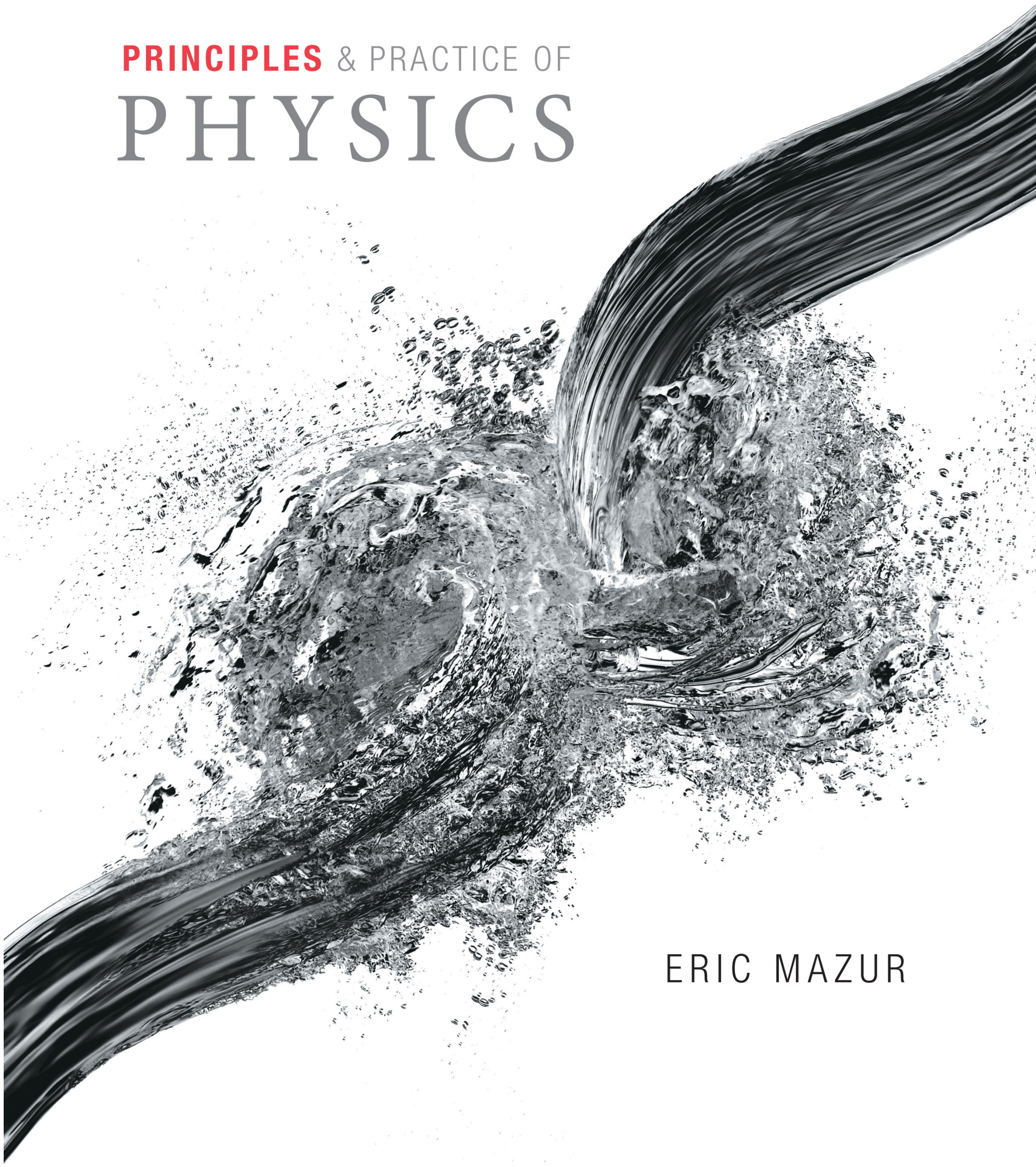


Chapter 3
Acceleration

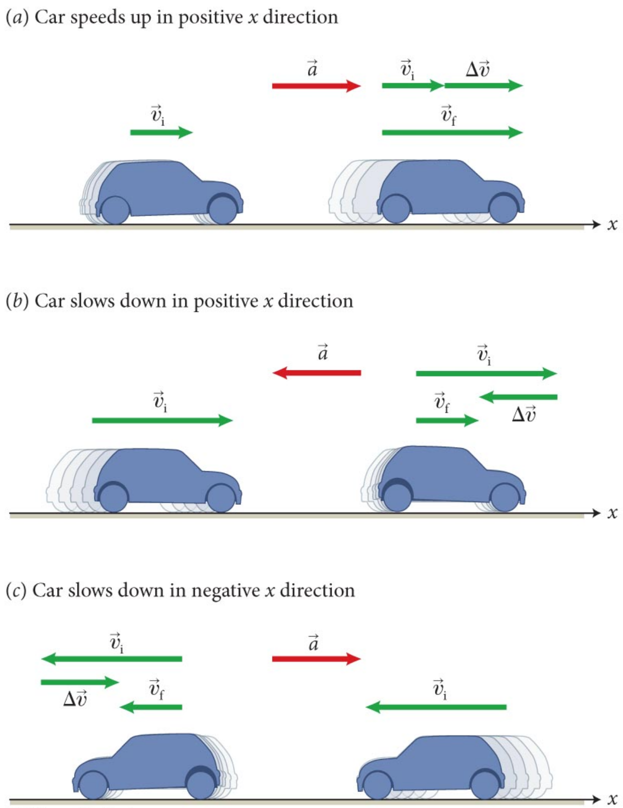
Chapter 3: Acceleration

Chapter Goal: To extend the description of motion in one dimension to include changes in velocity. This type of motion is called *acceleration*.

Chapter 3 Preview

Looking Ahead: Changes in velocity

- The non-uniform motion of an object can be described by the concept of acceleration.

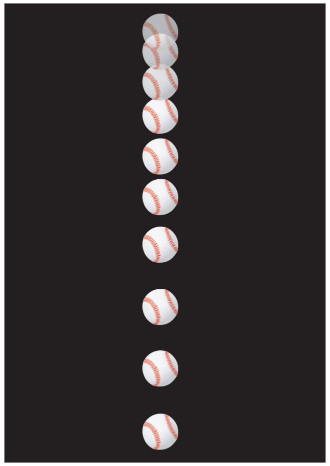


- You will learn how to describe acceleration both graphically and mathematically.

Chapter 3 Preview

Looking Ahead: Acceleration due to gravity

- An object speeding up as it falls:

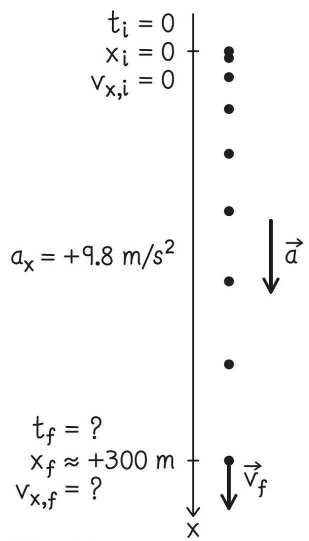


- Gravity is kind of a big deal
- Near the earth's surface, it is relatively simple
- You will learn how to account for the influence of gravity for objects moving near the surface of the Earth.

Looking Ahead: Motion diagrams

- Generalization of the "frame sequence" from Ch. 2:

  motion diagrams.



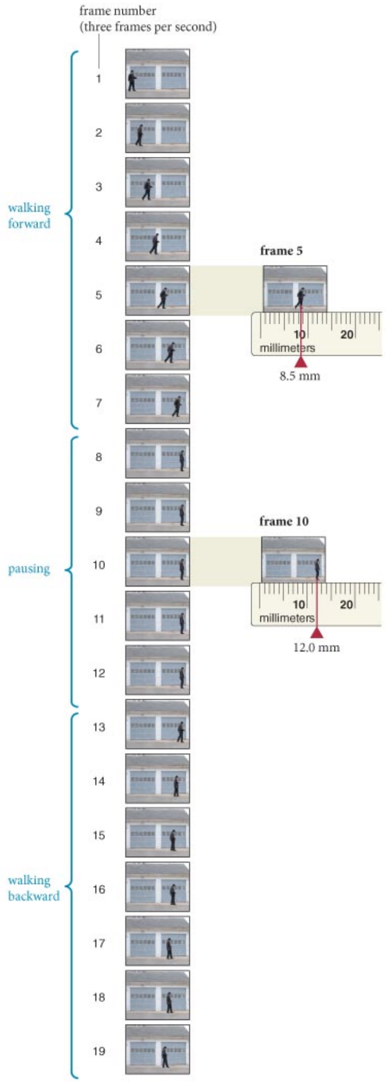
- Learn how to display and interpret motion using them
- How to determine position, displacement, speed, velocity, and acceleration from them

Chapter 3 Preview
Looking Back: Visualizing motion

- Ch. 2: analyzed motion by looking at individual frames of a film clip recorded at equally spaced times.

