University of Alabama
Department of Physics and Astronomy

PH 105 LeClair Summer 2012

Problem Set 4

Instructions:

1. Answer all questions below. All questions have equal weight.
2. All problems are due Tues 5 June 2012 at the start of lecture.
3. You may collaborate, but everyone must turn in their own work.

1. A 3.00 kg object is moving in a plane, with its x and y coordinates in meters given by $x = 5t^2 - 1$ and $y = 3t^3 + 2$, where t is in seconds. What is the magnitude of the net force acting on this object at $t = 2.00$ s?

Solution: If we can get the acceleration, we can get the force. Since we have the position as a function of time, finding the components of acceleration is no big deal:

$$a_x = \frac{d^2x}{dt^2} = 10 \tag{1}$$

$$a_y = \frac{d^2y}{dt^2} = 18t \tag{2}$$

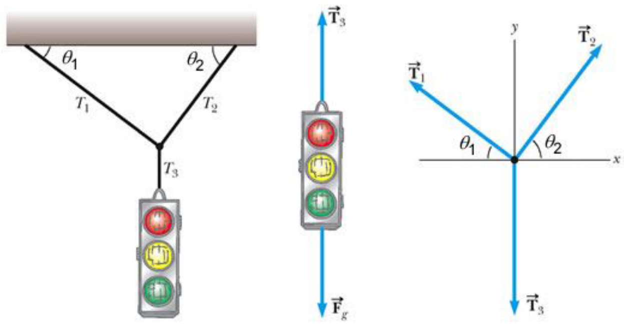
The *magnitude* of the force depends on the magnitude of the acceleration, so we'd better find it:

$$|\vec{\mathbf{a}}| = \sqrt{a_x^2 + a_y^2} = \sqrt{10^2 + 18^2t^2} \tag{3}$$

Newton's second law tells us that mass times acceleration is the net force, so:

$$|\vec{\mathbf{F}}| = m|\vec{\mathbf{a}}| = m\sqrt{10^2 + 18^2t^2} \approx 112\,\text{N} \tag{4}$$

2. A traffic light weighing $mg = 123$ N hangs from a cable tied to two other cables fastened to a support, as in the figure below. The upper cables make angles of $\theta_1 = 40°$ and $\theta_2 = 50°$ with the horizontal. Find the magnitudes of $\vec{\mathbf{T}}_1$, $\vec{\mathbf{T}}_2$, and $\vec{\mathbf{T}}_3$.



Solution: The cable T_3 has to support the traffic light's entire weight, so we must have $T_3 = 123\,\mathrm{N}$. A free-body diagram at the point where the three cables meet gives us horizontal and vertical forces, which much each sum to zero if the traffic light is to stay put.

$$\text{horizontal:} \quad \sum F_x = T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0 \tag{5}$$

$$\text{vertical:} \quad \sum F_y = T_1 \sin\theta_1 + T_2 \sin\theta_2 - T_3 = 0 \tag{6}$$

Two equations and two unknowns. Let's solve the first for T_1 an plug it in the second:

$$\text{from 5:} \quad T_2 = T_1 \frac{\cos\theta_1}{\cos\theta_2} \tag{7}$$

$$\text{plug into 6:} \quad 0 = T_1 \sin\theta_1 + \left(T_1 \frac{\cos\theta_1}{\cos\theta_2}\right) \sin\theta_2 - T_3 \tag{8}$$

Solving for T_1, since we know T_3 already,

$$T_3 = T_1 \left(\sin\theta_1 + \cos\theta_1 \tan\theta_2\right) \tag{9}$$

$$T_1 = \frac{T_3}{\sin\theta_1 + \cos\theta_1 \tan\theta_2} \approx 79.06\,\mathrm{N} \tag{10}$$

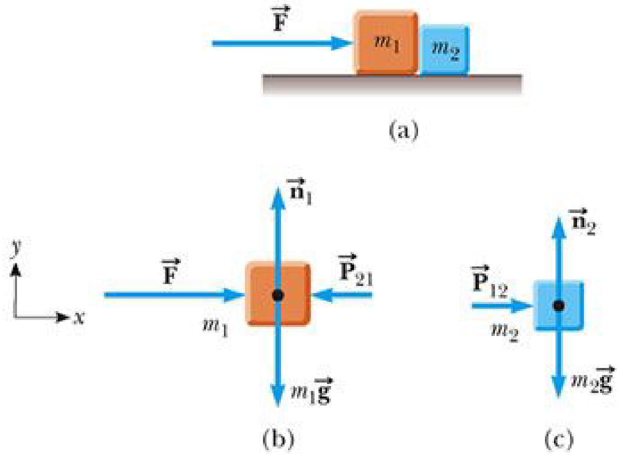
Using 7, we can find T_2 now:

$$T_2 = T_1 \frac{\cos\theta_1}{\cos\theta_2} = T_3 \frac{\cos\theta_1}{\sin\theta_1 \cos\theta_2 + \cos\theta_1 \cos\theta_2 \tan\theta_2} = T_3 \frac{\cos\theta_1}{\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2} \tag{11}$$

$$= \frac{T_3}{\tan\theta_1 \cos\theta_2 + \sin\theta_2} \approx 94.2\,\mathrm{N} \tag{12}$$

Note T_2 and T_1 are both less than T_3, as they must be.

3. Two blocks of masses m_1 and m_2 ($m_1 > m_2$) are placed in contact on a horizontal, frictionless surface, as shown in the figure below. A constant horizontal force of $\vec{\mathbf{F}} = 115\mathrm{N}$ is applied to m_1 as shown. Find the magnitude of the acceleration of the two blocks.



Solution: The blocks move together, so this is equivalent to a single mass $m_1 + m_2$ moving under the influence of F. The acceleration is thus $a = F/(m_1 + m_2)$.