Example force problems

1. An advertisement claims that a particular automobile can "stop on a dime." What net force would actually be necessary to stop a 850 kg automobile traveling initially at 45.0 km/h in a distance equal to the diameter of a dime, which is 1.8 cm. *Hint: watch the units!*

Solution: We know the initial velocity $v_o = 45\,\text{km/h} = 12.5\,\text{m/s}$, the final velocity $v_f = 0$, and the distance over which the car accelerated, $\Delta x = 0.018\,\text{m}$. This is enough to get us the net acceleration, which is then enough to get us the net force using Newton's second law. First the acceleration:

$$v_f^2 = v_o^2 + 2a\Delta x \tag{1}$$

$$a = -\frac{v_o^2}{2\Delta x} \tag{2}$$

The minus sign reminds us that the acceleration is in the direction opposite Δx. If this is the net acceleration, the net force causing it must follow $F = ma$, so the net force is in magnitude (i.e., we don't care about the sign)

$$|F_{\text{net}}| = ma = \frac{mv_o^2}{2\Delta x} \approx 3.7 \times 10^6\,\text{N} \approx 415\,\text{tons} \tag{3}$$

A scary, unsurvivable amount of force, equivalent to pulling about 440 g's (where 50 g's generally means serious injury or death).

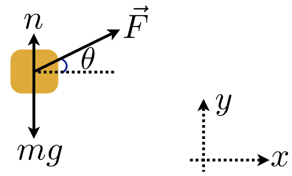
2. A block of mass $m = 5.00\,\text{kg}$ is pulled along a horizontal frictionless floor by a cord that exerts a force of magnitude $F = 12.0\,\text{N}$ at an angle of $65°$ with respect to horizontal. **(a)** What is the magnitude of the block's acceleration? **(b)** The force magnitude F is slowly increased. What is its value just before the block is lifted off the floor?

Solution: **Given:** A block pulled along a frictionless floor by a force making an angle θ with the horizontal.

Find: The block's acceleration, the maximum force before the block leaves the floor, and the block's acceleration at that point.

Sketch: Let the x and y axes be horizontal and vertical, respectively. We have only the block's weight, the normal force, and the applied force.
Relevant equations: We need only Newton's second law and geometry.

Symbolic solution: Along the vertical direction, a force balance must give zero for the block to



remain on the floor. This immediately yields the normal force.

$$\sum F_y = F_n - mg + F\sin\theta = 0 \qquad \Longrightarrow \qquad F_n = mg - F\sin\theta \tag{4}$$

A horizontal force balance gives us the acceleration:

$$\sum F_x = F\cos\theta = ma_x \qquad \Longrightarrow \qquad a_x = \frac{F}{m}\cos\theta \tag{5}$$

If the magnitude of the force is increased, **the block will leave the floor as the normal force becomes zero:**

$$F_n = mg - F\sin\theta = 0 \qquad \Longrightarrow \qquad F = \frac{mg}{\sin\theta} \tag{6}$$

At that point, its acceleration will be

$$a_x = \frac{F}{m}\cos\theta = \frac{g}{\tan\theta} \tag{7}$$

Numeric solution: Using the numbers given, the initial acceleration is

$$a_x = \frac{F}{m}\cos\theta = \frac{12.0\,\mathrm{N}}{5.00\,\mathrm{kg}}\cos 65^\circ \approx 1.01\,\mathrm{m/s^2} \tag{8}$$

At the point the block is about to leave the floor, the required force is

$$F = \frac{mg}{\sin\theta} = \frac{(5.00\,\mathrm{kg})\left(9.81\,\mathrm{m/s^2}\right)}{\sin 65^\circ} \approx 54\,\mathrm{N} \tag{9}$$

3. NOTE: you should draw free-body diagram for a mass on an inclined plane to show that $a = -g\sin\theta$. They should know this, but need to see the vector components work out.

A block is projected up a frictionless inclined plane with an initial speed of $v_o = 2.50\,\mathrm{m/s}$. The angle of incline is $\theta = 17.0^\circ$. (a) How far up the plane does the block go? (b) How long does it take to get there? (c) What is its speed when it gets back to the bottom?

