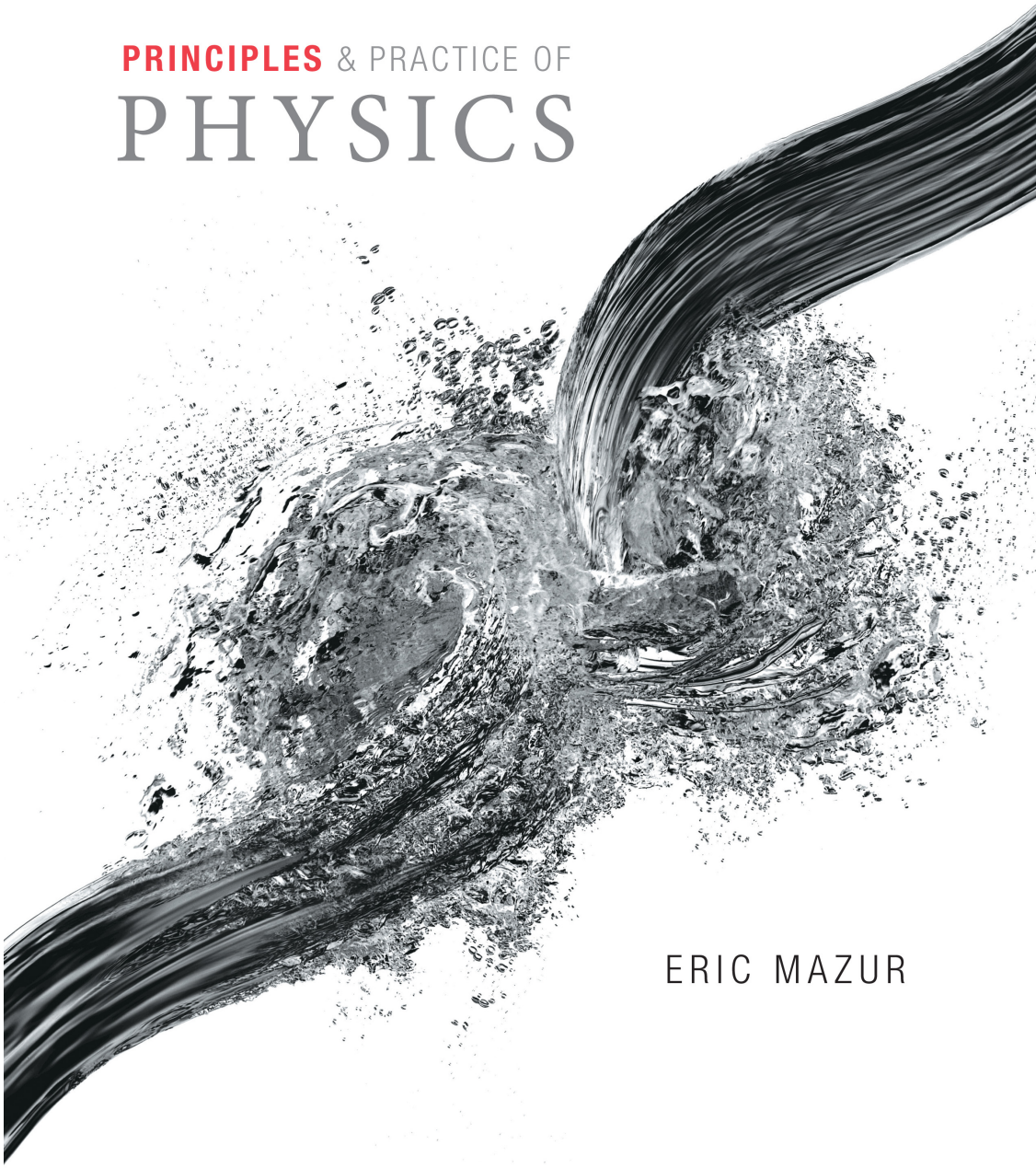
Chapter 2
Motion in One Dimension



MasteringPhysics, PackBack Answers

- You should be on both by now.
- MasteringPhysics problems?
  - Pearson publishing rep is here today
  - will end early so she can help you

- PackBack – should have email & be signed up
  - no email? check spam/promotions folders
  - webform to sign up now (link on MasteringPhysics)
  - $10 for Answers access
  - Book rental is ~$110

PackBack Answers

- Try to ask questions you are curious about
- Don't just use book discussion questions, ideally
  - unless that is what you are curious about …

- the volume of questions will be high
  - try to browse and see what has already been asked
  - provide answers & up/down vote as you browse

Labs

- first procedure – link on MasteringPhysics
- read ahead of time
- can print there (one per group!)

- I set the guidelines, but the TAs are in charge

Reading quiz

- Will normally go over it now …
- … but not this time

Chapter 2: Motion in One Dimension

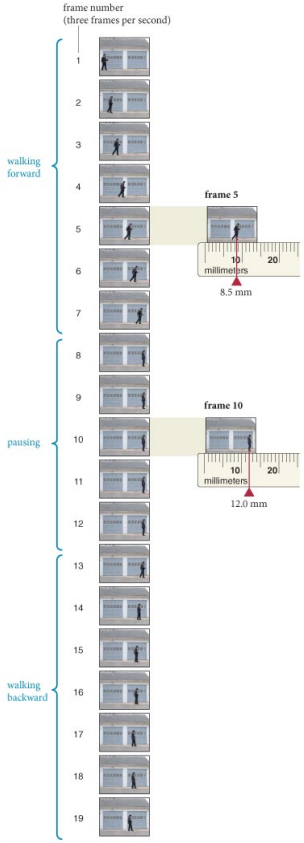
Describe motion in one dimension from both graphical and mathematical perspectives.

"kinematics"

Chapter 2 Preview

Looking Ahead: Kinematics, the language of motion

- The motion of an object can be observed by looking at it's location in individual frames of a film clip recorded at equally spaced times.



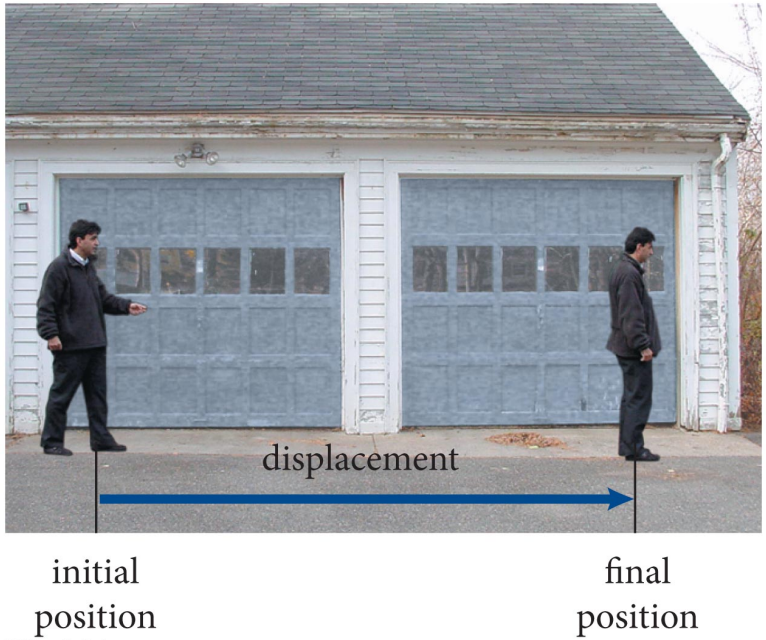
- You will learn about to describe the details of motion using concepts of position, displacement, speed, velocity, and acceleration.

Chapter 2 Preview

Looking Ahead: Scalars and vectors

- Both of the diagrams below imply that the object shown moves. However, only the second allows the direction of the movement to be determined.

(*a*)

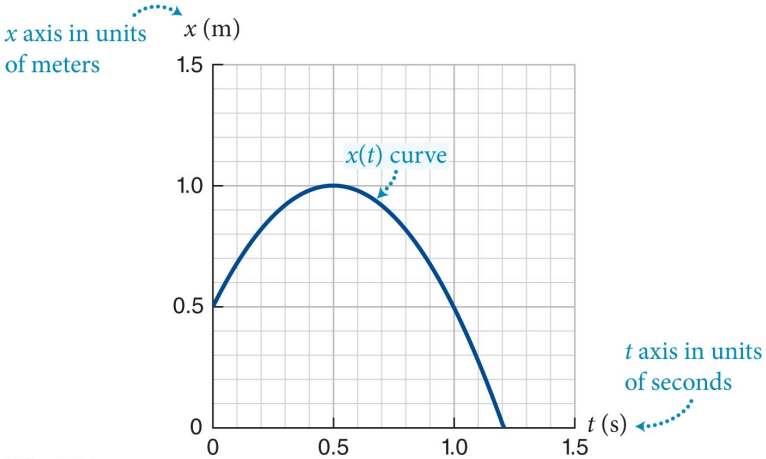


- All physical quantities studied by physicists can be classified as either scalars or as vectors.
- You will learn how distinguish scalars from vectors both geometrically and mathematically.

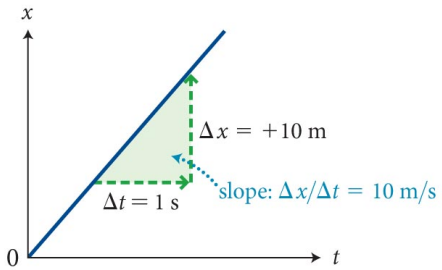
Chapter 2 Preview

Looking Ahead: Representations of motion

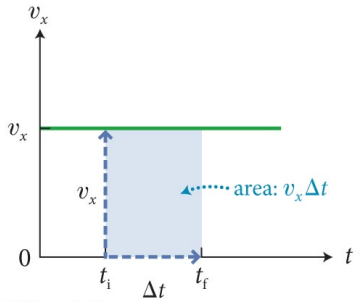
- The motion of an object can be represented in various graphical and mathematical ways.



(a) Position versus time: straight line with nonzero slope



(b) Velocity versus time: horizontal line



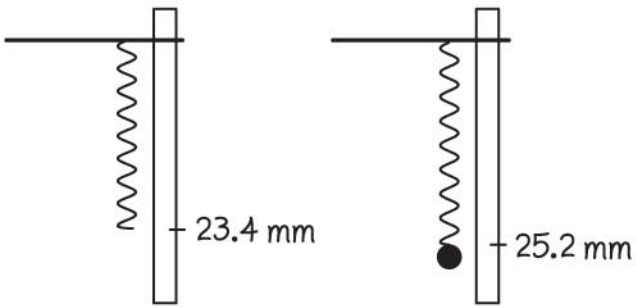
- You will learn how to correlate these various representations and how to determine the important kinematic quantities of position, displacement, speed, velocity, and acceleration from them.

Chapter 2 Preview

Looking Back: Representations in physics

- In Chapter 1 you learned how to use graphical and mathematical representations to describe the physics of a stretched spring.

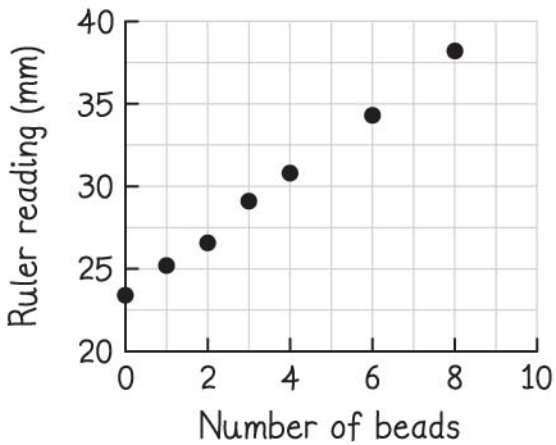
(a) Sketch



(b) Table

Number of beads	Ruler reading (mm)
0	23.4
1	25.2
2	26.5
3	29.1
4	30.8
6	34.3
8	38.2

(c) Graph



- You will generalize these methods in Chapter 2 to describe the physics of kinematics.

Concepts

Section 2.1: From reality to model

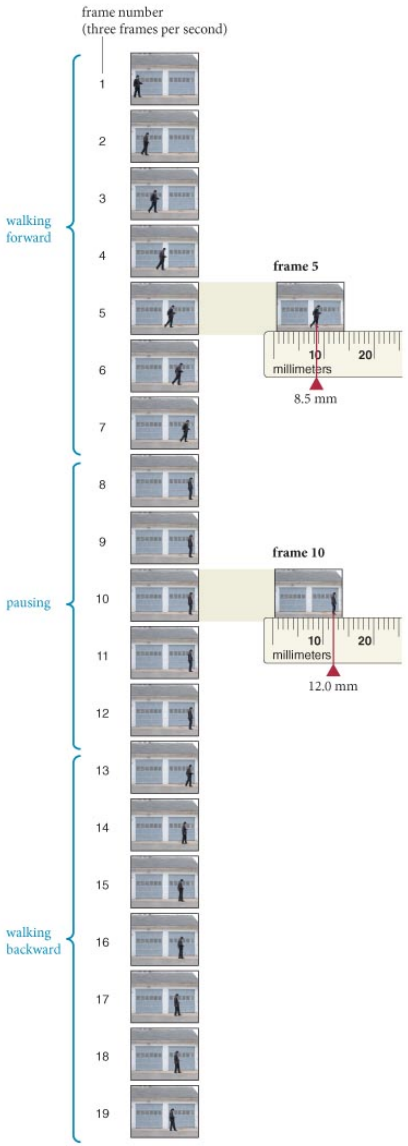
- Visualize the motion of objects in one dimension in several situations using the “frame sequence” diagram and by using motion graphs.
- Correlate the information about motion contained in “frame sequence” diagrams and motion graphs.
- Understand how a reference axis allows for the precise determination of physical information about motion.

Section 2.1: From reality to model

- The branch of physics that deals with the quantitative representation of motion is called **kinematics**.
- Kinematics provides a mathematical description of motions.
- It does not however, consider cause and effect of motion.

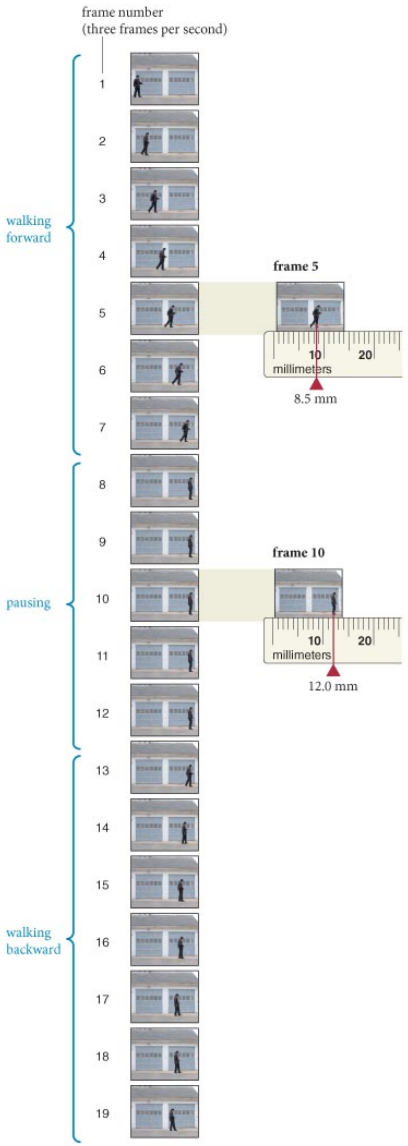
Section 2.1: From reality to model

- To analyze the motion of an object, we need to keep track of the object's position at different instants:
  - If the position does not change the object is at rest.
  - If the position changes the object is moving.



Section 2.1: From reality to model

- analyze the motion of a man walking by studying a film clip
- first need to establish
  - A **reference axis**—an imaginary straight line along the ground .
  - An **origin**—an arbitrarily chosen reference point: Choose the left edge of the frame in the figure.
- can now determine the position of the man in each frame measure from the origin.
- ***general problem solving:*** pick axes and origin



Section 2.1: From reality to model

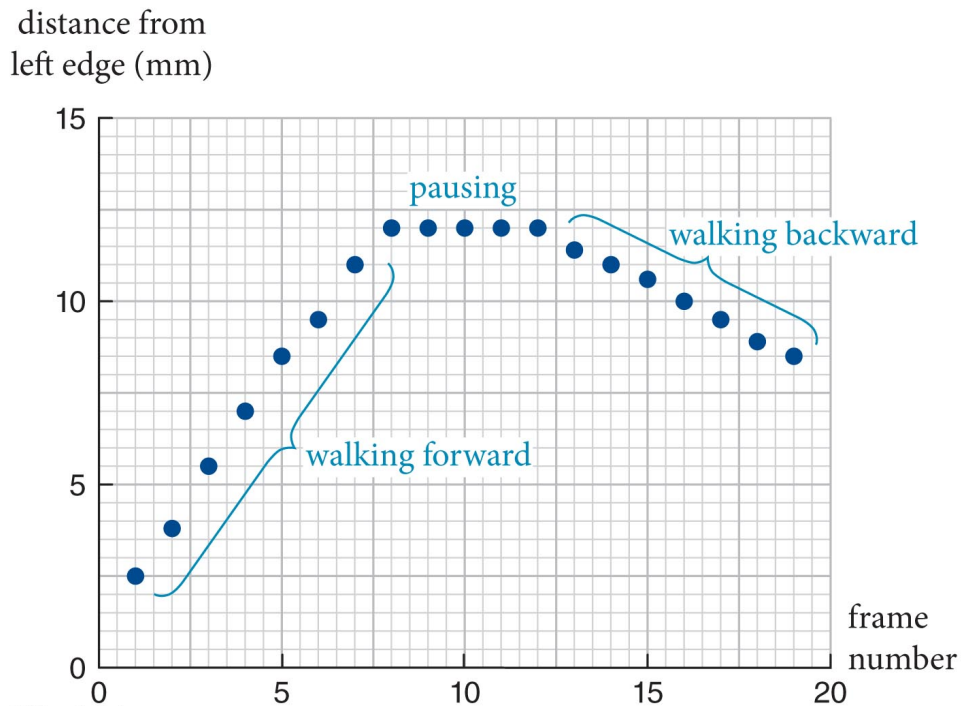
- Do this for every frame

Table 2.1 Position versus frame number from the film sequence in Figure 2.1

Frame number	Distance from left edge (mm)
1	2.5
2	3.8
3	5.5
4	7.0
5	8.5
6	9.5
7	11.0
8	12.0
9	12.0
10	12.0
11	12.0
12	12.0
13	11.4
14	11.0
15	10.6
16	10.0
17	9.5
18	8.9
19	8.5

Section 2.1: From reality to model

- The same information can be shown graphically:
  - The man's position relative to the origin is plotted along the vertical axis.
  - The horizontal axis represents the frame numbers.



