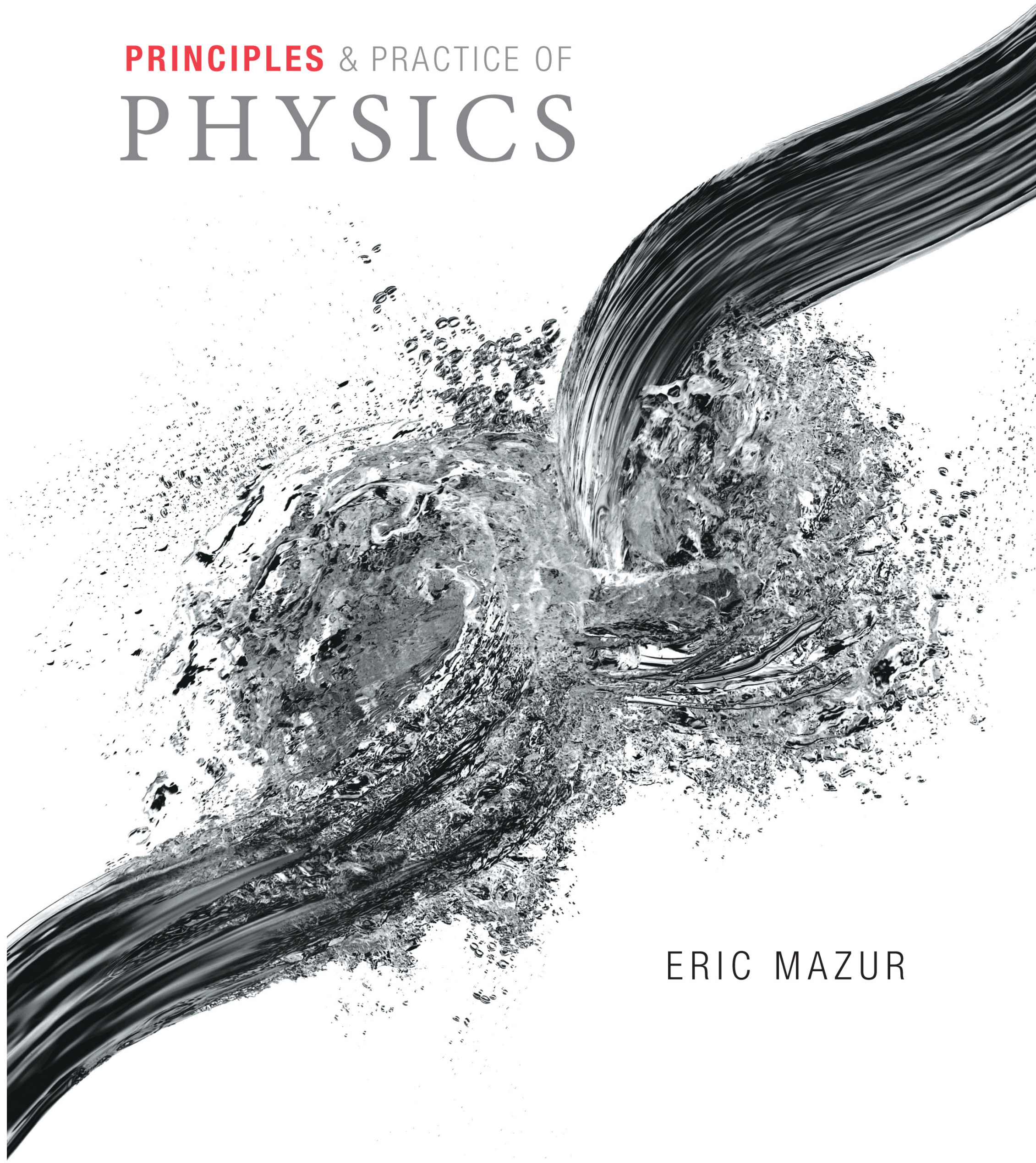
# Lecture Outline




## Chapter 17 Waves in Two and Three Dimensions

# Chapter 17: Waves in Two and Three Dimensions

**Chapter Goal: To extend the study of wave motion to two and three dimensions. Also, to study multidimensional interference and other new wave phenomena not observed in one dimension.**

## Looking Ahead: Characteristics of waves in two and three dimensions

- A **wavefront** is a curve or surface in a medium on which all points of a propagating wave have the same phase.
- A **planar wavefront** is a flat wavefront that is either a plane or a straight line.
- A **surface wave** is a wave that propagates in two dimensions and has circular wavefronts. A **spherical wave** is a wave that propagates in three dimensions and has spherical wavefronts.

## Looking Ahead: Characteristics of waves in two and three dimensions

- According to **Huygens' principle,** any wavefront may be regarded as a collection of many closely spaced, coherent point sources.
- **Diffraction** is the spreading out of waves either around an obstacle or beyond the edges of an aperture. The effect is more pronounced when the size of the obstacle or aperture is about equal to or smaller than the wavelength of the wave.
- You will learn about these characteristics of waves and the physical variables used to describe them.

# Chapter 17 Preview
## Looking Ahead: Sound waves

- **Sound** is a longitudinal *compressional* wave propagating through a solid, liquid, or gas. The wave consists of an alternating series of *compressions* (where the molecules of the medium are crowded together) and *rarefactions* (where the molecules are spaced far apart). The frequency range of *audible* sound is 20 Hz to 20 kHz.
- The speed of sound $c$ depends on the density and elastic properties of the medium. In dry air at 20° C, the speed of sound is 343 m/s.
- You will learn how to represent sound waves graphically and how the intensity of sound is represented mathematically.

# Chapter 17 Preview

## Looking Ahead: Interference effects

- Two or more sources emitting waves that have a constant phase difference are called **coherent sources**. If that constant phase difference is zero, the sources are said to be *in phase*.
- Along **nodal lines**, waves cancel each other, and so the displacement of the medium is zero. Along **antinodal lines,** the displacement of the medium is a maximum.
- The superposition of two waves of equal amplitude but slightly different frequencies results in a wave of oscillating amplitude. This effect is called **beating.**
- You will learn how to represent interference effects for sound graphically and mathematically.

# Chapter 17 Preview
## Looking Ahead: The effects of motion on sound

- The **Doppler effect** is a change in the observed wave frequency caused by the relative motion of a wave source and an observer.
- A **shock wave** is a conical (wedge-shaped) disturbance caused by the piling up of wavefronts from a source moving at a speed greater than or equal to the wave speed in the medium.
- You will learn how to represent the effects of motion on sound both graphically and mathematically.

# Chapter 17 Preview

## Looking Back: Representing waves

- A **wave** is a *disturbance* that propagates through material (the medium) or through empty space.
- A **wave pulse** is a single isolated propagating disturbance.
- The **wave function** represents the shape of a wave at any given instant and changes with time as the wave travels.

# Chapter 17 Preview
## Looking Back: Representing waves

- The **wave speed** $c$ is the speed at which a wave propagates. For a mechanical wave, $c$ is different from the speed $v$ of the particles of the medium and is determined by the properties of the medium.
- The **displacement** $\vec{D}$ of any particle of a medium through which a mechanical wave travels is a vector that points from the equilibrium position of the particle to its actual position.
- You learned about the characteristics of waves and the physical variables used to study wave motion.

# Looking Back: Representing waves

- In a **transverse** mechanical wave, the particles of the medium move perpendicular to the direction of the pulse movement.
- In a **longitudinal** mechanical wave, these particles move parallel to the direction of the pulse movement.
- In a **periodic wave,** the displacement at any location in the medium is a periodic function of time. A periodic wave is **harmonic** when the particle displacement can be represented by a sinusoidally varying function of space and time.
- You learned how to represent these types of waves both graphically and mathematically.

# Chapter 17 Preview

## Looking Back: Combining waves

- **Superposition of waves:** The resultant displacement of two or more overlapping waves is the algebraic sum of the displacements of the individual waves.
- **Interference** occurs when two waves overlap. The interference is *constructive* when the displacements due to the two waves are in the same direction and *destructive* when the displacements are in opposite directions.

# Chapter 17 Preview

## Looking Back: Combining waves

- If the displacement at a point in space remains zero as a wave travels through, that point is a **node.** The displacement at other points typically varies with time. If the displacement at a point in space varies over the greatest range as a wave travels through, that point is an **antinode.**
- You learned how to represent the overlap of waves in one dimension both graphically and mathematically.

## Looking Back: Combining waves

- When a wave pulse (the *incident* wave) reaches a boundary where the transmitting medium ends, the pulse is *reflected*, which means it reverses its direction.
- When a wave pulse is reflected from a fixed boundary, the reflected pulse is *inverted* relative to the incident pulse. When the reflection is from a boundary that is free to move, the reflected pulse is not inverted.

# Chapter 17 Preview

## Looking Back: Combining waves

- A **standing wave** is a pulsating stationary pattern caused by the interference of harmonic waves of equal amplitude and wavelengths traveling in opposite directions.
- You learned the physical properties of standing waves and their mathematical representation.

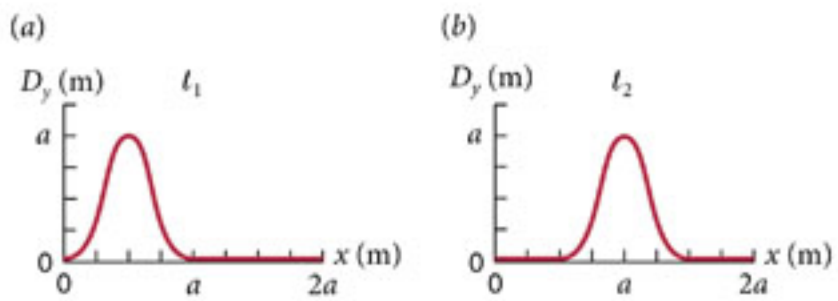
# Remaining schedule

- Today: waves in 2D, 3D (mostly qualitative)
- Tues next week: gravitation (Ch. 13)
- Thurs next week: thermal energy (Ch. 20)
- Lab next week: exam review

- Will put out practice problems for Ch. 13, 20 (not counted for a grade)
- Will put out list of sections covered on final

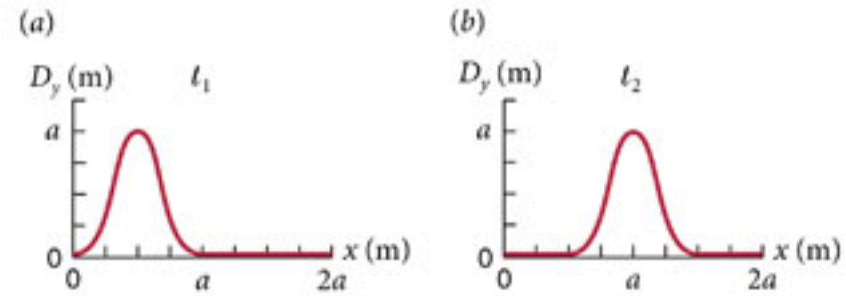
# Homework, Ch. 16

- 16.04 The graphs below show the displacement caused by a wave moving along a string at two instants, (a) $t_1$ and (b) $t_2$. Let $v_{av}$ denote the average speed of a piece of string during the time interval between $t_1$ and $t_2$.



# Homework, Ch. 16

- 16.04 cont'd




- Look at the vertical motion.
- How much distance covered between $t_1$ and $t_2$?
  - distance = $a$
- How about the horizontal motion?
  - distance = $a/2$
- Horizontal motion is the wave velocity.
- It covers half the distance in the same time …

# Homework, Ch. 16

- 16.10 A harmonic wave is made to travel along a string when you move your hand up and down. The wave has a specific period $T_1$, wavelength $\lambda_1$, amplitude $A_1$, and speed $c_1$, and also causes a certain transverse speed $v(x,t)$ of the particles that make up the rope. Then you repeat this up-and-down motion, this time completing the same motion twice as fast as before.

# Homework, Ch. 16

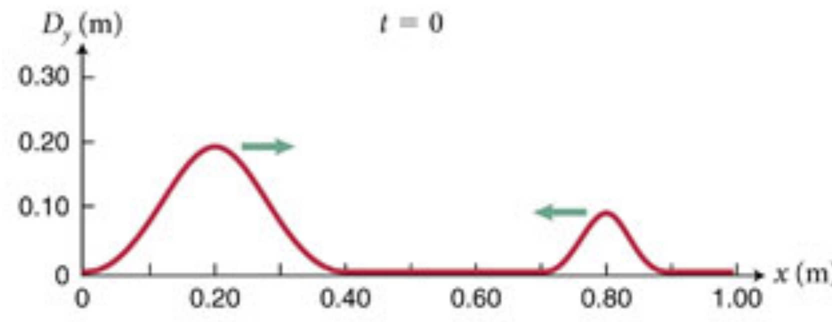
- A: Your hand is the source of the wave. If it moves at twice the speed, what happens to the period?
- B: speed only depends on the medium …
- C: $\lambda f = c = \lambda / T$   if $c$ is constant, what must $\lambda$ do if $T$ halves?
- D: Your hand is the source of the amplitude
- E: Transverse speed is the up and down oscillation, which is determined by your hand.

# Homework, Ch. 16

- 16.13

A: after time zero, what is max displacement at $x = 0$?

*Only 0.1m wave ever gets there*




B: How about at $x = 0.9$m?

*Only 0.2m wave ever gets there*

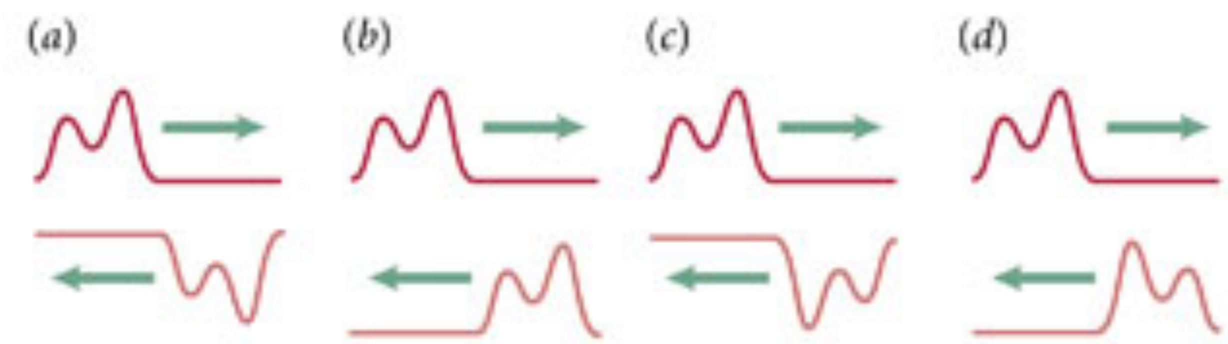
C: How about anywhere?

*Somewhere between 0.2 and 0.8m, the two overlap*

# Homework, Ch. 16

- 16.18 reflected pulse should be inverted & reflected about the vertical axis …

- Or first in = first out, but upside down



# Homework, Ch. 16

- 16.31 $f(x,t)=a\sin[bx+qt]$

- Motion? Solutions of the wave equation have the form $f(\mathrm{x},\mathrm{t}) = f(\mathrm{k}x \pm \omega t)$
- *Sign alone* determines direction of travel. + means along –x, – means along +x.

- Speed? Two ways
  - First: easiest, use the wave equation.

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

# Homework, Ch. 16

- Second? If the wave is to keep the same shape for any time, the argument of the function must stay constant.
- That is: $bx + \underline{qt} = \text{const}$

- Take a time derivative of each side …
  - $b(dx/dt) + q = 0$
  - $dx/dt = v$

- Either way, general result is
  - v = (time coefficient) / (x coefficient)

# Homework, Ch. 16

- Standing wave
- A: which way is the wave traveling? $f(\mathrm{k}x \pm \omega t)$, sign alone determines direction of travel …
- B: flip the sign
- C: superposition?
  - use $$\sin \mathrm{a} + \sin \mathrm{b} = 2 \sin\left(\frac{\mathrm{a}+\mathrm{b}}{2}\right) \cos\left(\frac{\mathrm{a}-\mathrm{b}}{2}\right)$$
  - Envelope is the x-dependent part
- D: do standing waves travel?

