University of Alabama
Department of Physics and Astronomy

PH 105 LeClair Summer 2012

Problem Set 5 Solutions

1. *HRW 5.31* A block is projected up a frictionless inclined plane with an initial speed of $v_o = 3.50\,\mathrm{m/s}$. The angle of incline is $\theta = 32.0°$. (a) How far up the plane does the block go? (b) How long does it take to get there? (c) What is its speed when it gets back to the bottom?

Solution: Let $\theta = 32°$. The acceleration down the ramp plane is $-g\sin\theta$. If it is to travel a distance d up the ramp given an initial velocity v_i, and reach final velocity $v_f = 0$:

$$v_f^2 - v_i^2 = 2ad = -2gd\sin\theta \tag{1}$$

$$d = \frac{v_i^2}{2g\sin\theta} \approx 1.18\,\mathrm{m} \tag{2}$$

Since we know the acceleration and initial velocity, we can find the time readily.

$$v(t) = v_i + at = v_i - gt\sin\theta = v_f = 0 \tag{3}$$

$$t = \frac{v_i}{g\sin\theta} \approx 0.673\,\mathrm{s} \tag{4}$$

What is the speed at the bottom? Same as it was on the way up. We can verify that, noting that moving down the ramp the acceleration is now $a = +g\sin\theta$, and the mass moves through distance d starting from rest:

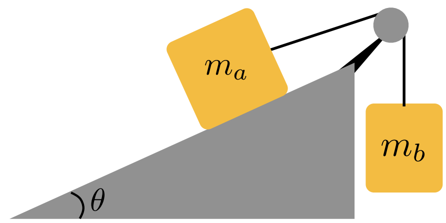
$$v_f^2 - v_i^2 = 2gd\sin\theta \tag{5}$$

$$v_f^2 = 2gd\sin\theta = 2g\left(\frac{v_i^2}{2g\sin\theta}\right)\sin\theta = v_i^2 \tag{6}$$

$$v_f = |v_i| \tag{7}$$

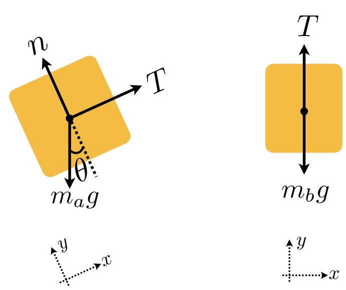
2. *HRW 5.57* A block of mass $m_a = 3.70\,\mathrm{kg}$ on a frictionless plane inclined at an angle $\theta = 30.0°$ is connected by a cord over a massless, frictionless pulley to a second block of mass $m_b = 2.30\,\mathrm{kg}$ (figure below). What are (a) the magnitude of the acceleration of each block, (b) the direction of the acceleration of the hanging block, and (c) the tension in the cord?

Solution:
Find: The tension in a cord connecting two blocks and the system's acceleration, with one block on a frictionless incline and the second on a flat surface with coefficient of kinetic friction μ_k.



Given: The mass of both blocks, the coefficient of friction for the block on the flat surface, and the angle of incline for the ramp.

Sketch: We need free-body diagrams for each mass. Note the axis definitions for each mass. We are not sure which way the masses will accelerate yet, but we will assume that the hanging mass m_b will fall, meaning the acceleration is in the $+x$ direction for mass m_a according to the sketches below. If we chose incorrectly, the acceleration will come out negative to let us know.



Since the rope is presumably taut the entire time of interest, the acceleration is the same for both blocks. For the same reason, the tension applied to both blocks is the same.

Relevant equations: Newton's second law and geometry will suffice. Along the y direction for m_a and along the x direction for m_b, the forces must sum to zero. Along x direction for m_a and y direction for m_b, the forces must give the acceleration for each mass.

$$\sum F_y = 0 \tag{8}$$

$$\sum F_x = ma_x \tag{9}$$

Symbolic solution: First consider mass A. The free body diagram above yields the following, noting that the acceleration will be purely along the x direction:

$$\sum F_y = n - m_a g\cos\theta = 0 \tag{10}$$
$$\sum F_x = T - m_a g\sin\theta = m_a a \qquad \Longrightarrow \qquad a = \frac{T}{m_a} - g\sin\theta \tag{11}$$

For mass B, things are simpler, but we should keep in mind that the acceleration is along $-y$:

$$\sum F_y = T - m_b g = -m_b a \qquad \Longrightarrow \qquad T = m_b(g-a) \tag{12}$$
$$\sum F_x = 0 \tag{13}$$

We have enough to find the acceleration in terms of known quantities now:

$$a = \frac{T}{m_a} - g\sin\theta = \frac{m_b}{m_a}(g-a) - g\sin\theta \tag{14}$$
$$a\left(1+\frac{m_b}{m_a}\right) = g\left(\frac{m_b}{m_a} - \sin\theta\right) \tag{15}$$
$$a = g\left(\frac{m_b - m_a\sin\theta}{m_a+m_b}\right) \tag{16}$$

Given this acceleration, the tension is found readily from $T = m_b(g-a)$.