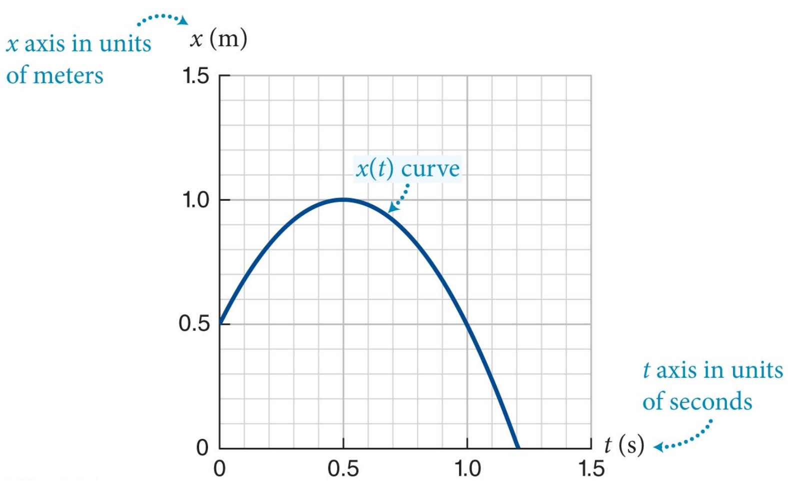
- Workable, but discrete approach is limited

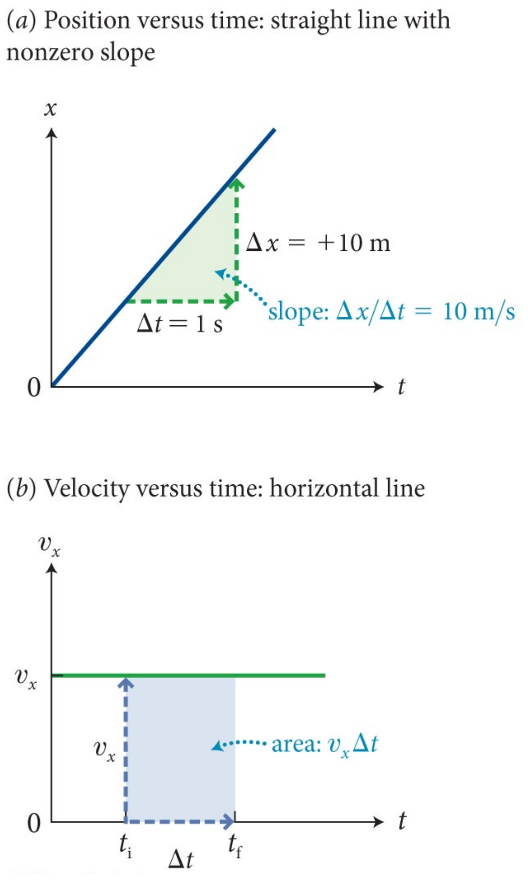
(if only we had some math for that)



Chapter 3 Preview
Looking Back: Representations of motion

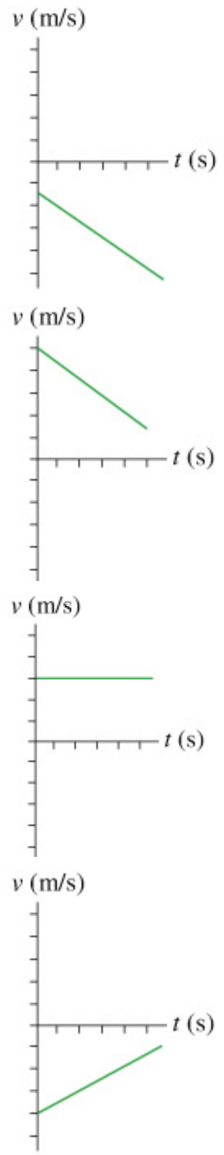
- Examined different graphical and mathematical ways of representing motion.





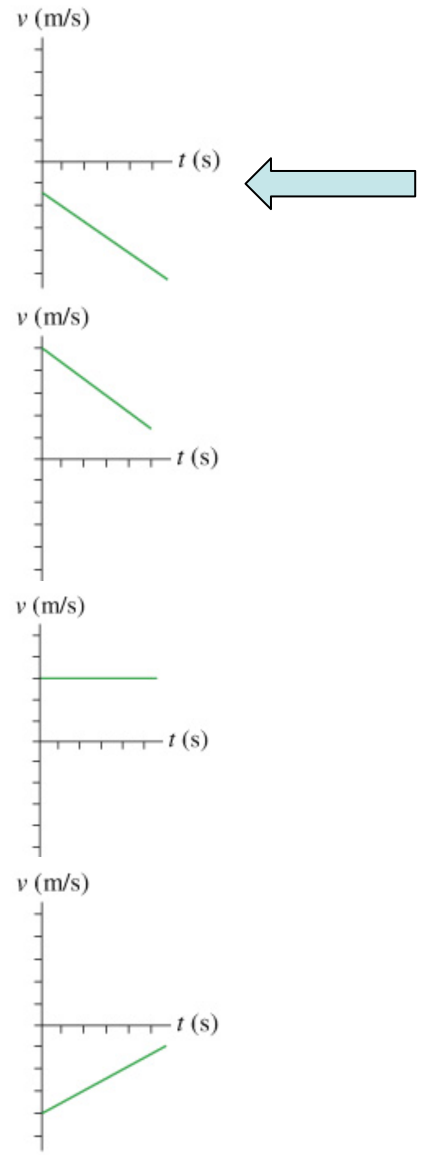
RQ 3.1

Which figure could represent the velocity versus time graph of a motorcycle whose *speed* is increasing?



RQ 3.1

Which figure could represent the velocity versus time graph of a motorcycle whose *speed* is increasing?



magnitude of *v* has to increase (sign not important)

RQ 3.2

Suppose that you toss a rock upward so that it rises and then falls back to the earth. If the acceleration due to gravity is 9.8 m/sec^2, what is the rock's acceleration at the instant that it reaches the top of its trajectory (where its velocity is momentarily zero)? Assume that air resistance is negligible.

1. The rock has a *downward* acceleration of 9.8 m/s^2.
2. The rock has a *downward* acceleration of 19.6 m/s^2.
3. The rock has an *upward* acceleration of 19.6 m/s^2.
4. The rock has an *upward* acceleration of 9.8 m/s^2.
5. The acceleration of the rock is zero.

RQ 3.2

Suppose that you toss a rock upward so that it rises and then falls back to the earth. If the acceleration due to gravity is 9.8 m/sec^2, what is the rock's acceleration at the instant that it reaches the top of its trajectory (where its velocity is momentarily zero)? Assume that air resistance is negligible.

1. **The rock has a *downward* acceleration of 9.8 m/s^2.**

This is the whole thing about gravity near earth's surface.

There is *always* nearly constant acceleration of 9.8 m/s^2

RQ 3.3

On a straight road, a car speeds up at a constant rate from rest to 20 m/s over a 5 second interval and a truck slows at a constant rate from 20 m/s to a complete stop over a 10 second interval. How does the distance traveled by the truck compare to that of the car?

1. The truck travels the same distance as the car.
2. There is not enough information to answer the question.
3. The truck travels twice as far as the car.
4. The truck travels half as far as the car.

RQ 3.3

On a straight road, a car speeds up at a constant rate from rest to 20 m/s over a 5 second interval and a truck slows at a constant rate from 20 m/s to a complete stop over a 10 second interval. How does the distance traveled by the truck compare to that of the car?

1. The truck travels the same distance as the car.
2. There is not enough information to answer the question.
3. **The truck travels twice as far as the car.**
4. The truck travels half as far as the car.

- Car: $a = (20\text{ m/s})/(5\text{ s}) = 4\text{ m/s}^2$

  $x_c(t) = x_{ic} + v_{ic}t + \frac{1}{2}at^2 = \frac{1}{2}at^2 = 50\text{ m}$
- Truck: $a = (-20\text{ m/s})/(10\text{ s}) = -2\text{ m/s}^2$

  $x_t(t) = x_{it} + v_{it}t + \frac{1}{2}at^2 = v_{ic}t - \frac{1}{2}at^2 = 100\text{ m}$

RQ 3.4

- On Mars, where air resistance is negligible, an astronaut drops a rock from a cliff and notes that the rock falls about d meters during the first t seconds of its fall. Assuming the rock does not hit the ground first, how far will it fall during the first $4t$ seconds of its fall?

RQ 3.4

- On Mars, where air resistance is negligible, an astronaut drops a rock from a cliff and notes that the rock falls about *d* meters during the first *t* seconds of its fall. Assuming the rock does not hit the ground first, how far will it fall during the first 4*t* seconds of its fall?

- Don't overthink it: distance ~ t^2
- If time is up 4x, distance is up 16x

Concepts

Section 3.1: Changes in velocity

Section Goals

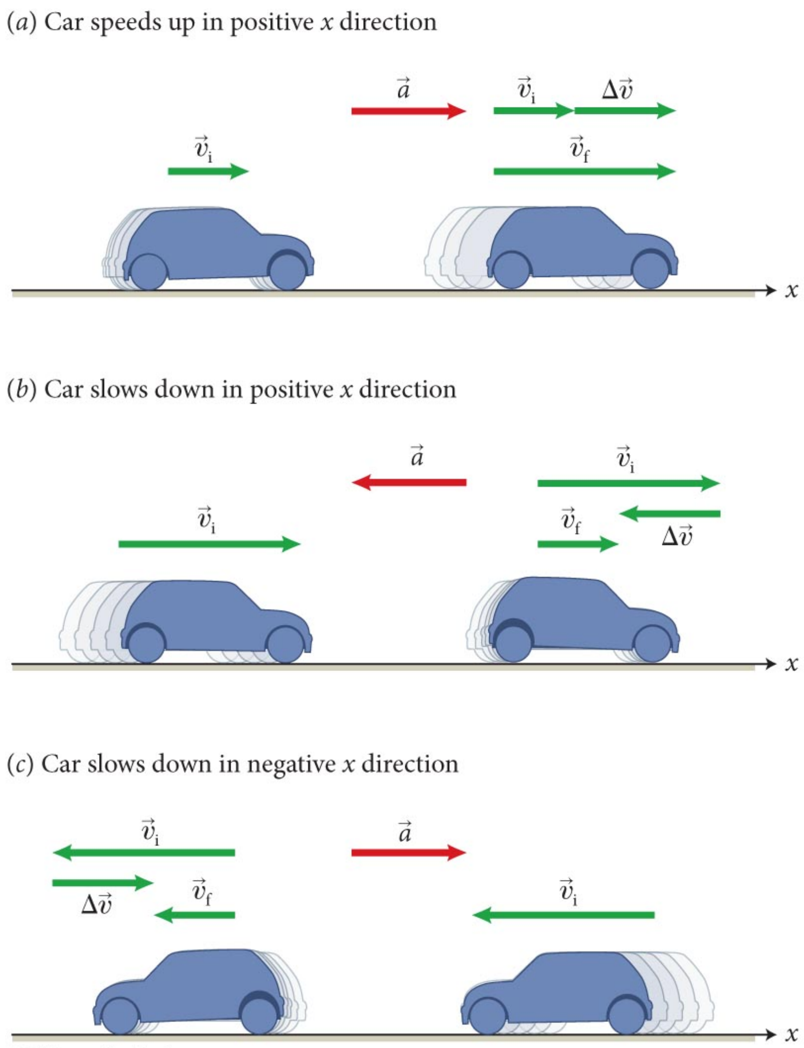
- Define acceleration from velocity.
- Identify if an object is accelerating from several different graphical representations of motion.
- Understand the vector relationships between velocity and acceleration.