# Homework, Ch. 16

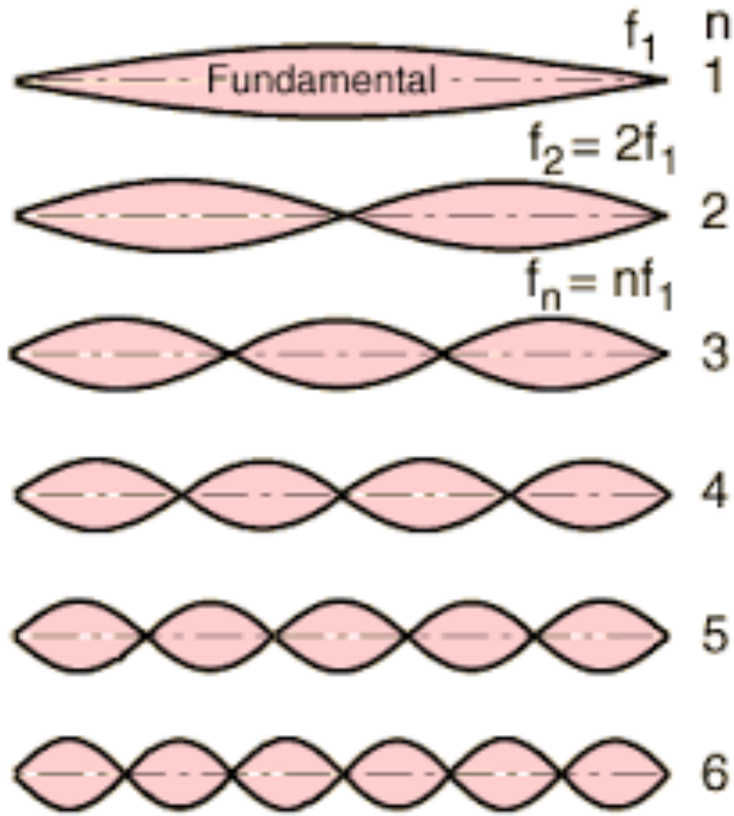
- E: if you have $y_e(x)$ right, what is it at $x = 0$? It is the same for all times.
- F: it will be straight when $y = 0$ for all $x$ at some particular $t$. This $t$ will be when the time-only function is zero …
- ... This will happen when its argument is either 0 or $\pi/2$, depending on the function you found ...
- G: It may be straight, but that only says something about potential energy ...

# Homework, Ch. 16

- Nodes
- A: normal modes are all about fitting oscillations within a given distance. Implies constraints ..
- B: Amplitude is irrelevant, wavelengths have to fit
- C: Minimum to satisfy boundary conditions at 0 and L? Fit $\lambda/2$, $\lambda$, *or* $3\lambda/2$ within distance L …
- D, E: use $f = c/\lambda$ ..

# Homework, Ch. 16

- Standing waves: fit half integer numbers of wavelengths in a certain spacing




http://hyperphysics.phy-astr.gsu.edu/hbase/waves/string.html

# Homework, Ch. 16

- 16.48 How long does it take a wave to travel the length of a cable?
- The speed of a wave in the cable is

$$v = \sqrt{\frac{T}{\mu}}$$

- Given a speed and distance, you can find the time well enough.
- So what is the tension? Weight of the cable is negligible … so what force does the cable counteract?

# Homework, Ch. 16

- 16.60 You hold one end of a string that is attached to a wall by its other end. The string has a linear mass density of 0.067 kg/m. You raise your end briskly at 11 m/s for 0.016s, creating a transverse wave that moves at 39 m/s

- A: Work: you have the formula, 16.33.
- B: Where did the energy come from?
- C,D: last time - energy is half kinetic, half potential …

# Homework, Ch. 16

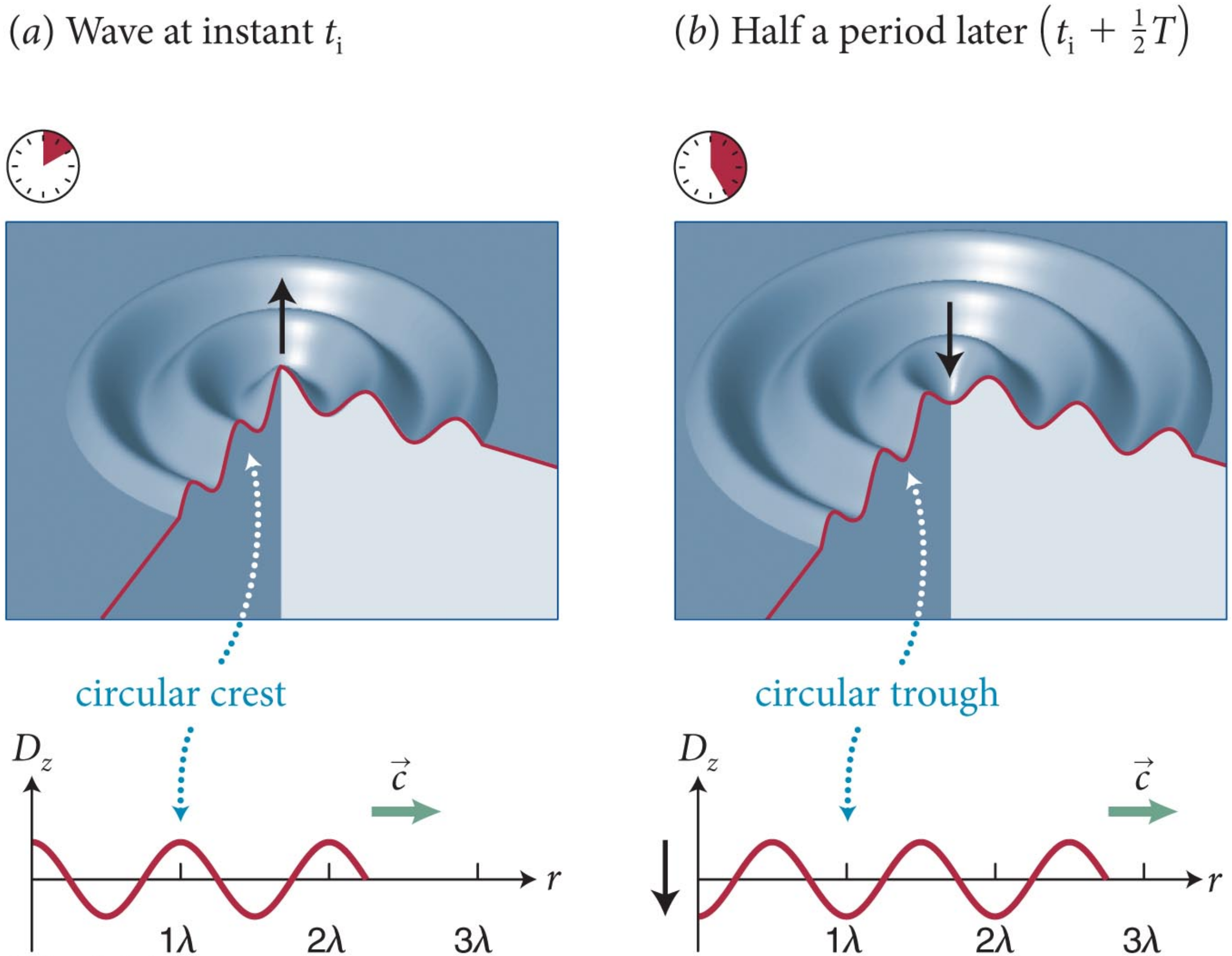
- Vibrating string: you've got this.

- Piano strings: $T = p_T A$, $\mu = \rho A$
  - Given that, you know the wave's speed.
  - $f = c/\lambda = c/2L$ for the lowest normal mode

# Concepts

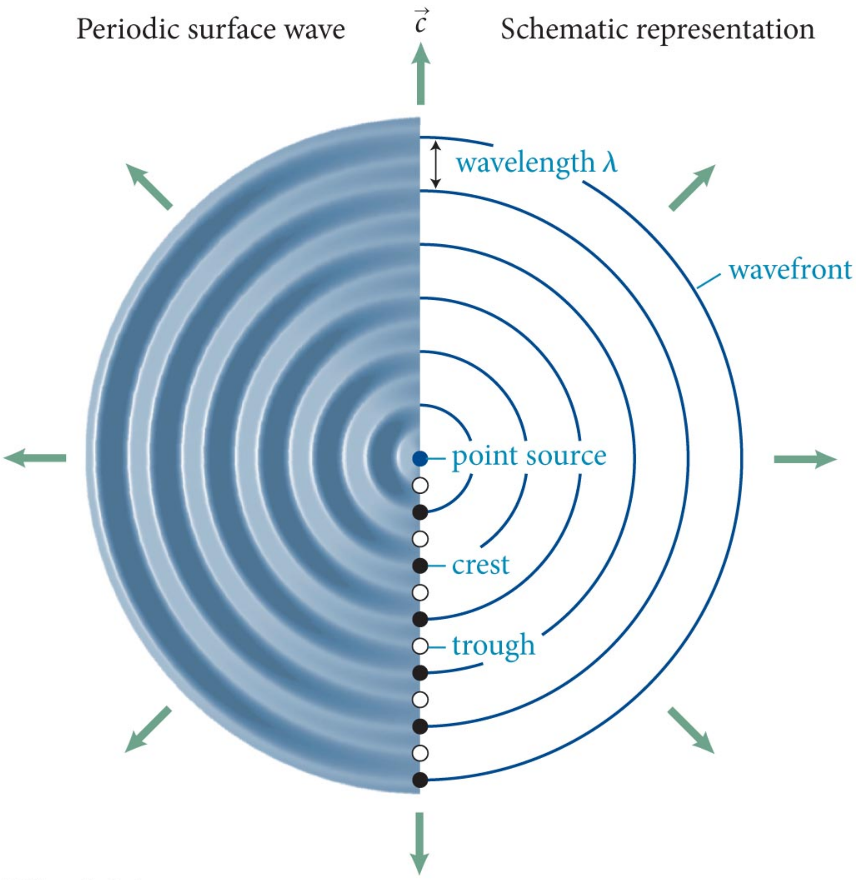
# Section 17.1: Wavefronts

- The figure shows cutaway views of a periodic **surface wave** at two instants that are half a period apart.



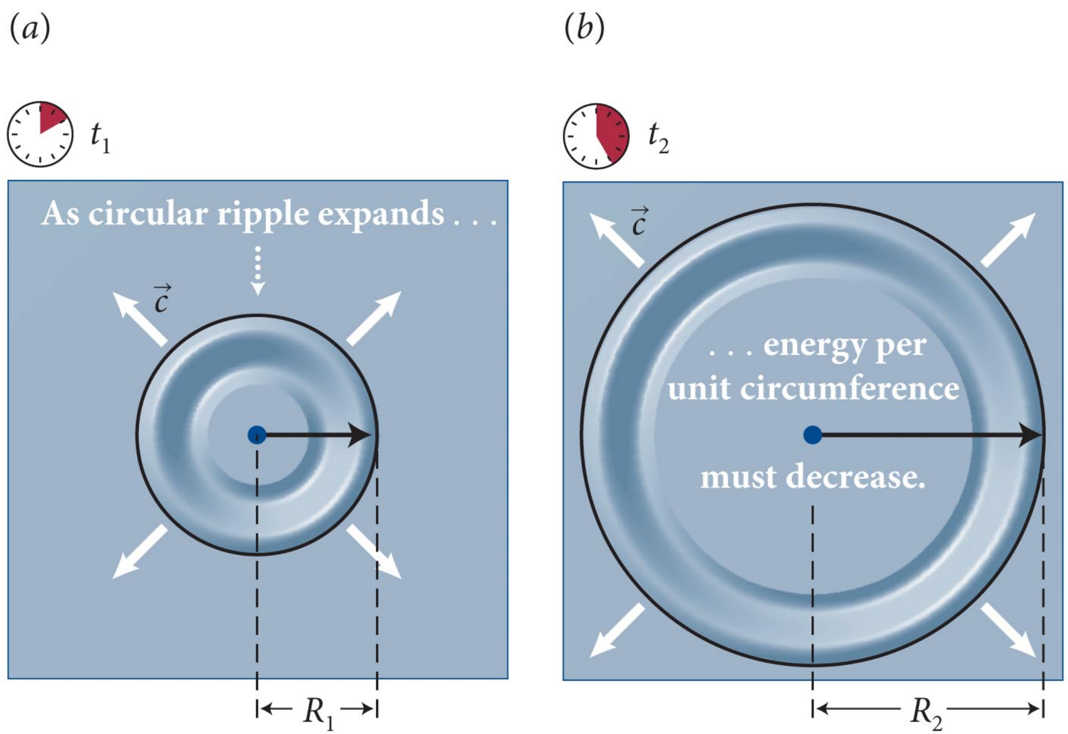
# Section 17.1: Wavefronts

- When the source of the wavefront can be localized to a single point, the source is said to be a **point source**.
- The figure shows a periodic surface wave spreading out from a point source.
- The curves (or surfaces) in the medium on which all points have the same phase is called a **wavefront**.



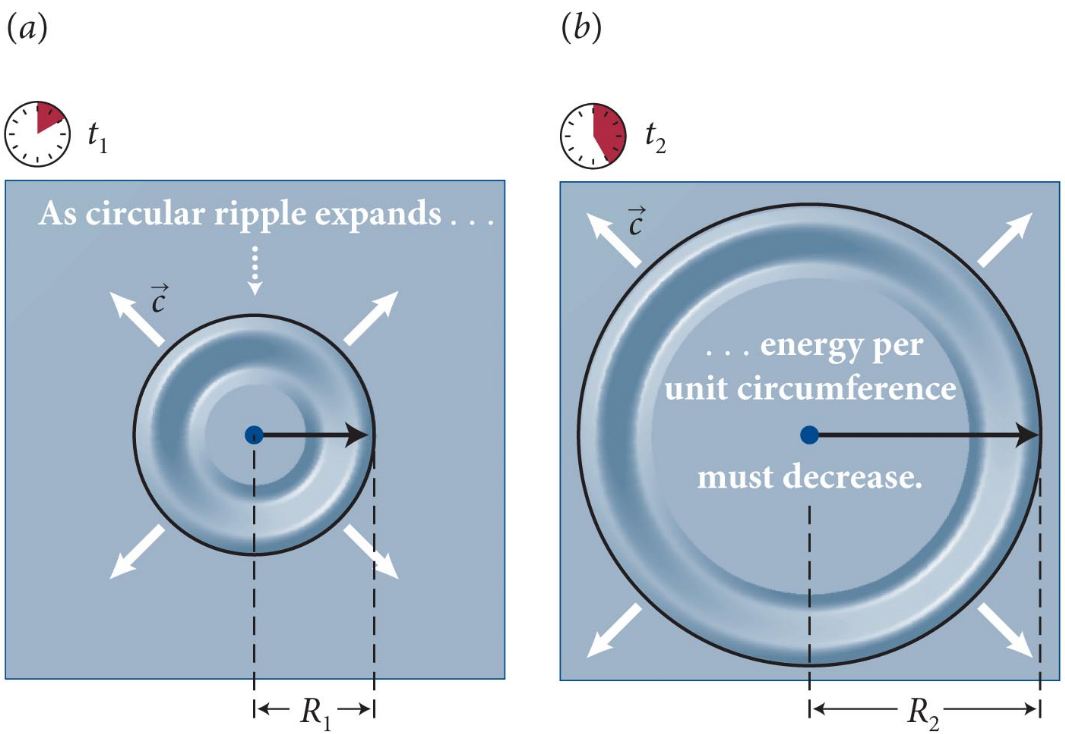
# Section 17.1: Wavefronts

- Consider the figure.
- If we assume that there is no energy dissipation, then there is no loss of energy as the wave moves outward.
- As the wavefront spreads, the circumference increases, and hence the energy per unit length decreases.



# Checkpoint 17.1

**17.1** Let $t_2 = 2t_1$ in Figure 17.3. (*a*) How does $R_1$ compare with $R_2$? (*b*) If the energy in the wave is $E$ and there is no dissipation of energy, what is the energy per unit length along the circumference at $R_1$? At $R_2$? (*c*) How does the energy per unit length along a wavefront vary with radial distance $r$?



# Checkpoint 17.1

**17.1** (a) The wave speed $c$ is constant, so in twice the time it covers twice the distance, $R_2 = 2R_1$

(b) Energy per unit length?