$$T = m_b g - m_b a = m_b g - m_b g\left(\frac{m_b - m_a\sin\theta}{m_a+m_b}\right) \tag{17}$$
$$= g\left[\frac{m_a m_b + m_b^2}{m_a+m_b} - \frac{m_b^2 + m_a m_b\sin\theta}{m_a+m_b}\right] = g\left[\frac{m_a m_b}{m_a+m_b}\right](1+\sin\theta) \tag{18}$$

Numeric solution: Given $m_a = 3.7\,\mathrm{kg}$, $m_b = 2.3\,\mathrm{kg}$, and $\theta = 30°$, the tension is

$$T = g\left[\frac{m_a m_b}{m_a+m_b}\right](1+\sin\theta) \approx 20.9\,\mathrm{N} \tag{19}$$

and for either block the acceleration is:

$$a = g\left(\frac{m_b - m_a\sin\theta}{m_a+m_b}\right) \approx 0.736\,\mathrm{m/s^2} \tag{20}$$

Since the acceleration is positive, we were correct in our original assumption - mass m_b moves downward, and mass m_a to the right.

3. *HRW 5.50* In the figure below, three ballot boxes are connected by cords, one of which wraps over a pulley having negligible friction on its axle and negligible mass. The three masses are $m_a = 30.0\,\mathrm{kg}$, $m_b = 40.0\,\mathrm{kg}$, and $m_c = 10.0\,\mathrm{kg}$. When the assembly is released from rest, (a) what is the tension in the cord connecting B and C, and (b) how far does A move in the first 0.250 s (assuming it does not reach the pulley and B and C do not reach the floor)?

Solution: Let the tension in the cord connecting B and C be T_{bc}, and the tension in the cord connecting B and A be T_{ba}. Mass C has only two forces acting on it: T_{bc} and its weight $m_c g$. Clearly the acceleration is downward, in the same direction as the weight and opposite the tension.

$$T_{bc} - m_c g = -m_c a \tag{21}$$

Mass A has only one force acting on it, the tension T_{ab}, giving

$$T_{ab} = m_a a \tag{22}$$

This is not quite enough information. However, since B and C are connected together, we may treat them, from the point of view of the upper cord, as a single mass $(m_b + m_c)$ connected to mass A. There are two forces acting on B and C connected together: their weight, and the tension T_{ab}. Thus,

$$T_{ab} - (m_b + m_c)\,g = -(m_b + m_c)\,a \tag{23}$$

Since we already know $T_{ab} = m_a a$,

$$m_a a - (m_b + m_c)\,g = -(m_b + m_c)\,a \tag{24}$$

$$a = \left[\frac{m_b + m_c}{m_a + m_b + m_c}\right] g \approx 6.13\,\mathrm{m/s^2} \tag{25}$$

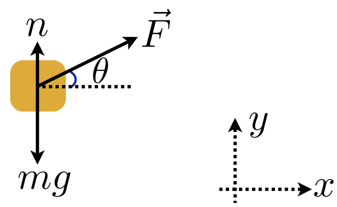
The desired tension is readily found now, since $T_{bc} = m_c\,(g - a)$

$$T_{bc} = m_c g - m_c \left[\frac{m_b + m_c}{m_a + m_b + m_c}\right] g = \left[\frac{m_c m_a + m_c m_b + m_c^2 - m_c m_b - m_c^2}{m_a + m_b + m_c}\right] \tag{26}$$

$$T_{bc} = g\left[\frac{m_c m_a}{m_a + m_b + m_c}\right] \approx 36.8\,\mathrm{N} \tag{27}$$

4. *HRW 6.30* A toy chest and its contents have a combined weight of 180 N. The coefficient of static friction between toy chest and floor is $\mu_s = 0.42$. A child attempts to move the chest across the floor by pulling on an attached rope. (a) If the rope makes an angle of $\theta = 42°$ with the horizontal, what is the magnitude of the force $\vec{\mathbf{F}}$ that the child must exert on the rope to pull the chest on the verge of moving? (b) Write an expression for the magnitude F required to pull the chest on the verge of moving as a function of the angle θ. Determine the value of θ for which F is (c) a minimum and (d) a maximum magnitude.

Solution: Here's a quick free-body diagram:



Along the y direction, the net force must be zero for the block to stay on the floor:

$$\sum F_y = N + F\sin\theta - mg = 0 \tag{28}$$
$$N = mg - F\sin\theta \tag{29}$$

Along the horizontal direction, we want the box to be on the verge of moving, so the point where acceleration is still zero:

$$\sum F_x = F\cos\theta - \mu_s N = F\cos\theta - \mu_s mg + \mu_s F\sin\theta = 0 \tag{30}$$
$$F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} \approx 74° \tag{31}$$

The force will be minimum when $dF/d\theta = 0$:

$$\frac{d}{d\theta}\left(\frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}\right) = \mu_s mg\left[\frac{\sin\theta - \mu_s\cos\theta}{(\cos\theta + \mu\sin\theta)^2}\right] = 0 \tag{32}$$

The pre-factor and denominator are irrelevant; the equation above will only be zero and the force at a minimum if $\sin\theta = \mu_s\cos\theta$, or when $\tan\theta = \mu_s$. In this case, that implies $\theta \approx 22.8°$ for minimum force.

What about the maximum? That is easier: it takes the most force when you are at 90° - you can apply as much force as you want at that point, and the box will never move sideways . . .