Solution: By considering the forces involved, we know that the acceleration of the block, pointing down the ramp, is $-g\sin\theta$. If it is to travel a distance d up the ramp given an initial velocity v_i, and reach final velocity $v_f = 0$, then:

$$\begin{align} v_f^2 - v_i^2 &= 2ad = -2gd\sin\theta \tag{10} \\ d &= \frac{v_i^2}{2g\sin\theta} \approx 1.09\,\mathrm{m} \tag{11} \end{align}$$

Since we know the acceleration and initial velocity, we can find the time readily.

$$\begin{align} v(t) &= v_i + at = v_i - gt\sin\theta = v_f = 0 \tag{12} \\ t &= \frac{v_i}{g\sin\theta} \approx 0.87\,\mathrm{s} \tag{13} \end{align}$$

What is the speed at the bottom? Same as it was on the way up (since we have no friction). We can verify that, noting that moving down the ramp the acceleration is now $a = +g\sin\theta$, and the mass moves through distance d starting from rest:

$$\begin{align} v_f^2 - v_i^2 &= 2gd\sin\theta \tag{14} \\ v_f^2 &= 2gd\sin\theta = 2g\left(\frac{v_i^2}{2g\sin\theta}\right)\sin\theta = v_i^2 \tag{15} \\ v_f &= |v_i| \tag{16} \end{align}$$

4. In the figure below, three ballot boxes are connected by cords, one of which wraps over a pulley having negligible friction on its axle and negligible mass. The three masses are $m_a = 30.0\,\mathrm{kg}$, $m_b = 40.0\,\mathrm{kg}$, and $m_c = 10.0\,\mathrm{kg}$. When the assembly is released from rest, **(a)** what is the tension in the cord connecting B and C, and **(b)** how far does A move in the first $0.250\,\mathrm{s}$ (assuming it does not reach the pulley)? The table may be assumed to be frictionless.

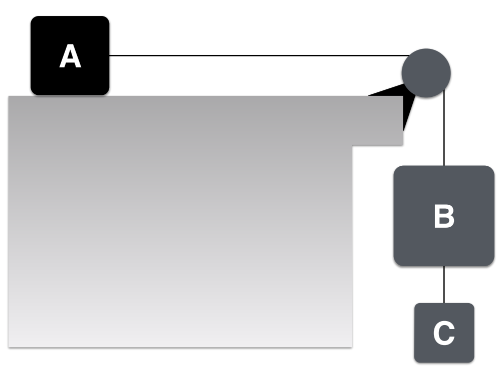


Figure 1: *Three boxes connected by cords, one of which wraps over a pulley.*

Solution: Let the tension in the cord connecting B and C be T_{bc}, and the tension in the cord connecting B and A be T_{ba}. Mass C has only two forces acting on it: T_{bc} and its weight $m_c g$. Clearly the acceleration is downward, in the same direction as the weight and opposite the tension.

$$T_{bc} - m_c g = -m_c a \tag{17}$$

Mass A has only one force acting on it, the tension T_{ab}, giving

$$T_{ab} = m_a a \tag{18}$$

This is not quite enough information. However, since B and C are connected together, we may treat them, from the point of view of the upper cord, as a single mass $(m_b + m_c)$ connected to mass A. There are two forces acting on B and C connected together: their weight, and the tension T_{ab}. Thus,

$$T_{ab} - (m_b + m_c)\,g = -\,(m_b + m_c)\,a \tag{19}$$

Since we already know $T_{ab} = m_a a$,

$$m_a a - (m_b + m_c)\,g = -\,(m_b + m_c)\,a \tag{20}$$

$$a = \left[\frac{m_b + m_c}{m_a + m_b + m_c}\right] g = \frac{5}{8} g \approx 6.13\,\mathrm{m/s^2} \tag{21}$$

The desired tension is readily found now, since $T_{bc} = m_c\,(g - a)$

$$T_{bc} = m_c g - m_c \left[\frac{m_b + m_c}{m_a + m_b + m_c}\right] g = \left[\frac{m_c m_a + m_c m_b + m_c^2 - m_c m_b - m_c^2}{m_a + m_b + m_c}\right] \tag{22}$$

$$T_{bc} = g\left[\frac{m_c m_a}{m_a + m_b + m_c}\right] \approx 36.8\,\mathrm{N} \tag{23}$$

Given an acceleration a, the distance traveled in time t is readily found.

$$\Delta x = \frac{1}{2} a t^2 \approx 0.192\,m \tag{24}$$

5. A projectile is launched with initial velocity $\vec{\mathbf{v}}_i$ from the start of a ramp, with the ramp making an angle φ with respect to the horizontal. The projectile is launched with an angle $\theta > \varphi$ with respect to the horizontal. At what position along the ramp does the projectile land?