At 1: $E/2\pi R_{1.}$

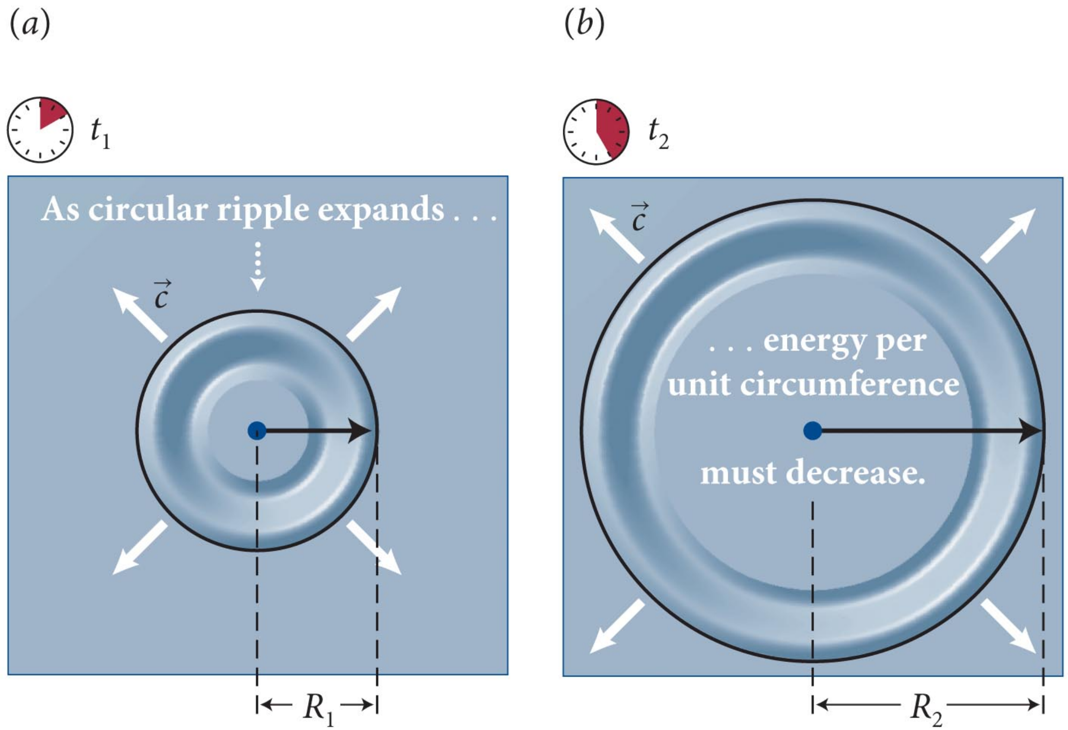
At 2: If the radius doubles, so does the circumference. Now at point 2, same energy but double the circumference, so

$E/2\pi R_2 = E/4\pi R_1$

(c) Energy per unit length goes as $1/r$ since circumference increases with $r$

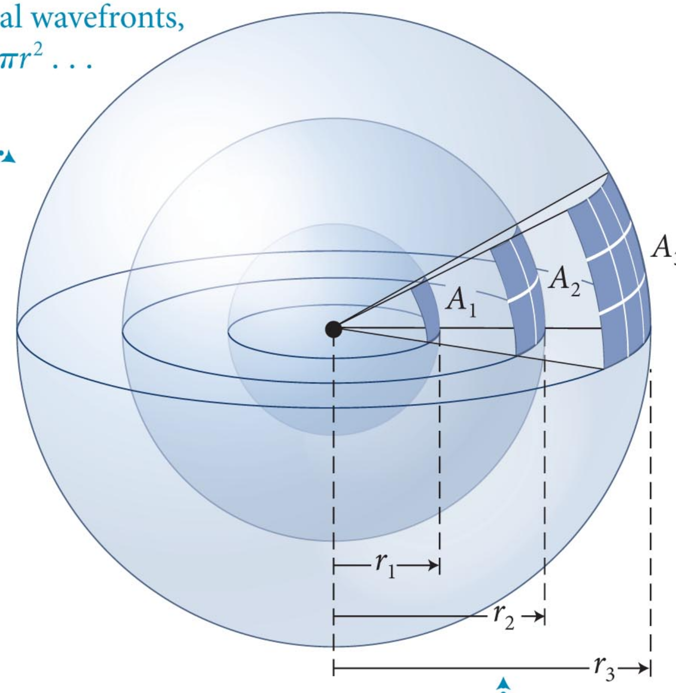
# Section 17.1: Wavefronts

- The expansion of the circular wavefronts causes the energy per unit length along the wavefront to decrease as $1/r$.
- In Chapter 6 we saw $E_\lambda = ½(\mu\lambda)\omega^2 A^2$ (Eq. 16.41),
- Therefore, it follows that for waves in two dimensions $A \sim 1/\sqrt{r}$.
  - *e.g., water waves*



# Section 17.1: Wavefronts

- The waves that spread out in three dimensions are called **spherical waves**.
- The energy carried by a spherical wavefront is spread out over a spherical area of $A = 4\pi r^2$.
- So, for waves in three dimensions, $E \sim 1/r^2$, and therefore $A \sim 1/r$.



# Example 17.1 Ripple amplitude

The amplitude of a surface wave for which $\lambda = 0.050$ m is 5.0 mm at a distance of 1.0 m from a point source. What is the amplitude of the wave

(a) 10 m from the source and

(b) 100 m from the source

## Example 17.1 Ripple amplitude (cont.)

❶ GETTING STARTED I am given that the amplitude $A = 5.0$ mm at $r = 1.0$ m. As the wave spreads out, its amplitude diminishes, and I need to calculate the amplitude at $r = 10$ m and $r = 100$ m. In addition I need to determine by how much the wave attenuates as it propagates over a 100-period time interval past these two positions.

## Example 17.1 Ripple amplitude (cont.)

❷ DEVISE PLAN Because the wave is a 2D surface wave, the amplitude is proportional to $1/\sqrt{r}$ .

I know the amplitude $A_{1.0\text{ m}}$ at $r = 1.0$ m, so I can use this dependence to determine the amplitude at other distances from the source.

For parts *a* and *b*, I need to determine $A_{10\text{ m}}$ and $A_{100\text{ m}}$ at $r = 10$ m and $r = 100$ m.

## Example 17.1 Ripple amplitude (cont.)

❸ EXECUTE PLAN (*a*) The ratio of the amplitudes must go as the square root of the ratio of the distances. At 1.0 m and 10 m we have

$$\sqrt{(1.0\ \mathrm{m})}\Big/\sqrt{(10\ \mathrm{m}} = \sqrt{(1.0/10)} = 0.32,$$

and so the amplitude at 10 m is 0.32 × (5.0 mm) = 1.6 mm. ✔

(*b*) At 100 m, $\sqrt{(1.0\ \mathrm{m})}\Big/\sqrt{(100\ \mathrm{m}} = 0.10$, and so the amplitude is 0.10 × (5.0 mm) = 0.50 mm. ✔

## Example 17.1 Ripple amplitude (cont.)

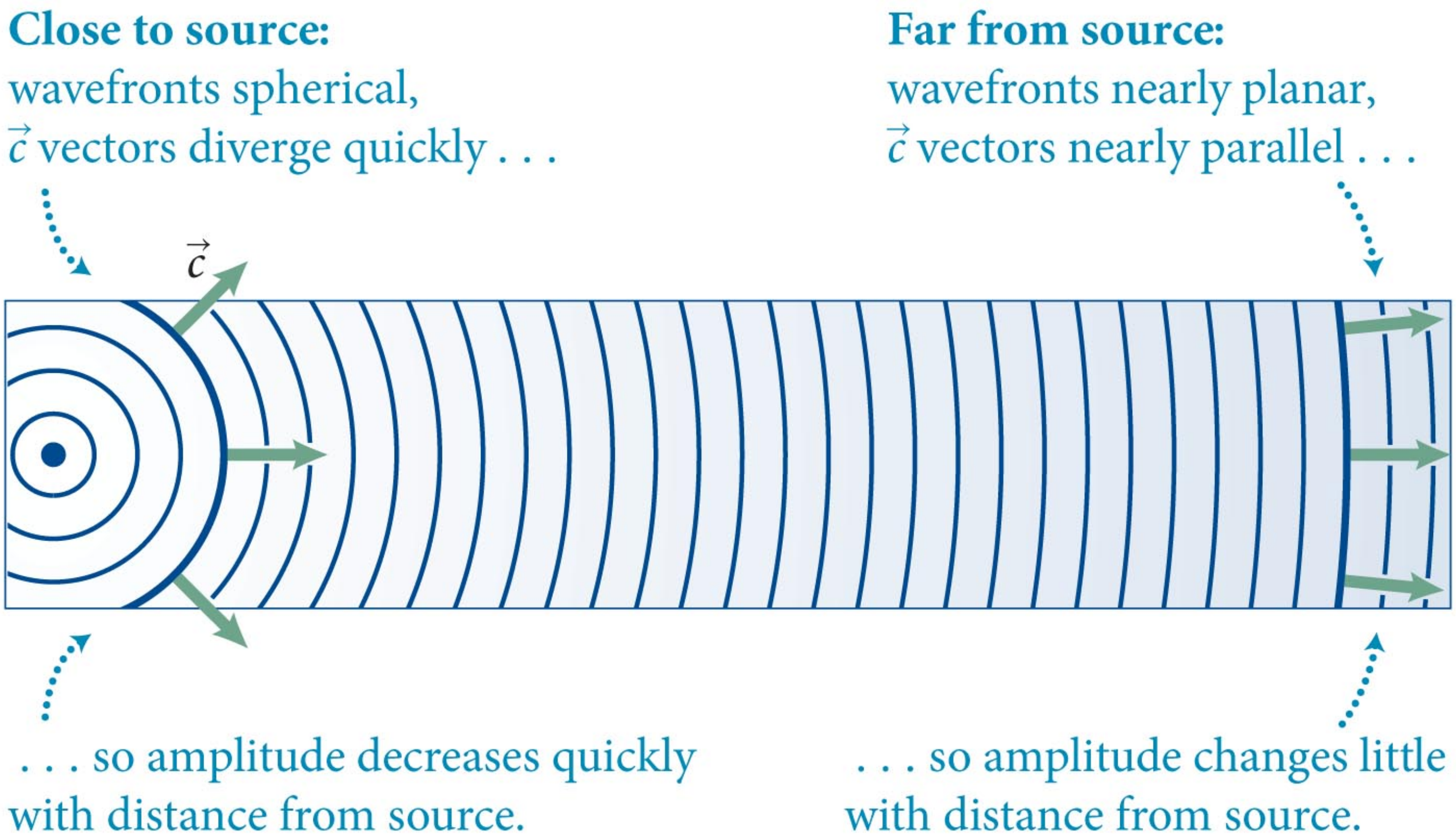
❹ EVALUATE RESULT The amplitudes at 10 m and 100 m are both smaller than the amplitude at 1.0 m, which is what I expect.

From 1 m to 10 m, factor 3 decrease. F*rom 10 m to 100 m also a factor of three*, even though distance is 10 times larger.

Amplitude decays more slowly than linear

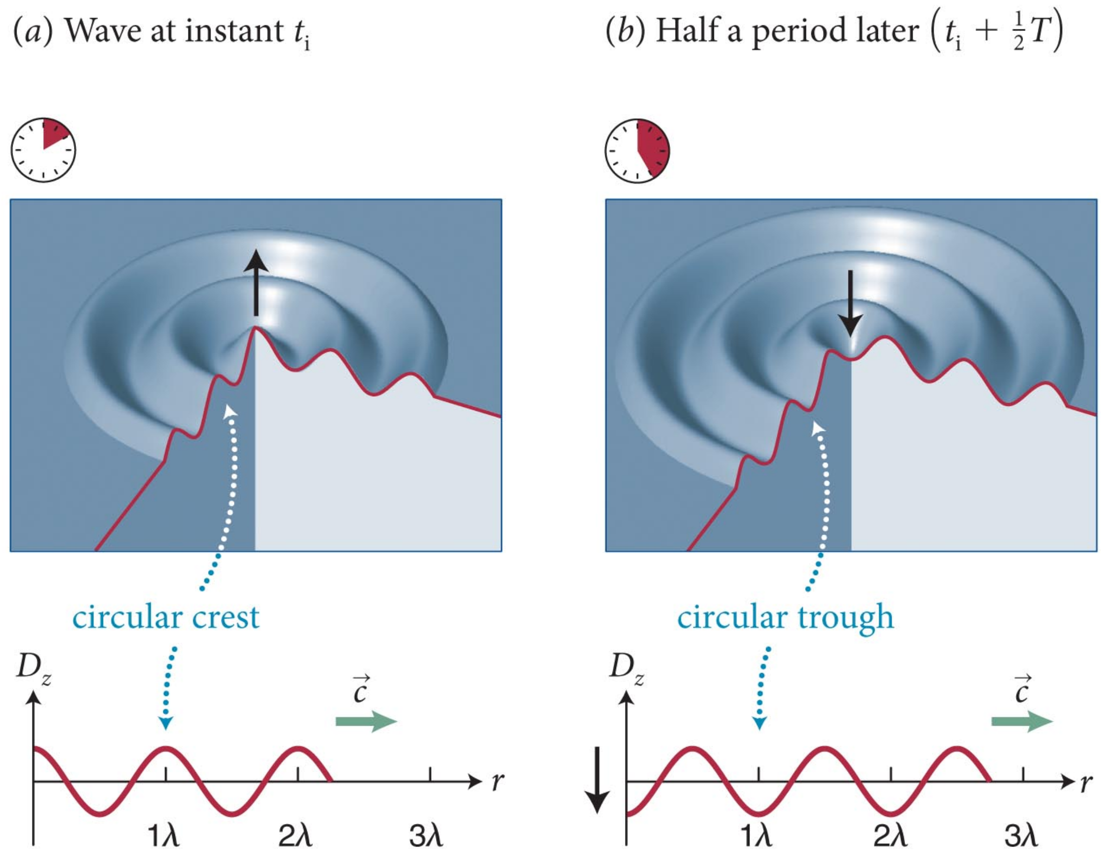
# Section 17.1: Wavefronts

- Far from a point source, the spherical wavefronts essentially become a two-dimensional flat wavefront called a **planar wavefront**.



# Checkpoint 17.2

**17.2** Notice that in the views of the surface wave in Figure 17.1 the amplitude does not decrease with increasing radial distance $r$. How could such waves be generated?



# Checkpoint 17.2

**17.2** Would work to *decrease* the source amplitude as a function of time.

First wave out is diminished when the second one is created, so make the second one smaller to compensate.

By the time the third one comes out, both the first and second are smaller (but still equal), so make the third one even smaller …

Makes it uniform over space, but not in time – uniformly decreases over entire wave pattern.

# Section 17.1
# Question 1

Which of the following factors plays a role in how much a wave's amplitude decreases as the wave travels away from its source? Answer all that apply.

1. Dissipation of the wave's energy
2. Dimensionality of the wave
3. Destructive interference by waves created by other sources

# Section 17.1
# Question 1

Which of the following factors plays a role in how much a wave's amplitude decreases as the wave travels away from its source? Answer all that apply.

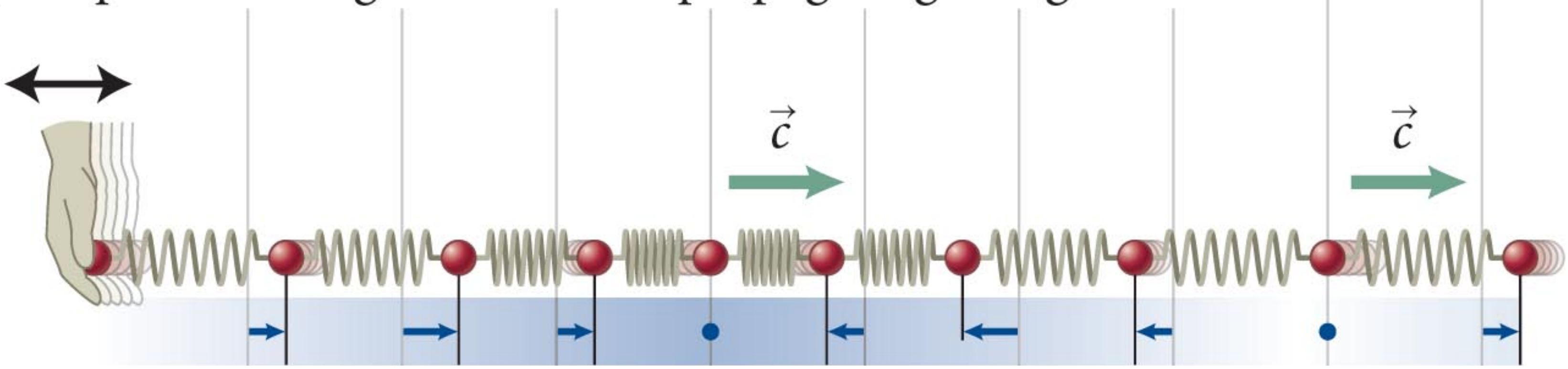
✔ 1. Dissipation of the wave's energy
✔ 2. Dimensionality of the wave
3. Destructive interference by waves created by other sources (don't lose any energy/amplitude this way!)