Section 3.1: Changes in velocity

- If an object's velocity is changing, the object is accelerating.
- The *x* component of the average acceleration of an object is the change in the *x* component of the velocity divided by the time interval during which this change took place.
- The SI unit of acceleration is m/s^2.

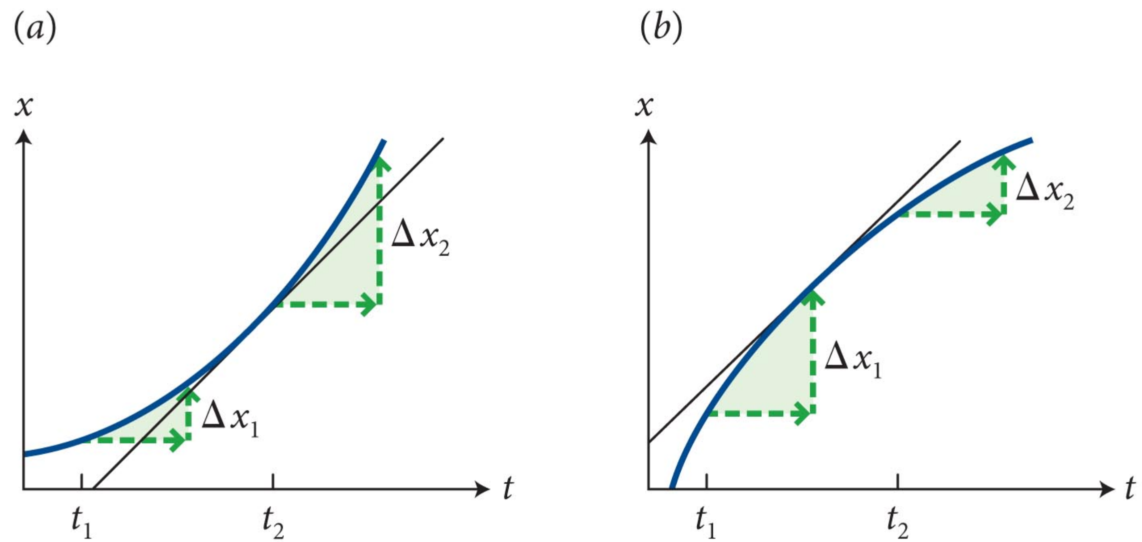
Section 3.1: Changes in velocity

- Whenever an object's velocity vector $\vec{v}$ and acceleration vector $\vec{a}$ point in the same direction, the object speeds up.
- If $\vec{v}$ and $\vec{a}$ point in the opposite direction, the object slows down.



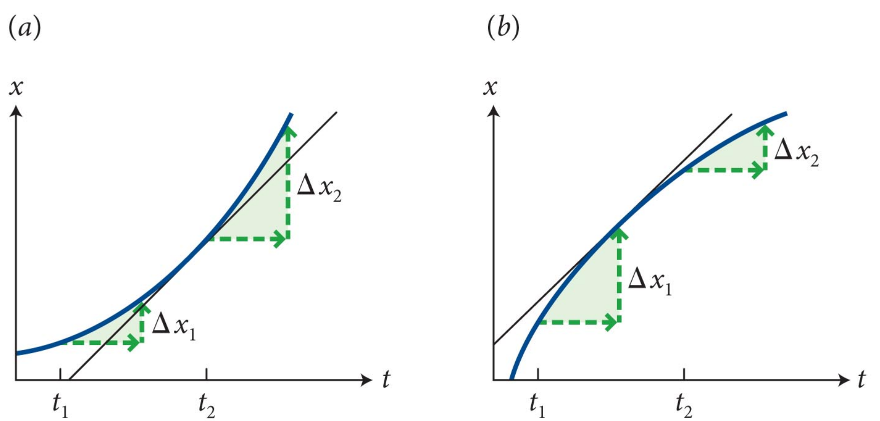
Section 3.1: Changes in velocity

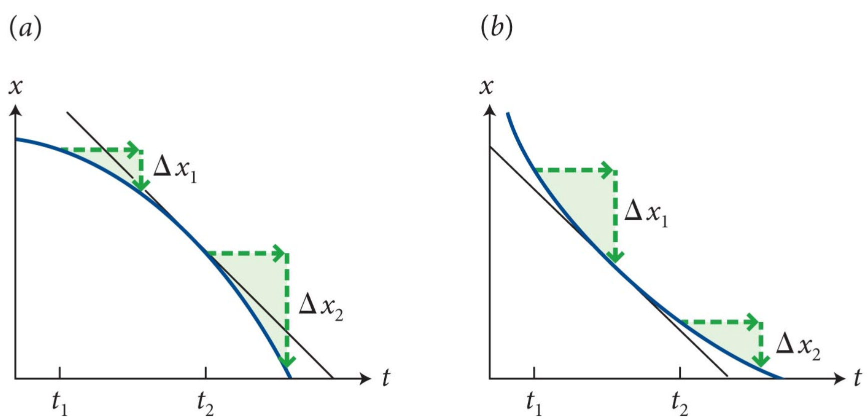
- For accelerating objects, the $x(t)$ curve is a *not* a straight line.
- The figure shows the $x(t)$ curve for two accelerating objects:
  - For each object, consider the displacements Δx_1 and Δx_2 during two equal time intervals (Δt) at two different times.
  - If the displacement increases with time then the velocity is increasing (for example, $\Delta x_2 > \Delta x_1$).
  - If the displacement decreases then velocity is decreasing.



Section 3.1: Changes in velocity

- The curvature of the $x(t)$ curve is a measure of the x component of acceleration (a_x).
- An upward curvature corresponds to a positive a_x:
  - The curve lies above the tangent; **faster than linear**
- A downward curvature corresponds to a negative a_x:
  - The curve lies below the tangent; **slower than linear**





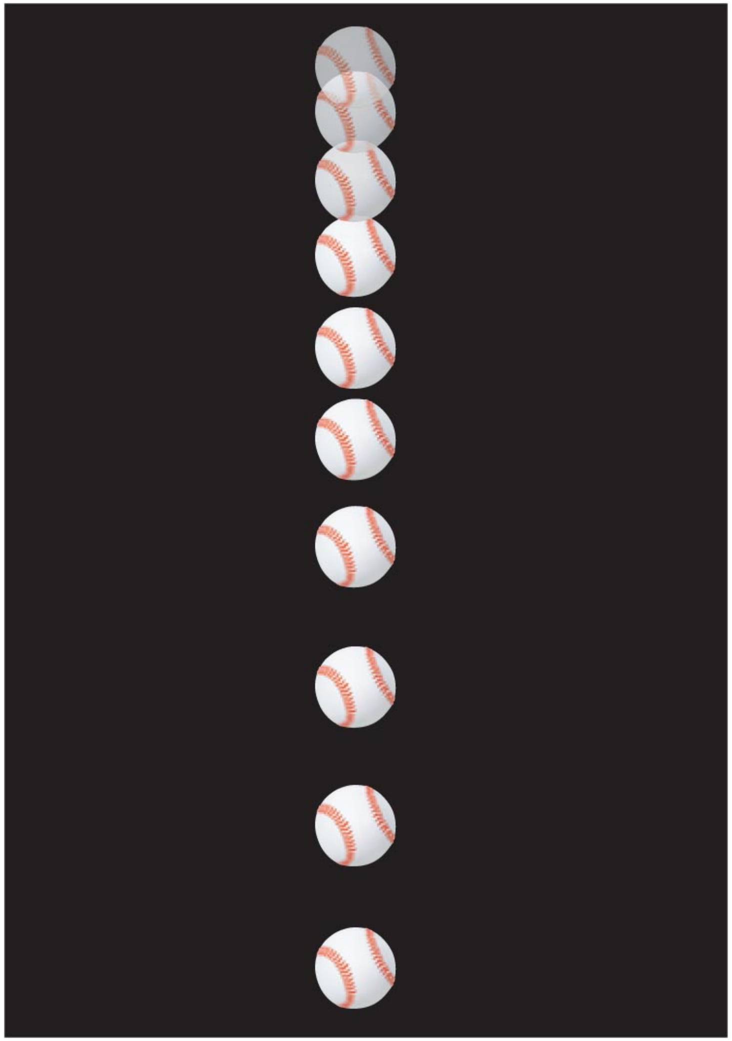
Section 3.2: Acceleration due to gravity

Section Goals

- Identify gravity as the cause of the vertical acceleration of an object moving near the surface of the Earth.
- Display the effects of gravity of an object using motion diagrams.
- Model the idealized case of vertical motion in the absence of other influences, such as air resistance, using the concept of free fall.

Section 3.2: Acceleration due to gravity

- An object falling in a straight line toward the Earth's surface: **accelerated motion**.
- A falling ball recorded at equal time intervals of 0.05 s.
- The increasing displacements tells us that the speed increases as it falls: the ball accelerates.