Solution: **Find:** The point at which a projectile launched from the base of a ramp inclined at angle φ hits the ramp.

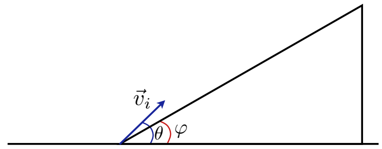


Figure 2: *A projectile is launched onto a ramp.*

Given: The projectile's launch speed and angle, the ramp angle.

Sketch: Let the origin be at the projectile's launch position, with the x and y axes of a cartesian coordinate system aligned as shown below.

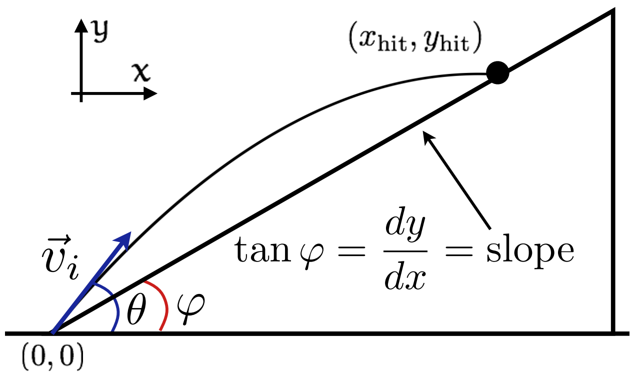


Figure 3: *Where does the projectile hit the ramp?*

Thus ramp begins at position $(0, 0)$, and the projectile is launched from $(0, 0)$. We seek the intersection of the projectile's trajectory with the surface of the ramp at position $(x_{\text{hit}}, y_{\text{hit}})$, subject to the condition that $y_{\text{hit}} \geq 0$, *i.e.,* the projectile actually reaches the ramp.

Relevant equations: We have already derived the trajectory $y(x)$ for a projectile launched from the origin:

$$y_p = x \tan\theta - \frac{gx^2}{2|\vec{\mathbf{v}}_i|^2 \cos^2\theta} \tag{25}$$

The ramp itself can be described by a simple line. We know the slope m of the ramp is $m = \Delta x/\Delta y = \tan\varphi$, and we know it intersects the point $(x_o, y_o) = (0, 0)$. This is sufficient to derive an equation of the line describing the ramp's surface, $y_r(x)$, using point-slope form:

$$y_r - y_o = m\,(x - x_o)$$
$$y_r = (\tan\varphi)\,x$$

The distance l the projectile goes along the ramp surface is found simply from $(x_{\text{hit}}, y_{\text{hit}})$ or x_{hit} and φ:

$$l = \sqrt{x_{\text{hit}}^2 + y_{\text{hit}}^2} \qquad \text{or} \qquad l = \frac{x}{\cos\varphi} \tag{26}$$

We also need a sanity condition to check that the projectile actually hits the ramp, which means we require $y_{\text{hit}} > 0$ and $x_{\text{hit}} > 0$. Finally, the point of intersection must occur when $y_r = y_p \equiv y_{\text{hit}}$.

Symbolic solution: We need only impose the condition $y_r = y_p$ to begin our solution. The resulting x value is the x_{hit} we desire.

$$y_r = x\tan\varphi = y_p = x\tan\theta - \frac{gx^2}{2|\vec{\mathbf{v}}_i|^2\cos^2\theta} \qquad \text{note} \quad x = l\cos\varphi \tag{27}$$

$$l\cos\varphi\tan\varphi = l\cos\varphi\tan\theta - \frac{gl^2\cos^2\varphi}{2|\vec{\mathbf{v}}_i|^2\cos^2\theta} \tag{28}$$

$$\tan\varphi = \tan\theta - \frac{gl\cos\varphi}{2|\vec{\mathbf{v}}_i|^2\cos^2\theta} \tag{29}$$

$$l = \frac{2|\vec{\mathbf{v}}_i|^2\cos^2\theta}{g\cos\varphi}\,(\tan\theta - \tan\varphi) = \frac{2|\vec{\mathbf{v}}_i|^2\cos\theta}{g\cos\varphi}\,(\sin\theta - \tan\varphi\cos\theta) \tag{30}$$

$$l = \frac{2|\vec{\mathbf{v}}_i|^2\cos\theta}{g\cos^2\varphi}\,(\sin\theta\cos\varphi - \sin\varphi\cos\theta) = \frac{2|\vec{\mathbf{v}}_i|^2\cos\theta\sin(\theta - \varphi)}{g\cos^2\varphi} \tag{31}$$

$$\tag{32}$$

As we should expect, the distance up the ramp depends on the relative angle between the ramp and launch, $\theta - \varphi$. We could have found this result a bit quicker if we had noted the identity (which we basically just derived).