## Section Goals

You will learn to

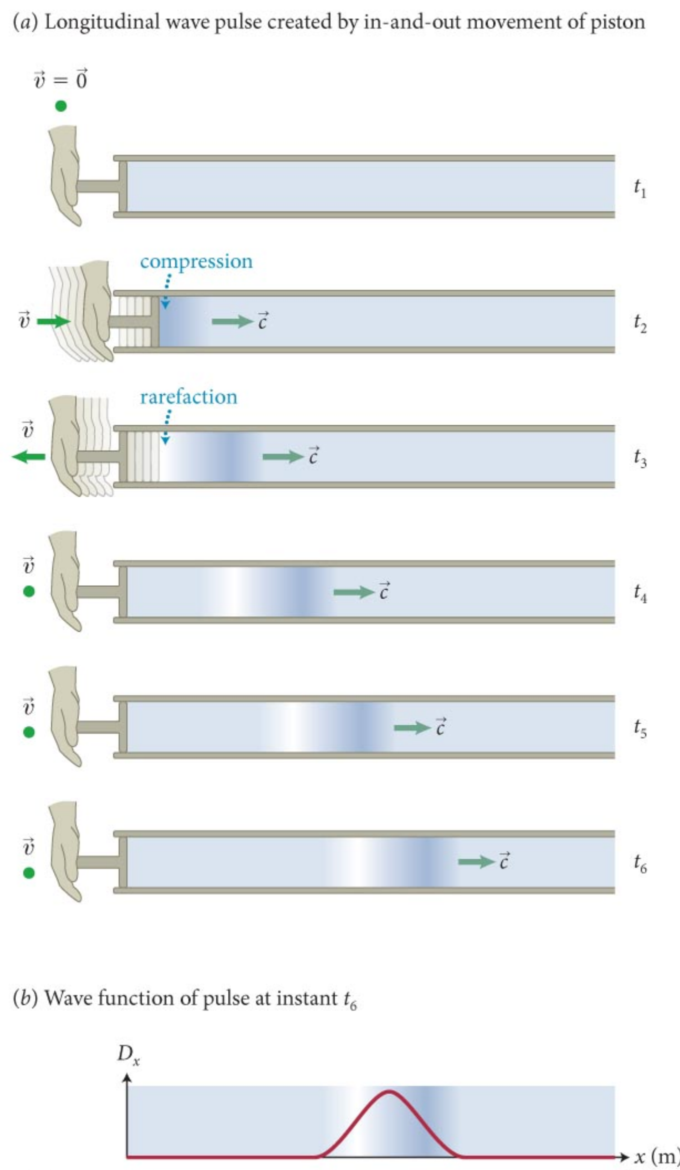
- Define the **physical characteristics** of sound.
- Represent sound graphically.

(*b*) Snapshot of longitudinal wave propagating along chain



# Section 17.2: Sound

- Longitudinal waves propagating through any kind of material is what we call **sound**.
- The human ear can detect longitudinal waves at frequencies from 20 Hz to 20 kHz.
- Sound waves consist of an alternating series of *compressions* and *rarefactions*.
- For dry air at 20° C, the speed of sound is 343 m/s.



## Exercise 17.2 Wavelength of audible sound

Given that the speed of sound waves in dry air is 343 m/s, determine the wavelengths at the lower and upper ends of the audible frequency range (20 Hz–20 kHz).

# Exercise 17.2 Wavelength of audible sound (cont.)

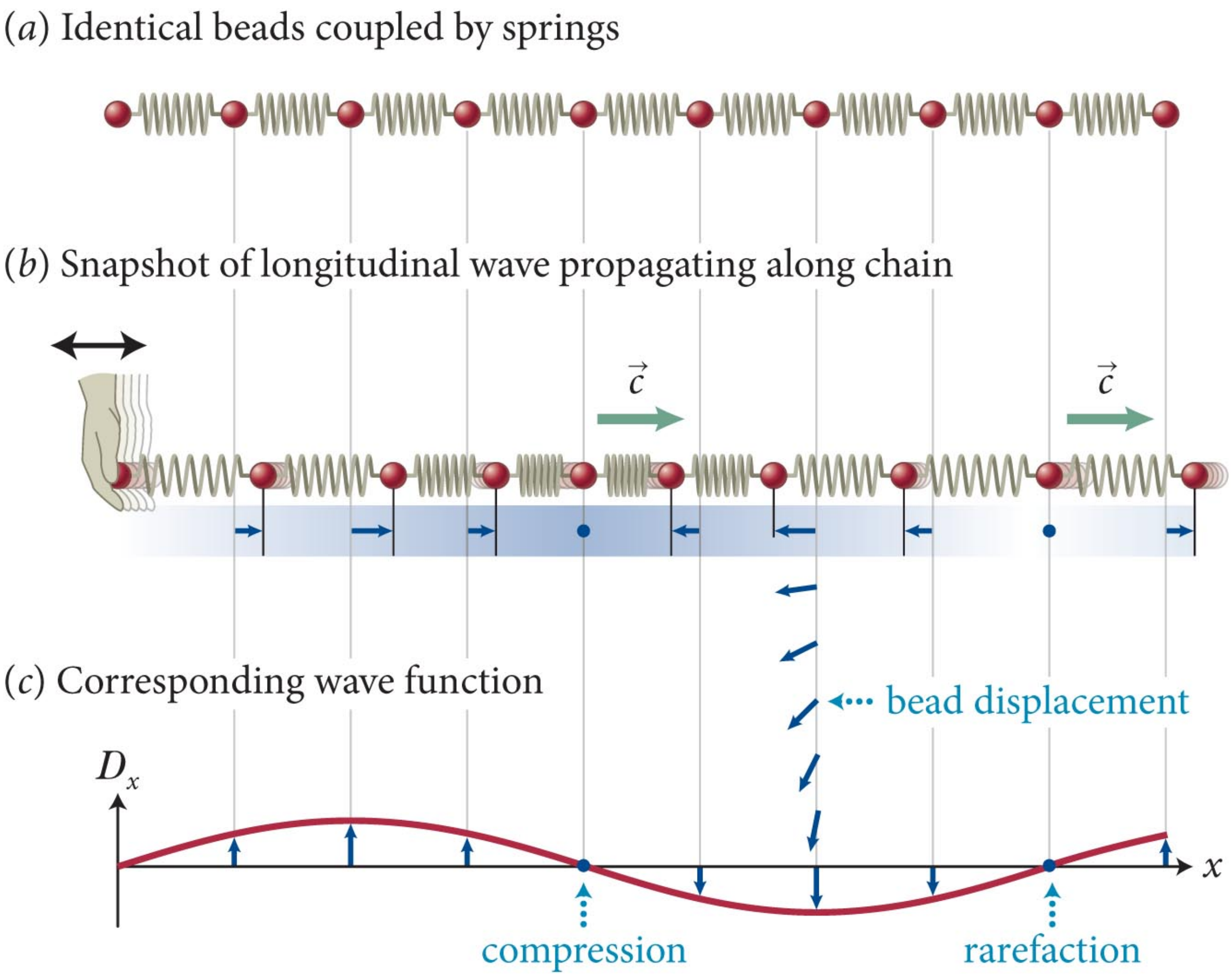
**SOLUTION** The wavelength is equal to the distance traveled in one period. At 20 Hz, the period is 1/(20 Hz) = 1/(20 $s^{-1}$) = 0.050 s, so the wavelength is (343 m/s)(0.050 s) = 17 m. ✔

The period of a wave of 20 kHz is 1/(20,000 Hz) = $5 \times 10^{-5}$ s, so the wavelength is (343 m/s)($5.0 \times 10^{-5}$ s) = 17 mm. ✔

Conveniently, the size of everyday objects …

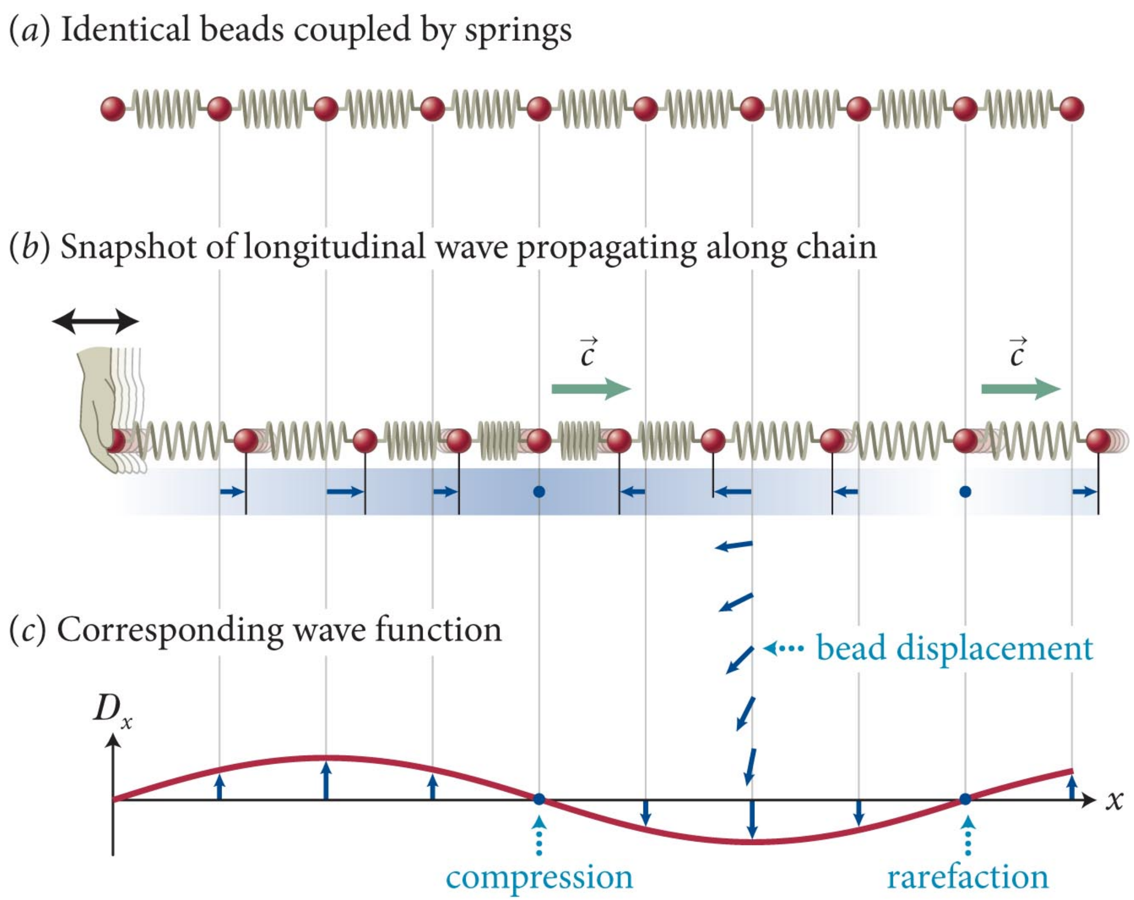
# Section 17.2: Sound

- The figure illustrates a mechanical model for a longitudinal waves.



# Checkpoint 17.3

**17.3** Does the wave speed along the chain shown in Figure 17.9 increase or decrease when (*a*) the spring constant of the springs is increased and (*b*) the mass of the beads is increased?



# Checkpoint 17.3

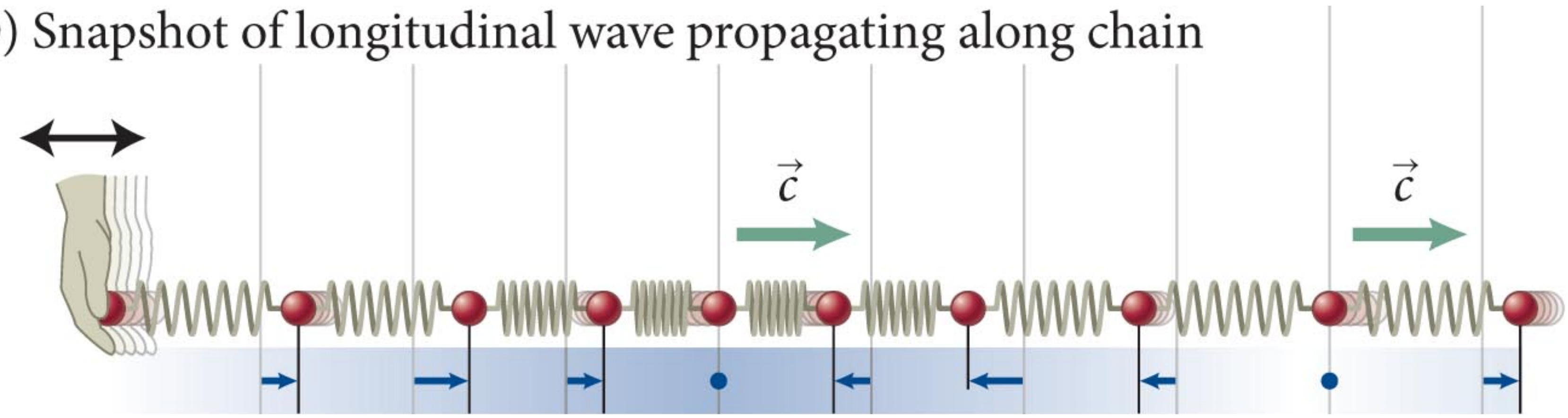
**17.3** (a) Increase – the greater spring constant, the faster any disturbance is passed along. Just like increasing tension in a string (same mechanical model)!

(b) Decrease – greater mass slows down the transmission of the wave just like with beads on a string.

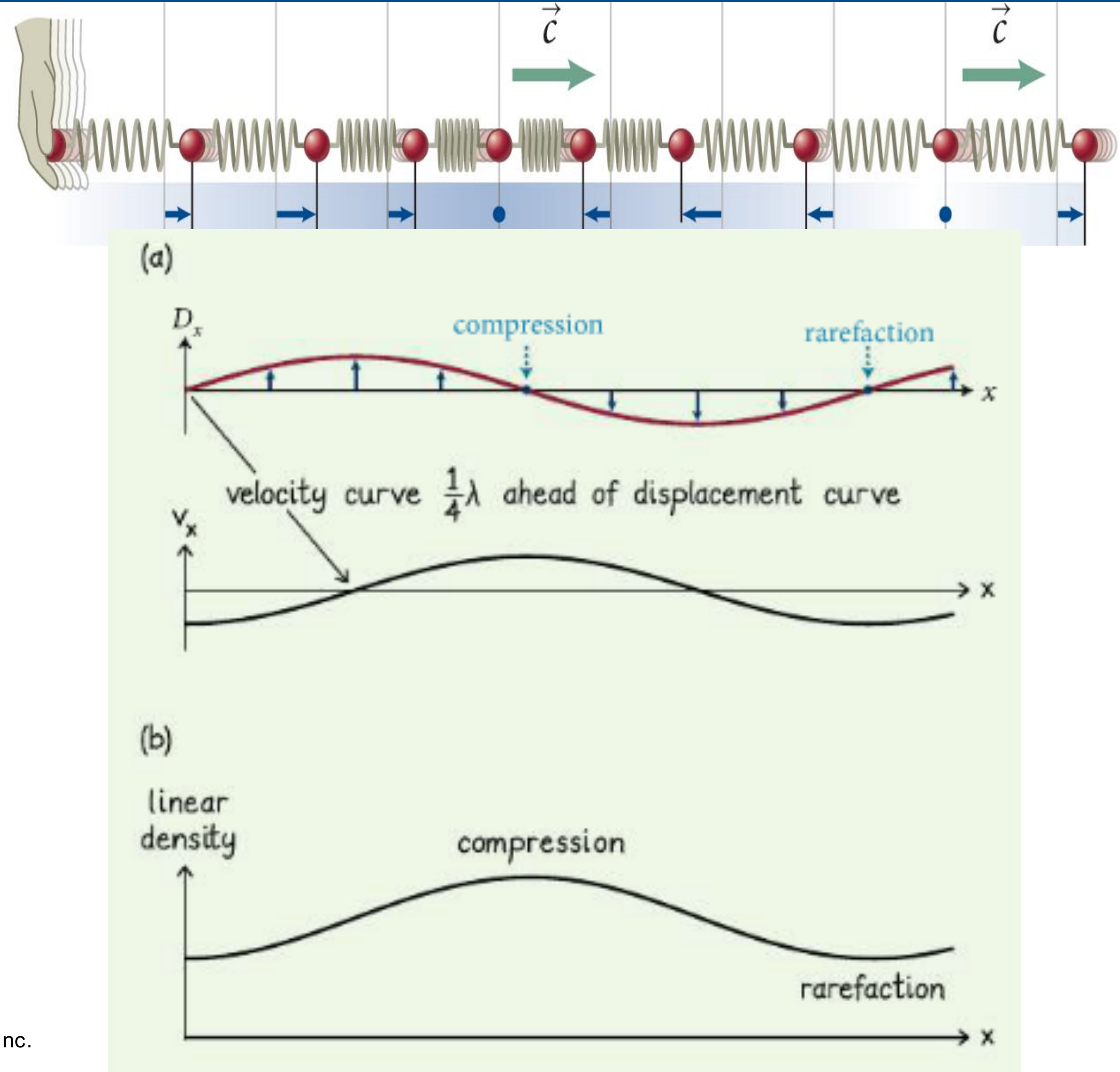
# Checkpoint 17.4

**17.4** (*a*) Plot the velocity of the beads along the chain in Figure 17.9*b* as a function of their equilibrium position *x*. (*b*) Plot the linear density (number of beads per unit length) as a function of *x*.