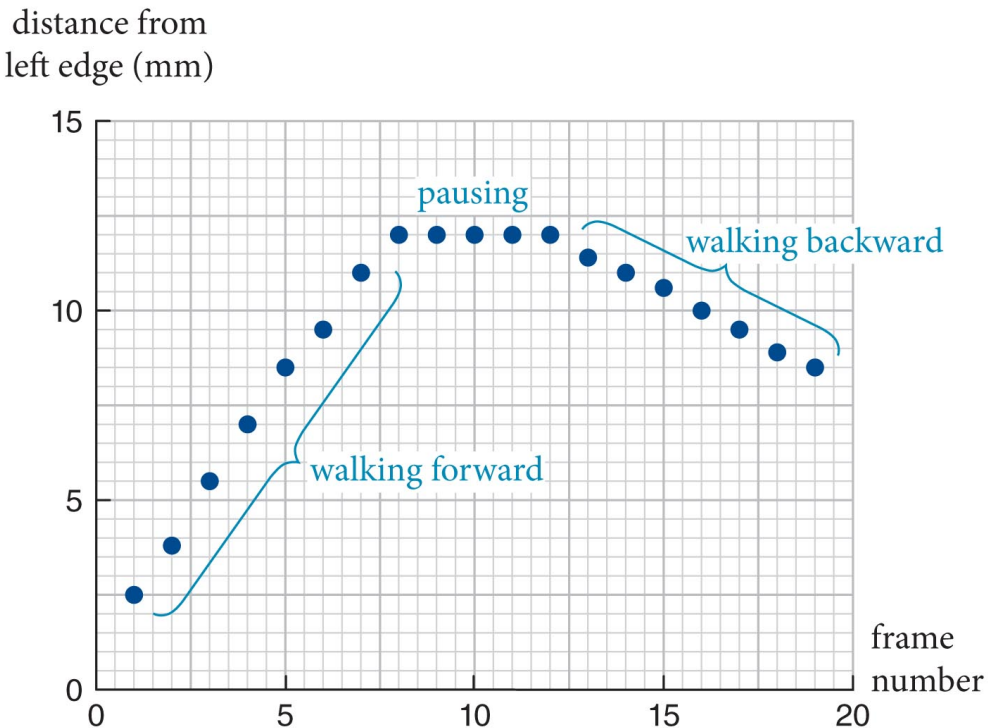
always label axes!

Checkpoint 2.2

(a) The data points for frames 8–12 are all at position 12.0 mm. What must be happening?

(*b*) The data points for frames 7 and 14 are aligned horizontally. What must be happening?

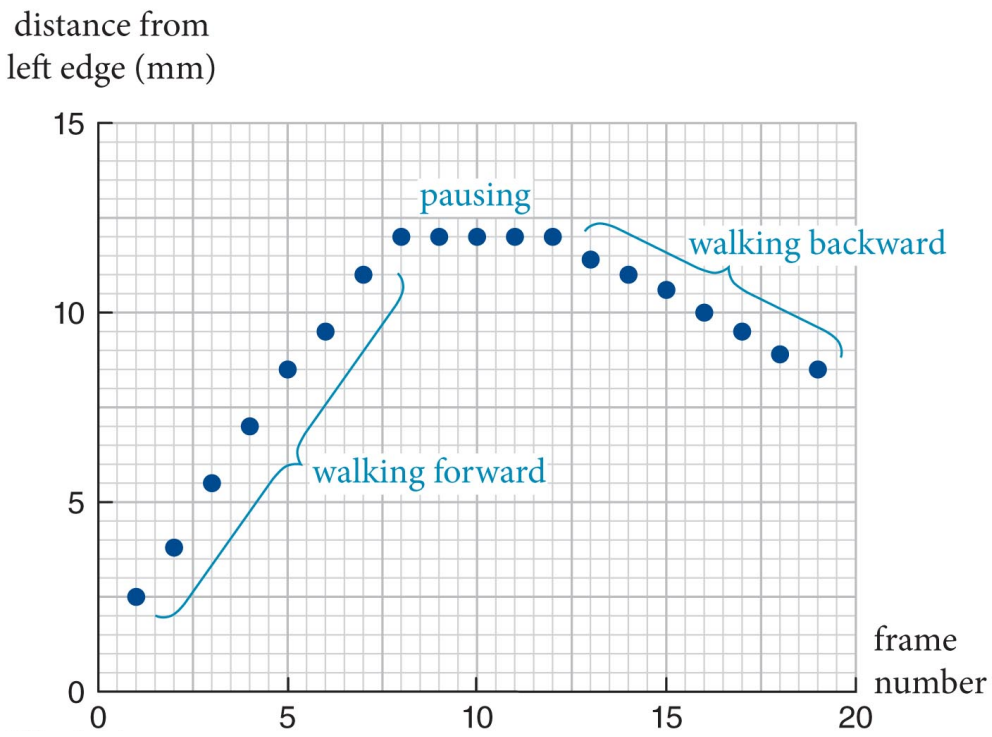
(*c*) What if two data points were aligned vertically? Could this happen?



Checkpoint 2.2

(a) The data points for frames 8–12 are all at position 12.0 mm.

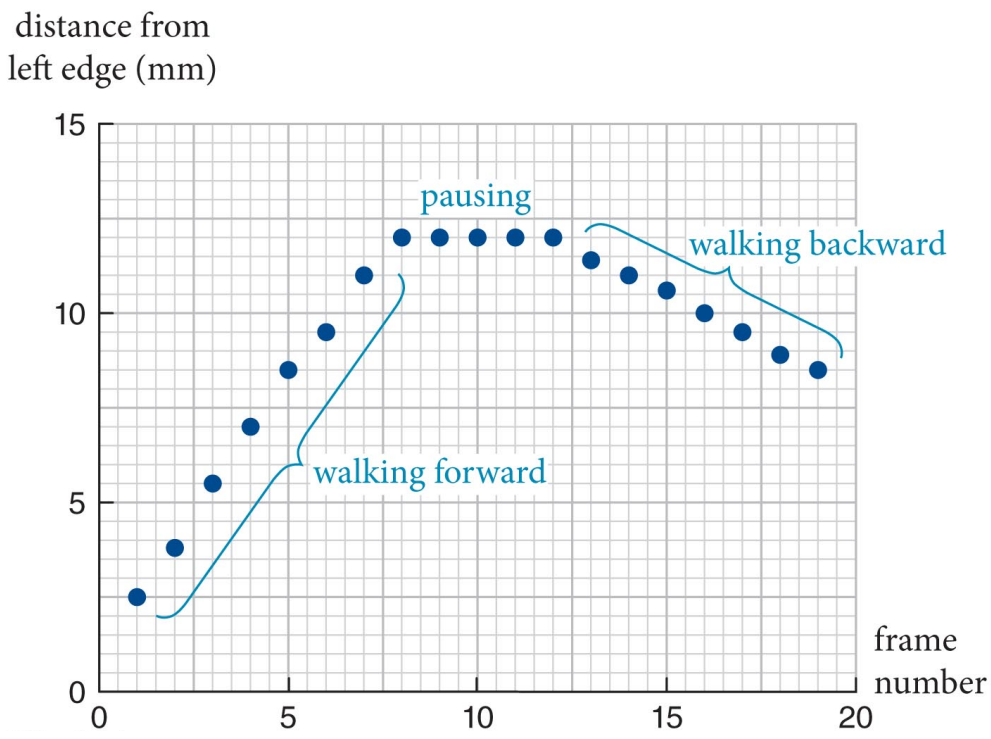
The man is not moving – position isn't changing



Checkpoint 2.2

(*b*) The data points for frames 7 and 14 are aligned horizontally.

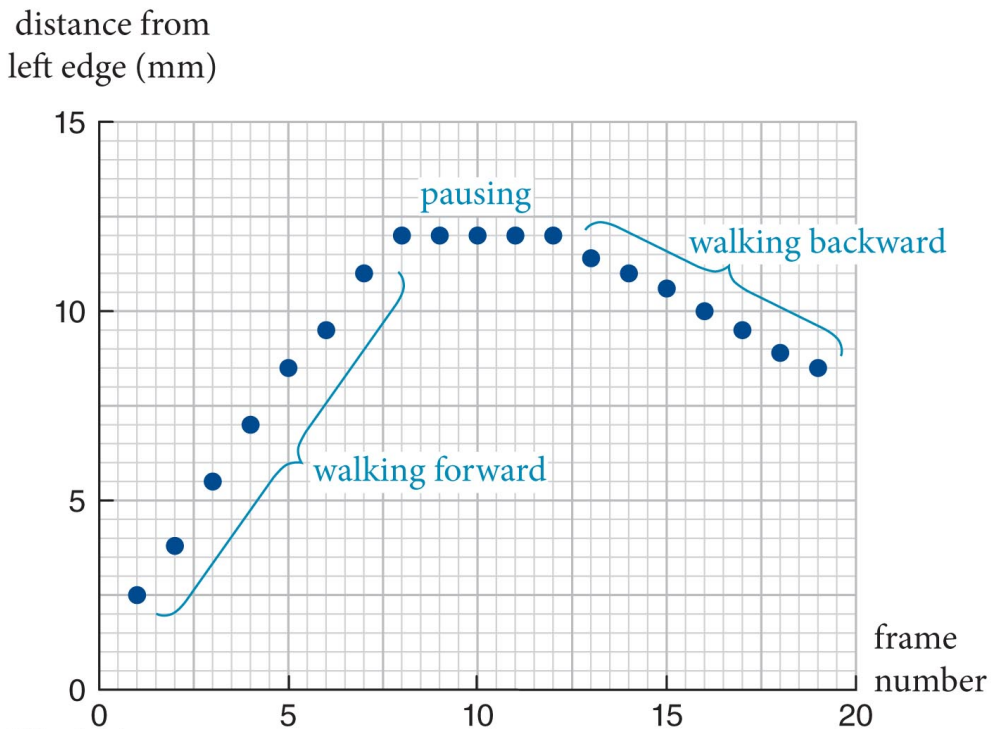
In frame 14 the man returned to the same position he had in frame 7



Checkpoint 2.2

(*c*) What if two data points were aligned vertically?

That would imply an object has two different positions at a single instant in time. This is not a thing that happens.



Section 2.2: Position and displacement

You will learn to

- Differentiate the concepts of position and displacement for motion in one dimension.
- Recognize the physical significance of the algebraic signs for position and displacement.
- Distinguish between the distance traveled by an object in one dimensional motion and its displacement.

Section 2.2: Position and displacement

- These are initial steps towards creating our first **model**:
  - A model is a simplified abstract representation of a real-world phenomenon.
  - What is really important in the video?
  - Developing models is one of the most important skills in physics.

- To make this model useful: calibrate the data points
  - need real-world distances and time intervals.

Section 2.2: Position and displacement

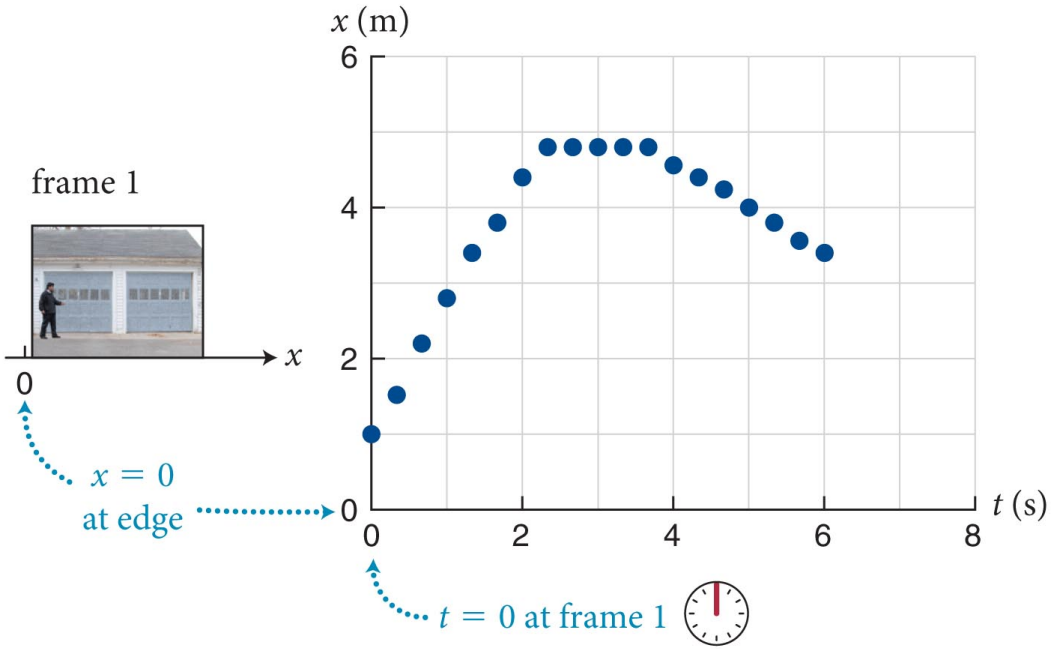
- To calibrate the axes in our video:
  - Put markers on the ground and a clock in the camera's field of view

  or

  - Use an object of known height already in the video and the frame rate of the camera.
    - E.g., 1.8 m for the height of the man and 30 frames per second for the frame rate

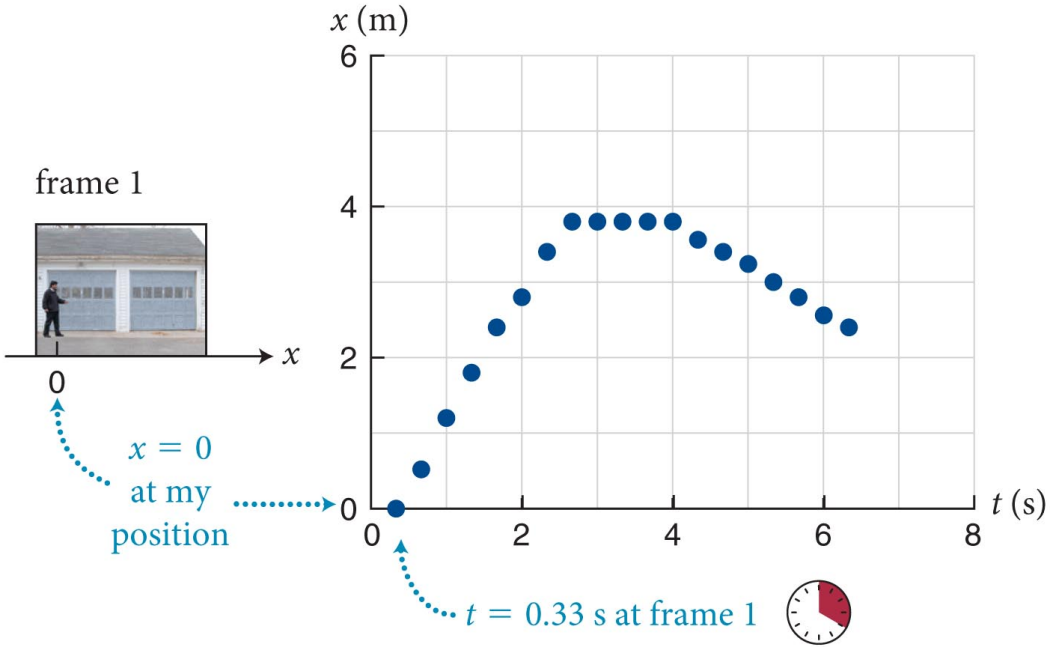
Section 2.2: Position and displacement

- The resulting calibrated data as a *position-versus-time* graph.
  - The vertical axis is position (x) in meters
  - The horizontal axis is time (t) in seconds.
  - The choice of origin is arbitrary (for x or t).



Section 2.2: Position and displacement

- Origin was arbitrarily chosen to be the left edge of frame.
- Suppose, now we choose the man's position in frame 1 to be the origin and the clock reading at frame 1 to be $t = 0.33$ s.
  - Doing so *does not* affect relative positions of data points as seen in the figure
  - It just shifts the whole thing

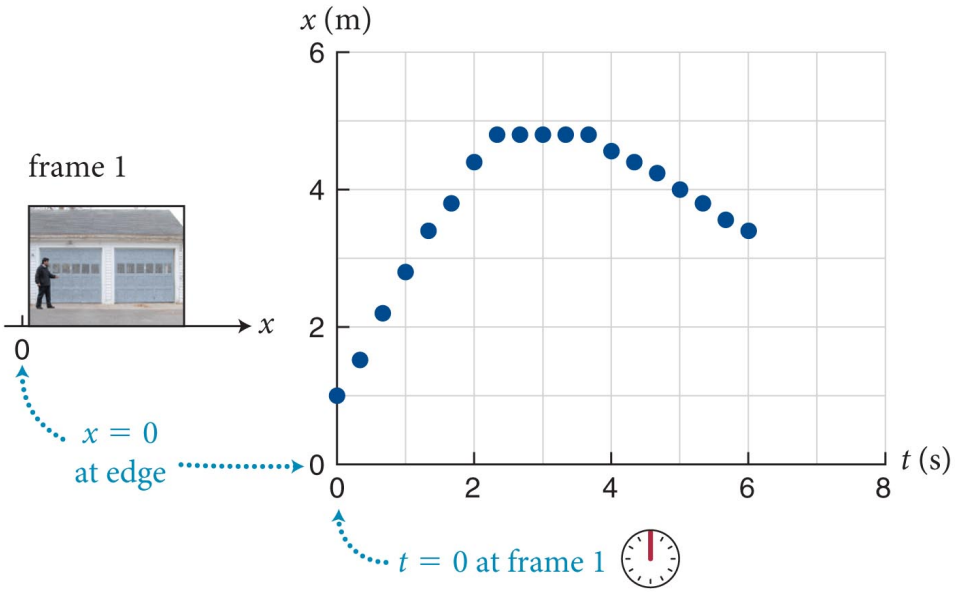


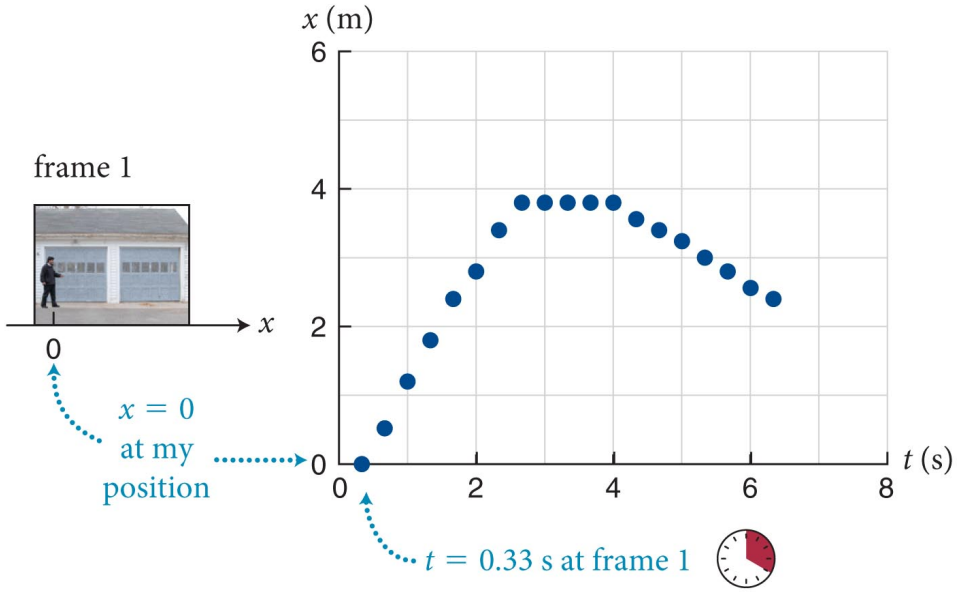
Checkpoint 2.4

The *absolute positions* are completely different in each graph.