$$\tan\theta - \tan\varphi = \frac{\sin(\theta - \varphi)}{\cos\theta\cos\varphi} \tag{33}$$

6. A 3.00 kg object is moving in a plane, with its x and y coordinates in meters given by $x(t) = 5t^2 - 1$ and $y(t) = 3t^3 + 2$, where t is in seconds. What is the magnitude of the net force acting on this object at $t = 2.00\,\text{s}$?

Solution: If we can get the acceleration, we can get the force. Since we have the position as a function of time, finding the components of acceleration is no big deal:

$$a_x = \frac{d^2x}{dt^2} = 10 \tag{34}$$

$$a_y = \frac{d^2y}{dt^2} = 18t \tag{35}$$

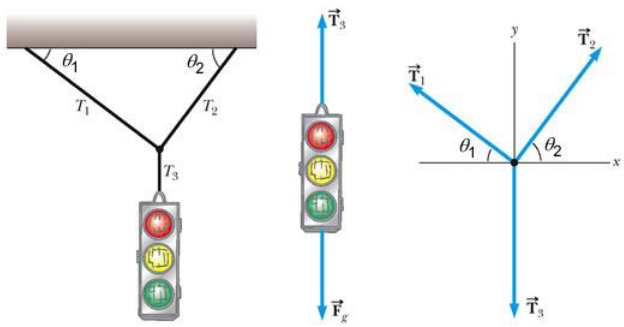
The *magnitude* of the force depends on the magnitude of the acceleration, so we'd better find it:

$$|\vec{\mathbf{a}}| = \sqrt{a_x^2 + a_y^2} = \sqrt{10^2 + 18^2t^2} \tag{36}$$

Newton's second law tells us that mass times acceleration is the net force, so:

$$|\vec{\mathbf{F}}| = m|\vec{\mathbf{a}}| = m\sqrt{10^2 + 18^2t^2} \approx 112\,\mathrm{N} \tag{37}$$

7. A traffic light weighing $mg = 123\,\mathrm{N}$ hangs from a cable tied to two other cables fastened to a support, as in the figure below. The upper cables make angles of $\theta_1 = 40°$ and $\theta_2 = 50°$ with the horizontal. Find the magnitudes of $\vec{\mathbf{T}}_1$, $\vec{\mathbf{T}}_2$, and $\vec{\mathbf{T}}_3$.



Solution: The cable T_3 has to support the traffic light's entire weight, so we must have $T_3 = 123\,\mathrm{N}$. A free-body diagram at the point where the three cables meet gives us horizontal and vertical forces, which much each sum to zero if the traffic light is to stay put.

$$\text{horizontal:} \qquad \sum F_x = T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0 \tag{38}$$

$$\text{vertical:} \qquad \sum F_y = T_1 \sin\theta_1 + T_2 \sin\theta_2 - T_3 = 0 \tag{39}$$

Two equations and two unknowns. Let's solve the first for T_1 an plug it in the second:

$$\text{from 38:} \qquad T_2 = T_1 \frac{\cos\theta_1}{\cos\theta_2} \tag{40}$$

$$\text{plug into 39:} \qquad 0 = T_1 \sin\theta_1 + \left(T_1 \frac{\cos\theta_1}{\cos\theta_2}\right) \sin\theta_2 - T_3 \tag{41}$$

Solving for T_1, since we know T_3 already,

$$T_3 = T_1 \left(\sin\theta_1 + \cos\theta_1 \tan\theta_2\right) \tag{42}$$

$$T_1 = \frac{T_3}{\sin\theta_1 + \cos\theta_1 \tan\theta_2} \approx 79.06\,\mathrm{N} \tag{43}$$