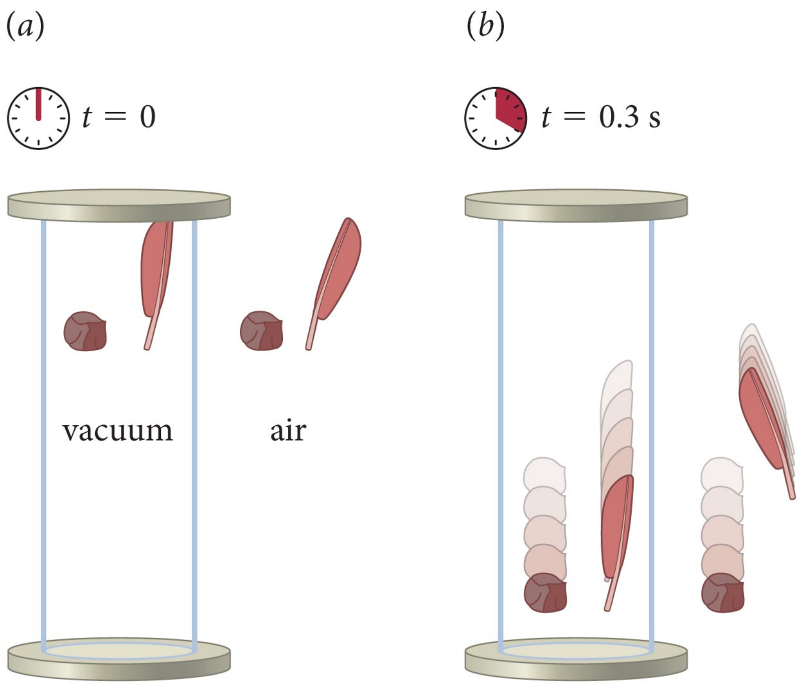
Checkpoint Question 3.5

- Does the acceleration of an object as it falls depend on the physical characteristics of the object?

3.5 Hold a book and a sheet of paper, cut to the same size as the book, side by side 1 m above the floor. Hold the paper parallel to the floor and the book with its covers parallel to the floor, and release them at the same instant. Which hits the floor first? Now put the paper on top of the book and repeat the experiment. What do you notice?

Section 3.2: Acceleration due to gravity

- What is the magnitude of acceleration due to gravity as an object falls?
  - feather vs stone
  - in a vacuum, no air resistance.
  - the acceleration due to gravity *does not* depend on the physical characteristics of object
    - The motion of an object under the influence of gravity only is called **free fall**.

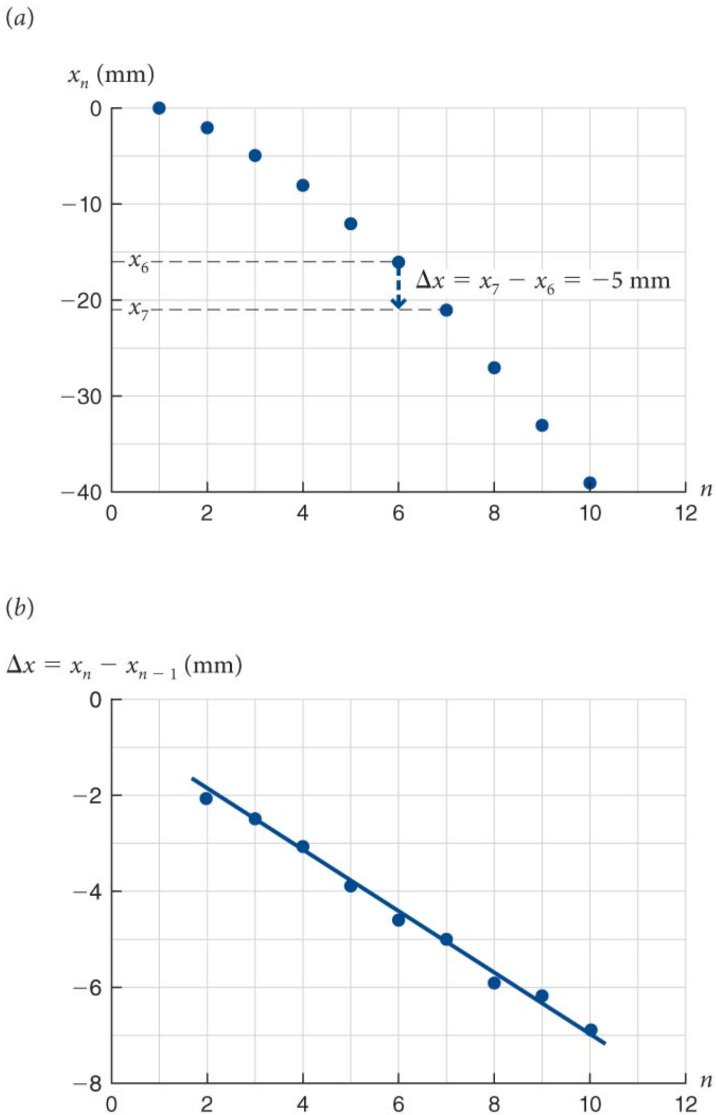


- what properties are most fundamental?
- which are circumstantial?

https://youtu.be/frZ9dN_ATew

Section 3.2: Acceleration due to gravity

- What is the magnitude of acceleration due to gravity as an object falls?
  - like last time, position vs time changes faster than linearly
  - *displacement* between adjacent times is linear
    - therefore, the velocity increases at a constant rate.



Section 3.2: Acceleration due to gravity

- What is the magnitude of acceleration due to gravity as an object falls?
  - without air resistance, the magnitude of the acceleration of all falling objects is 9.8 m/s^2.
  - Means? The amount of time it takes to fall from a certain height is the same for all falling objects.

Section 3.3: Projectile motion

Section Goals

- Define the motion of objects that are launched but not self-propelled as projectile motion.
- Model the vertical trajectory of projectiles as objects that are in free fall.
- Represent projectile motion graphically using motion diagrams and motion graphs.

Section 3.3: Projectile motion

- An object that is launched but not self-propelled is called a **projectile**.
- Its motion is called **projectile motion**.
- The path the object follows is called its **trajectory**.

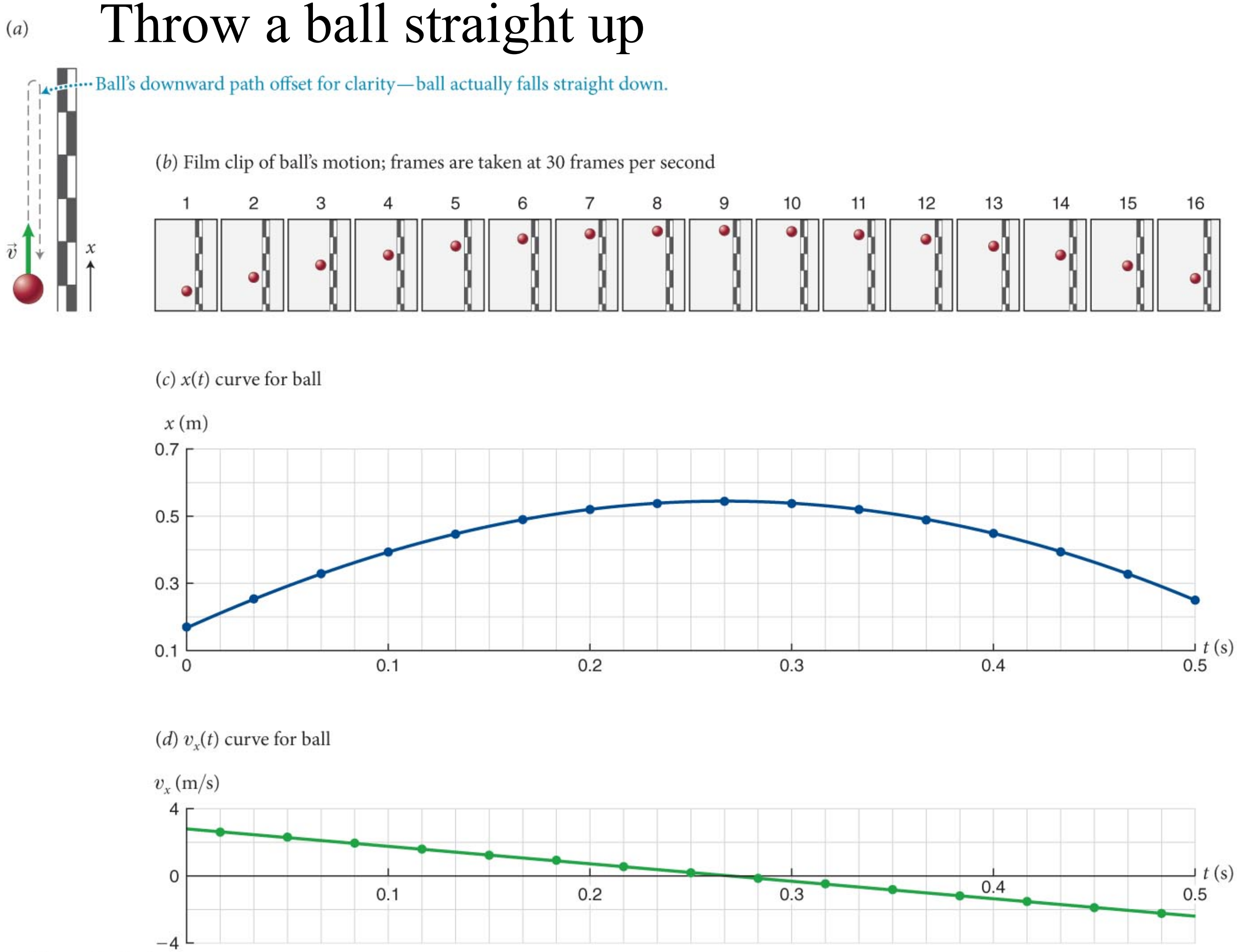
(This is not an example of physicists being careful about terminology.)

(There is no possibility of ambiguity here.)

(This is an example of physicists being pedantic sometimes.)