The *relative positions* from one time to another are the same

Which do you think really matters?





the physics can't depend on arbitrary choices we make

QUESTION ….

An object goes from one point in space to another. After it arrives at its destination, its displacement is

1. either greater than or equal to
2. always greater than
3. always equal to
4. either smaller than or equal to
5. always smaller than
6. either smaller than or larger than

the distance it traveled.

We try to use words in very specific ways

An object goes from one point in space to another. After it arrives at its destination, its displacement is

1. either greater than or equal to
2. always greater than
3. always equal to
4. ✔ either smaller than or equal to
5. always smaller than
6. either smaller than or larger than

the distance it traveled.

Displacement

- The **arrow** pointing from the initial position of an object to its final position represents a physical quantity called the **displacement** of the object.
- Displacement *does not* depend on the choice of axis or origin.
- This is how you give directions – series of displacements

initial position

final position

Section 2.2: Position and displacement

- The x component of an object's displacement is the change in its x coordinate.
- Find by subtracting the initial x coordinate from the final x coordinate
- Independent of the choice of origin:
  - The x component of displacement is measured along some specific x axis.
  - The x component of displacement can be negative or positive.

Checkpoint 2.5

2.5 Suppose you walk in a straight line from a point P to a point Q, 2 m away from P, and then walk back along the same line to P.

(a) What is the x component of your displacement for the round trip?

(b) What distance did you travel during the round trip?

(c) Is *distance traveled* the same thing as the *x component of the displacement*?

Checkpoint 2.5

2.5 Suppose you walk in a straight line from a point P to a point Q, 2 m away from P, and then walk back along the same line to P.

(a) What is the x component of your displacement for the round trip?

Displacement is how far you went *in net*, not the length of your journey.

You didn't go anywhere in net – displacement is 0.

Checkpoint 2.5

2.5 Suppose you walk in a straight line from a point P to a point Q, 2 m away from P, and then walk back along the same line to P.

(b) What distance did you travel during the round trip?

You went nowhere in the end, but walked 4m to do it.

Checkpoint 2.5

2.5 Suppose you walk in a straight line from a point P to a point Q, 2 m away from P, and then walk back along the same line to P.

(c) Is *distance traveled* the same thing as the *x component of the displacement*?

No – displacement doesn't care about backtracking, but distance does!

Distance

- The **distance** traveled is the length covered by a moving object along the path of its motion.
- In contrast to the x component of displacement, distance traveled is always positive.

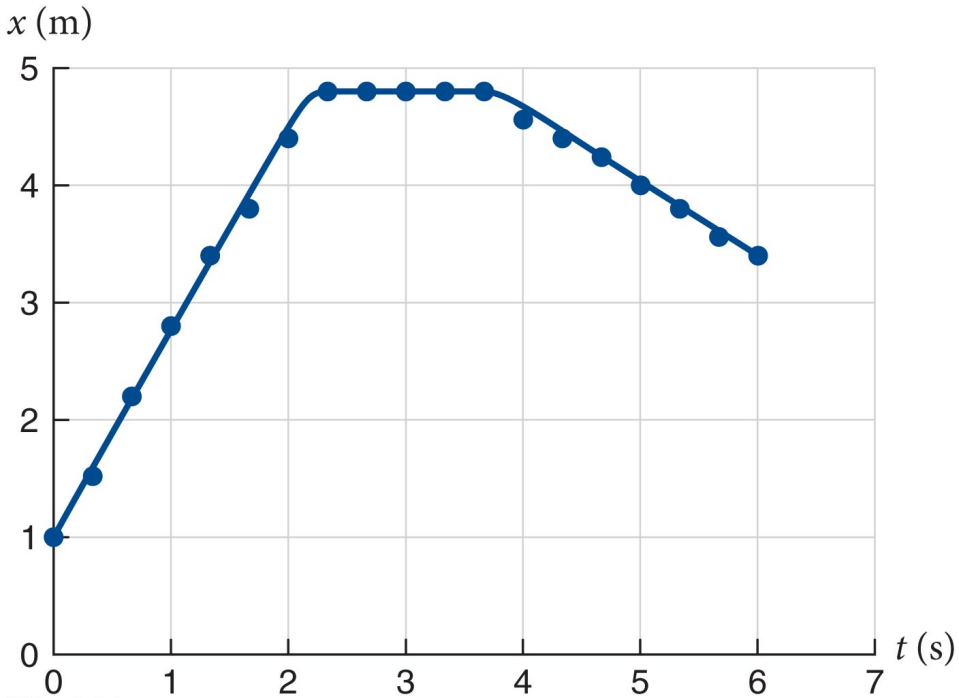
- Your *displacement* to drive to class might be 1mi, but you might drive 2mi **distance** to accomplish it.

Section 2.3: Representing motion

- Represent the motion of an object in several graphical and mathematical ways.
- Correlate these various representations.
- Determine the important kinematic quantities of position, displacement, speed, and velocity from graphical and mathematical representations.

Section 2.3: Representing motion

- A complete representation of the motion of the man should allow us to determine his positions between frames.
- This can be achieved by **interpolating** data points in the *position-versus-time* graph

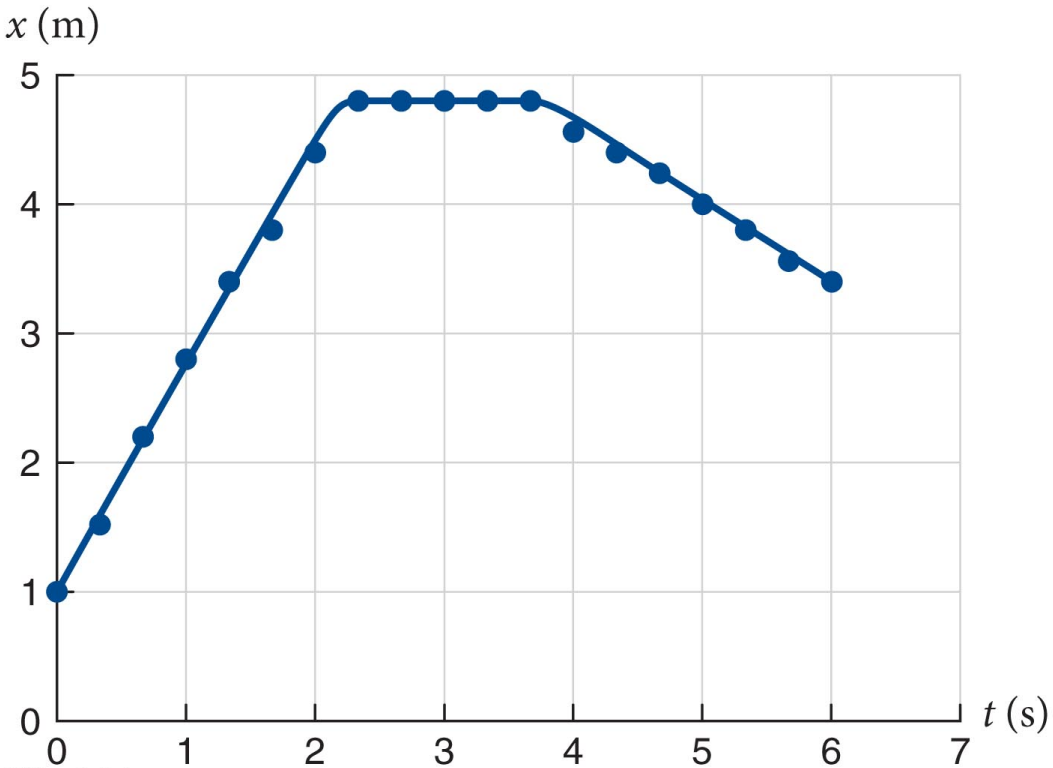


Checkpoint 2.6

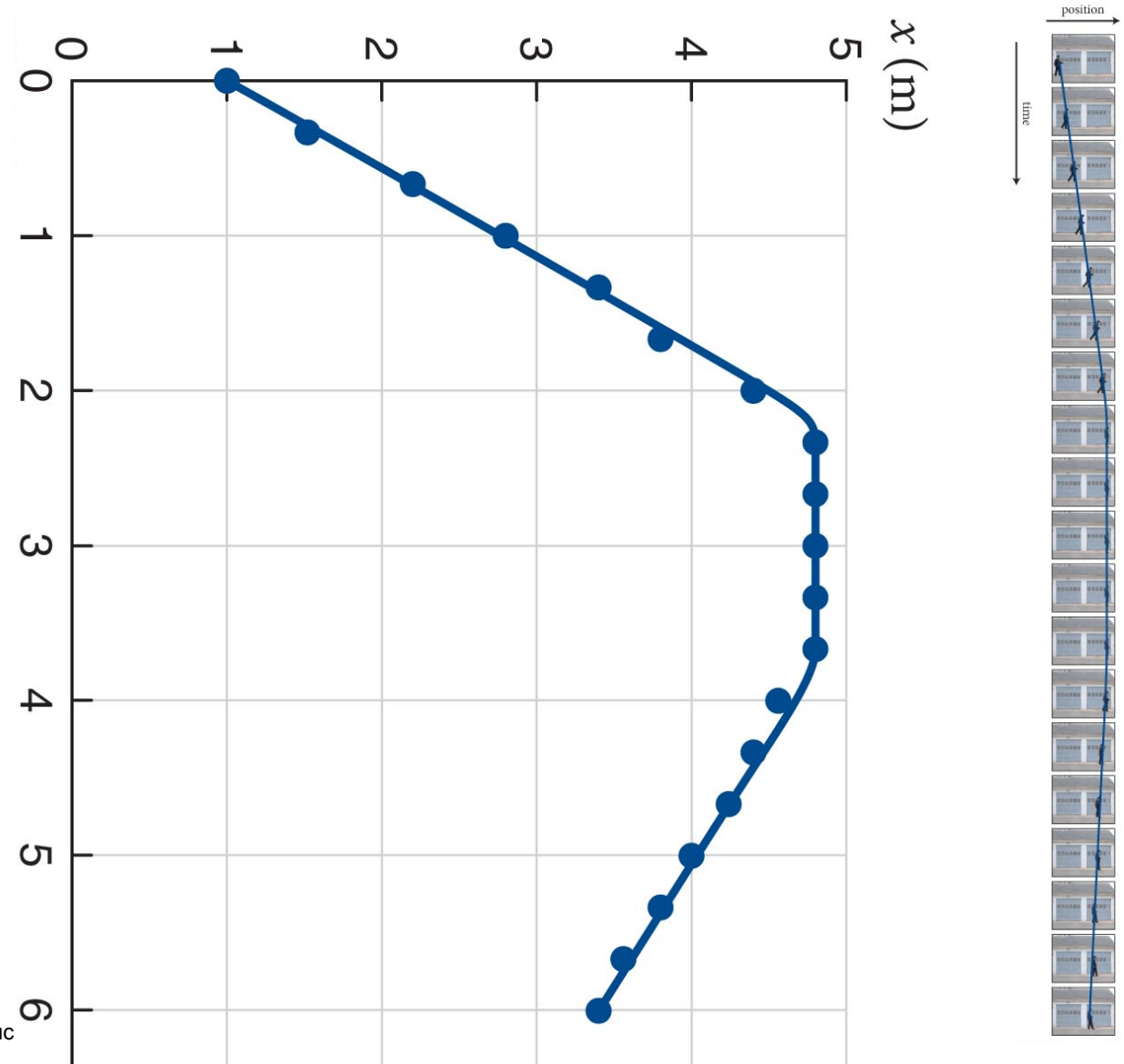
(*a*) How long did it take to go from $x = +1.0$ m to $x = +4.0$ m?

(*b*) From $x = +2.0$ m to $x = +3.0$ m?

(*c*) For how long the man at $x = +4.8$ m?

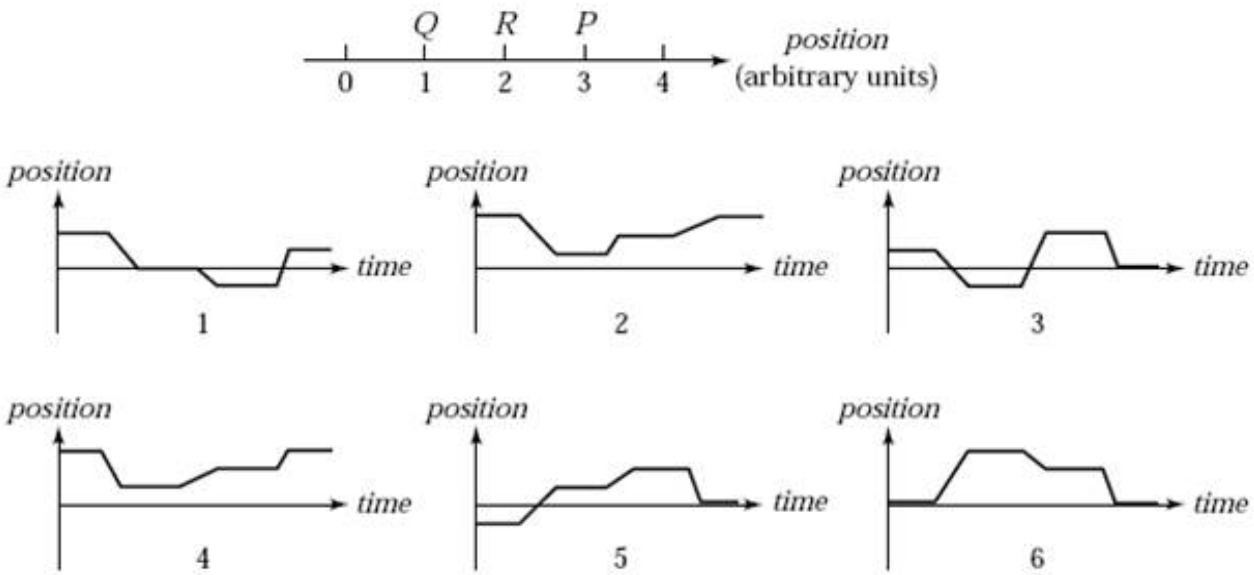


How the *position-versus-time* graph relates to the film clip



Section 2.3

A person initially at point P in the illustration stays there a moment. She then moves along the axis to Q and stays there a moment. She then runs quickly to R, stays there a moment, and then strolls slowly back to P. Which of the position-versus-time graphs below correctly represents this motion?



Section 2.3

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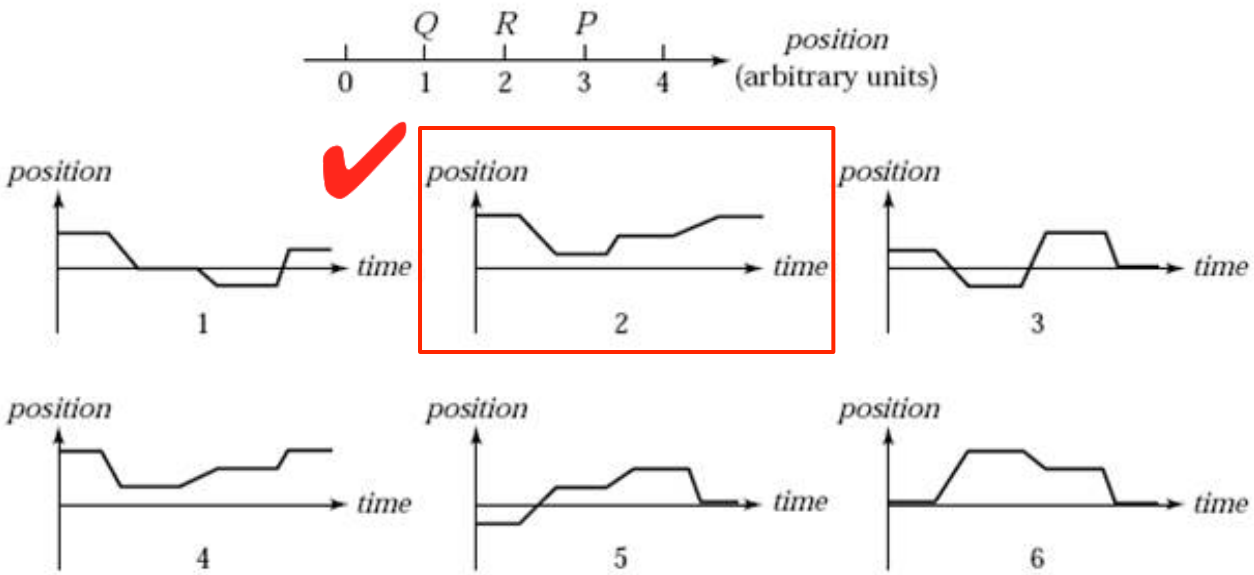
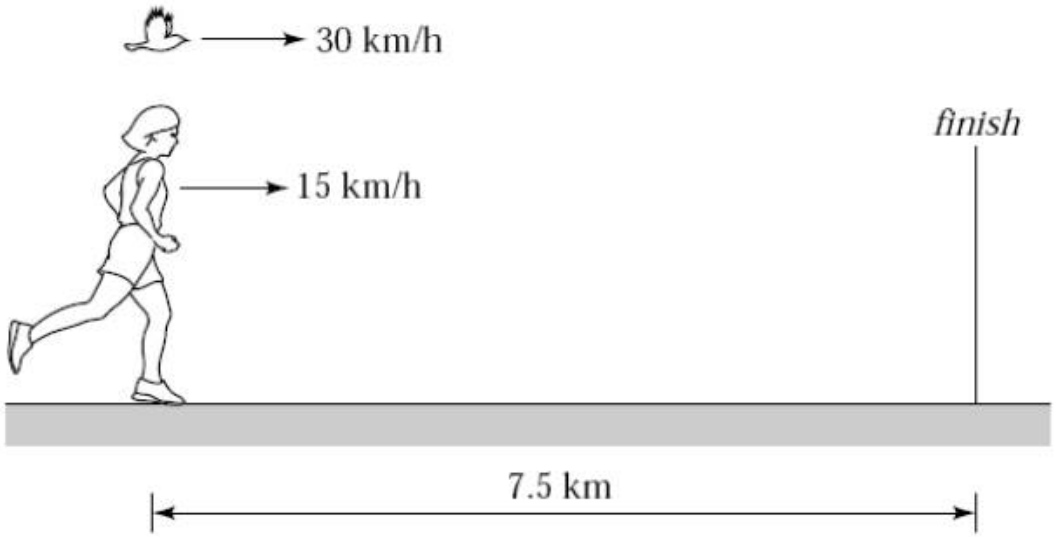


Section 2.3

A marathon runner runs at a steady 15 km/hr. When the runner is 7.5 km from the finish, a bird begins flying from the runner to the finish at 30 km/hr. When the bird reaches the finish line, it turns around and flies back to the runner, and then turns around again, repeating the back-and-forth trips until the runner reaches the finish line. How many kilometers does the bird travel?