(*b*) Snapshot of longitudinal wave propagating along chain

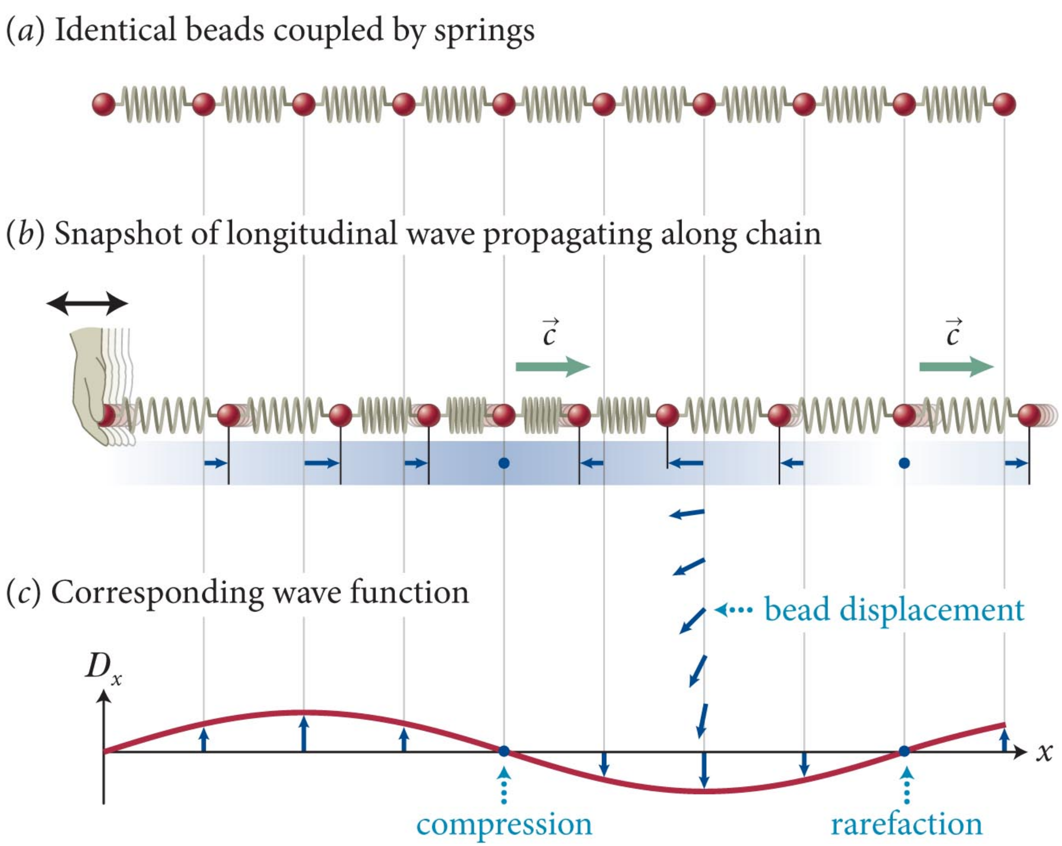


# Checkpoint 17.4



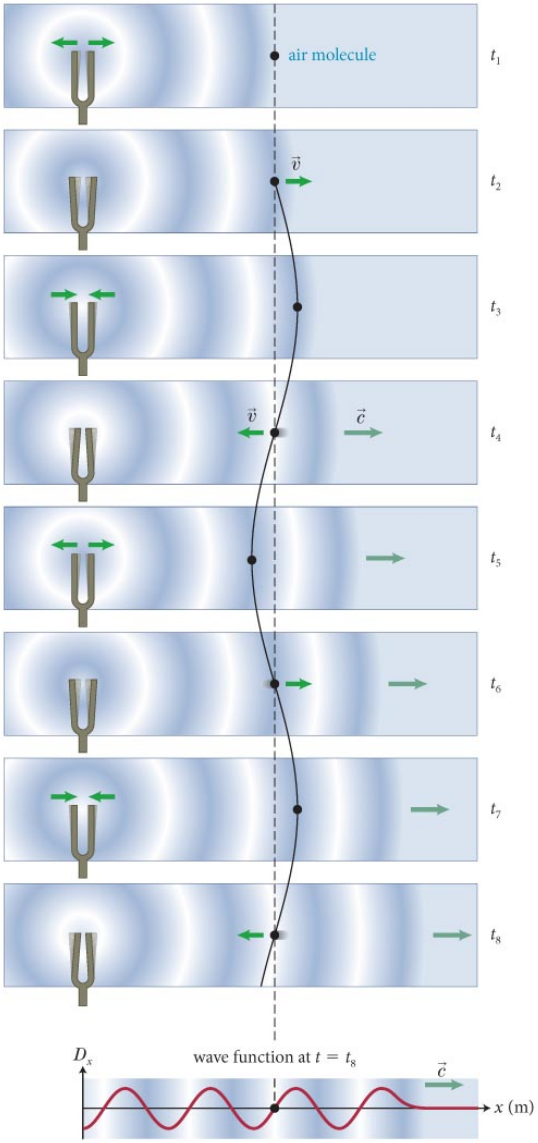
# Section 17.2: Sound

- Longitudinal waves can also be represented by plotting the linear density of the medium as a function of position.
- **The compressions and rarefactions in longitudinal waves occur at the locations where the medium displacement is zero.**



# Section 17.2: Sound

- The figure shows a sound wave generated by an oscillating tuning fork.
- At any fixed position: oscillates in time
- At any given time: spatial oscillation

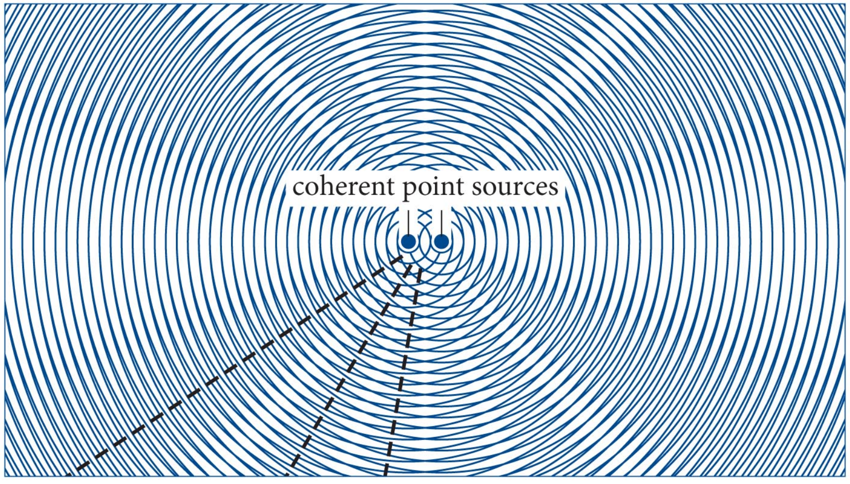


# Section 17.3: Interference

## Section Goals

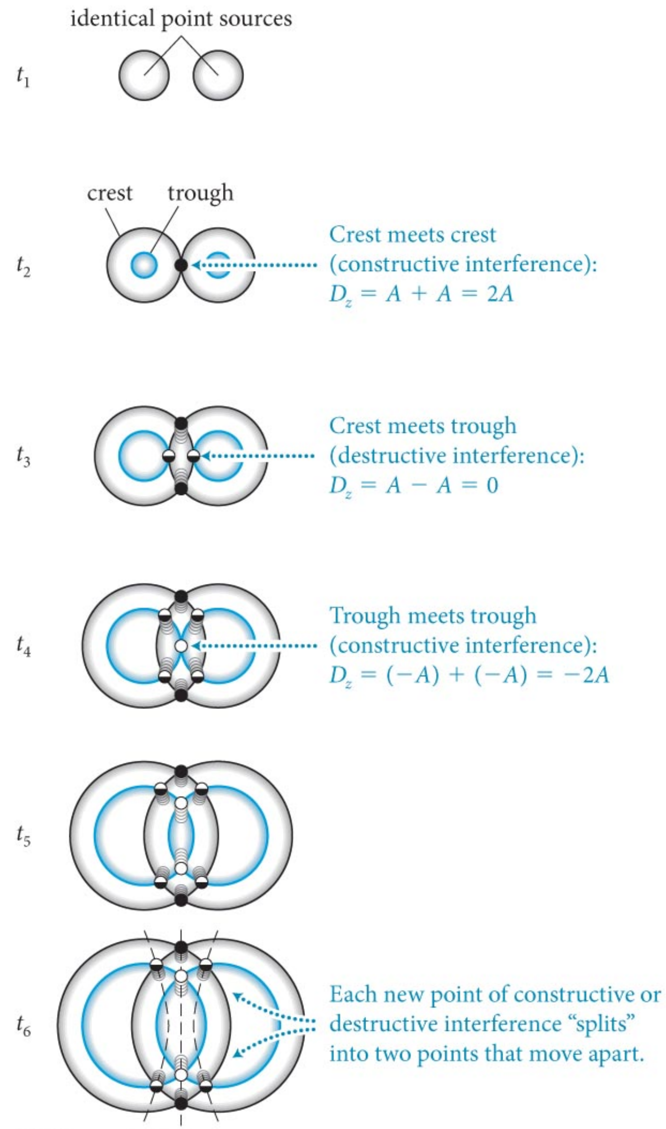
You will learn to

- Visualize the superposition of two or more two- or three-dimensional waves traveling through the same region of a medium at the same time.
- Define and represent visually the nodal and antinodal lines for interference in two dimensions.



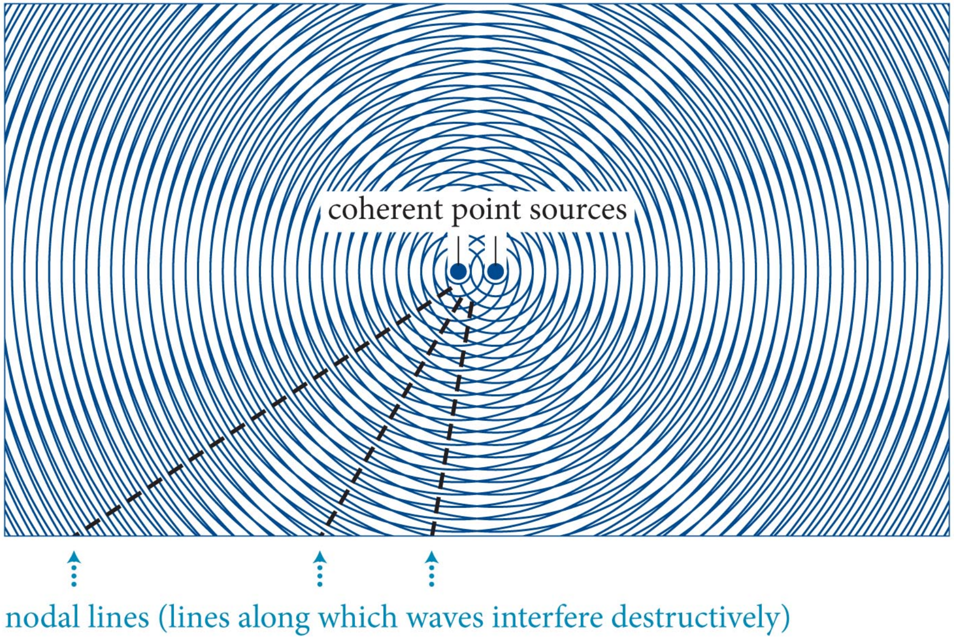
# Section 17.3: Interference

- Let us now consider the superposition of overlapping waves in two and three dimensions.
- The figure shows the interference of two identical circular wave pulses as they spread out on the surface of a liquid.

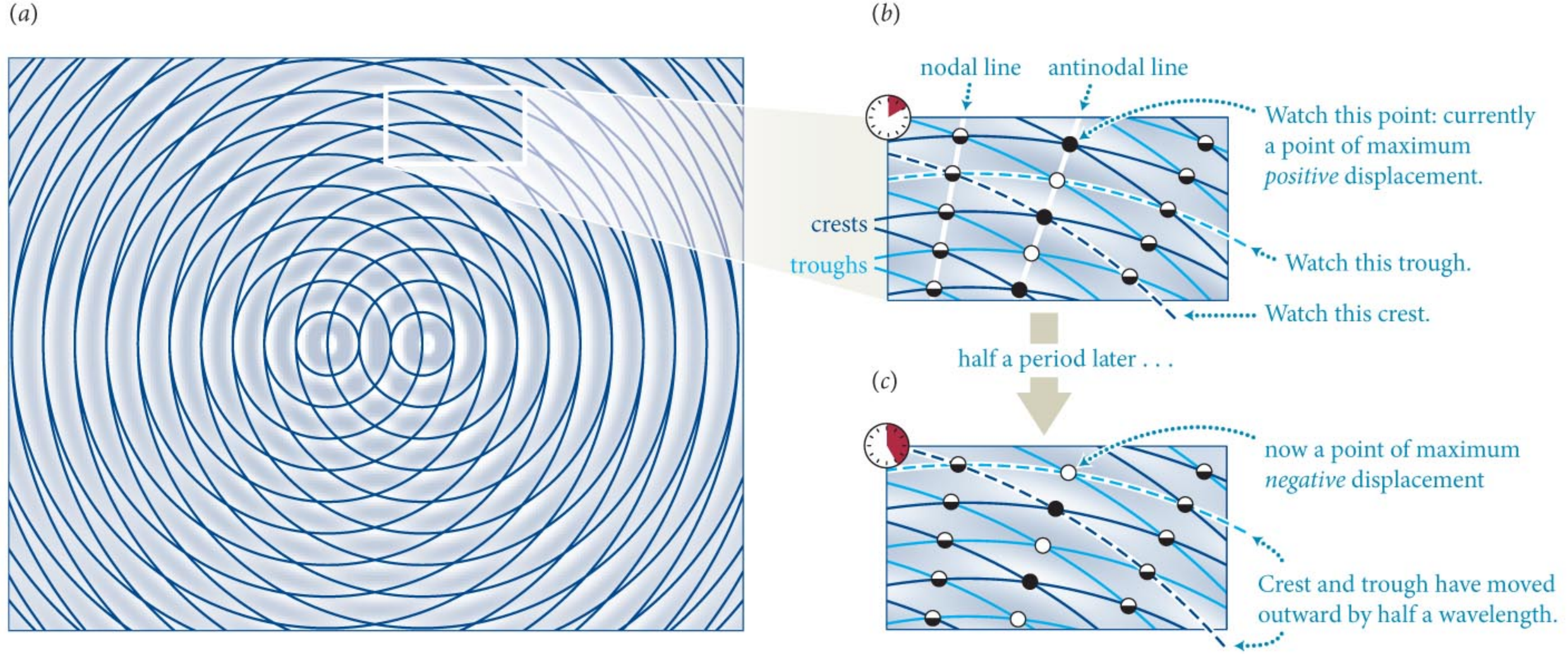


# Section 17.3: Interference

- Sources that emit waves having a constant phase difference are called **coherent sources**.
- The pattern produces by overlapping circular wavefronts is called a *Moiré pattern*.
- Along **nodal lines** the two waves cancel each other and the vector sum of the displacement is always zero.