Section 3.3: Projectile motion

Throw a ball straight up



Section 3.3: Projectile motion

- Consider $x(t)$ and $v(t)$ curves:
  - **As the ball moves upward** it slows down
    - v and a are in opposite directions
    - since v is up, a must be down
  - **As the ball moves down** it speeds up
    - v and a must be in the same direction
    - since v is down, a is down
  - the **v (*t*) curve is a straight line** *for the whole motion*
    - slope approximately the *acceleration due to gravity*.

  - once the object is released, the rest of its motion is determined by gravity alone (free fall).

Checkpoint 3.8

3.8 Imagine throwing a ball downward so that it has an initial speed of 10 m/s.

What is its speed 1 s after you release it?

2 s after?

Constant acceleration: gain/lose same speed each second

- launched downward, so it speeds up
- ~$10 m/s^2$, 1 second later: gain 10m/s ➔ 20m/s
- 2 seconds later: gain another 10m/s ➔ 30m/s

Section 3.3: Projectile motion

- What happens at the very top of the trajectory of a ball launched upward?
  - At the top, velocity changes from up to down, which means that acceleration must be nonzero.
  - At the very top, the instantaneous velocity is zero.
  - Acceleration, however, is nonzero.

  - *Acceleration is always ~*$9.8 m/s^2$

Section 3.4: Motion diagrams

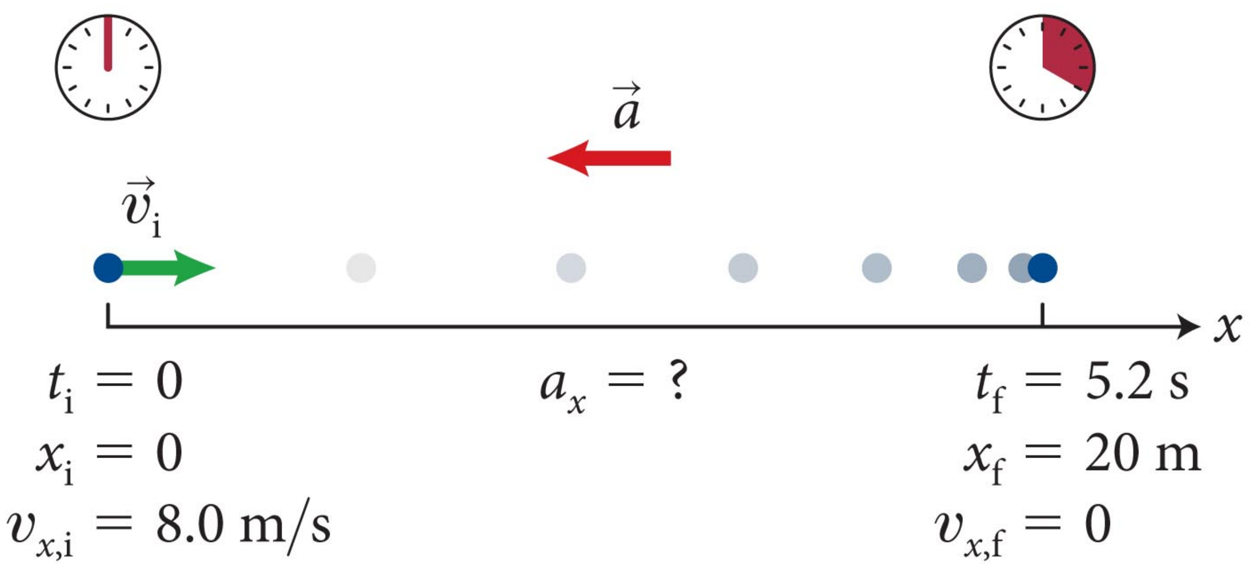
Section Goals

You will learn to

- Generalize the "frame sequence" diagram introduced in Chapter 2 to a new visual representation called a **motion diagram**.
- Represent and correlate the kinematic quantities, position, displacement, velocity, and acceleration on motion diagrams.

Section 3.4: Motion diagrams

- Motion diagrams are pictorial representations of objects in motion:
  - visualize the motion of an object described in a problem.
  - they show an object's x, v, and a at several equally spaced instances (including at the start and end).
  - it is basically a cartoon
- Below: a motion diagram for a bicycle with an initial velocity of 8.0 m/s slowing down to a stop.



Procedure: Analyzing motion using motion diagrams

Solving motion problems: a diagram summarizing what you have & what you want may all but solve the problem

1. Use dots to represent the moving object at equally spaced time intervals. If the object moves at constant speed, the dots are evenly spaced; if the object speeds up, the spacing between the dots increases; if the object slows down, the spacing decreases.
2. Choose an x (position) axis that is convenient for the problem. Most often this is an axis that (*a*) has its origin at the initial or final position of the object and (*b*) is oriented in the direction of motion or acceleration.

Procedure: Analyzing motion using motion diagrams (cont.)

3. Specify x & v at all relevant instants. **Particularly, specify**
   - **the *initial conditions*** - position and velocity at the beginning of the time interval of interest
   - **the *final conditions*** - position and velocity at the end of that time interval.
   - also note where v reverses direction or a changes.
   - unknown parameters = question mark.
4. Indicate the acceleration of the object between all the instants specified

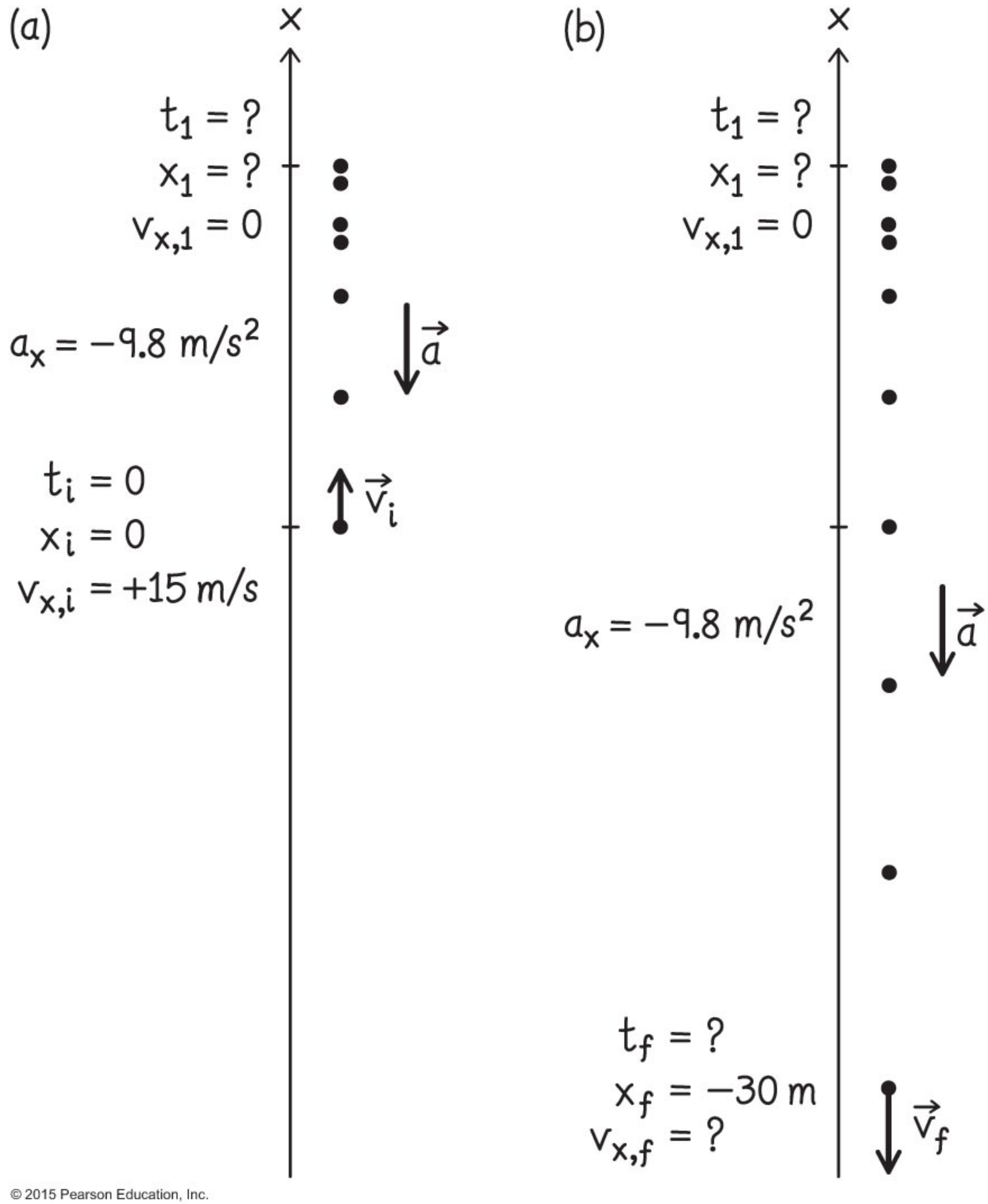
Procedure: Analyzing motion using motion diagrams (cont.)

5. With more than one object, draw separate diagrams side by side, using one common x axis.

6. If the object reverses direction, separate the motion diagram into two parts, one for each direction

Checkpoint 3.9

3.9 Make a motion diagram for the following situation: A seaside cliff rises 30 m above the ocean surface, and a person standing at the edge of the cliff launches a rock vertically upward at a speed of 15 m/s. After reaching the top of its trajectory, the rock falls into the water.


(a)
x
$t_1 = ?$
$x_1 = ?$
$v_{x,1} = 0$
$a_x = -9.8\ m/s^2$
$\vec{a}$
$t_i = 0$
$x_i = 0$
$v_{x,i} = +15\ m/s$
$\vec{v}_i$
(b)
x
$t_1 = ?$
$x_1 = ?$
$v_{x,1} = 0$
$a_x = -9.8\ m/s^2$
$\vec{a}$
$t_f = ?$
$x_f = -30\ m$
$v_{x,f} = ?$
$\vec{v}_f$
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Example 3.2 Can this be?