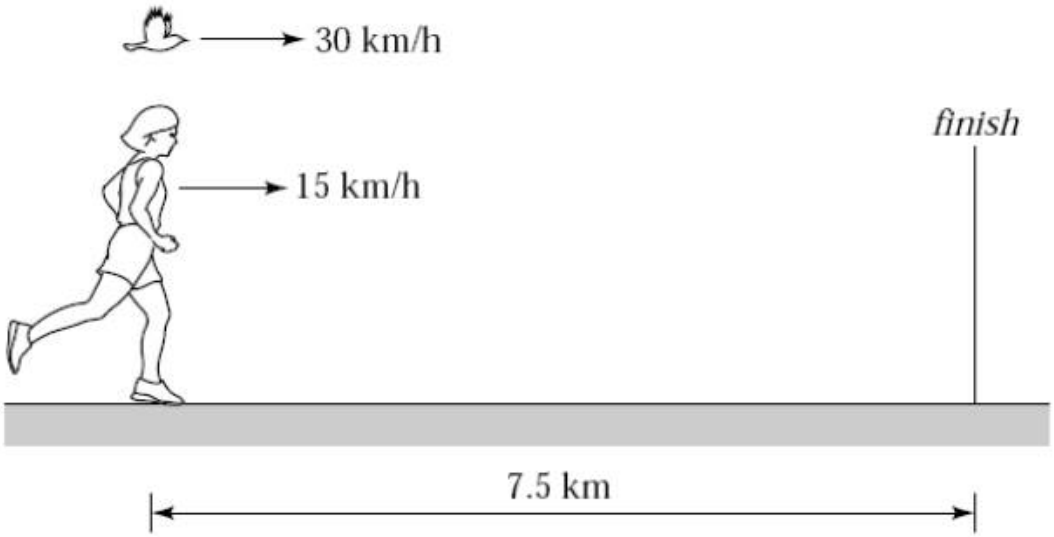
1. 10 km
2. 15 km
3. 20 km
4. 30 km



Section 2.3
Clicker Question 3

The runner takes half an hour to run 7.5km. The bird thus flies for half an hour, and covers (½ h)(30 km/h) = 15km

1. 10 km
2. ✔ 15 km
3. 20 km
4. 30 km



Example 2.1 Graphical representation

The curve on the next slide is a graphical representation of the motion of a certain object.

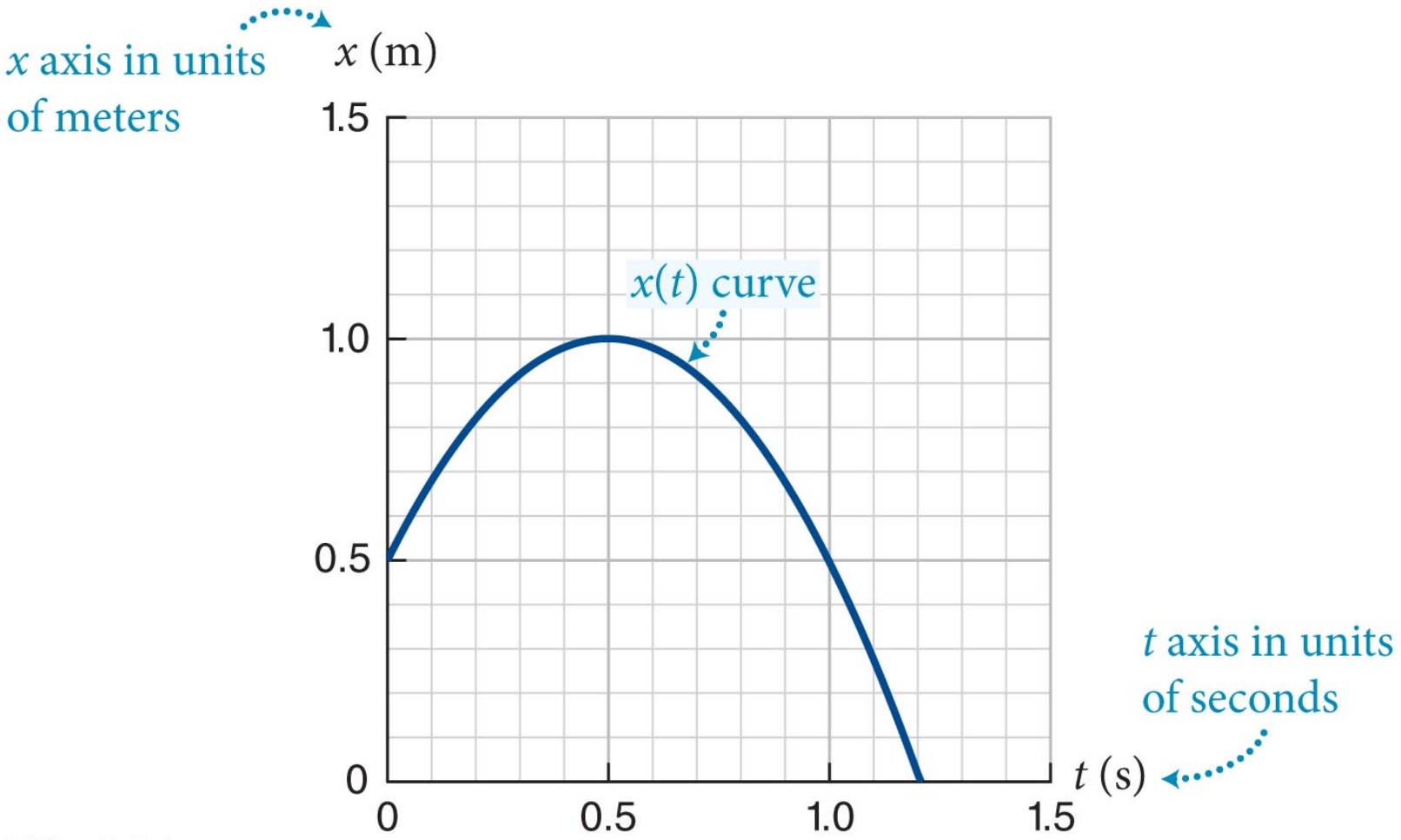
(a) What was the x coordinate of the object at $t = 0.50$ s?

(b) At what instant(s) did the object reach $x = +0.80$ m?

(c) What distance did the object travel between $t = 0.80$ s and $t = 1.2$ s?

(d) How long did it take for the object to move from $x = +1.0$ m to $x = 0$?

Section 2.3: Representing motion



Example 2.1 Graphical representation (cont.)

❶ GETTING STARTED

The problem boils down to reading off values from the graph.

Example 2.1 Graphical representation (cont.)

❷ DEVISE PLAN

To find the object's position x at a given instant t, draw a vertical line at the value on the t axis corresponding to the given instant in time and read off the value of x at which that line intercepts the curve in the graph.

Likewise, to find the instant t at which the object is at a given position x, I draw a horizontal line at the given value of x and read off the value of t at which that horizontal line intercepts the curve.

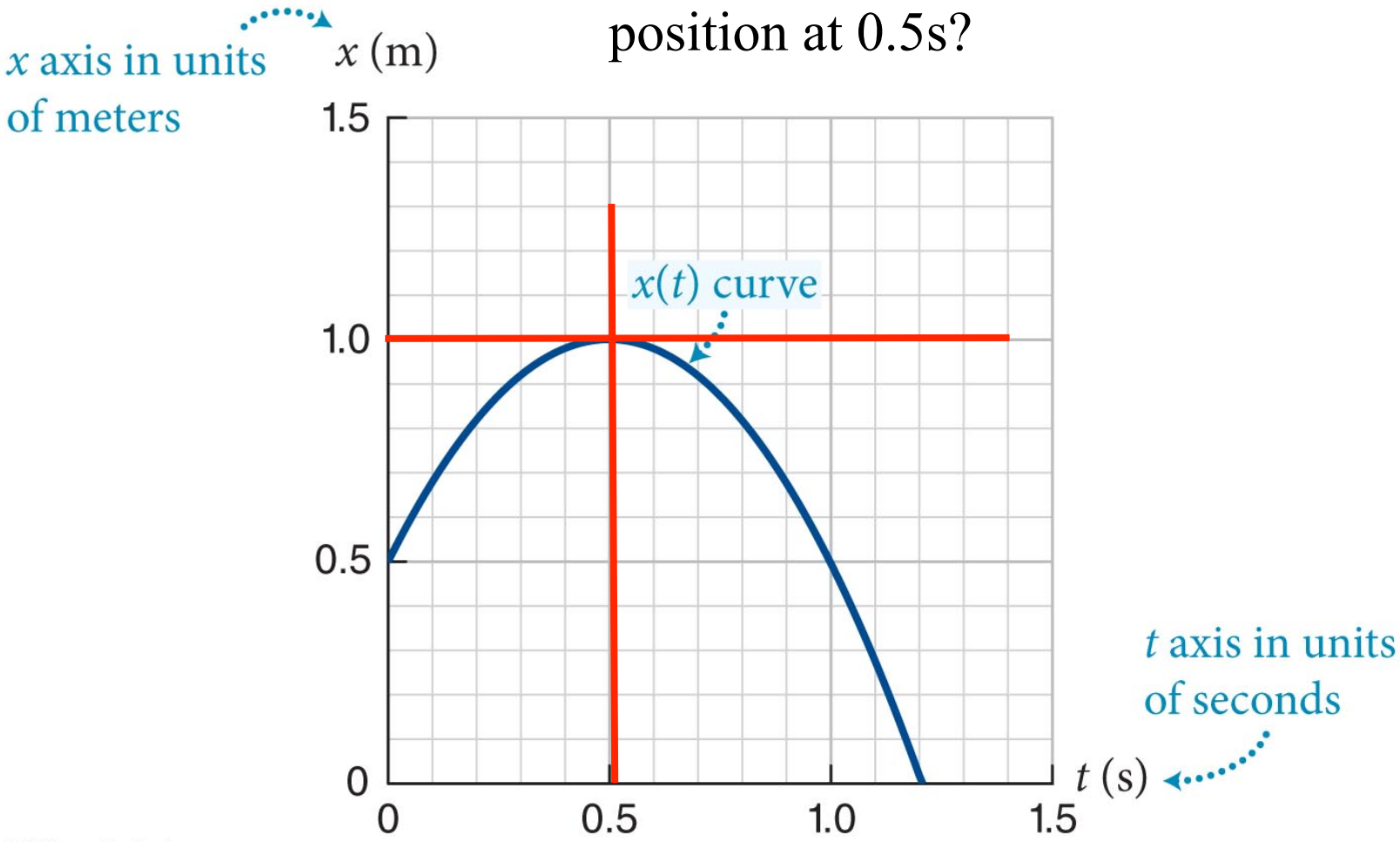
Example 2.1 Graphical representation (cont.)

❸ EXECUTE PLAN

(a) A vertical line drawn at $t = 0.50$ s in Figure 2.8 intercepts the curve at $x = 1.0$ m. ✔

Section 2.3: Representing motion

position at 0.5s?



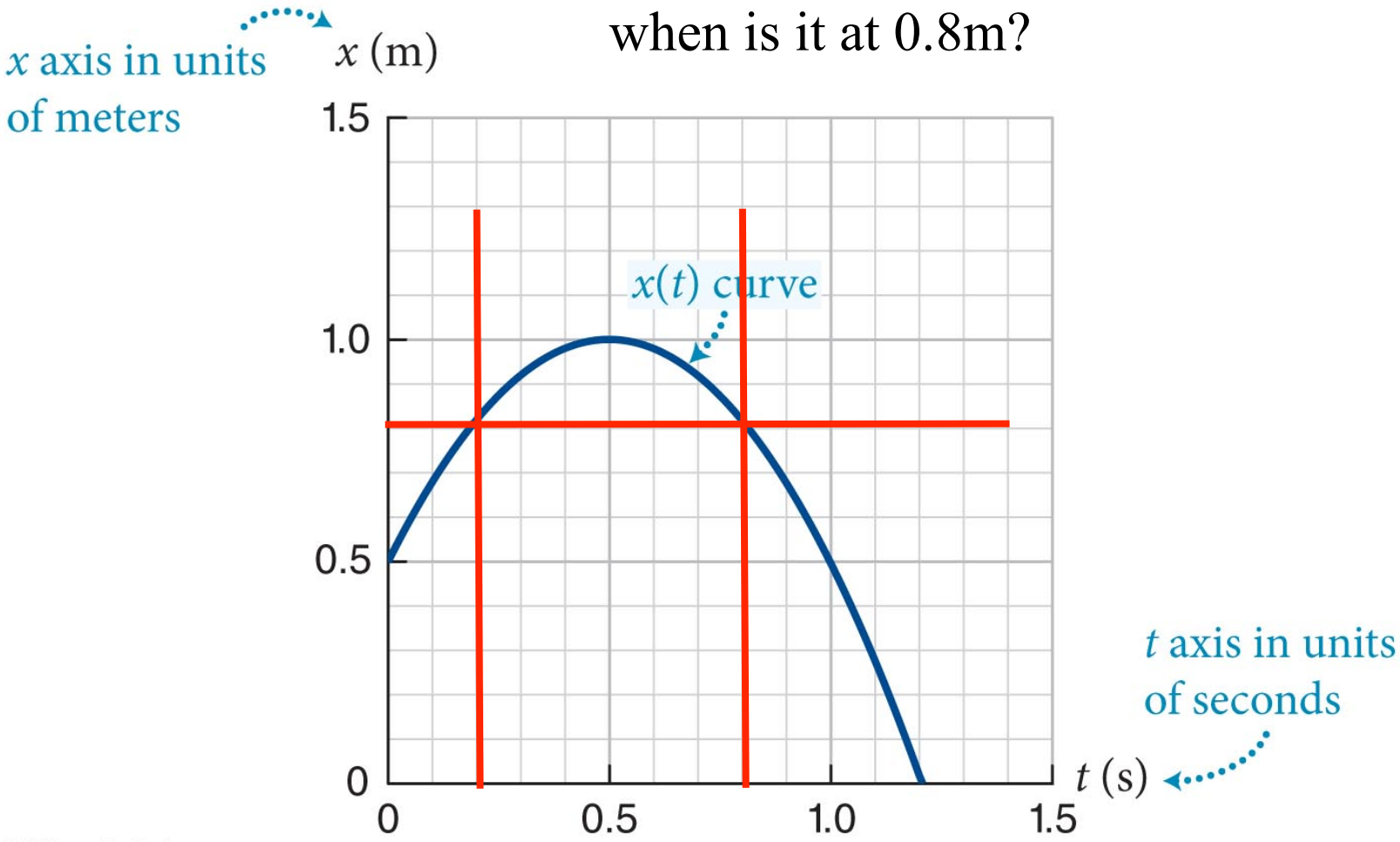
Example 2.1 Graphical representation (cont.)

❸ EXECUTE PLAN

(*b*) A horizontal line at $x = +0.80$ m intercepts the curve at $t = 0.20$ s and at $t = 0.80$ s. ✔

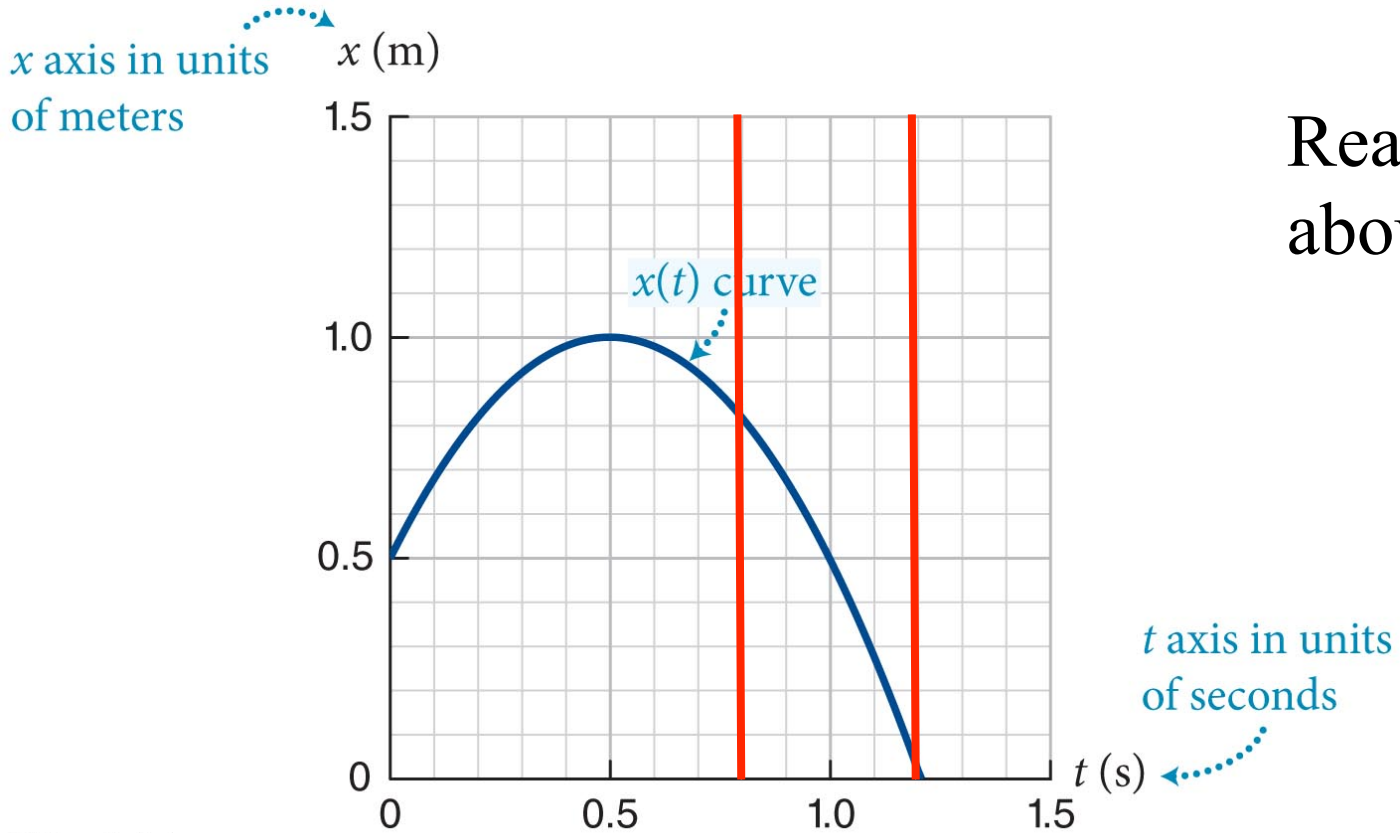
Section 2.3: Representing motion

when is it at 0.8m?