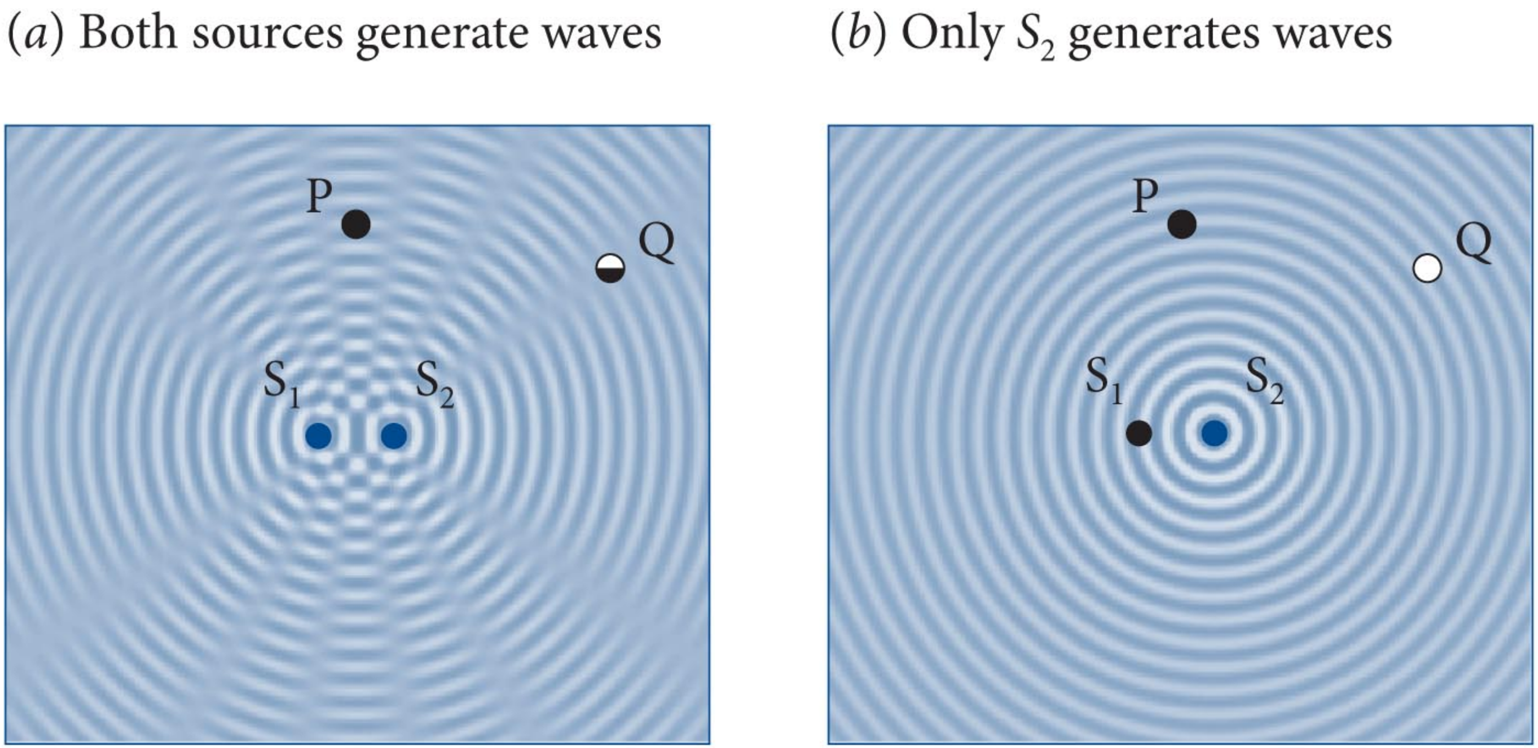


- The figure shows a magnified view of the interference pattern seen on the previous slide.
- Along **antinodal** lines the displacement is a maximum.



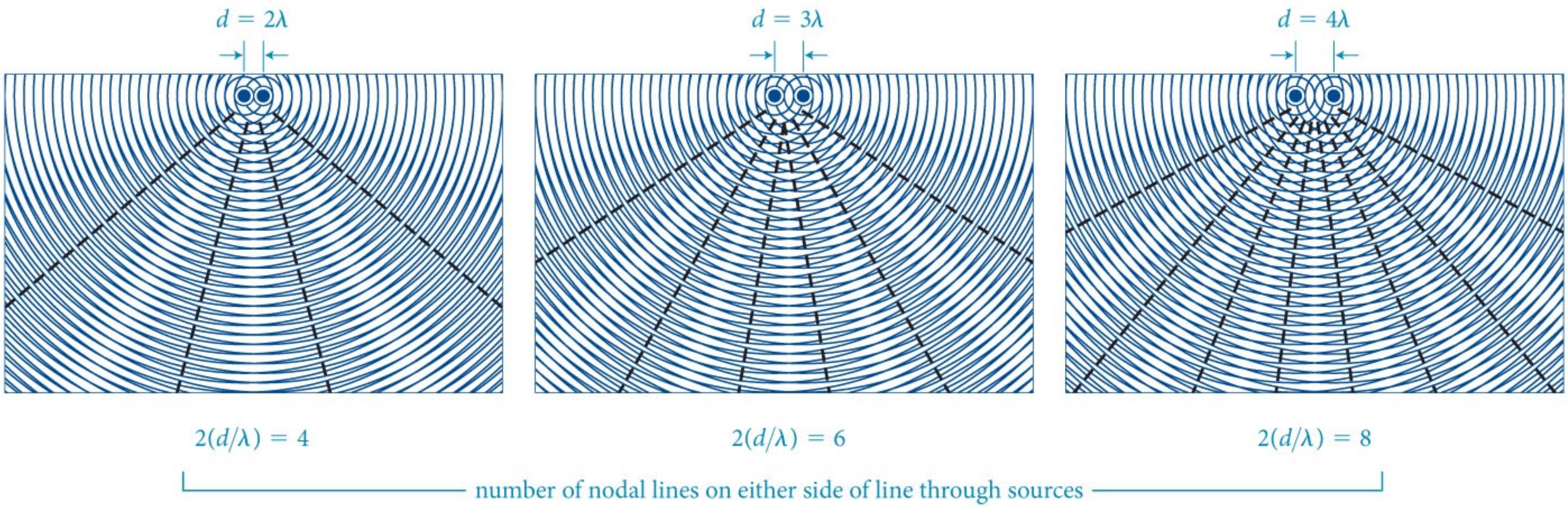
# Section 17.3: Interference

- One consequence of nodal regions is illustrated in the figure.
  - **When the waves from two coherent sources interfere, the amplitude of the sum of these waves in certain directions is less than that of a single wave.**



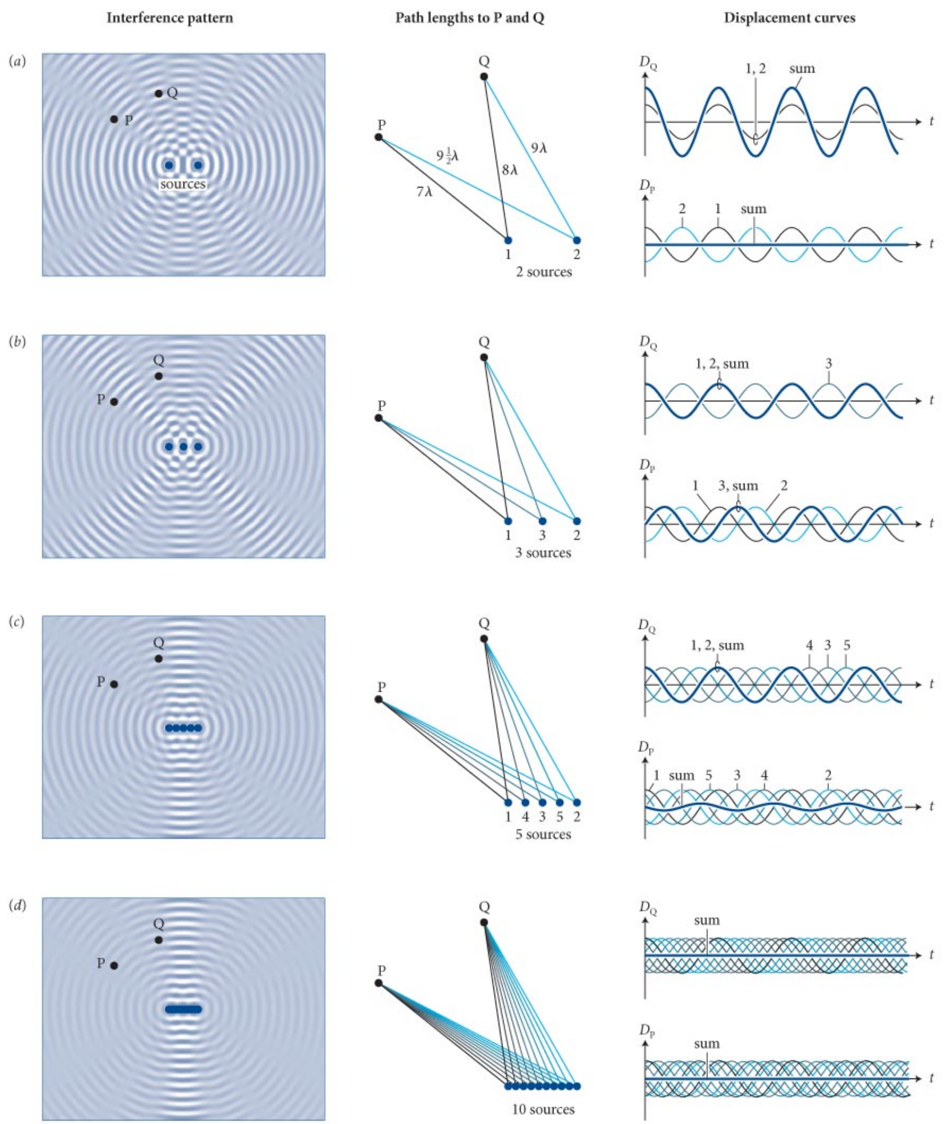
# Section 17.3: Interference

- The effect that the separation between the two point sources have on the appearance of nodal lines is shown in the figure.
  - **If two coherent sources located a distance *d* apart emit identical waves of wavelength $\lambda$, then the number of nodal lines on either side of a straight line running through the centers of the sources is the greatest integer smaller than or equal to $2(d/\lambda)$.**



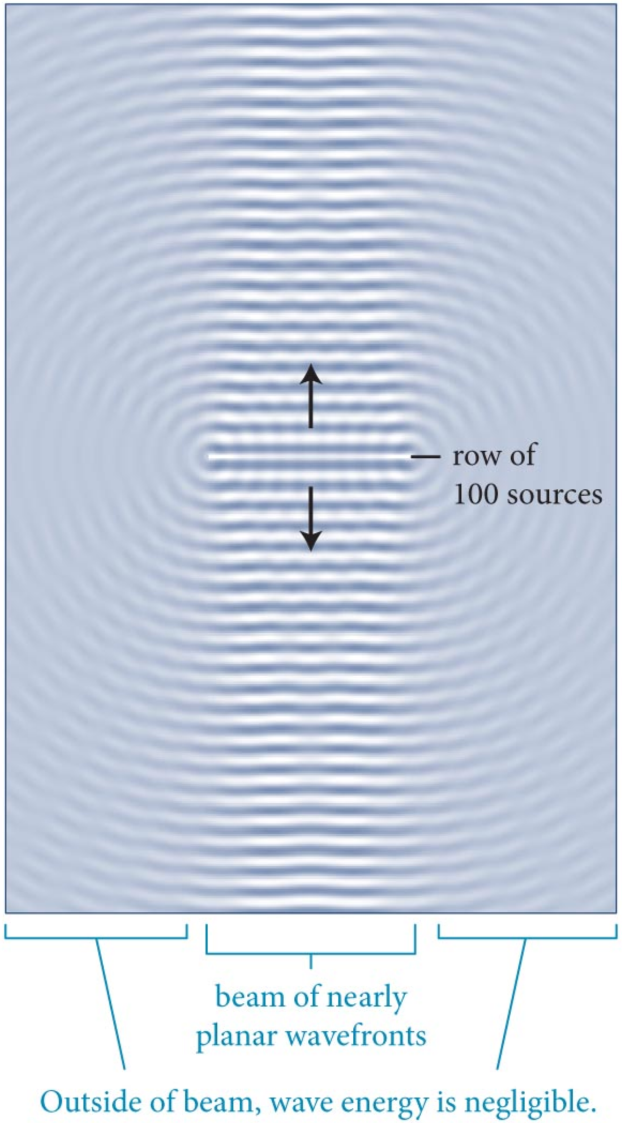
# Section 17.3: Interference

- With more than two coherent sources?
- Do one pair first, then add a third source to the resultant of that pair. Repeat.
- Find the path lengths from either source
- Difference is ½ **integer: destructive**
- Difference is **integer: constructive**



# Section 17.3: Interference

- The figure shows what happens when 100 coherent sources are placed close to each other:
  - **When many coherent point sources are placed close together along a straight line, the waves nearly cancel out in all directions except the direction perpendicular to the axis of the sources.**

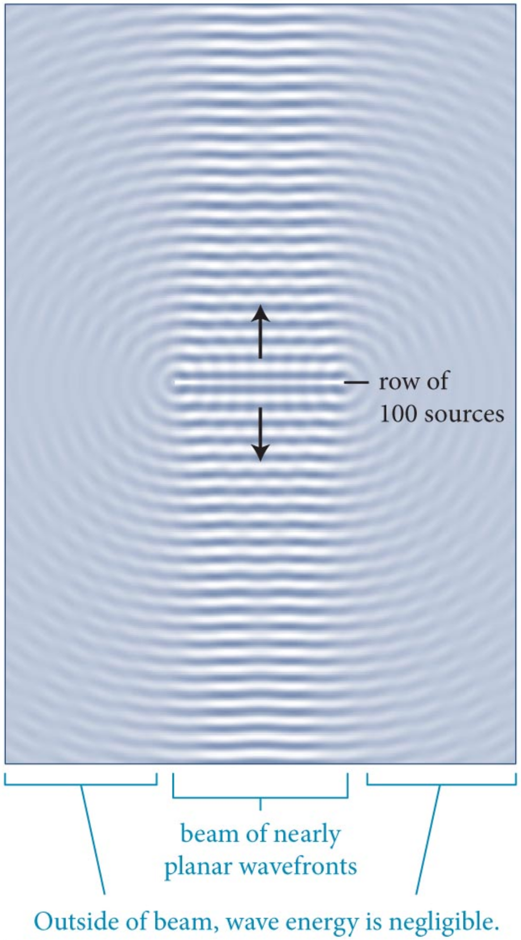


# Checkpoint 17.11

**17.11** How does the wave amplitude along the beam of wavefronts in Figure 17.20 change with distance from the row of sources?

It doesn't very much!

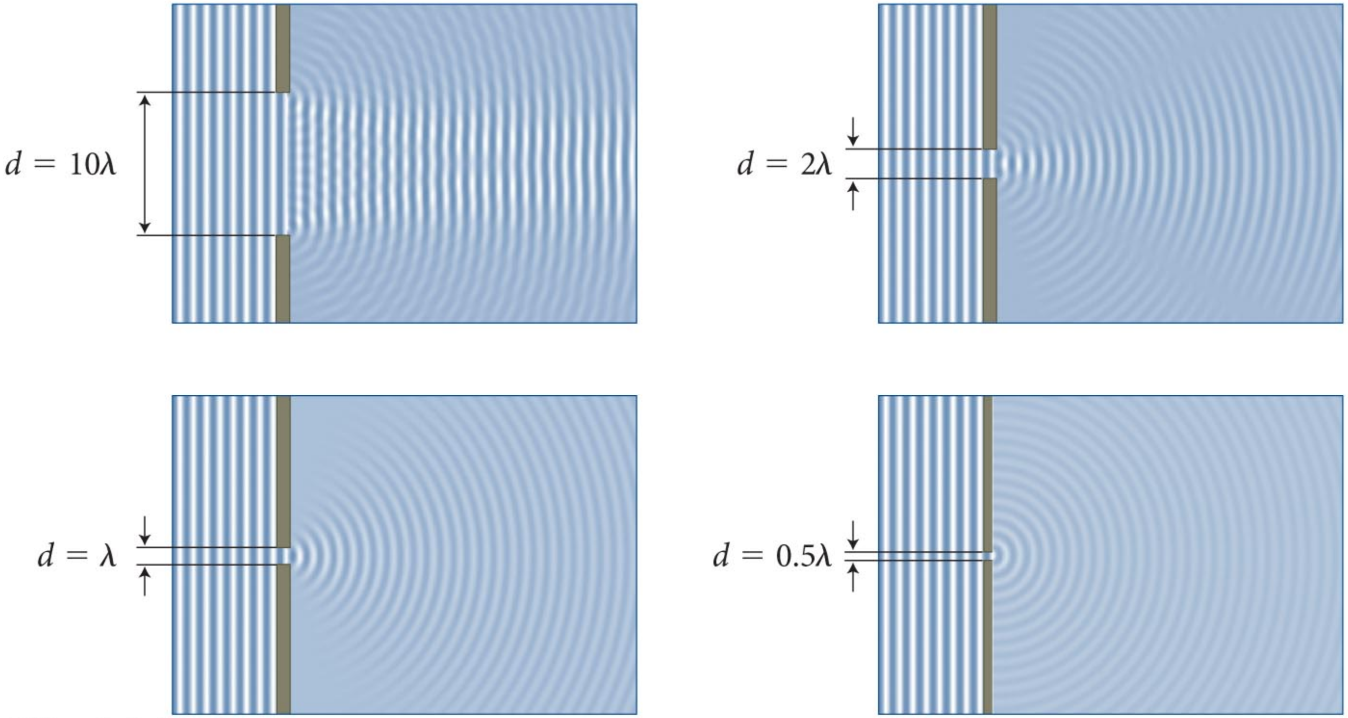
Neighboring sources 'shore each other up'



