University of Alabama
Department of Physics and Astronomy

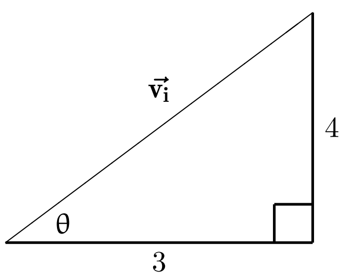
PH 105 LeClair Summer 2012

Quiz 2

1. A projectile is launched on level ground with a velocity of $\vec{v_i} = 3.00\,\hat{\imath} + 4.00\,\hat{\jmath}$ in units of m/s. What is the launch angle θ_i, relative to the x (or $\hat{\imath}$) axis?

- ● 53.1°
- □ 36.9°
- □ 45.0°
- □ 69.3°

First, what does this velocity vector actually specify? Our definition of unit vectors means that it specifies a vector which is formed by moving 3 units in the $\hat{\imath}$ (or x) direction, and then 4 units in the $\hat{\jmath}$ (or y) direction: as shown below:



The angle of launch is the angle between $\vec{v_i}$ and the x axis. If we call that angle θ:

$$\tan\theta = \frac{y}{x} = \frac{4}{3} \qquad \Rightarrow \qquad \theta = \arctan\frac{4}{3} = 53.1^\circ$$

2. How far in the x (or $\hat{\imath}$) direction does the projectile in question 1 travel before impact? Recall

$$h = y_{max} = \frac{{v_i}^2\sin^2\theta_i}{2g} \qquad\qquad R = x_f - x_i = \frac{{v_i}^2\sin 2\theta_i}{g}$$

- □ 0.816 m
- □ 1.57 m
- ● 2.45 m

- ☐ 0.882 m

In this case, we can directly use the formula for range given above. We just need to know the *magnitude* of the initial velocity (the speed), and the initial angle found in question 1.

First, we find v_i:

$$v_i = |\vec{v}_i| = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = 5$$

Now that we know v_i and θ_i, we can find the range:

$$R = x_f - x_i = \frac{{v_i}^2 \sin 2\theta_i}{g} = \frac{5^2 \sin(2 \cdot 53.1°)}{9.8} = 2.45\,\text{m}$$

3. A particle has a trajectory that follows $\vec{r} = (3.2\,\hat{\imath} + 1.5\,\hat{\jmath})t + \frac{1}{2}(4.9\,\hat{\imath} + 9.8\,\hat{\jmath})t^2$, where t is in seconds, and r is in meters. What is the velocity in the y (or $\hat{\jmath}$) direction at $t = 17.2\,\text{s}$? Note

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\,\hat{\imath} + \frac{dy}{dt}\,\hat{\jmath} = v_x\,\hat{\imath} + v_y\,\hat{\jmath}$$

- ☐ 258 m/s
- ☐ 137 m/s
- ☐ 312 m/s
- ● 170 m/s

We know that we can write any position vector $\vec{r}$ as $\vec{r} = \vec{x} + \vec{y} = x\,\hat{\imath} + y\,\hat{\jmath}$. Thus, first we can group all of the $\hat{\jmath}$ terms together and find an expression for y:

$$\vec{y} = y\,\hat{\jmath} = 1.5t\,\hat{\jmath} + \frac{1}{2}\left(9.8t^2\right)\hat{\jmath} = \left(1.5t + 4.9t^2\right)\hat{\jmath} \qquad \Rightarrow \qquad y = 1.5t + 4.9t^2$$

Given this expression for y, the velocity in the $\hat{\jmath}$ or y direction is just:

$$\vec{v}_y = v_y\,\hat{\jmath} = \frac{d}{dt}[y]\,\hat{\jmath} = (1.5 + 9.8t)\,\hat{\jmath} \qquad \Rightarrow \qquad v_y = (1.5 + 9.8t) = [1.5 + 9.8\,(17.2)] = 170$$

4. How far has the particle in question 3 traveled *in the x or $\hat{\imath}$ direction* from $t = 0$ to $t = 17.2$ sec?

- ☐ 2250 m
- ● 780 m

- ☐ 1480 m
- ☐ 2920 m

What we are really asking here is 'what is the difference in x position from $t = 0$ to $t = 17.2\,\text{sec}$'? To answer this, we first need the x position as a function of time, which can be found by grouping the $\hat{\imath}$ terms together like we did above:

$$\vec{x} = x\,\hat{\imath} = 3.2t\,\hat{\imath} + \frac{1}{2}\left(4.9t^2\right)\,\hat{\imath} = \left(3.2t + 2.45t^2\right)\,\hat{\imath} \qquad \Rightarrow \qquad x = 3.2t + 2.45t^2$$

Now we can easily calculate the change in x position between the two time points:

$$\Delta x = x(17.2) - x(0) = \left[3.2 \cdot (17.2) + 2.45 \cdot (17.2)^2\right] - \left[3.2 \cdot (0) + 2.45 \cdot (0)^2\right] = 780$$