Example 2.1 Graphical representation

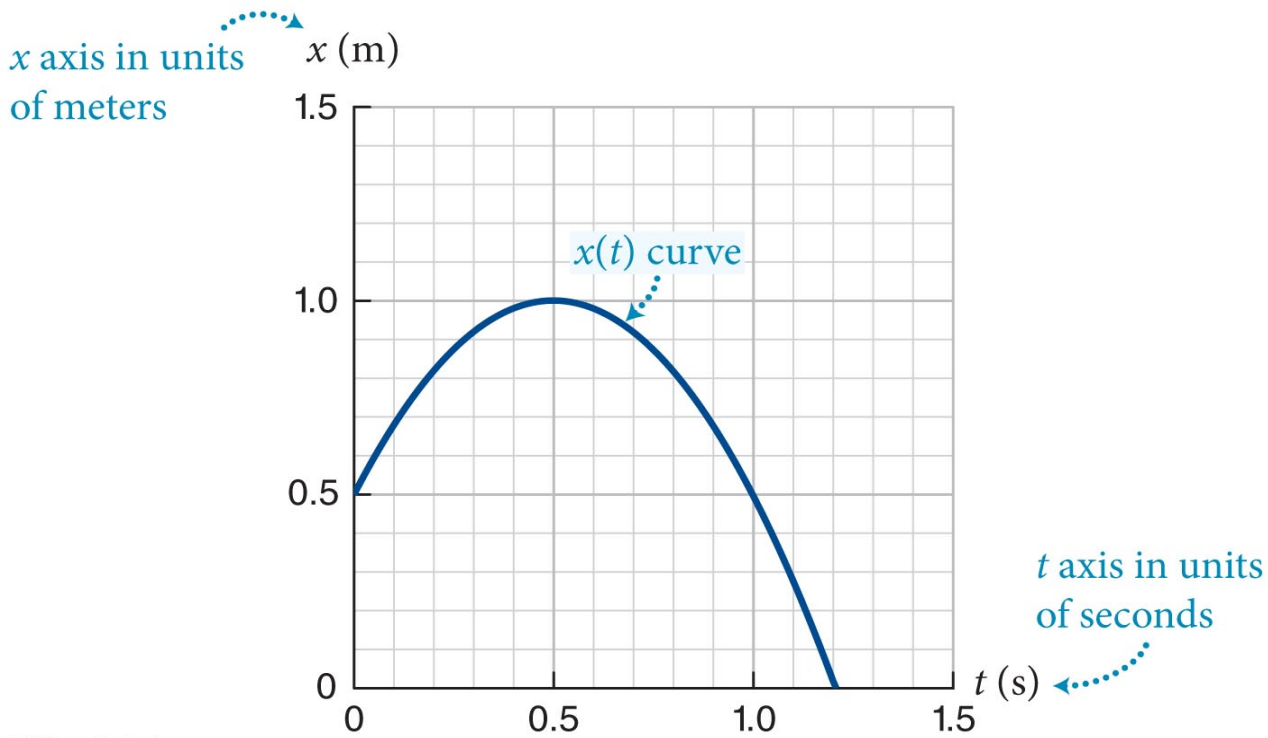
❸ EXECUTE PLAN (*c*) What distance did the object travel between $t = 0.80$ s and $t = 1.2$ s?



Reading the graph, about 0.8m

Example 2.1 Graphical representation (cont.)

❸ EXECUTE PLAN (*d*) How long did that take? The object was at $x = +1.0$ m at $t = 0.50$ s and at $x = 0$ at $t = 1.2$ s. Thus it took 0.7 s to move between these points.



Example 2.1 Graphical representation (cont.)

❹ EVALUATE RESULT

That I obtain two instants in part *b* is OK because the object can return to a certain position.

The answers to parts *c* and *d* (0.80 m and 0.7 s) are within the range of the curve (max 1.0 m and max 1.2 s), so my answers are not crazy.

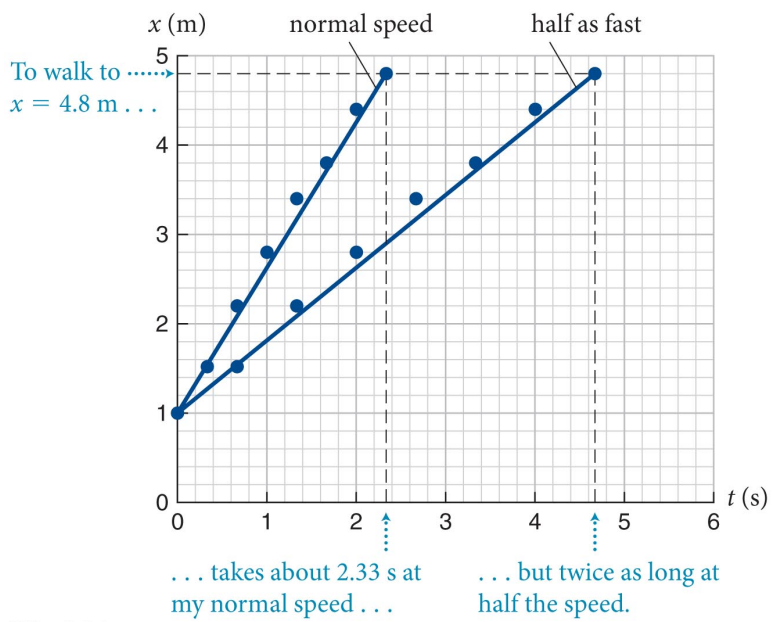
Section 2.4: Average speed and average velocity

You will learn to

- Define the average speed and average velocity of an object in one-dimensional motion.
- Compute the average speed and the average velocity of an object in one-dimensional motion from motion graphs.
- Correlate the geometry of motion graphs with the average speed and the average velocity of an object in one-dimensional motion.

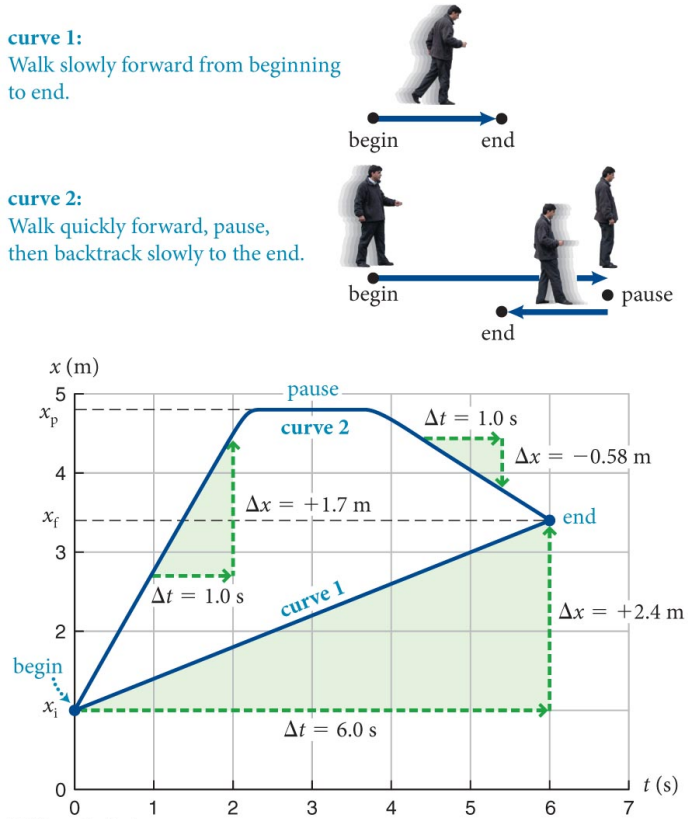
Average Distance

- The next step in describing the motion of the man is to determine the **average speed** of the motion.
- The rate at which the *x-versus-t* curve rises with increasing time is called the slope of the curve:
  - Notice how the slope becomes less steep when the speed decreases.



Section 2.4: Average speed and average velocity

- An objects average speed is the **distance traveled** divided by the time interval required to travel that distance.
- Curve 2 shows that the man travels a total distance of 5.2 m in 6.0 s:
  - Average speed = (5.2 m)/(6.0 s) = 0.87 m/s.



Average speed

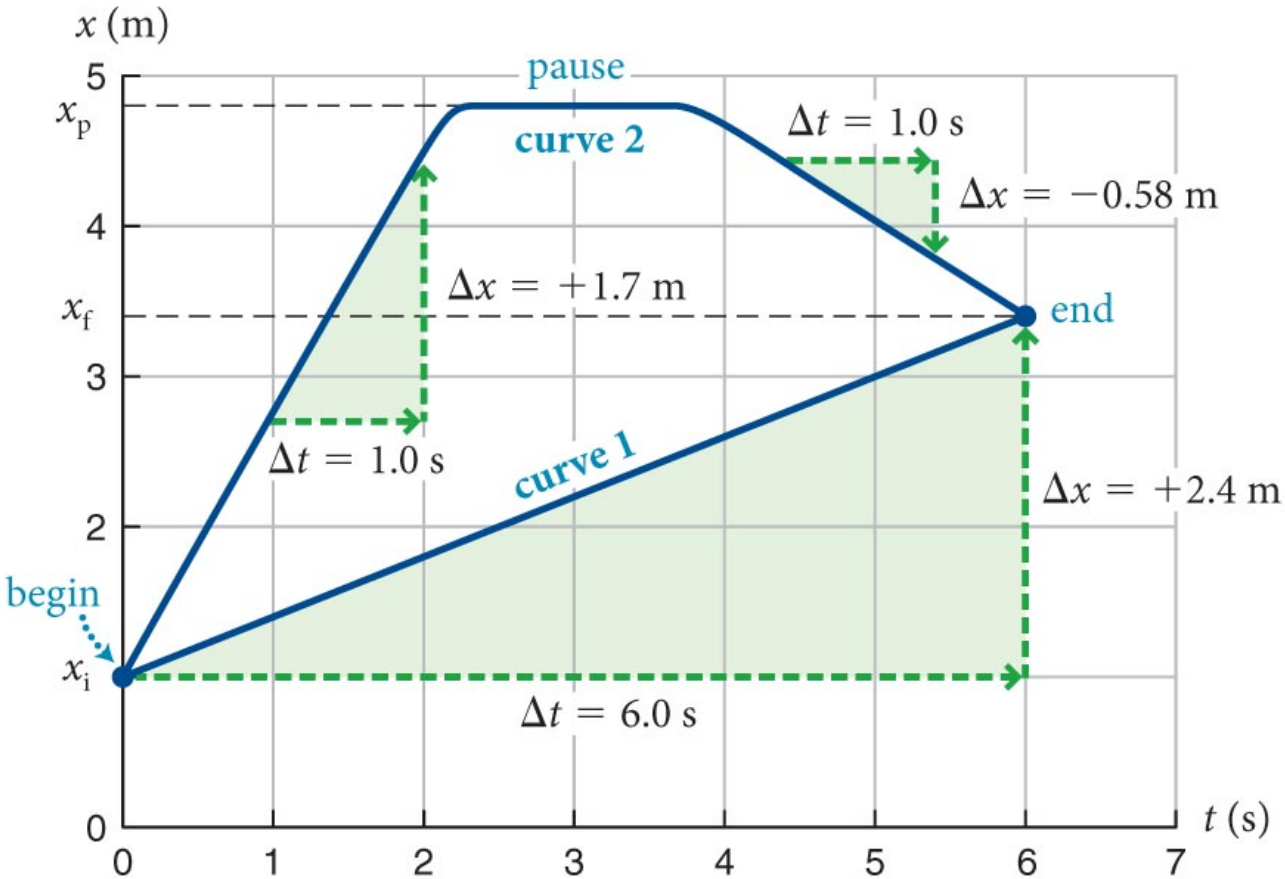
- Average speed would be dividing your odometer reading by how long the trip took
- This is *not* the same as your speed at any given instant

- If position vs time is a straight line:
  - constant slope → constant speed
  - all intervals along the line have same average speed
- If position vs time is a flat line:
  - object is not moving

Checkpoint 2.8

2.8 (*a*) Using curve 2, at what instant did the man first pass the position $x_f = +3.4$ m

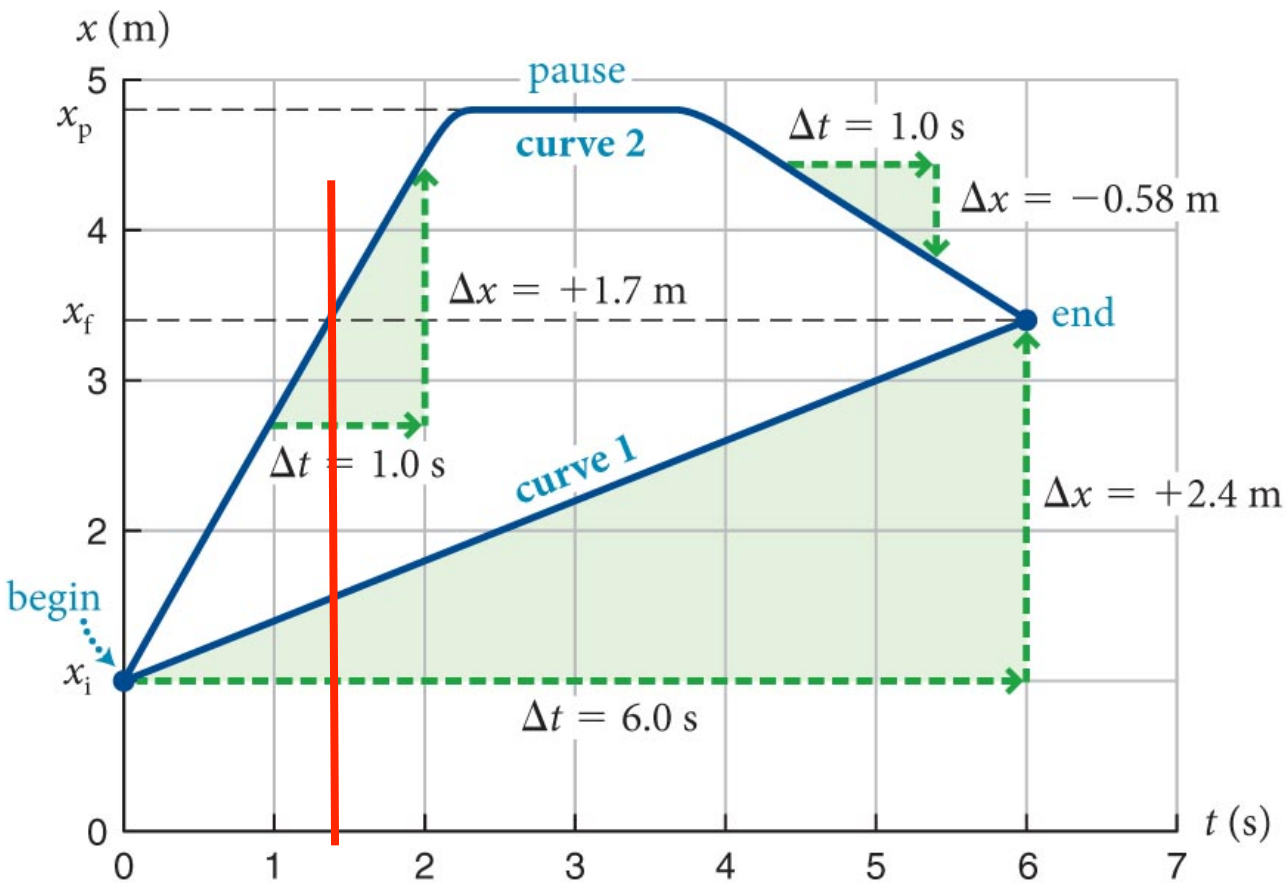
(*b*) What was his average speed up to that instant?



Checkpoint 2.8

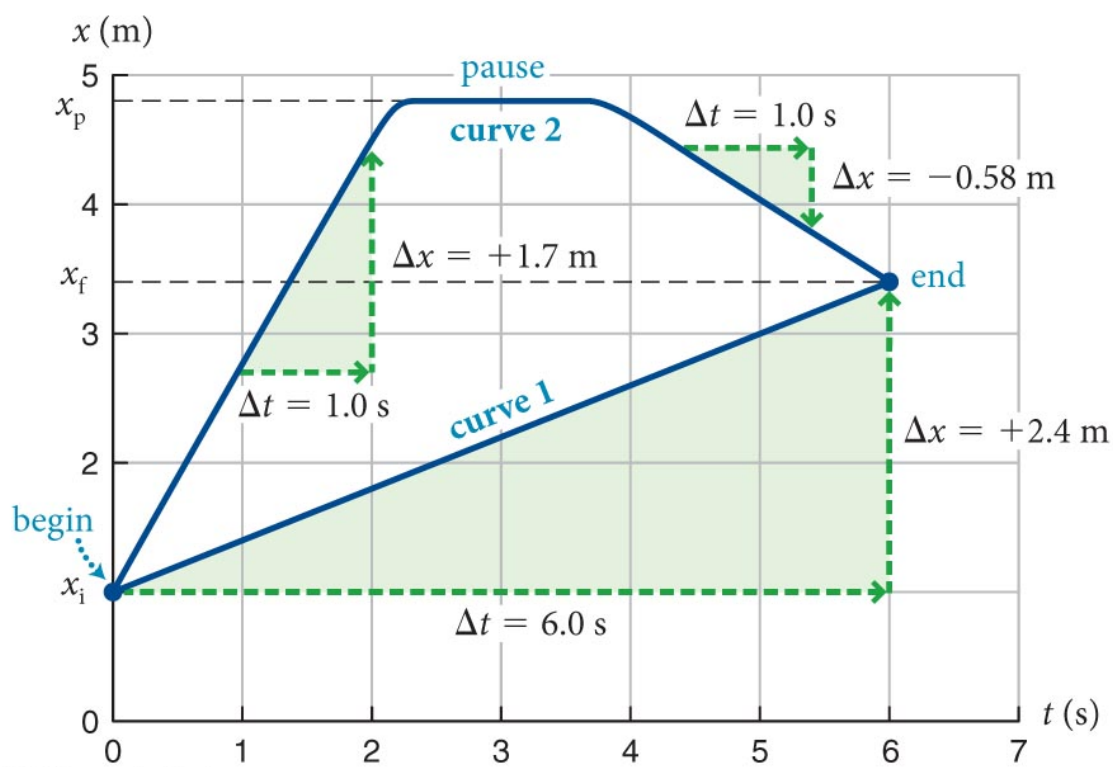
2.8 First passes x_f at around 1.4 seconds or so

Average speed at that point is the slope of the line, 1.7 m/s



Section 2.4: Average speed and average velocity

- In a graph of position versus time, the steeper the slope of the curve, the higher the speed.
- Compare the average speeds during the first and third segments of the motion represented by curve 2 in the figure.



Section 2.4: Average speed and average velocity

Velocity

- The quantity that gives both the speed and the direction of travel is **velocity**.
- The x component of an object's average velocity is the x component of its displacement divided by the length of the time interval taken to travel that displacement.