## Section Goals

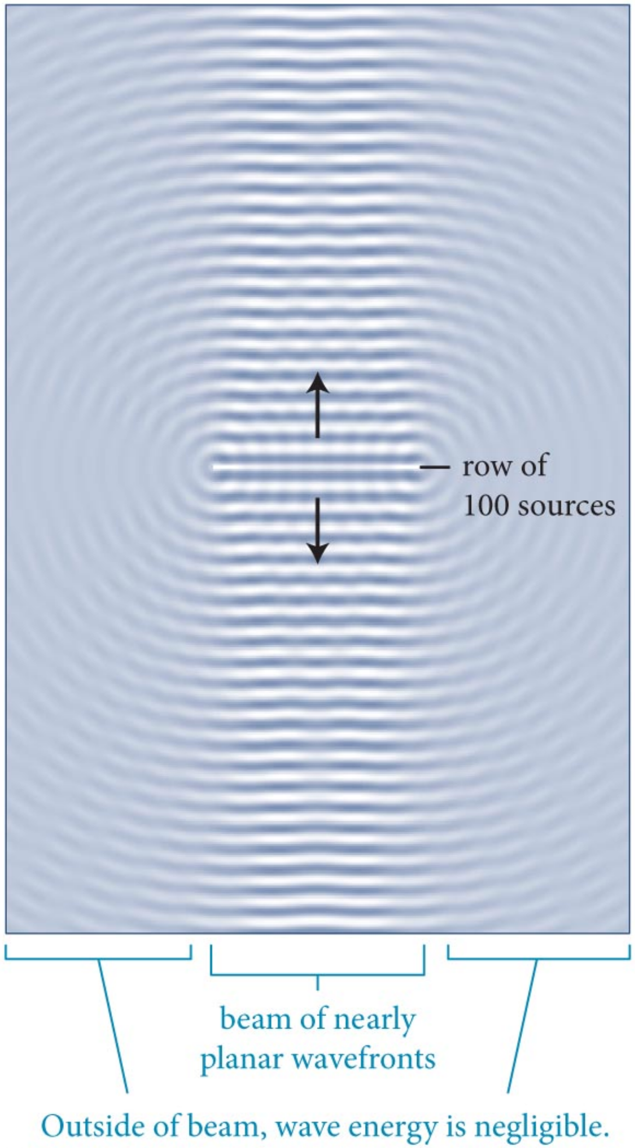
You will learn to

- Define the physical causes of **diffraction**.
- Represent diffraction **graphically**.



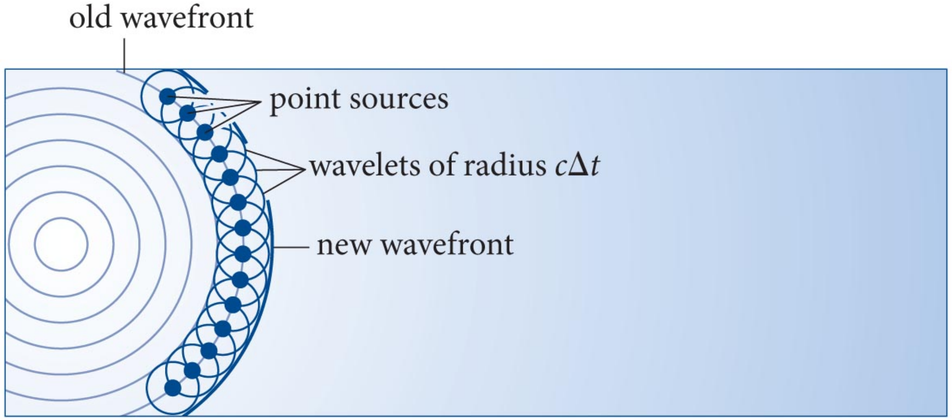
# Section 17.4: Diffraction

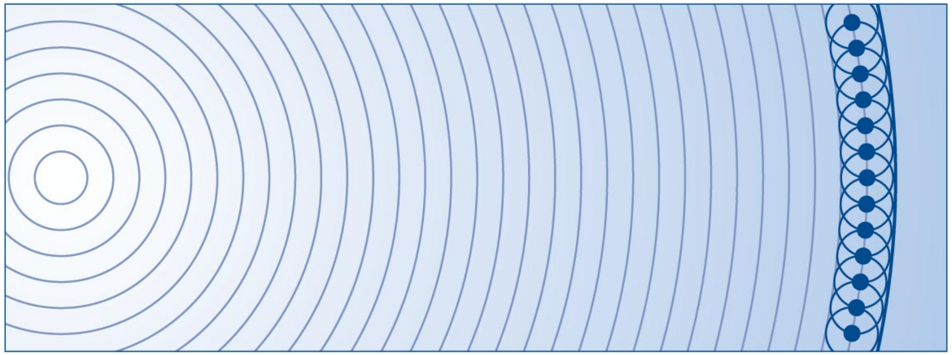
- **Huygens' principle** states that any wavefront can be regarded as a collection of closely spaced, coherent point sources.
- All these point sources emit wavelets, and these forward-moving wavelets combine to form the next wavefront.



# Section 17.4: Diffraction

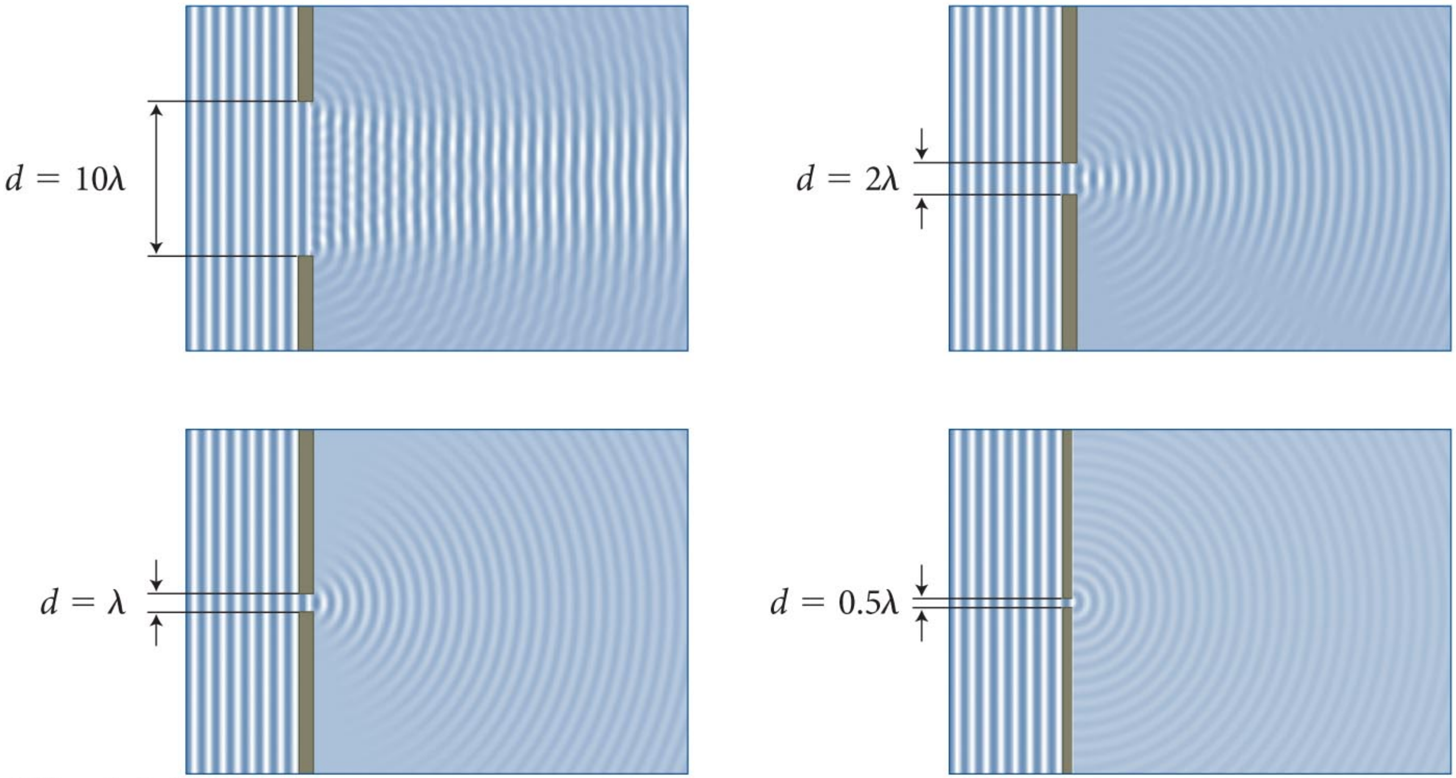
- The figure shows planar wavefronts incident on gaps of varying size.
- **Obstacles or apertures whose width is smaller than the wavelength of an incident wave give rise to considerable spreading of that wave.**
- The spreading is called **diffraction**.



# Checkpoint 17.12

**17.12** Suppose the barriers in Figure 17.22 were held at an angle to the incident wavefronts. Sketch the transmitted wavefronts for the case where the width of the gap is much smaller than the wavelength of the incident waves.



# Checkpoint 17.12

**17.12** Doesn't make a difference: the gap causes the same diffraction regardless. Only relies on incident waves causing the gap to become a point source.

# Section 17.4
# Question 4

Rank the relative extent of spreading of sound waves after they pass through a gap in a barrier. Consider three possibilities: The wavelength is (1) much smaller than the gap width, (2) comparable to the gap width, and (3) much greater than the gap width.

1. 1 > 2 > 3
2. 1 > 3 > 2
3. 2 > 1 > 3
4. 2 > 3 > 1
5. 3 > 2 > 1
6. 3 > 1 > 2

# Section 17.4
# Question 4

Rank the relative extent of spreading of sound waves after they pass through a gap in a barrier. Consider three possibilities: The wavelength is (1) much smaller than the gap width, (2) comparable to the gap width, and (3) much greater than the gap width.

1. 1 > 2 > 3
2. 1 > 3 > 2
3. 2 > 1 > 3
4. 2 > 3 > 1
5. 3 > 2 > 1 small wavelength = very directional.
6. 3 > 1 > 2

# Chapter 17: Self-Quiz #5

Because sound waves diffract around an open doorway, you can hear sounds coming from outside the doorway. You cannot, however, see objects outside the doorway unless you are directly in line with them. What does this observation imply about the wavelength of light?

# Chapter 17: Self-Quiz #5

## Answer

Because light does not diffract as it travels through the doorway, this observation implies that the wavelength of the light must be smaller than the width of the doorway.

Given that visible light has wavelengths between $4 \times 10^{-7}$ m and $7 \times 10^{-7}$ m and most doorways are about 1 m wide and 2 m tall, this is indeed the case.

# Quantitative Tools

# Section 17.5: Intensity

## Section Goals

You will learn to

- Define the **intensity** of a wave.
- Calculate the intensity of a wave using the **decibel scale**.

# Section 17.5: Intensity

- For waves in three dimensions, **intensity** $I$ is defined as

$$I \equiv \frac{P}{A}$$

  - $P$ is the power delivered by the wave over an area $A$.
  - SI units: $\mathrm{W/m^2}$
- If the power delivered by a point source is $P_s$, the intensity at a distance $r$ from the source is

$$I = \frac{P_s}{A_{\text{sphere}}} = \frac{P_s}{4\pi r^2} \quad \text{(uniformly radiating point source)}$$

- For two-dimensional surface waves, the intensity is

$$I_{\text{surf}} \equiv \frac{P}{L}$$

  - SI units: W/m

# Section 17.5: Intensity

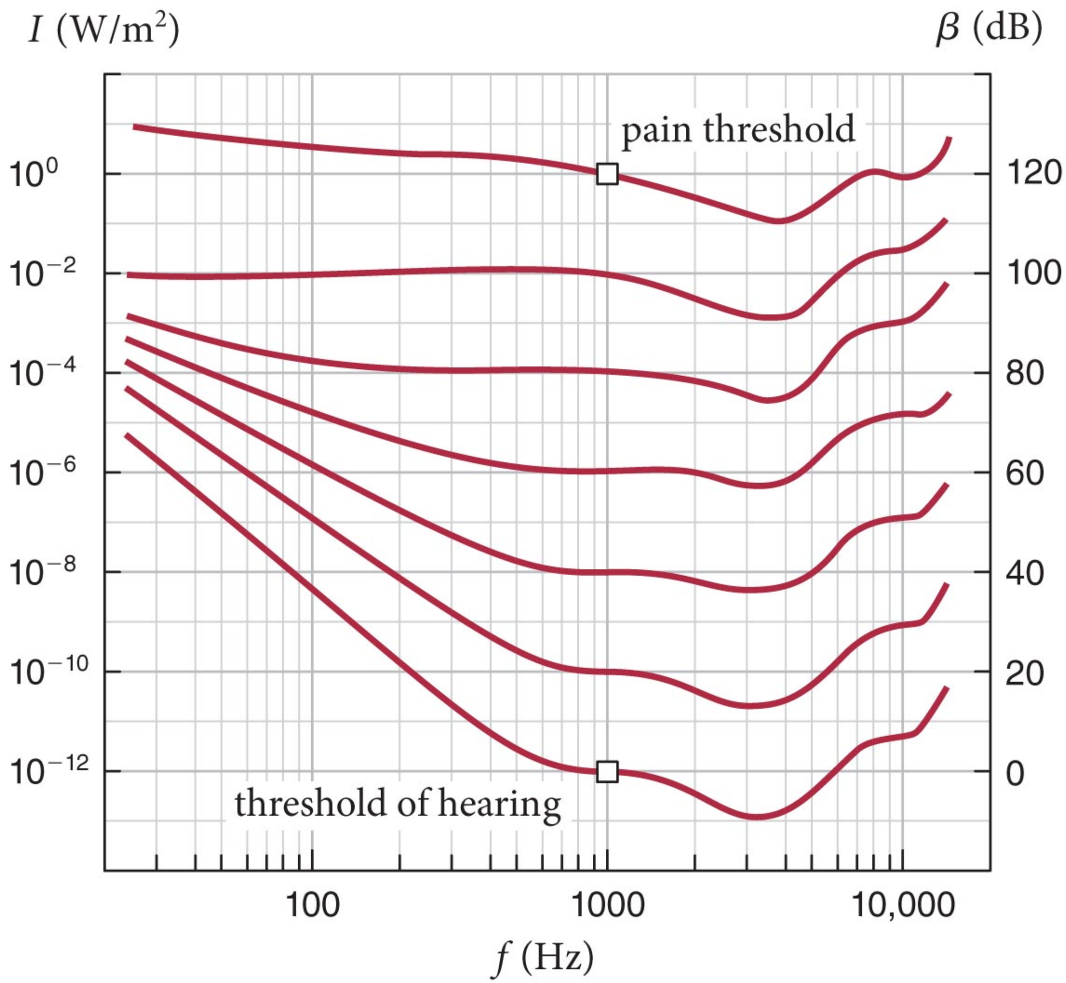
- The human ear can handle an extremely wide range of intensities, from the *threshold of hearing* $I_{th} \approx 1 \times 10^{-12}$ W/m² to the *threshold of pain* at $\approx 1.0$ W/m².
- To deal with this vast range of intensities, it's convenient to use a logarithmic scale and it's logical to place the zero of the scale at the threshold of hearing.
- To do so, we define the **intensity level** $\beta$, expressed in **decibels** (dB), as

$$\beta \equiv (10\ \text{dB}) \log\left(\frac{I}{I_{\text{th}}}\right)$$

where $I_{th} = 1 \times 10^{-12}$ W/m².

# Section 17.5: Intensity

Average auditory response of the human ear. Most sensitive at 3kHz. (lower magnitude means more sensitive. 3kHz is very annoying.)



# Section 17.5: Intensity

**Table 17.1** Approximate intensity levels

| Source | distance (m) | $\beta$ (dB) | Description |
|---|---|---|---|
| Jet engine | 50 | 140 | pain |
| Pneumatic hammer | 10 | 110 | |
| Shout | 1.5 | 100 | very loud |
| Car horn | 10 | 90 | |
| Hair dryer | 0.2 | 80 | loud |
| Automobile interior | | 70 | |
| Conversation | 1 | 60 | moderate |
| Office background | | 50 | |
| Library background | | 40 | |
| Suburban bedroom | | 30 | quiet |
| Whisper | 1 | 20 | |
| Normal breathing | 5 | 10 | barely audible |

## Exercise 17.5 Doubling the intensity

A clarinet can produce about 70 dB of sound. By how much does the intensity level increase if a second clarinet is played at the same time?