A newspaper article you read claims that by the time it reaches the ground, a stone dropped from the top of the Empire State Building (which has approximately 100 floors) "travels the length of a window faster than you can say *Watch out! A stone!*" Estimate whether this is true.

Example 3.2 Can this be? (cont.)

❶ GETTING STARTED At first this problem appears ill-defined.

1. What am I supposed to calculate?
2. Where to begin?

1st isn't clear, 2nd is: use a motion diagram.

Organize what you have.

Example 3.2 Can this be? (cont.)

❶ GETTING STARTED

x axis with:

origin at top of building

+x pointing down

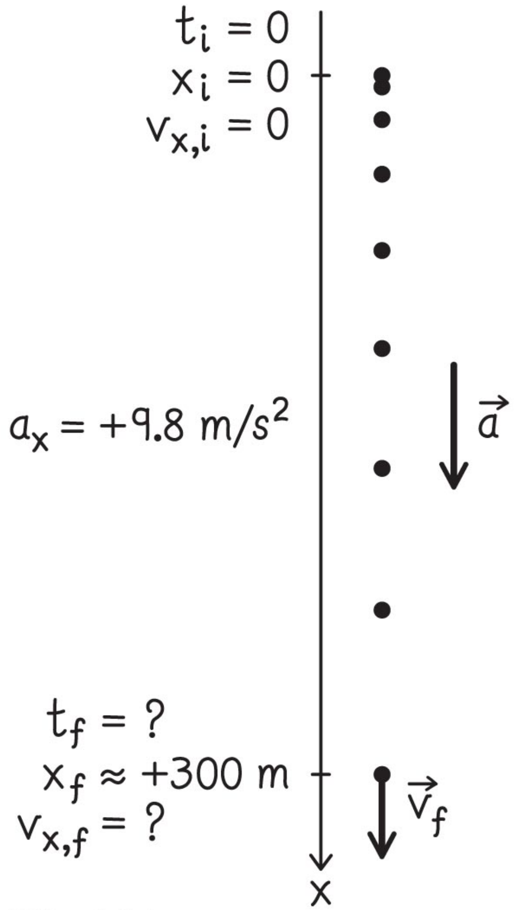
x increases as stone falls

initial conditions:

$$t_\mathrm{i} = 0, \;\; x_\mathrm{i} = 0, \;\; v_{x,\mathrm{i}} = 0$$

OMG. +x is downward? You monster.

Why would it matter?



Example 3.2 Can this be? (cont.)

❶ GETTING STARTED

initial time t_i and position x_i are zero by choice of origin

the x component of the initial velocity is zero

assumption based on “dropped” not “thrown”

Example 3.2 Can this be? (cont.)

❶ GETTING STARTED

Interested in speed just as stone hits.

final position = ground

Distance between the initial and final positions?

each floor is about 3 m high, net 300 m high

how have you not googled this yet? (381 m)

Final conditions?

$$t_{\mathrm{f}} = ?,\ \ x_{\mathrm{f}} = +300\ \mathrm{m},\ \ \upsilon_{x,\mathrm{f}} = ?$$

Example 3.2 Can this be? (cont.)

❶ GETTING STARTED

Between $x_i = 0$ and $x_f = +300$ m, the stone accelerates downward because of gravity.

Assume air resistance is negligible (why?)

write "$a_x = +9.8 \text{ m/s}^2$" between the initial and final positions.

Example 3.2 Can this be? (cont.)

❷ DEVISE PLAN

How fast does it have to be going to travel the length of a window in the same time interval it takes to say "Watch out! A stone!"

Are we going that fast or not?

Because no values are given, use estimates

Example 3.2 Can this be? (cont.)

❸ EXECUTE PLAN

Takes roughly 2 s to say those words

A window is about 2 m tall.

As long as the stone is faster than (2 m)/(2 s) = 1 m/s just before it hits the ground, it is true

Acceleration in free fall is about 10 m/s^2, so the speed of the stone increases by 10 m/s each second.

It takes way longer than 1 s to fall.

It is like 1 s from *this* building

The statement must be true. ✔

Example 3.2 Can this be? (cont.)

❹ EVALUATE RESULT

Even if I account for some slowing down due to air resistance, I know from experience that a stone dropped from even a much smaller height travels more than 1 m in 1 s just before it reaches the ground.

Chapter 3: Self-Quiz #1

Two stones are released from rest at a certain height, one 1 s after the other.

(a) Once the second stone is released, does the difference in their speeds increase, decrease, or stay the same?

(b) Does their separation increase, decrease, or stay the same?

(c) Is the time interval between the instants at which they hit the ground less than, equal to, or greater than 1 s? (Use $x(t)$ curves to help you visualize this problem.)

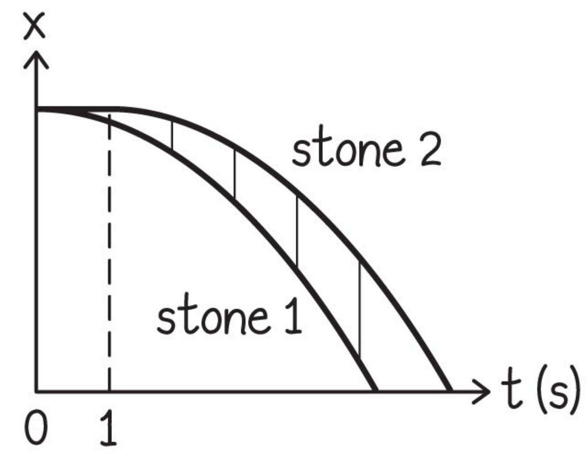
Chapter 3: Self-Quiz #1

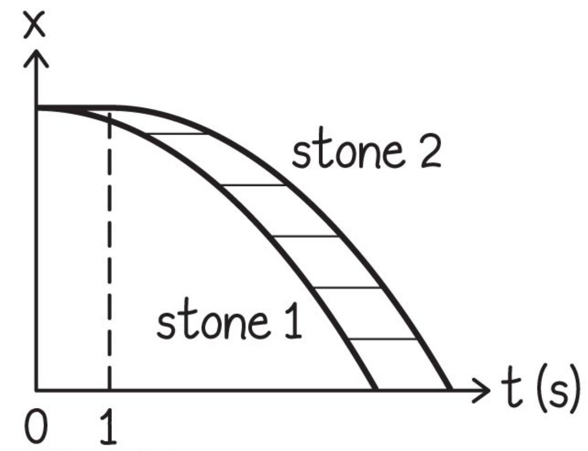
Answer

(a) Both stones accelerate at about 10 m/s^2, so the speeds increase at the same rate, the **difference in the speeds remains the same.**

(b) The separation increases because the speed of the first stone is always greater. Position goes as *v* times *t*

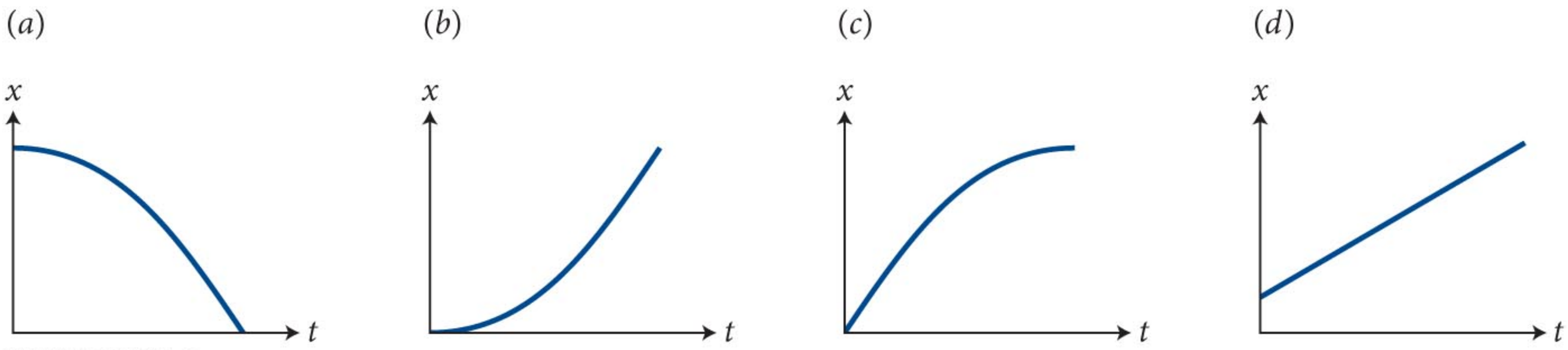
(c) the second stone always remains 1 s behind, this is how time works





Chapter 3: Self-Quiz #2

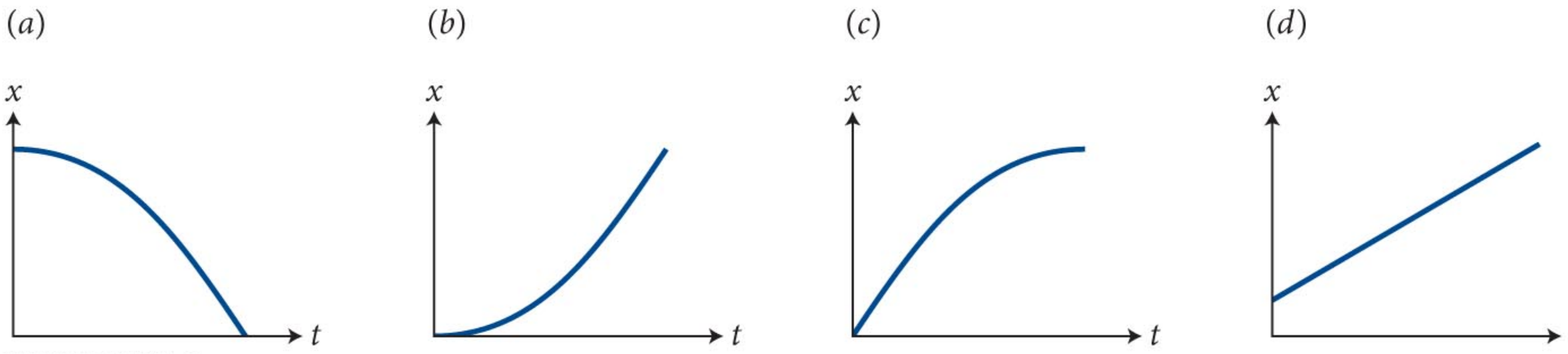
Which of the graphs in Figure 3.12 depict(s) an object that starts from rest at the origin and then speeds up in the positive *x* direction?



