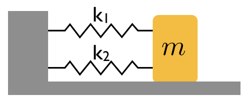
UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 105 LeClair Summer 2012

Problem Set 6 Solutions

1. A mass m is connected to two springs k_1 and k_2 as shown below. If the mass is pushed to the left such that the springs both compress by a distance x from equilibrium and released from rest, what is the velocity of the object as it passes through its original equilibrium position?



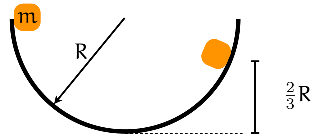
Solution: The easy approach here is to use work and energy. Using force will get you the acceleration, but since the acceleration will vary with position (since the spring force is not constant), this is not a trivial approach. Remember, our kinematic equations are for the most part valid only for constant accelerations.

The work required to compress spring 1 by a through a displacement x is $\frac{1}{2}k_1x^2$, and it takes an additional $\frac{1}{2}k_2x^2$ to compress spring 2. The total work is the sum of these two. Once we let the mass go, this work will be converted to kinetic energy. At the equilibrium point, no work is required to hold the springs, so the entirety of the original work must be in the form of kinetic energy. Since the mass starts at rest, we need not worry about any initial kinetic energy. Thus,

$$W = \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2 = \Delta K = \frac{1}{2}mv^2 \tag{1}$$

$$v = x\sqrt{\frac{k_1 + k_2}{m}} \tag{2}$$

2. A particle of mass m is released from rest at the rim of a smooth bowl of radius R as shown below. It slides without friction down the bowl, and up the other side. When the particle is at a height $\frac{2}{3}R$ from the base of the bowl, what is its speed? Assume the particle is a point mass, and has no moment of inertia.

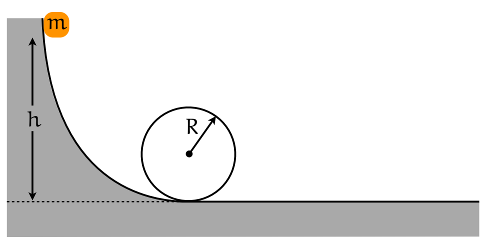


Solution: The work done by the gravitational force in moving the mass from its starting to ending height must equal the change in kinetic energy. Since the object starts at rest, the change in kinetic energy is the same as the final kinetic energy. The work done by the gravitational force is just the mass' weight mg times the change in height, which is $\frac{1}{3}R$. Thus,

$$W = mg\Delta y = \frac{1}{3}mgR = \Delta K = \frac{1}{2}mv_f^2 \tag{3}$$

$$v = \sqrt{\frac{2}{3}gR} \tag{4}$$

3. A *point mass* m moves along the frictionless track shown below. It starts from rest at a height h above the bottom of the loop of radius R, where R is much larger than r. What is the minimum value of h (in terms of R) such that the object completes the loop? *Hint: work-energy will get you the velocity at the top of the loop, but centripetal force tells you if you can stay on the loop.*



Solution: There are two factors at play here. First, the work done by the gravitational force in moving the mass from its starting to ending height must give a change in kinetic energy. Since the mass starts out at rest and moves to a lower height, it will acquire kinetic energy just like the last problem. This will give us the actual velocity of the mass *if* it makes it to the top of the loop.

Does the mass reach the top of the loop? We solved this problem already, and found that in order to satisfy the constraint imposed by circular motion we needed a velocity of at least $v_r = \sqrt{Rg}$. This came from noting that in the case of a vanishing normal force, the minimum condition to stay on the track, the centripetal force was provided entirely by the weight: $mg = mv^2/R$. The actual velocity we find from energy considerations must be equal to or greater than v_r.

Just like the last problem, change in height times weight gives gravitational work, which will equal the final kinetic energy since we start from rest:

$$W = mg\Delta y = mg\left(h - 2R\right) = \Delta K = \frac{1}{2}mv_a^2 \tag{5}$$

$$v_a = \sqrt{2g\left(h - 2R\right)} \tag{6}$$

Comparing the actual and required velocities:

$$v_a \geqslant v_r \tag{7}$$
$$\sqrt{2g\left(h - 2R\right)} \geqslant \sqrt{gR} \tag{8}$$
$$2h - 4R \geqslant R \tag{9}$$
$$h \geqslant \frac{5}{2}R \tag{10}$$

Thus, we need to be a bit higher than the loop itself, not because of the energy required, but because we need to have a sufficiently high speed to stay on the circular loop.