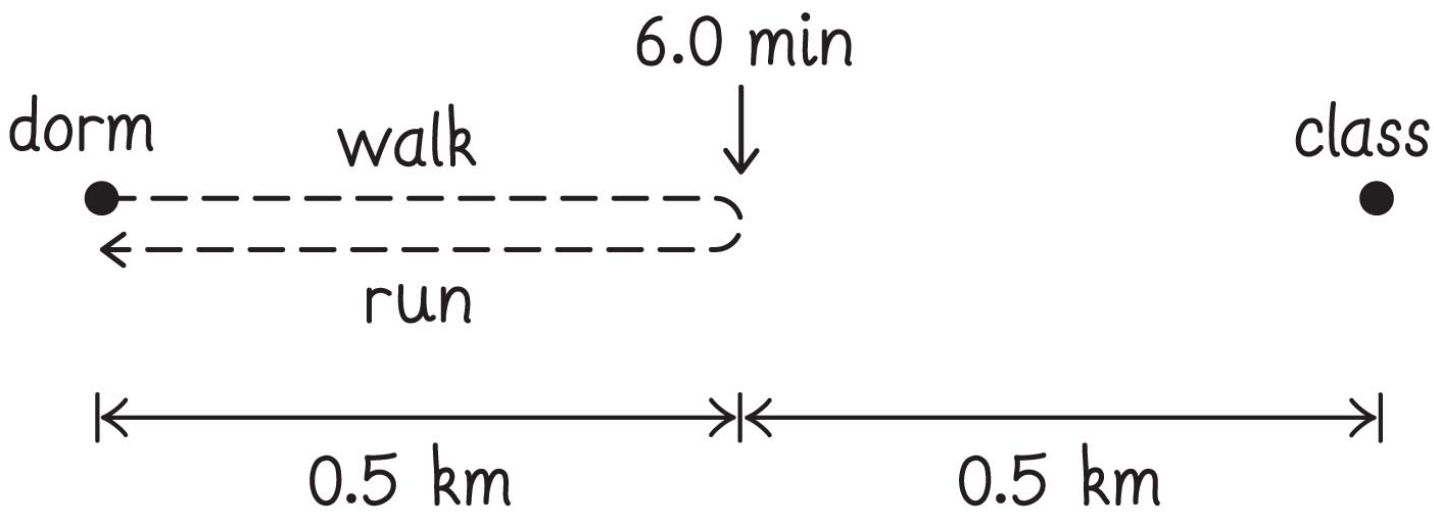
- speed: "The car is going 100 km/h"
- velocity: "The car is going 100 km/h due west"

Example 2.3 Speed and velocity

- With about 20 min to spare, you walk leisurely from your dorm to class, which is 1.0 km away.
- Halfway there, you realize you have forgotten your notebook and run back to your dorm.
- You walked 6.0 min before turning around, and you travel three times as far per unit of time running as you do when walking.
- What are (*a*) the x component of your average velocity and (*b*) your average speed over the entire trip?

Example 2.3 Speed and velocity (cont.)

❶ GETTING STARTED To begin this problem, it helps to make a sketch to visualize the situation



$$v_{run} = 3v_{walk}$$

Example 2.3 Speed and velocity (cont.)

❷ DEVISE PLAN To find the x component of your average velocity, I need to divide the x component of your displacement by the time interval over which this displacement took place. To find your average speed, I must divide the distance traveled by the time interval.

Example 2.3 Speed and velocity (cont.)

❸ EXECUTE PLAN

(a) When you are back at the dorm, the x component of your displacement is zero because your initial and final positions are the same. Therefore the x component of your average velocity is zero too. ✔

average velocity =

(x component of the displacement)/(time interval)

Example 2.3 Speed and velocity (cont.)

❸ (*b*) You traveled halfway to class and back, and the distance between your dorm and the classroom is 1 km. You traveled 0.5km twice for a total of 1.0 km.

You walked for 6.0 min. Because you run three times as fast as you walk, it only takes 2.0 min to return. The total round-trip time is 8.0 min, or 480 s. Your average speed therefore is

(distance traveled)/(time interval) =

(1000 m)/(480 s) = 2.1 m/s. ✔

Example 2.3 Speed and velocity (cont.)

❹ EVALUATE RESULT

If you start and end at the same place, your displacement is zero and so your average velocity is zero no matter what.

For (b): average speed is 2.1m/s ~ 4.7 mi/h

In the right ballpark for a person doing a combination of walking and running.

Guided Problem 2.2 City driving

You need to drive to a store that is 1.0 mi west of your house on the same street on which you live. There are 5 traffic lights between your house and the store, and on your trip you hit all five of them just as they change to red. While you are moving, your average speed is 20 mi/h, but you have to wait 1 min at each light.

(*a*) How long does it take you to reach the store? (*b*) What is your average velocity for the trip? (*c*) What is your average speed?

Guided Problem 2.2 City driving (cont.)

❶ GETTING STARTED

1. Draw a diagram that helps you visualize all the driving and stopped segments and your speed in each segment. Does it matter where the traffic lights are located?
2. You start and stop, speed up and slow down. What does the average speed signify in this case, and how is it related to your displacement in each segment?

Guided Problem 2.2 City driving (cont.)

❷ DEVISE PLAN

3. During how long a time interval are you moving? During how long a time interval are you stopped at the lights?
4. What is your displacement for the trip?
5. How can you apply the answers to questions 3 and 4 to obtain your average velocity?
6. What is the distance traveled, and how is it related to your average speed?
7. How are average velocity and average speed related in this case?

Guided Problem 2.2 City driving (cont.)

❸ EXECUTE PLAN

total time is 3 min (1 mi at 20mi/hr) plus 5 min waiting

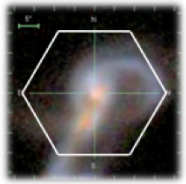
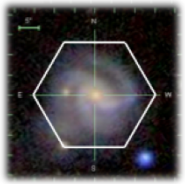
total distance is 1 mi

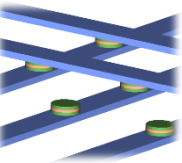
average speed = (1 mi)/(8 min) = 7.5 mi/h

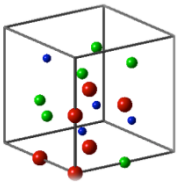
❹ EVALUATE RESULT

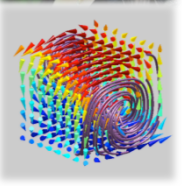
8. Are your answers plausible, and is your result within the range of your expectations?







Women in Physics and Astronomy

Department of Physics & Astronomy / MINT

We invite all women interested in physics & astronomy (undergraduate and graduate students, postdocs and research assistants associated with science & engineering) to our

Fall Semester Opening Meeting

Meet and Greet

Wednesday, September 2nd, 5:00 PM

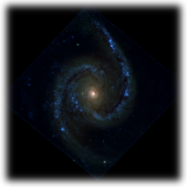
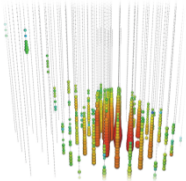
223 Gallalee Building

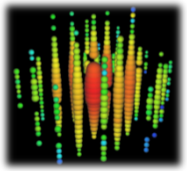
Please join us for some interesting discussions with cookies and tea. Everybody is welcome!

We look forward to meeting you!

Prof. Dawn Williams, Prof. Claudia Mewes and Prof. Preethi Nair

Fall semester schedule at http://physics.ua.edu/wphys/





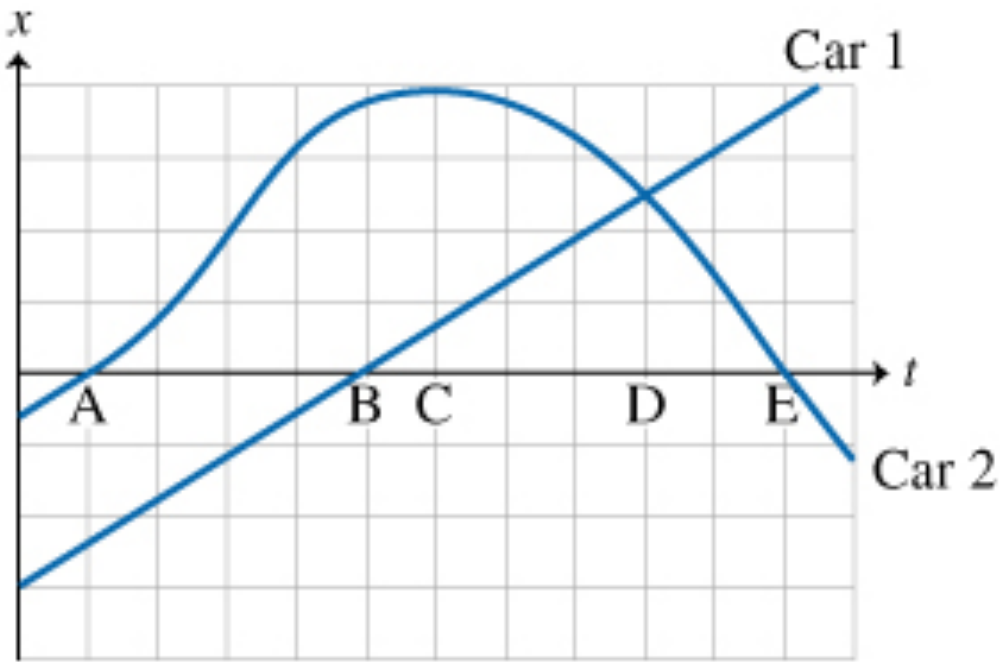



Things

- Packback:
  - 2Q / 1A is probably too many questions
  - this is new, so some tuning will happen
  - let's make it 3 posts per week total
    - can be all answers, 2A/1Q, 2Q/1A, or all questions
    - hopefully makes for more discussion
- MasteringPhysics
  - numbers are still a bit low … are you on it?

Reading quiz 2

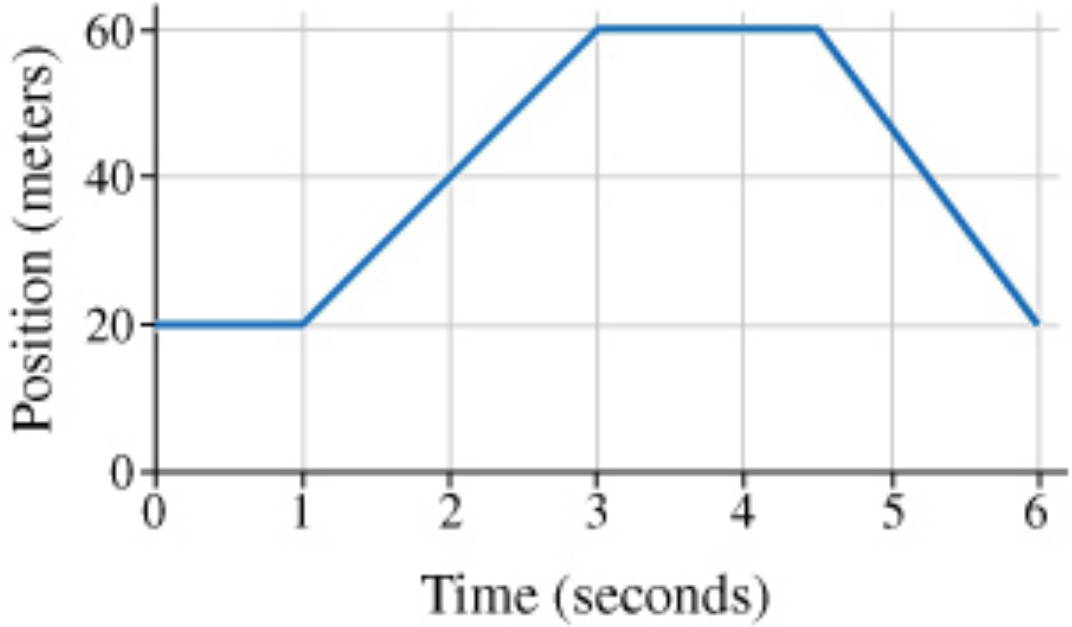
Position vs time graphs



- When do they pass?
- Are they going in the same direction then?
- When does car 1 stop?
- When does car 2 stop?
- When do they have nearly the same speed?

Reading quiz 2

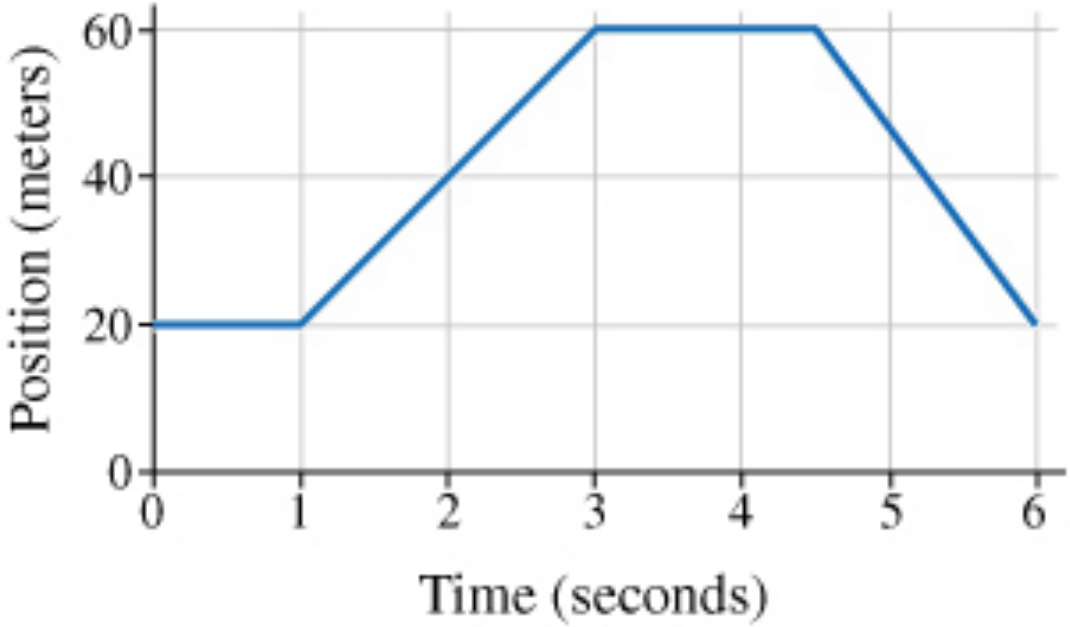
Average velocity from a graph



- avg velocity over [0,1s]?
- over [1,3s]?
- over [0,3s]?
- over [3,6s]?
- whole graph?

Reading quiz 2

Average velocity from a graph



- avg velocity over [0,1s]?

  0 m/s
- over [1,3s]?

  40m/2s = 20m/s
- over [0,3s]?

  40m/3s = 13.3m/s
- over [3,6s]?

  -40m/3s = -13.3m/s
- whole graph?

  0 m/s

Reading quiz 2

What depends on choice of origin?

- Average velocity
- Position
- Displacement
- Speed
- Instantaneous velocity

Reading quiz 2

What depends on choice of origin?

- Average velocity
- Position
- Displacement
- Speed
- Instantaneous velocity

The rest only depend on differences in position

Quantitative Tools

Section 2.5: Scalars and vectors

- **Scalars:** just numbers. Can be + or -, may have units.
  - Examples: Temperature, volume
  - Scalars follow ordinary arithmetic

- **Vectors:** Physical quantities that are completely specified by a magnitude and a direction in space.
  - vectors are things that need arrows to be properly represented
  - they have their own arithmetic

Checkpoint 2.11

2.11 Are the following quantities vectors or scalars:

(i) the price of a movie ticket,

(ii) the average velocity of a ball launched vertically upward,

(iii) the position of the corner of a rectangle

(iv) the length of a side of that rectangle?

Checkpoint 2.11

2.11 Are the following quantities vectors or scalars:

(i) the price of a movie ticket,

scalar – just a number (with units)

(ii) the average velocity of a ball launched vertically upward,

vector – needs direction

(iii) the position of the corner of a rectangle

vector – needs a direction (which corner?)

(iv) the length of a side of that rectangle?

scalar – just a number (with units)

Section 2.5: Scalars and vectors

- For now we will only study one-dimensional (1-D) motion.
  - only move forward or backward in 1-D.
  - direction indicated by sign (+ or -)
- To distinguish vector from scalars: notation

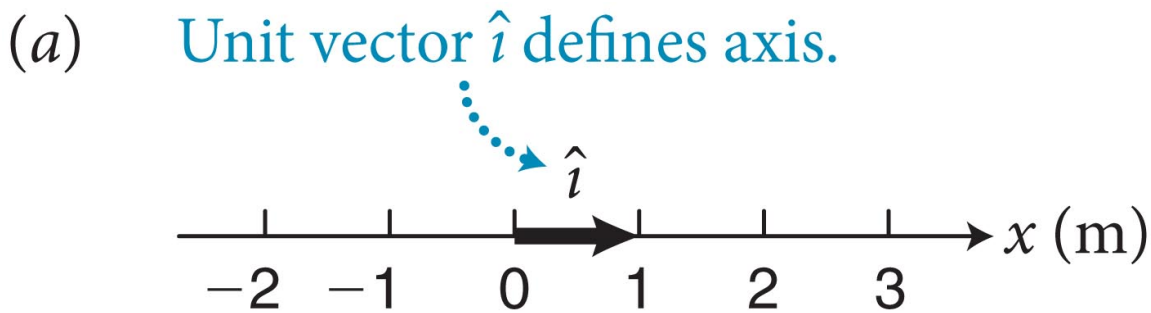
$$\vec{a} \quad \text{vector}$$

$$b \quad \text{scalar}$$

Section 2.5: Scalars and vectors

- The **magnitude of a vector** is how long it is
  - magnitude is always positive.
  - magnitude is denoted by $|\vec{b}|$ or b

- To specify mathematically, we introduce **unit vectors**:
  - a unit vector pointing in the $+x$ direction is denoted by $\hat{\imath}$
  - unit vectors have a magnitude of 1. $|\hat{\imath}| \equiv 1$

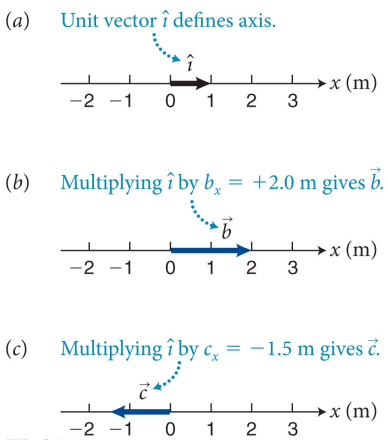


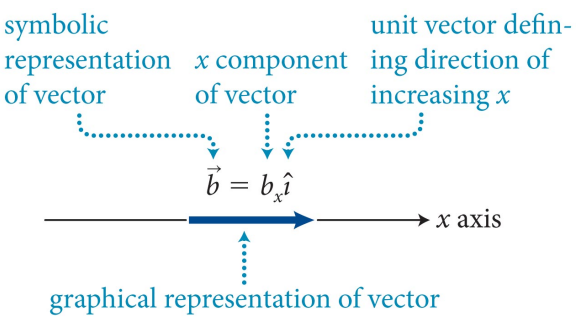
Section 2.5: Scalars and vectors

- Any vector along the x axis can be written in unit vector notation.

$$\vec{b} = b_x \hat{\imath}$$

  - b_x is a scalar called the x **component** of the vector
  - b_x is negative if the vector points in the -x direction.
  - this notation says "move b_x units along x"
  - in 1-D, the magnitude of $\vec{b}$ is the absolute value of b_x.