Chapter 3: Self-Quiz #2

Answer

Choice *b* is the correct answer because its initial position is zero and the slope is initially zero but then increasing, indicating that the object speeds up.



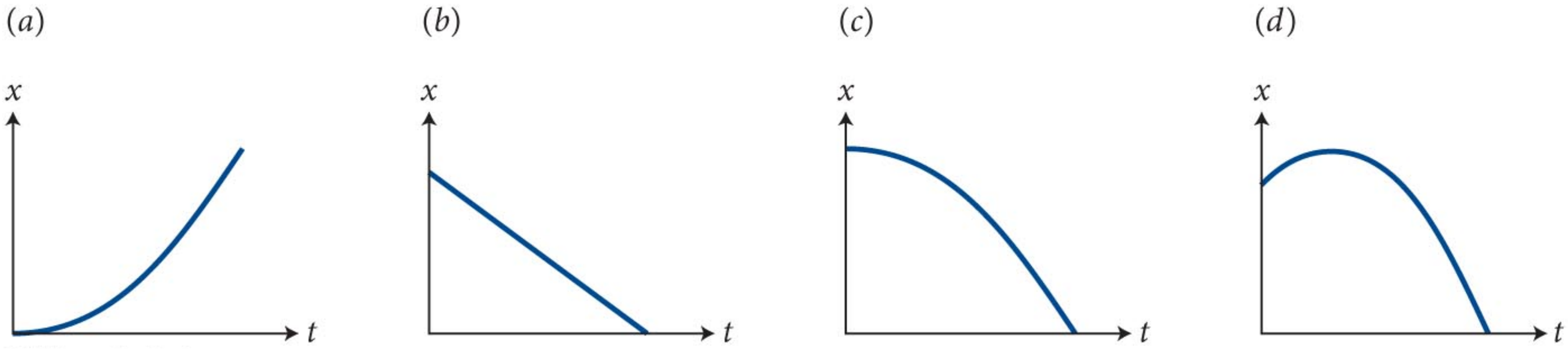
speeds in –x
no start at origin

slows

no accel
no start at origin

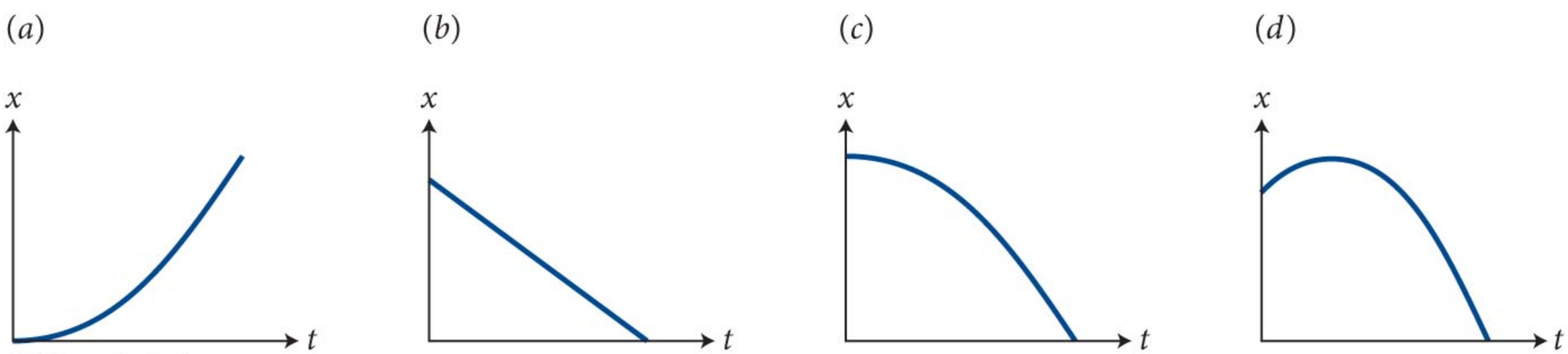
Chapter 3: Self-Quiz #3

Which of the graphs in Figure 3.13 depict(s) an object that starts from a positive position with a positive x component of velocity and accelerates in the negative x direction?



Answer

Choice *d*. Choice *a* does not have a positive initial position. Choice *b* represents zero acceleration. Choice *c* represents zero initial velocity.



Quantitative Tools

Section 3.5: Motion with constant acceleration

Section Goals

You will learn to

- Represent motion with constant acceleration using motion graphs and mathematics.
- Construct self-consistent position-versus-time, velocity-versus-time, and acceleration-versus-time graphs for specific motion situations.

Section 3.5: Motion with constant acceleration

- We can write down the definition for the x component of *average acceleration*:

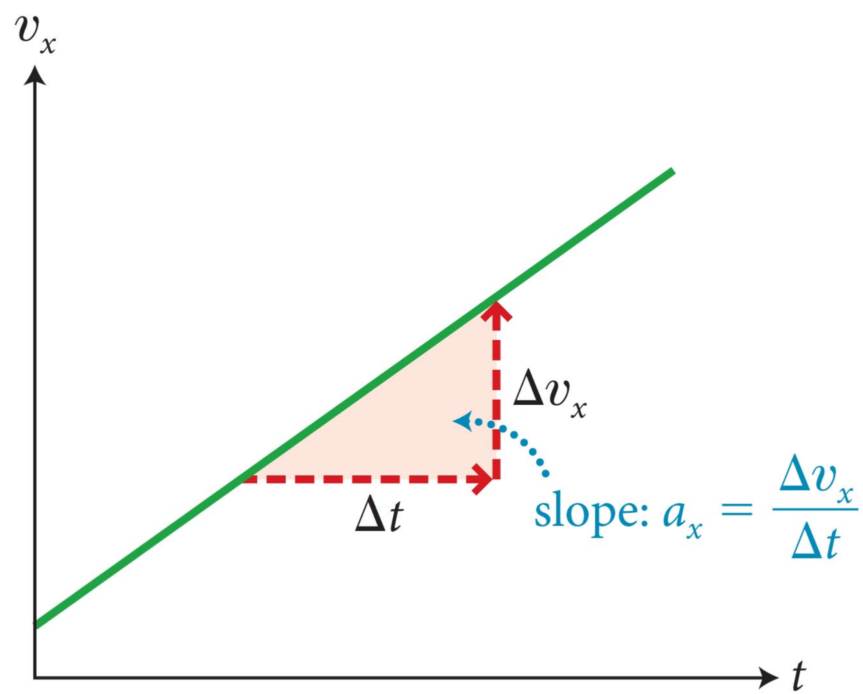
$$a_{x,\text{av}} \equiv \frac{\Delta \upsilon_x}{\Delta t} = \frac{\upsilon_{x,\text{f}} - \upsilon_{x,\text{i}}}{t_\text{f} - t_\text{i}}$$

- Notice the similarity between this definition and the definition of average velocity in chapter 2:

$$\upsilon_{x,\text{av}} \equiv \frac{\Delta x}{\Delta t} = \frac{x_\text{f} - x_\text{i}}{t_\text{f} - t_\text{i}}$$

Section 3.5: Motion with constant acceleration

- Now let us consider the motion of an object with **constant acceleration**:
  - For motion with constant acceleration, $a_{x,\text{av}} = a_x$ and $v_x(t)$ curve is a straight line.
  - Rewriting Equation we can get the x-component of final velocity:



$$v_{x,\text{f}} = v_{x,\text{i}} + a_x \Delta t \quad \text{(constant acceleration)}$$

Section 3.5: Motion with constant acceleration

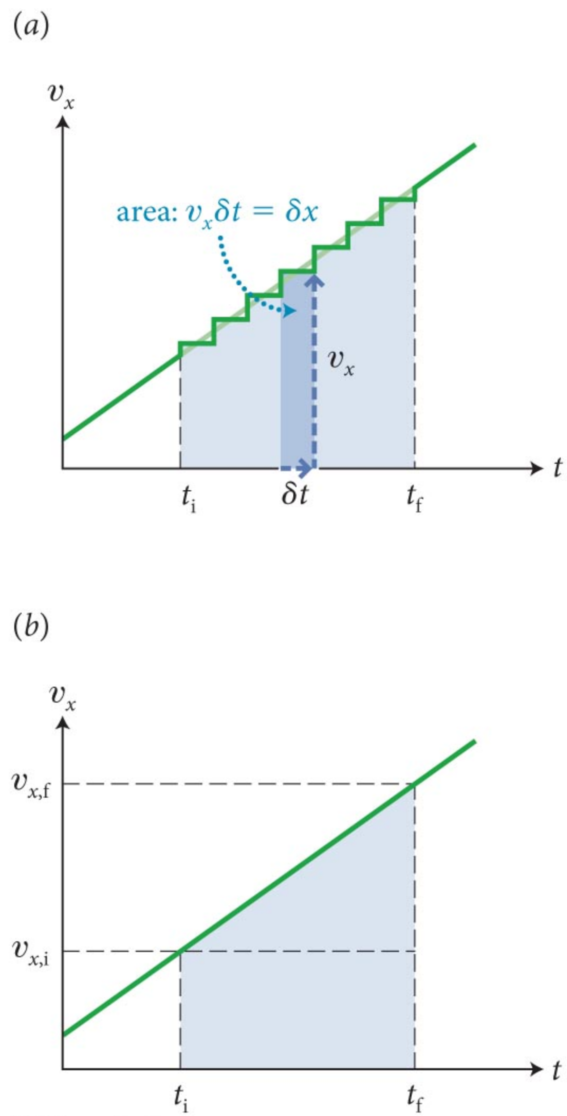
- displacement is the area under the $v_x(t)$ curve.