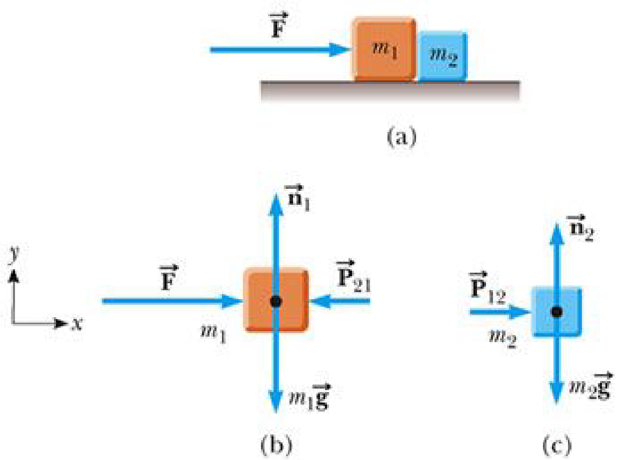
Using 40, we can find T_2 now:

$$T_2 = T_1 \frac{\cos\theta_1}{\cos\theta_2} = T_3 \frac{\cos\theta_1}{\sin\theta_1\cos\theta_2 + \cos\theta_1\cos\theta_2\tan\theta_2} = T_3 \frac{\cos\theta_1}{\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2} \tag{44}$$

$$= \frac{T_3}{\tan\theta_1\cos\theta_2 + \sin\theta_2} \approx 94.2\,\mathrm{N} \tag{45}$$

Note T_2 and T_1 are both less than T_3, as they must be.

8. Two blocks of masses m_1 and m_2 ($m_1 > m_2$) are placed in contact on a horizontal, frictionless surface, as shown in the figure below. A constant horizontal force of $\vec{\mathbf{F}} = 115\mathrm{N}$ is applied to m_1 as shown. Find the magnitude of the acceleration of the two blocks.

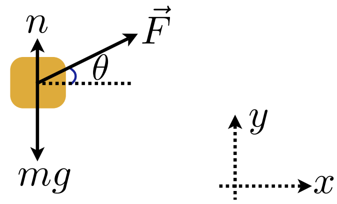


Solution: The blocks move together, so this is equivalent to a single mass $m_1 + m_2$ moving under the influence of F. The acceleration is thus $a = F/(m_1 + m_2)$.

9. *HRW 6.30* A toy chest and its contents have a combined weight of 180 N. The coefficient of static friction between toy chest and floor is $\mu_s = 0.42$. A child attempts to move the chest across the floor by pulling on an attached rope. (a) If the rope makes an angle of $\theta = 42°$ with the horizontal, what is the magnitude of the force $\vec{\mathbf{F}}$ that the child must exert on the rope to pull the chest on the verge of moving? (b) Write an expression for the magnitude F required to pull the chest on the verge of moving as a function of the angle θ. Determine the value of θ for which F is (c) a minimum and (d) a maximum magnitude.

Solution: Here's a quick free-body diagram:

Along the y direction, the net force must be zero for the block to stay on the floor:



$$\sum F_y = N + F\sin\theta - mg = 0 \tag{46}$$
$$N = mg - F\sin\theta \tag{47}$$

Along the horizontal direction, we want the box to be on the verge of moving, so the point where acceleration is still zero:

$$\sum F_x = F\cos\theta - \mu_s N = F\cos\theta - \mu_s mg + \mu_s F\sin\theta = 0 \tag{48}$$
$$F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} \approx 74° \tag{49}$$

The force will be minimum when $dF/d\theta\!=\!0$:

$$\frac{d}{d\theta}\left(\frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}\right) = \mu_s mg\left[\frac{\sin\theta - \mu_s\cos\theta}{(\cos\theta + \mu\sin\theta)^2}\right] = 0 \tag{50}$$

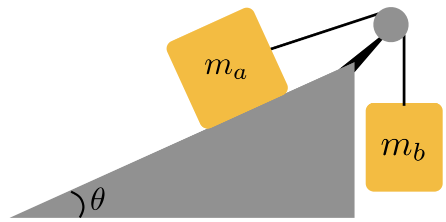
The pre-factor and denominator are irrelevant; the equation above will only be zero and the force at a minimum if $\sin\theta\!=\!\mu_s\cos\theta$, or when $\tan\theta\!=\!\mu_s$. In this case, that implies $\theta\!\approx\!22.8°$ for minimum force.