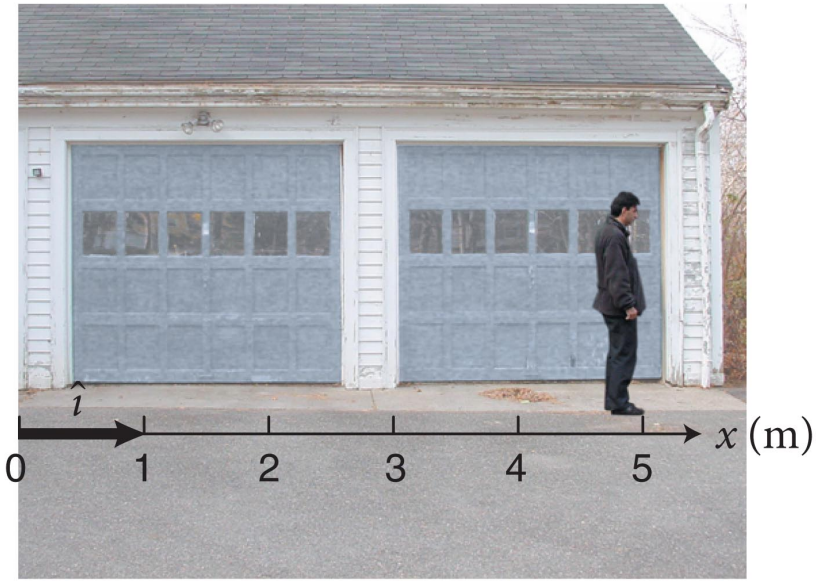


- magnitude of vector: $b = |b_x|$
- If $b_x > 0$: $\vec{b}$ is in same direction as $\hat{\imath}$
- If $b_x < 0$: $\vec{b}$ is in opposite direction to $\hat{\imath}$

Checkpoint 2.12

2.12 Consider the axis and unit vector below.

- Define a vector pointing from the origin to tip of the man's feet
- Write it in terms of an x component and unit vector
- What does the vector represent?



Checkpoint 2.12

2.12 Consider the axis and unit vector below.

- Define a vector pointing from the origin to tip of the man's feet
- Write it in terms of an x component and unit vector

$$\vec{r} = (5\mathrm{m})\,\hat{\imath}$$

- What does the vector represent? **His position!**

Section 2.6: Position and displacement vectors

Section Goals

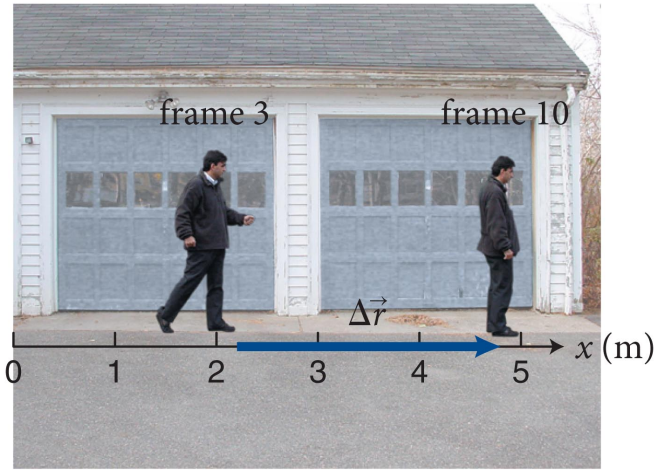
You will learn to

- Use “delta” notation to represent the change in a physical quantity.
- Correlate the distance traveled and displacement for an object moving in one dimension.
- Represent the distance traveled and displacement of an object mathematically.

Section 2.6: Position and displacement vectors

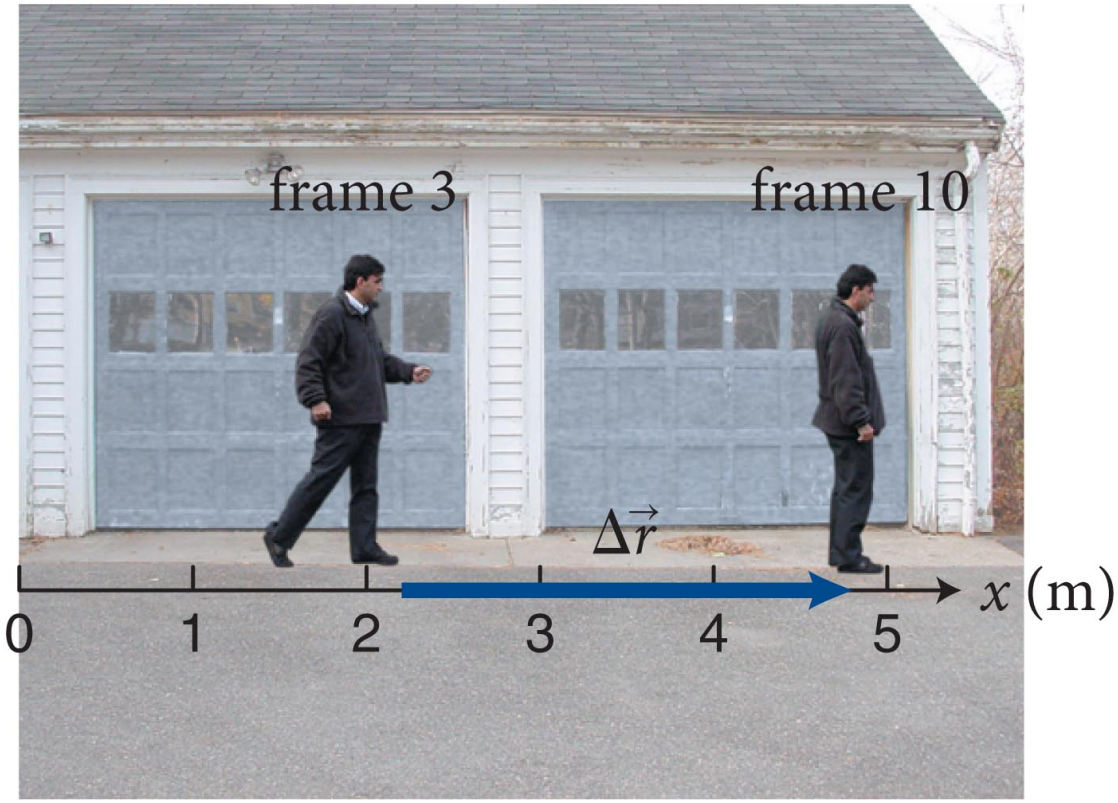
- **Displacement** can be represented graphically by an arrow that points from an object's initial to final position.
  - We denote displacement by $\Delta\vec{r}$
  - Δ means 'change in'
  - The x component of displacement is $\Delta r_x \equiv \Delta x$
  - $\Delta x = x_f - x_i$, where x_i and x_f are initial and final positions.
  - The **distance** between initial and final points is

$$d \equiv |x_f - x_i| \text{ (1D only)}$$



Checkpoint 2.13

2.13 (*a*) What is the *x* component of the displacement in the figure below? (*b*) Write the displacement in terms of its *x* component and the unit vector.



Section 2.6: Position and displacement vectors

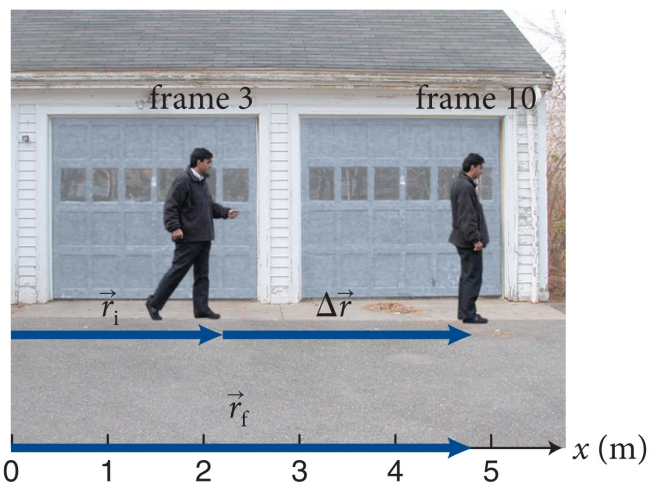
- Since displacement is a vector,

  $\Delta x\,\hat{\imath} = (x_f - x_i)\hat{\imath} = x_f\,\hat{\imath} - x_i\,\hat{\imath}$, where $\Delta x\,\hat{\imath} = \Delta r_x\,\hat{\imath} = \Delta\vec{r}$.
- Using vector notation displacement can be written as

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

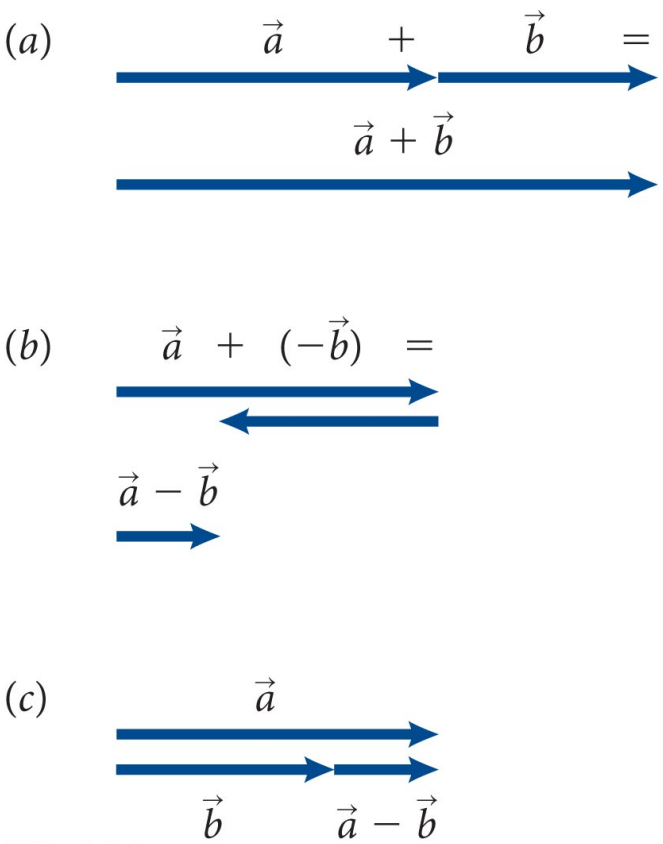
- Rearranging we get the final position vector:

$$\vec{r}_f = \vec{r}_i + \Delta\vec{r}$$



Graphical prescription of adding and subtracting vectors

- To add vectors, connect them head to tail.
- To subtract a vector from another vector, reverse the direction of the vector being subtracted and connect head to tail.



Section 2.7: Velocity as a vector

Section Goals

You will learn to

- Define average speed and average velocity mathematically.
- Correlate the graphical and mathematical representations for average speed and average velocity.

Section 2.7: Velocity as a vector

- From our discussion yesterday, the x component of average velocity is

$$\upsilon_{x,\mathrm{av}} \equiv \frac{\Delta x}{\Delta t} = \frac{x_{\mathrm{f}} - x_{\mathrm{i}}}{t_{\mathrm{f}} - t_{\mathrm{i}}}$$

- Since $\Delta x\,\hat{\imath} = \Delta r_x\,\hat{\imath} = \Delta\vec{r}$, average velocity can also be written using vector notation as

$$\vec{\upsilon}_{\mathrm{av}} \equiv \upsilon_{x,\mathrm{av}}\,\hat{\imath} = \frac{\Delta\vec{r}}{\Delta t}$$

- subtract final & initial position vectors

Section 2.8: Motion at constant velocity

Section Goals

You will learn to

- Represent motion at constant velocity using motion graphs and mathematics.
- Construct position-versus-time and velocity-versus-time graphs for motion at constant velocity.
- Adapt the mathematical definition of average velocity into an equation that allows the future motion of a moving object to be predicted from its present motion.

Section 2.8: Motion at constant velocity

- If an object moves at a constant velocity, then the x(t) graph is a straight line.
  - b/c slope is velocity …

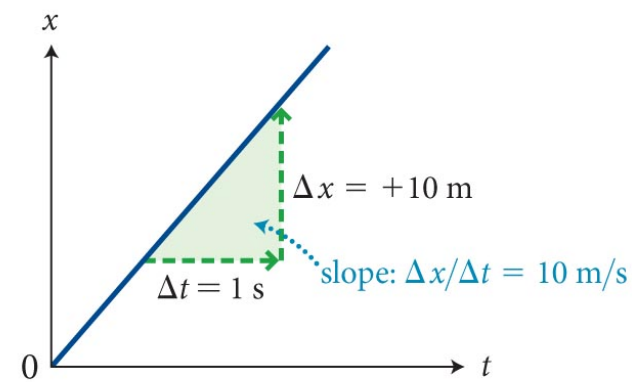
$$\Delta x = v_x \Delta t \quad \text{(constant velocity)}$$

- equivalently:

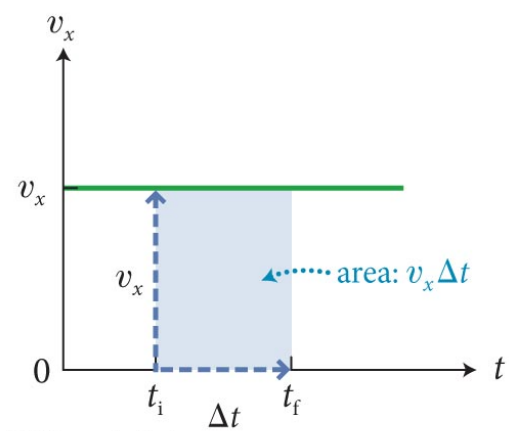
$$x_f = x_i + v_x \Delta t \quad \text{(constant velocity)}$$

- Δx during any interval is the **area under the $v_x(t)$ curve** over that interval.

(*a*) Position versus time: straight line with nonzero slope



(*b*) Velocity versus time: horizontal line



Example 2.10 Motion at constant velocity

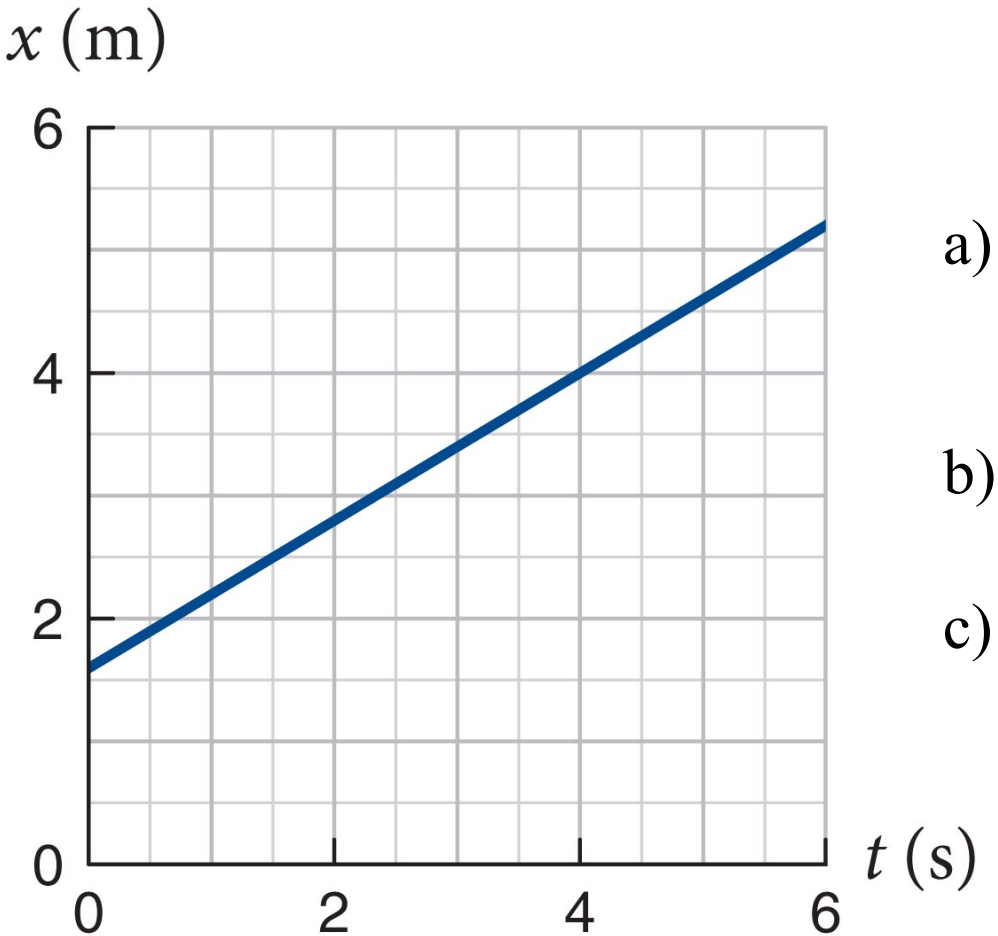
I'll show you a position-versus-time graph for part of the motion of an object moving at constant velocity.

(a) What is the x component of the object's velocity?

(b) Write an expression for $x(t)$, the x coordinate of the position of the object at an arbitrary time t.

(c) What is the object's position at $t = 25$ s?

Example 2.10 Motion at constant velocity



a) What is the x component of the object's velocity?

b) Write an expression for $x(t)$

c) What is the x coordinate of the object at $t = 25$ s?

Example 2.10 Motion at constant velocity (cont.)

❶ GETTING STARTED – Clearly $x(t)$ is a straight line.

I need a the slope to get velocity.

I need the y-intercept to write $x(t)$.

Example 2.10 Motion at constant velocity (cont.)

❷ DEVISE PLAN The slope is the x component of the object's velocity, $\Delta x/\Delta t$. Since $x(t)$ is a line, this is the same everywhere.

I can use $x_f = x_i + v_x \Delta t$ to write $x(t)$

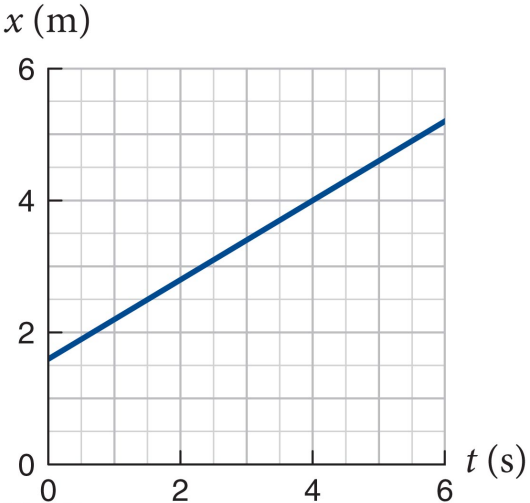
I can use the x(t) equation to find position at $t = 25$ s, presuming the motion does not change

Example 2.10 Motion at constant velocity (cont.)

