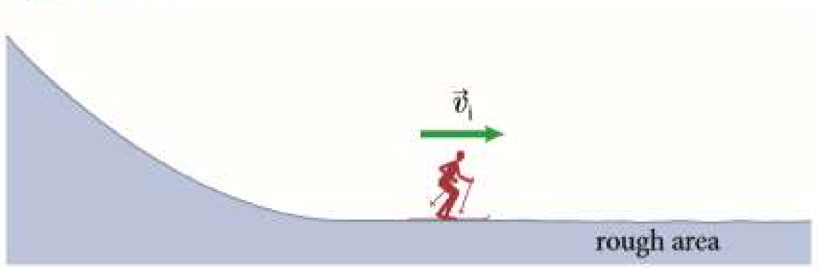
35. In Figure P8.35, a 50-kg skier heads down a slope, reaching a speed of 35 km/h. She then slides across a horizontal snow field but hits a rough area. Assume the snow before the rough area is so slippery that you can ignore any friction between the skis and the snow. If the frictional force exerted by the snow in the rough area is 40 N, how far across the rough area does the skier travel before stopping? ••

Figure P8.35



8.35. Call the direction of motion the $+x$ axis. We find the acceleration of the skier using $a_x = \frac{1}{m}\sum F_x = \frac{F^f_{\text{ice skier }x}}{m}$.

Because the acceleration is constant we can use a kinematic equation to write $\Delta x = \frac{v^2_{\text{s}x,\text{f}} - v^2_{\text{s}x,\text{i}}}{2a_x} = \frac{m(v^2_{\text{s}x,\text{f}} - v^2_{\text{s}x,\text{i}})}{2F^f_{\text{ice skier }x}}$.

Because we want to know at what point the skier stops, we insert $v_{\text{s}x,\text{f}} = 0$ and find $\Delta x = \frac{(50\text{ kg})((0) - (9.72\text{ m/s})^2)}{2(-40\text{ N})} = 59\text{ m}$.

48. Picture an object in free fall. If the leading face of the object (which means the face closest to the ground) has a large surface area, air resistance becomes important. For low speeds, the force due to air resistance may be approximated as proportional to the object's velocity according to the expression $\vec{F}^{a} = -b\vec{v}$, where b is a number that depends on the shape and volume of the object and on the density of the air. (*a*) Show that, for an object of inertia m, there is a limiting speed (called the *terminal speed*) given by $v_t = mg/b$. (*b*) Show that, if the object is released from rest, its speed as a function of time is given by

$$v(t) = v_t\left[1 - e^{-(b/m)t}\right].$$ 54.

(*c*) What are the units of m/b? Sketch a graph of v as a function of time, with the horizontal axis in units of multiples of m/b. ●●●

8.48. (*a*) Writing the sum of all forces in the y direction yields $\sum F_y = -mg + bv_y = ma_y$. Under the influence of gravity alone, the object would continue to speed up. But as it does so, the resistance from the air increases. This trade-off will continue until the acceleration is zero. At that time $mg = bv_y$, and because the acceleration is zero, the speed at this time will remain unchanged. We call it the terminal speed $v_t = mg/b$. (*b*) Starting from the sum of all forces on the object and rewriting the acceleration as the time derivative of the velocity yields $m\dot{v}(t) = -(b/m)v(t) + mg$. This is an ordinary differential equation that has the general solution $v(t) = \alpha e^{-\lambda t} + \beta$, where α, β, and λ are constant to be determined. If the object is dropped, then $v(t=0) = 0$, which tells us that $\alpha = -\beta$. In the limit as $t \to \infty$ we know the speed has to approach the terminal speed. Hence $\beta = mg/b$ and $\alpha = -mg/b$. Finally, at the instant of release, there should not be any air resistance yet, such that the acceleration should simply be that due to gravity. Inserting the results for α and β and taking the time derivative yields

$$v(t) = \frac{mg}{b}(e^{-\lambda t} - 1)$$

$$\dot{v}(t=0) = -\lambda\frac{mg}{b} = -g$$

$$\Rightarrow \lambda = \frac{m}{b}$$

Finally, putting all constants back in, one obtains $v(t) = (mg/b)(e^{-(b/m)t}\ \ 1)$. (*c*) The units of m/b are seconds.