text{weight of water pulled up} = \rho h A g \tag{13}$$

What is the best Superman could do? He could reduce the pressure in the straw to zero. There is no lower pressure! In this case, $P_o = 0$, equating the two forces gives:

$$P_a A = \rho h A g \tag{14}$$

$$h = \frac{P}{\rho g} \approx 10.3\,\mathrm{m} \tag{15}$$

The best he can do is to cause the atmospheric pressure P_a to push up on the water with all its might, but no more. The power of suction is fundamentally limited by the surrounding pressure, Superman or otherwise.