❸ EXECUTE PLAN (*a*) read the graph. We can use positions at 0 and 6s

$$v_x = \frac{x_f - x_i}{t_f - t_i} = \frac{+5.2\ \text{m} - (+1.6\ \text{m})}{6.0\ \text{s} - 0} = \frac{+3.6\ \text{m}}{6.0\ \text{s}}$$

$$= +0.60\ \text{m/s.}$$ ✔

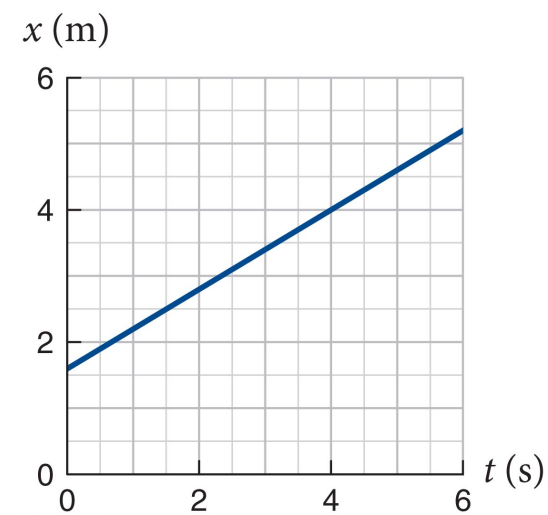


Section 2.8: Motion at constant velocity

❸ EXECUTE PLAN

(*b*) Take the object's position at $t = 0$ as the initial position. With $t_i = 0$, $x_i = +1.6$ m, and $v_x = +0.60$ m/s:

$$x_f = x_i + v_x \Delta t = +1.6 \text{ m} + (+0.60 \text{ m/s})(t_f - 0)$$
$$= +1.6 \text{ m} + (+0.60 \text{ m/s})t_f \,.$$



t_f is arbitrary, so this works for any time:

$$x(t) = +1.6 \text{ m} + (+0.60 \text{ m/s})t. \checkmark$$

Example 2.10 Motion at constant velocity (cont.)

❸ EXECUTE PLAN (c) at $t = 25$ s

$$x(t = 25\text{ s}) = +1.6\text{ m} + (+0.60\text{ m/s})(25\text{ s})$$
$$= +1.6\text{ m} + 15\text{ m} = +17\text{ m.}$$ ✔

- *17 b/c significant digits*
- *assumes particle is still moving at constant velocity then!*

Example 2.10 Motion at constant velocity (cont.)

❹ EVALUATE RESULT

- The x component of the object's velocity is positive, as I would expect from the positive slope of the $x(t)$ curve.
- I can double-check the value (0.60 m/s) by reading off the increase in x over a 1-s interval in the graph.
- I see that the object starts near $x = 2$ m and the x coordinate increases by 3 m every 5 s. In 25 s, the object displacement should therefore be 5(3 m) = 15 m, and so the object's final position should be near (2 m) + (15 m) = 17 m, which is what I got.

Section 2.9: Instantaneous velocity

Section Goals

You will learn to

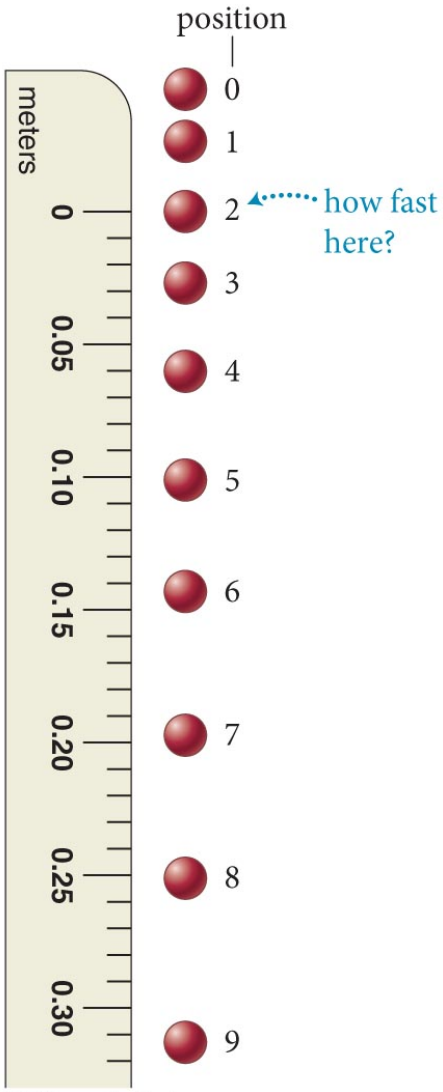
- Represent motion with changing velocity using motion graphs and mathematics.
- Relate the concept of a tangent line with instantaneous velocity on a position-versus-time graph.
- Generalize mathematically the definition of the average velocity of a moving object to its instantaneous velocity by the use of a limiting process.

Section 2.9: Instantaneous velocity

The figure shows the successive position of a ball falling at 0.0300-s intervals.

- We want to be able to find the velocity as the ball passes any position in its motion, that is the instantaneous velocity $\vec{v}$.
- Let us start by first finding the average velocity between positions 2 and 9:

$$v_{x,29} = \frac{x_9 - x_2}{t_9 - t_2}$$



Section 2.9: Instantaneous velocity

By reducing the time interval Δt you can get closer to the velocity at point 2.

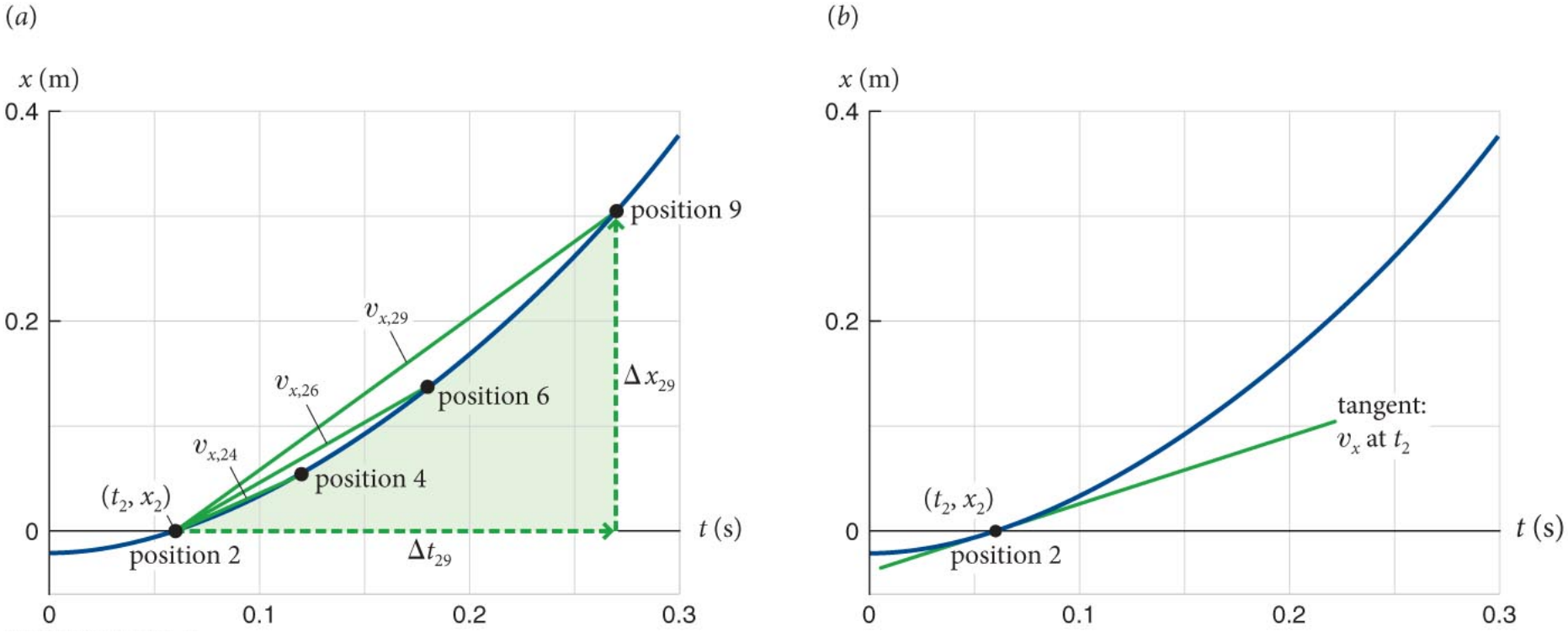
- Letting Δt approach zero, we obtain the x component of the velocity at instant t:

$$\upsilon_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

- The right side of the equation is the definition for the derivative, so

$$\upsilon_x \equiv \frac{dx}{dt}$$

Section 2.9: Instantaneous velocity



Section 2.9
Quick check

The slope of the curve in the position vs. time graph for a particle's motion gives

1. the particle's speed.
2. the particle's acceleration.
3. the particle's average velocity.
4. the particle's instantaneous velocity.

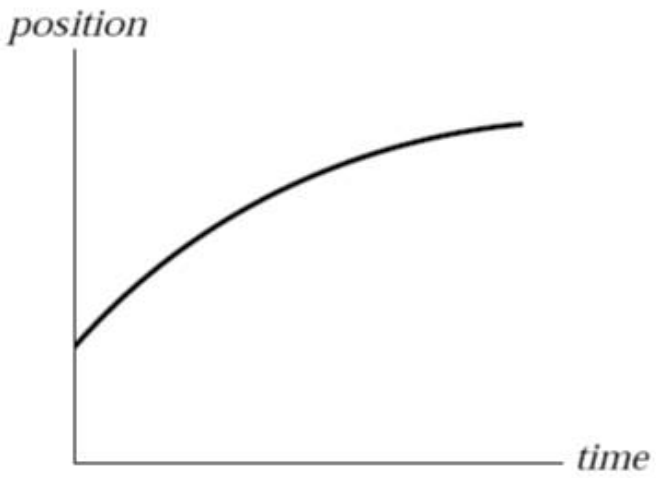
Section 2.9
Clicker Question 4

The slope of the curve in the position vs. time graph for a particle's motion gives

1. the particle's speed.
2. the particle's acceleration.
3. the particle's average velocity.
4. ✔ the particle's instantaneous velocity.

Section 2.9
Clicker Question 5

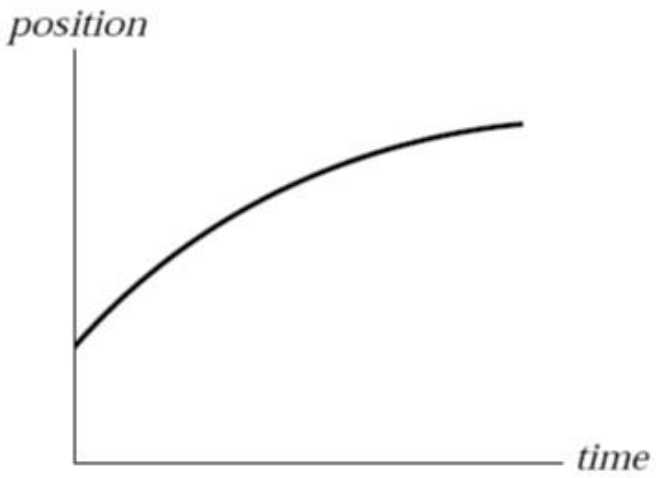
A train car moves along a long straight track. The graph shows the position as a function of time for this train. The graph shows that the train



1. speeds up all the time.
2. slows down all the time.
3. speeds up part of the time and slows down part of the time.
4. moves at a constant velocity.

Section 2.9
Clicker Question 5

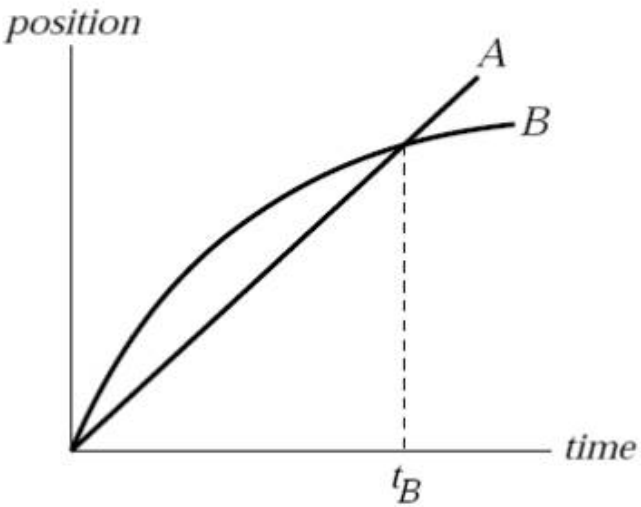
A train car moves along a long straight track. The graph shows the position as a function of time for this train. The graph shows that the train



1. speeds up all the time.
2. ✔ slows down all the time.
3. speeds up part of the time and slows down part of the time.
4. moves at a constant velocity.

Section 2.9 Question

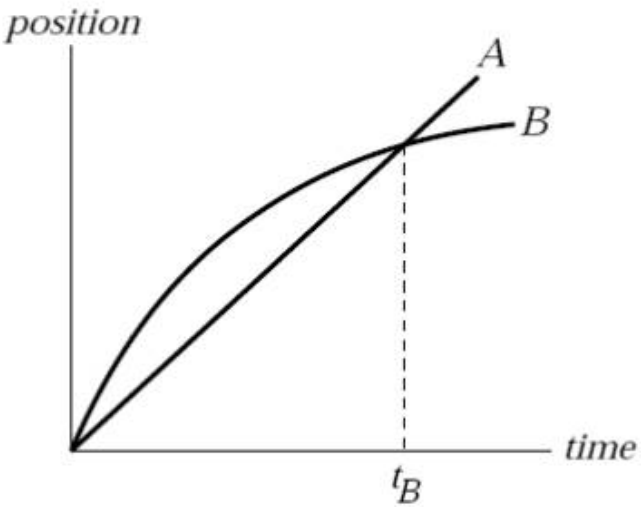
The graph shows position as a function of time for two trains running on parallel tracks. Which is true?



1. At time t_B, both trains have the same velocity.
2. Both trains speed up all the time.
3. Both trains have the same velocity at some time before t_B.
4. Somewhere on the graph, both trains have the same acceleration.

Section 2.9
Clicker Question 6

The graph shows position as a function of time for two trains running on parallel tracks. Which is true?



1. At time t_B, both trains have the same velocity.
2. Both trains speed up all the time.
3. ✔ Both trains have the same velocity at some time before t_B.
4. Somewhere on the graph, both trains have the same acceleration.

Chapter 2: Summary

Concepts: Average speed and average velocity

- The **distance traveled** is the accumulated distance covered by an object along the path of its motion, without regard to direction. The x component of an object's **displacement** is the change in its x coordinate.
- The **average speed** of an object is the distance it travels divided by the time interval it takes to travel that distance.

Chapter 2: Summary

Concepts: Average speed and average velocity

- The x component of an object's **average velocity** is the x component of its displacement divided by the time interval taken to travel that displacement. The magnitude of the average velocity is not necessarily equal to the average speed.

Chapter 2: Summary

Quantitative Tools: Average speed and average velocity

- The **distance** d between two points x_1 and x_2 is

$$d = |x_1 - x_2|$$

- The x component of the **displacement** Δx of an object that moves from a point x_i to a point x_f is

$$\Delta x = x_f - x_i$$

- The x component of the object's **average velocity** is

$$v_{x,\text{av}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Chapter 2: Summary

Concepts: Vectors and scalars

- A **scalar** is a physical quantity that is completely specified by a number and a unit of measure.
- A **vector** is a physical quantity that is completely specified by a number, a unit of measure, and a direction.
- The number and unit of measure together are called the **magnitude** of the vector.
- A **unit vector** has magnitude one and no units.

Chapter 2: Summary

Concepts: Position, displacement, and velocity as vectors

- The **position vector,** or **position,** of a point is drawn from the origin of a coordinate system to the point.
- The **displacement vector** of an object is the change in its position vector. It is the vector that points from the tip of the original position vector to the tip of the final position vector.

Chapter 2: Summary

Quantitative Tools: Position, displacement, and velocity as vectors

- The position $\vec{r}$ of a point that has x coordinate x is

$$\vec{r} = x\hat{\imath}$$

- The displacement vector $\Delta\vec{r}$ of an object moving along the x axis is

$$\Delta\vec{r} = \vec{r}_{\mathrm{f}} - \vec{r}_{\mathrm{i}} = (x_{\mathrm{f}} - x_{\mathrm{i}})\hat{\imath}$$

- The average velocity of an object moving along the x axis is

$$\vec{\upsilon}_{\mathrm{av}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{x_{\mathrm{f}} - x_{\mathrm{i}}}{t_{\mathrm{f}} - t_{\mathrm{i}}}\hat{\imath}$$

Chapter 2: Summary

Concepts: Additional properties of velocity

- When the velocity of an object is constant, its **position-versus-time graph** is a straight line, **velocity versus-time graph** is a horizontal line.
- The **instantaneous velocity** of an object is its velocity at any particular instant.
- The slope of an object's $x(t)$ curve at a given instant is numerically equal to the x component of the object's velocity at that instant.
- The area under the object's $v_x(t)$ curve during a time interval is the x component of the object's displacement during that interval.

Quantitative Tools: Additional properties of velocity

- The x component of an object's instantaneous velocity is the derivative of the object's x coordinate with respect to time:

$$\upsilon_x = \frac{dx}{dt}$$

A data-driven approach

watch an object fall

- record position vs time
- what can we find out?

t (s)	x (m)
0	0
1	5
2	20
3	45
4	80
5	125

- (clearly not real data)

A data-driven approach

what's the plan?

- see what we can figure out from data alone

- need an abstraction – a **model** for how it falls
  - come up with a hypothesis from data
  - what does it predict?
  - test it

A data-driven approach

what's the hypothesis?

- we don't have great everyday intuition
  - falling happens too fast (hang time?)
  - no accurate timing
- let's try something that seems plausible

A data-driven approach

possible hypothesis 1

for every increment of time Δt, the object falls the same distance Δx.

Mathematically: $\Delta x \propto \Delta t$, or Δx vs Δt is a straight line

A data-driven approach

nice idea, but wrong

Possible hypothesis 1: for every increment of time Δt, the object falls the same distance Δx.

If this were true, the Δx should be the same for identical time intervals, or $\Delta x/\Delta t$ should be constant

(We know now this means constant velocity.)

A data-driven approach

data says:

Hypothesis 1 is wrong. The object speeds up as it falls.

time t (s)	distance x (m)	rate distance changed $\Delta x/\Delta t$ (m/s)
0	0	–
1	5	5
2	20	15
3	45	25
4	80	35
5	125	45

A data-driven approach

look carefully

- The rate of position change also increases
- That is: the velocity also increases
  - but it increases by the same amount each second!
  - suggests its rate of change is constant.

time t (s)	distance x (m)	rate distance changed $\Delta x/\Delta t$ (m/s)
0	0	–
1	5	5
2	20	15
3	45	25
4	80	35
5	125	45

A data-driven approach

a new hypothesis

Hypothesis 2: The rate at which the velocity changes is constant.

Finding a constant of motion is a big deal.

Like, no matter what happens, this thing doesn't change. It must be kind of a big deal.

A data-driven approach

try it

Indeed! The position and velocity continuously increase, but the rate of velocity change is constant.

This is *acceleration:* $$\frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2x}{dt^2}$$

time t (s)	distance x (m)	average velocity $\bar{v}$ (m/s)	rate velocity changes $\Delta\bar{v}/\Delta t$ (m/s^2)
0	0	–	–
1	5	5	–
2	20	15	10
3	45	25	10
4	80	35	10
5	125	45	10

A data-driven approach

what did we figure out?

- The rate of change of velocity is (essentially) constant for a falling object
- We call this rate of change **acceleration**
- Velocity increases linearly with time
- Since $x = dv/dt$, this means x(t) increases quadratically

Wrap-up

What do you need to do

- finish your homework by 5pm.
- reading quiz for Tuesday (Ch. 3)

- make sure you did PackBack
  - have relaxed the rules
  - any combination of questions and answers that adds to 3
  - can all be answers – want better signal to noise …