## Exercise 17.5 Doubling the intensity (cont.)

**SOLUTION** If the intensity of the sound produced by one clarinet is $I_c$, the intensity level of one clarinet is

$$\beta_1 = (10\ \text{dB})\log\left(\frac{I_c}{I_{th}}\right) = 70\ \text{dB}.$$

## Exercise 17.5 Doubling the intensity (cont.)

**SOLUTION** The second clarinet doubles the intensity, so the intensity level becomes

$$\beta_2 = (10\text{ dB})\log\left(\frac{2I_\text{c}}{I_\text{th}}\right) = (10\text{ dB})\left[\log 2 + \log\left(\frac{I_\text{c}}{I_\text{th}}\right)\right]$$
$$= (10\text{ dB})\log 2 + \beta_1,$$

where I have used the logarithmic relationship log $AB = \log A + \log B$. Because $\log 2 \approx 0.3$, the intensity level increases to $\beta_2 \approx (10\text{ dB})(0.3) + 70\text{ dB} = 73\text{ dB}$. So, even though the intensity doubles, the intensity *level* increases by only 3 dB. ✔

# Checkpoint 17.13

**17.13** In Exercise 17.5, how many clarinets must play at the same time in order to increase the intensity level from 70 dB to 80 dB?

10 dB means a factor of 10 increase in intensity (log scale!), so we need 10 clarinets playing at the same time.

# Section 17.5
# Question 5

Increasing your distance from a point source of spherical waves by a factor of 10 reduces the intensity level $\beta$ by how many decibels?

1. 2 dB
2. 4 dB
3. 5 dB
4. 10 dB
5. 20 dB

# Section 17.5
# Question 5

Increasing your distance from a point source of spherical waves by a factor of 10 reduces the intensity level $\beta$ by how many decibels?

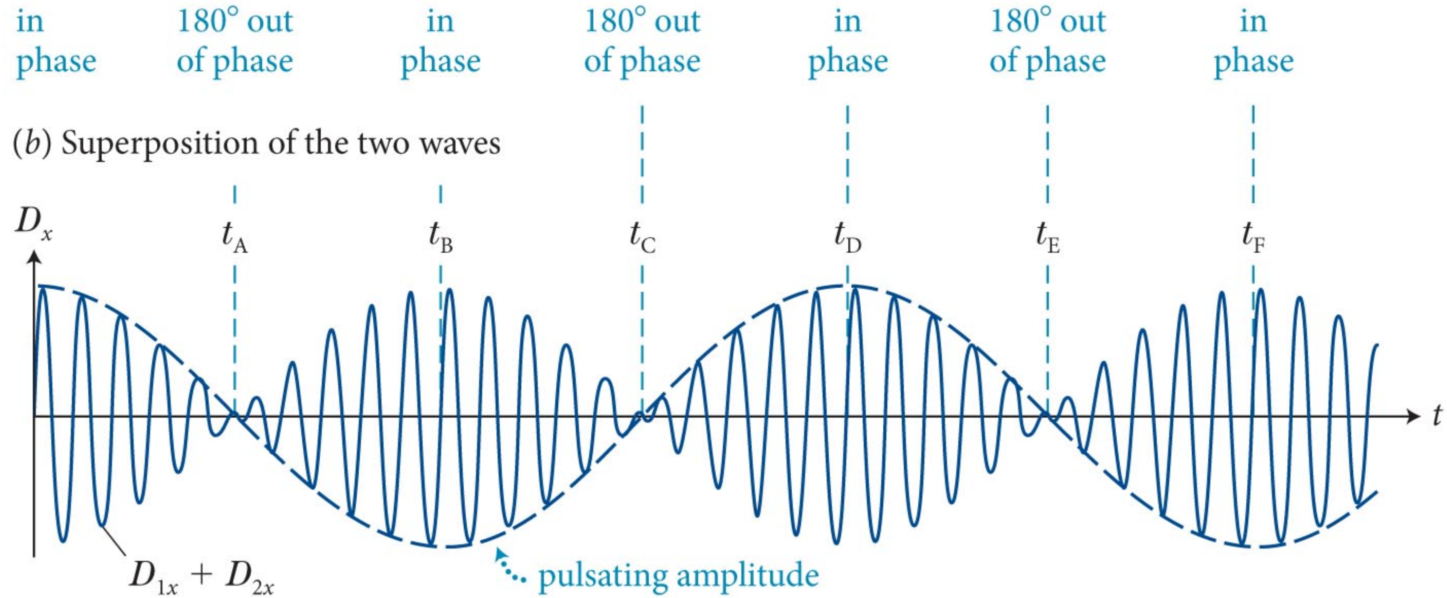
1. 2 dB
2. 4 dB
3. 5 dB
4. 10 dB
5. ✔ 20 dB – intensity goes as $1/r^2$, a factor of 1/100.

   $\text{dB} = 10 \log(1/100) = -20$

## Section Goals

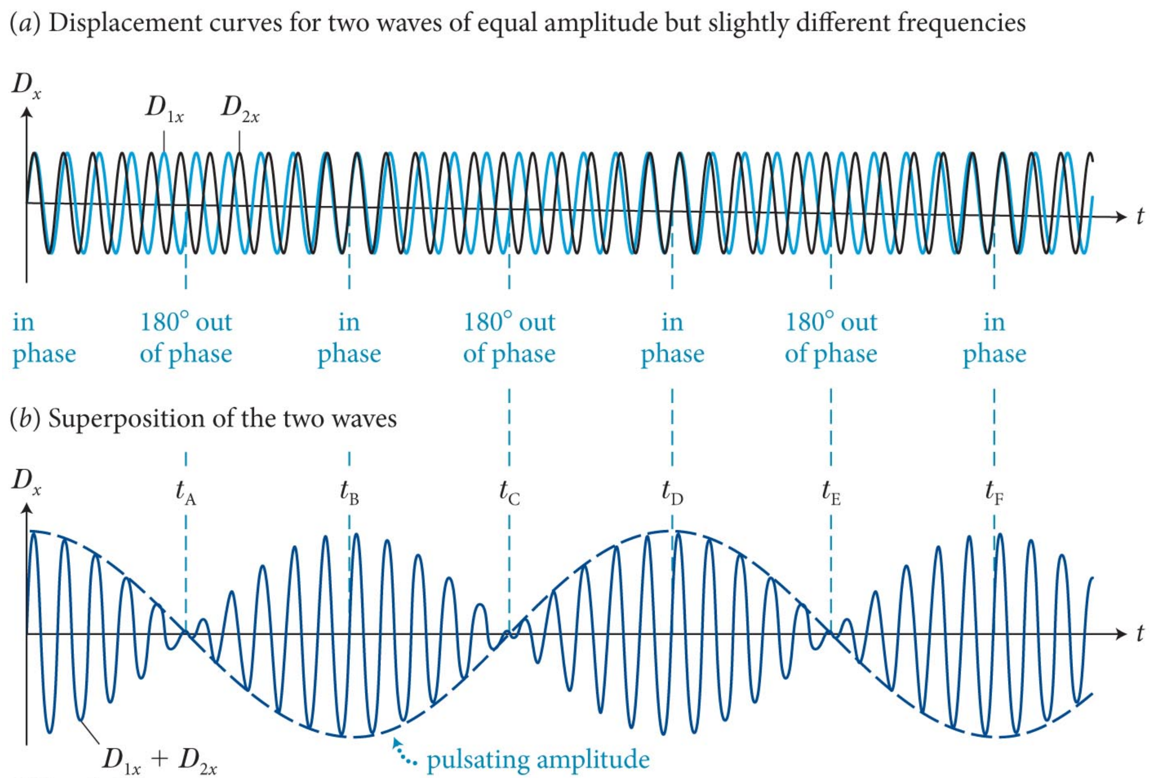
You will learn to

- Establish the concept of **beats**, which arises from the overlap of equal amplitude waves with slightly **different frequency**.
- Derive the mathematical formula that relates the frequency of the beats to the frequencies of the overlapping waves.



# Section 17.6: Beats

- Part (*a*) shows the displacement curves for two waves of equal amplitude *A*, but slightly different frequencies.
- The superposition of the two waves result in a wave of oscillating amplitude as shown in part (*b*).
- This effect is called **beating**.



# Section 17.6: Beats

- The displacement caused by the two individual waves at some fixed point is given by

$$D_{1x} = A\ \sin(2\pi f_1 t)$$

$$D_{2x} = A\ \sin(2\pi f_2 t)$$

- The superposition of the two waves gives us

$$D_x = D_{1x} + D_{2x} = A(\sin 2\pi f_1 t + \sin 2\pi f_2 t)$$

- Using trigonometric identities, we can simplify the equation to

$$D_x = 2A\ \cos\tfrac{1}{2}\left[2\pi(f_1 - f_2)t\right]\sin\tfrac{1}{2}\left[2\pi(f_1 - f_2)t\right]$$

# Section 17.6: Beats

- Using $\Delta f = |f_1 - f_2|$ and $f_{av} = ½\,(f_1 + f_2)$, we can write

$$D_x = 2A\ \cos\left[2\pi(\tfrac{1}{2}\Delta f)t\right]\sin(2\pi f_{av}t)$$

- We can see that the resulting wave has a frequency of $f_{av}$.
- The frequency of the amplitude variation is $½\Delta f$.
- However, since two beats occur in each cycle of this amplitude variation, the **beat frequency** is twice that

$$f_{beat} \equiv \left|f_1 - f\right|$$

## Exercise 17.7 Tuning a piano

Your middle-C tuning fork oscillates at 261.6 Hz. When you play the middle-C key on your piano together with the tuning fork, you hear 15 beats in 10 s. What are the possible frequencies emitted by this key?

## Exercise 17.7 Tuning a piano

**SOLUTION** The beat frequency—the number of beats per second—is equal to the difference between the two frequencies (Eq. 17.8).

I am given the frequency of the tuning fork, $f_t = 261.6$ Hz, and the beat frequency, $f_B = (15 \text{ beats})/(10 \text{ s}) = 1.5$ Hz.

I do not know, however, whether the frequency $f_p$ of the struck middle-C piano key is higher or lower than that of the tuning fork.

## Exercise 17.7 Tuning a piano

**SOLUTION** If it is higher, I have $f_B = f_p - f_t$.

If it is lower, then $f_B = f_t - f_p$.

So

$$f_p = f_t \pm f_B = 261.6 \text{ Hz} \pm 1.5 \text{ Hz}$$

and the possible frequencies emitted by the out-of-tune middle-C key are 260.1 Hz and 263.1 Hz. ✔

# Section 17.6
# Question 6

One way to tune a piano is to strike a tuning fork (which emits only one specific frequency), then immediately strike the piano key for the frequency being sounded by the fork, and listen for beats. In making an adjustment, a piano tuner working this way causes the beat frequency to increase slightly. Is she going in the right direction with that adjustment?

1. Yes
2. No

# Section 17.6
# Question 6

One way to tune a piano is to strike a tuning fork (which emits only one specific frequency), then immediately strike the piano key for the frequency being sounded by the fork, and listen for beats. In making an adjustment, a piano tuner working this way causes the beat frequency to increase slightly. Is she going in the right direction with that adjustment?

1. Yes
2. No – faster beating means larger difference in freq.

# Section 17.7: Doppler effect

It is interesting, but easy enough for you to read about on your own.

And, you already understand it intuitively. Really!

# Section 17.7
# Question 7

A train (or ambulance) approaches as you wait at a crossing. Is the whistle frequency you hear

1. higher than
2. lower than, or
3. the same as

the frequency you would hear if the train were stationary?

## Section 17.7
## Question 7

A train (or ambulance) approaches as you wait at a crossing. Is the whistle frequency you hear

✔ 1. higher than
2. lower than, or
3. the same as

the frequency you would hear if the train were stationary?

# Chapter 17: Summary

## Concepts: Characteristics of waves in two and three dimensions

- A **wavefront** is a curve or surface in a medium on which all points of a propagating wave have the same phase.
- A **planar wavefront** is a flat wavefront that is either a plane or a straight line.
- A **surface wave** is a wave that propagates in two dimensions and has circular wavefronts.
- A **spherical wave** is a wave that propagates in three dimensions and has spherical wavefronts.

## Concepts: Characteristics of waves in two and three dimensions

- According to **Huygens' principle**, any wavefront may be regarded as a collection of many closely spaced, coherent point sources.
- **Diffraction** is the spreading out of waves either around an obstacle or beyond the edges of an aperture. The effect is more pronounced when the size of the obstacle or aperture is about equal to or smaller than the wavelength of the wave.

## Quantitative Tools: Characteristics of waves in two and three dimensions

- If no energy is dissipated, the amplitude $A$ of a wave originating at a point source decreases with increasing distance $r$ from the source as

$$A \propto \frac{1}{\sqrt{r}} \quad \text{(surface wave)}$$

or

$$A \propto \frac{1}{r} \quad \text{(spherical wave).}$$

- The **intensity** $I$ (in $\mathrm{W/m^2}$) of a spherical wave that delivers power $P$ to an area $A$ oriented normal to the direction of propagation is

$$I \equiv \frac{P}{A}.$$

## Quantitative Tools: Characteristics of waves in two and three dimensions

- If a point source emits waves uniformly in all directions at a power $P_s$ and no energy is dissipated, the intensity a distance $r$ from the source is

$$I = \frac{P_s}{4\pi r^2}.$$

- The intensity $I_{surf}$ (in W/m) of a surface wave that delivers power $P$ to a length $L$ oriented normal to the direction of propagation is

$$I_{surf} \equiv \frac{P}{L}.$$

## Concepts: Sound waves

- **Sound** is a longitudinal *compressional* wave propagating through a solid, liquid, or gas. The wave consists of an alternating series of *compressions* (where the molecules of the medium are crowded together) and *rarefactions* (where the molecules are spaced far apart). The frequency range of *audible* sound is 20 Hz to 20 kHz.
- The speed of sound $c$ depends on the density and elastic properties of the medium. In dry air at 20° C, the speed of sound is 343 m/s.

## Quantitative Tools: Sound waves

- The *threshold of hearing* $I_{\text{th}}$ is the minimum sound intensity audible to humans. For a 1.0-kHz sound,

$$I_{\text{th}} \approx 10^{-12}\ \text{W/m}^2.$$

- For a sound of intensity $I$, the **intensity level** $\beta$ in decibels is

$$\beta \equiv (10\ \text{dB}) \log\left(\frac{I}{I_{\text{th}}}\right).$$

## Concepts: Interference effects

- Two or more sources emitting waves that have a constant phase difference are called **coherent sources**. If that constant phase difference is zero, the sources are said to be *in phase*.
- Along **nodal lines,** waves cancel each other, and so the displacement of the medium is zero. Along **antinodal lines,** the displacement of the medium is a maximum.
- The superposition of two waves of equal amplitude but slightly different frequencies results in a wave of oscillating amplitude. This effect is called **beating.**

# Chapter 17: Summary

## Quantitative Tools: Interference effects

- When two waves of frequencies $f_1$ and $f_2$ result in **beating**, the **beat frequency** is

$$f_{\text{beat}} \equiv |f_1 - f_2|,$$

and the displacement $D_x$ of the particles of the medium is

$$D_x = 2A\cos\left[2\pi(\tfrac{1}{2}\Delta f)t\right]\sin(2\pi f_{\text{av}}t),$$

where $\Delta f = \left|f_1 - f_2\right|$ and $f_{\text{av}} = \tfrac{1}{2}(f_1 + f_2)$.

# Chapter 17: Summary

## Concepts: The effects of motion on sound

- The **Doppler effect** is a change in the observed wave frequency caused by the relative motion of a wave source and an observer.
- A **shock wave** is a conical (wedge-shaped) disturbance caused by the piling up of wavefronts from a source moving at a speed greater than or equal to the wave speed in the medium.

## Quantitative Tools: The effects of motion on sound

- If a source moving with speed $\upsilon_s$ relative to the medium produces sound of frequency $f_s$, the **Doppler effect** causes an observer moving with speed $\upsilon_o$ relative to the medium to observe the sound as having a frequency $f_o$ given by

$$\frac{f_o}{f_s} = \frac{c \pm \upsilon_o}{c \pm \upsilon_s}.$$

- The $\pm$ signs are chosen so that $f_o > f_s$ when the source and observer approach each other and $f_o < f_s$ when they move apart.

## Quantitative Tools: The effects of motion on sound

- As a shock wave propagates at speed $c$, the angle $\theta$ it makes with the direction in which the source moves is given by

$$\sin\theta = \frac{c}{\upsilon_s} \quad (\upsilon_s > c),$$

where $\upsilon_s$ is the speed of the source relative to the medium. The ratio $\upsilon_s/c$ is the *Mach number*.