University of Alabama
Department of Physics and Astronomy

PH 105 LeClair Summer 2012

Problem Set 8

Instructions:

1. Answer all questions below. All questions have equal weight.
2. Due Wed 13 June 2012 at the start of lecture.
3. You may collaborate, but everyone must turn in their own work.

1. A 12.0 g wad of sticky clay is hurled horizontally at a 100 g wooden block initially at rest on a horizontal surface. The clay sticks to the block. After impact, the block slides 7.50 m before coming to rest. If the coefficient of friction between the block and the surface is 0.650, what was the speed of the clay immediately before impact?

Solution: The initial collision is purely inelastic, but we can use conservation of momentum. Let the block have mass M and the clay mass m with initial velocity v_i, and let the velocity of wad and clay after the collision be v_f. Conservation of momentum before and after the collision gives:

$$m v_i = (m+M)\, v_f \tag{1}$$

$$v_f = \frac{m}{M+m} v_i \tag{2}$$

Conservation of energy now relates the kinetic energy of the block plus clay after the collision to the work done against friction. The normal force on the clay and block together is just $N = (m+M)g$, as you can easily verify, so the work done against friction over the stopping distance x is just $\mu_k N x$. Thus,

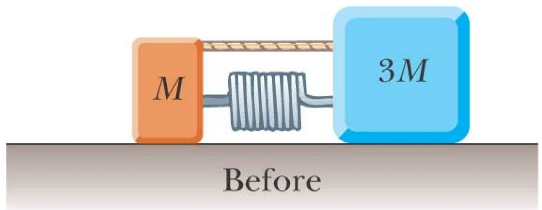
$$\frac{1}{2}(m+M)\, v_f^2 = \mu_k (m+M)\, g x \tag{3}$$

$$x = \frac{v_f^2}{2\mu_k g} = \frac{m_1^2 v_i^2}{2\mu_k g\,(m+M)^2} \tag{4}$$

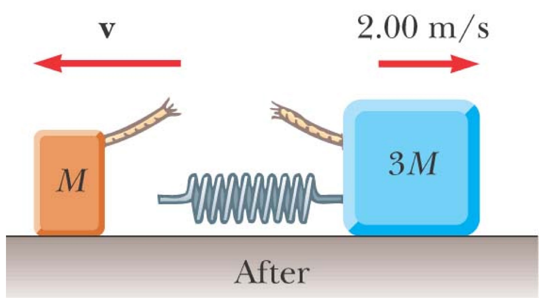
$$v_i = \left(\frac{m+M}{m}\right)\sqrt{2\mu_k g x} \approx 91.2\,\text{m/s} \tag{5}$$

2. Two blocks of masses M and $3M$ are placed on a horizontal, frictionless surface. A light spring is attached to one of them, and the blocks are pushed together with the spring between them (see below). A cord initially holding the blocks together is burned; after this, the block of mass $3M$ moves to the right and the block of mass M moves to the left.

If the original elastic potential energy in the spring is 6 J, find the speed of the block of mass M.



(a)



(b)

Solution: Conservation of energy relates the initial potential energy in the spring to the final potential energy of the two blocks. Conservation of momentum before and after the string is cut adds another constraint. Let the block of mass M have velocity v_1 after the collision, and the block of mass $3M$ have velocity v_o after the collision.

$$E_i = \frac{1}{2}kx^2 = E_f = \frac{1}{2}Mv_1^2 + \frac{1}{2}(3M)\,v_o^2 \tag{6}$$

$$p_i = 0 = p_f = 3mv_o - mv_1 \qquad \Longrightarrow \qquad v_1 = 3v_o \tag{7}$$

In fact, we can see now that since the figure gives you $v_o = 2\,\text{m/s}$, we need not proceed further; $v_1 = 6\,\text{m/s}$. Were you not given v_o, plugging the momentum conservation result into the energy conservation equation would yield, after some algebra,

$$v_o = \sqrt{\frac{kx^2}{12M}} \tag{8}$$

3. A 1500 kg car is traveling at 15 m/s and hits a 2500 kg SUV head-on. The SUV was at rest before the collision, but left in neutral, and the car and SUV are stuck together after the collision (purely inelastic collision).

What is the final velocity of the car and SUV stuck together?

Solution: Conservation of momentum is enough to solve this one. Let the mass of the car be m_1 and the mass of the SUV be m_2, the initial velocity of the car v_i, and the final velocity of both together v_f. Then

$$p_i = m_1 v_i = p_f = (m_1 + m_2)\, v_f \tag{9}$$

$$v_f = \left(\frac{m_1}{m_1 + m_2}\right) v_i \approx 5.6\,\text{m/s} \tag{10}$$