What about the maximum? That is easier: it takes the most force when you are at 90° - you can apply as much force as you want at that point, and the box will never move sideways . . .

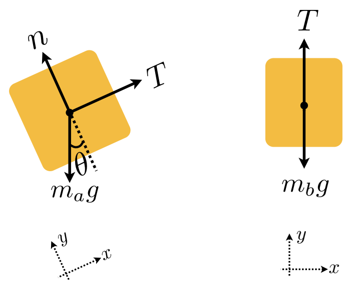
10. *HRW 5.57* A block of mass $m_a\!=\!3.70$ kg on a frictionless plane inclined at an angle $\theta\!=\!30.0°$ is connected by a cord over a massless, frictionless pulley to a second block of mass $m_b\!=\!2.30$ kg (figure below). What are (a) the magnitude of the acceleration of each block, (b) the direction of the acceleration of the hanging block, and (c) the tension in the cord?

Solution:
Find: The tension in a cord connecting two blocks and the system's acceleration, with one block on a frictionless incline and the second on a flat surface with coefficient of kinetic friction μ_k.

Given: The mass of both blocks, the coefficient of friction for the block on the flat surface, and the angle of incline for the ramp.



Sketch: We need free-body diagrams for each mass. Note the axis definitions for each mass. We are not sure which way the masses will accelerate yet, but we will assume that the hanging mass m_b will fall, meaning the acceleration is in the $+x$ direction for mass m_a according to the sketches below. If we chose incorrectly, the acceleration will come out negative to let us know.



Since the rope is presumably taut the entire time of interest, the acceleration is the same for both blocks. For the same reason, the tension applied to both blocks is the same.

Relevant equations: Newton's second law and geometry will suffice. Along the y direction for m_a and along the x direction for m_b, the forces must sum to zero. Along x direction for m_a and y direction for m_b, the forces must give the acceleration for each mass.

$$\sum F_y = 0 \tag{51}$$

$$\sum F_x = ma_x \tag{52}$$

Symbolic solution: First consider mass A. The free body diagram above yields the following, noting that the acceleration will be purely along the x direction:

$$\sum F_y = n - m_a g \cos\theta = 0 \tag{53}$$

$$\sum F_x = T - m_a g \sin\theta = m_a a \qquad \Longrightarrow \qquad a = \frac{T}{m_a} - g\sin\theta \tag{54}$$

For mass B, things are simpler, but we should keep in mind that the acceleration is along $-y$:

$$\sum F_y = T - m_b g = -m_b a \qquad \Longrightarrow \qquad T = m_b(g-a) \tag{55}$$
$$\sum F_x = 0 \tag{56}$$

We have enough to find the acceleration in terms of known quantities now:

$$a = \frac{T}{m_a} - g\sin\theta = \frac{m_b}{m_a}(g-a) - g\sin\theta \tag{57}$$
$$a\left(1+\frac{m_b}{m_a}\right) = g\left(\frac{m_b}{m_a} - \sin\theta\right) \tag{58}$$
$$a = g\left(\frac{m_b - m_a\sin\theta}{m_a+m_b}\right) \tag{59}$$

Given this acceleration, the tension is found readily from $T = m_b(g-a)$.

$$T = m_b g - m_b a = m_b g - m_b g\left(\frac{m_b - m_a\sin\theta}{m_a+m_b}\right) \tag{60}$$
$$= g\left[\frac{m_a m_b + m_b^2}{m_a+m_b} - \frac{m_b^2 + m_a m_b\sin\theta}{m_a+m_b}\right] = g\left[\frac{m_a m_b}{m_a+m_b}\right](1+\sin\theta) \tag{61}$$

Numeric solution: Given $m_a = 3.7\,\mathrm{kg}$, $m_b = 2.3\,\mathrm{kg}$, and $\theta = 30°$, the tension is

$$T = g\left[\frac{m_a m_b}{m_a+m_b}\right](1+\sin\theta) \approx 20.9\,\mathrm{N} \tag{62}$$

and for either block the acceleration is:

$$a = g\left(\frac{m_b - m_a\sin\theta}{m_a+m_b}\right) \approx 0.736\,\mathrm{m/s^2} \tag{63}$$

Since the acceleration is positive, we were correct in our original assumption - mass m_b moves downward, and mass m_a to the right.