- for an object in motion with constant acceleration, the displacement ($\Delta x = x_f - x_i$) in time interval ($\Delta t = t_f - t_i$) is given by the area of the shaded trapezoid
- Setting $t_i = 0$, the object's final position can be written as

$$x_f = x_i + \upsilon_{x,i} t_f + \tfrac{1}{2} a_x t_f^2 \quad \text{(constant acceleration)}$$

- we can determine the object's final velocity

$$\upsilon_{x,f} = \upsilon_{x,i} + a_x t_f \quad \text{(constant acceleration)}$$



Section 3.5: Motion with constant acceleration

- Since t_f is an arbitrary instant in time in the object's motion, we can drop the subscript *f* and rewrite as

$$x(t) = x_i + \upsilon_{x,i} t + \tfrac{1}{2} a_x t^2 \quad \text{(constant acceleration)}$$

$$\upsilon_x(t) = \upsilon_{x,i} + a_x t \quad \text{(constant acceleration)}$$

- This is easier with calculus, continuous time

$$a = \frac{dv}{dt} \implies v = \int a\, dt = at + C$$

- C is v(t=0) or v_i $\quad v(t) = v_i + at$

- Once more:

$$x = \frac{dv}{dt} \implies x = \int v\, dt = v_i t + \frac{1}{2}at^2 + C'$$

- C' is x(t=0) or x_i

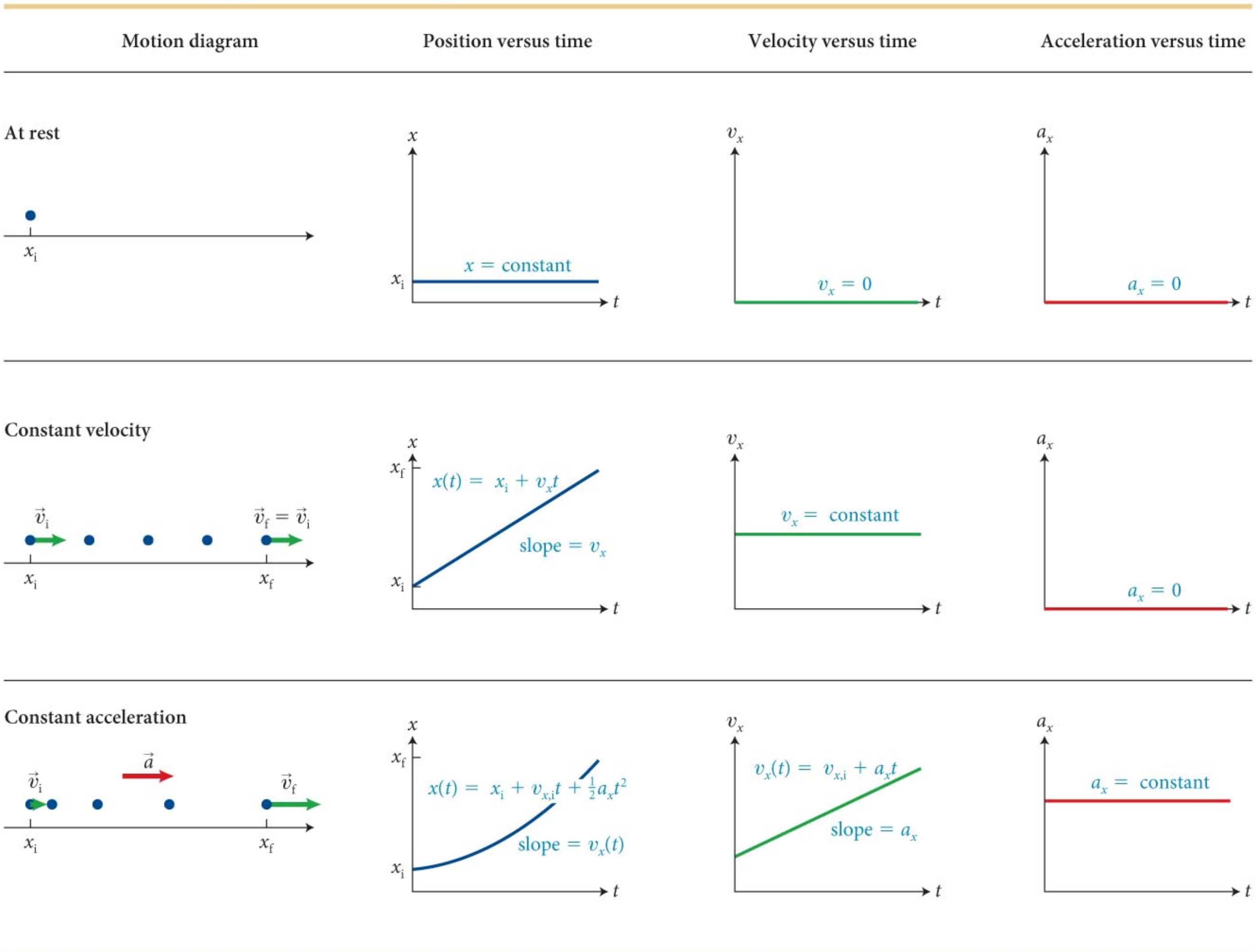
$$x(t) = x_i + v_i t + \frac{1}{2}at^2$$

In terms of displacement

$$\Delta x = x_f - x_i = v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$

Section 3.5: Motion with constant acceleration

Table 3.1 Kinematics graphs for three basic types of motion



Example 3.4 Collision or not?

You are bicycling at a steady 6.0 m/s when someone suddenly walks into your path 2.5 m ahead. You immediately apply the brakes, which slow you down at 6.0 m/s^2. Do you stop in time to avoid a collision?

Example 3.4 Collision or not? (cont.)

❶ GETTING STARTED

In order to avoid a collision, you must come to a stop in less than 2.5 m.

Need to calculate the distance traveled under the given conditions. Is it more or less than 2.5 m?

Example 3.4 Collision or not? (cont.)

❷ DEVISE PLAN I have equations for displacement, but I don't know the time interval Δt.

From the definition of acceleration:

$$\Delta t = (\upsilon_{x,\mathrm{f}} - \upsilon_{x,\mathrm{i}})/a_x$$

which contains no unknowns on the right side.

Example 3.4 Collision or not? (cont.)

❸ EXECUTE PLAN Substituting the expression for the time interval gives the x component of the displacement necessary to stop:

$$\Delta x = \upsilon_{x,\mathrm{i}} \underbrace{\boxed{\frac{\upsilon_{x,\mathrm{f}} - \upsilon_{x,\mathrm{i}}}{a_x}}}_{\Delta t} + \tfrac{1}{2} a_x \Bigg(\underbrace{\boxed{\frac{\upsilon_{x,\mathrm{f}} - \upsilon_{x,\mathrm{i}}}{a_x}}}_{\Delta t} \Bigg)^2 = \frac{\upsilon_{x,\mathrm{f}}^2 - \upsilon_{x,\mathrm{i}}^2}{2a_x} \qquad (1)$$

Example 3.4 Collision or not? (cont.)

❸ EXECUTE PLAN

With $+x$ along the direction of the motion

$$v_{x,\mathrm{i}} = +6.0\ \mathrm{m/s}$$
$$v_{x,\mathrm{f}} = 0$$
$$a_x = -6.0\ \mathrm{m/s^2}.$$

$$\Delta x = \frac{0 - (+6.0\ \mathrm{m/s})^2}{2(-6.0\ \mathrm{m/s^2})} = +3.0\ \mathrm{m}$$

more than the 2.5 m required. You will totally collide. ✔

Section 3.5: Motion with constant acceleration

- Notice: rearranging we can find the final velocity of an object under constant acceleration over a certain displacement (Δx):

$$\upsilon_{x,\mathrm{f}}^2 = \upsilon_{x,\mathrm{i}}^2 + 2a_x \Delta x \quad \text{(constant acceleration)}$$

- advantage: don’t need to know time!

Checkpoint 3.10

3.10 Determine the velocity of the stone dropped from the top of the Empire State Building in Example 3.2 just before the stone hits the ground.

$$\upsilon_{x,\mathrm{f}}^2 = \upsilon_{x,\mathrm{i}}^2 + 2a_x \Delta x \quad \text{(constant acceleration)}$$

initial velocity is zero

displacement ~ 300 m

acceleration ~ 10 m/s^2

$$v_{\mathrm{x,f}} \sim 80 \text{ m/s}$$

Section 3.6: Free-fall equations

Section Goals

- Model free-fall motion using the concept of gravity and the definitions of velocity and acceleration.
- Manipulate the equations for free-fall into a form that allows the prediction of the future motion of an object from its present state of motion.

Section 3.6: Free-fall equations

- The magnitude of the **acceleration due to gravity** is designated by the letter g:

$$g \equiv \left|\vec{a}_{\text{free fall}}\right|$$

  - Near Earth's surface $g = 9.8\ \text{m/s}^2$.
  - The direction of the acceleration is downward, and if we chose a positive axis pointing upward, $a_x = -g$.
- If an object is dropped from a certain height with zero velocity along an **upward-pointing *x*-axis**, then

$$x_\text{f} = x_\text{i} - \tfrac{1}{2} g t_\text{f}^2$$

$$\upsilon_{x,\text{f}} = -g t_\text{f}$$

Example 3.5 Dropping the ball

Suppose a ball is dropped from height $h = 20$ m above the ground. How long does it take to hit the ground, and what is its velocity just before it hits?

Example 3.5 Dropping a ball (cont.)

❶ GETTING STARTED

x axis that points upward

origin at the initial position of the ball

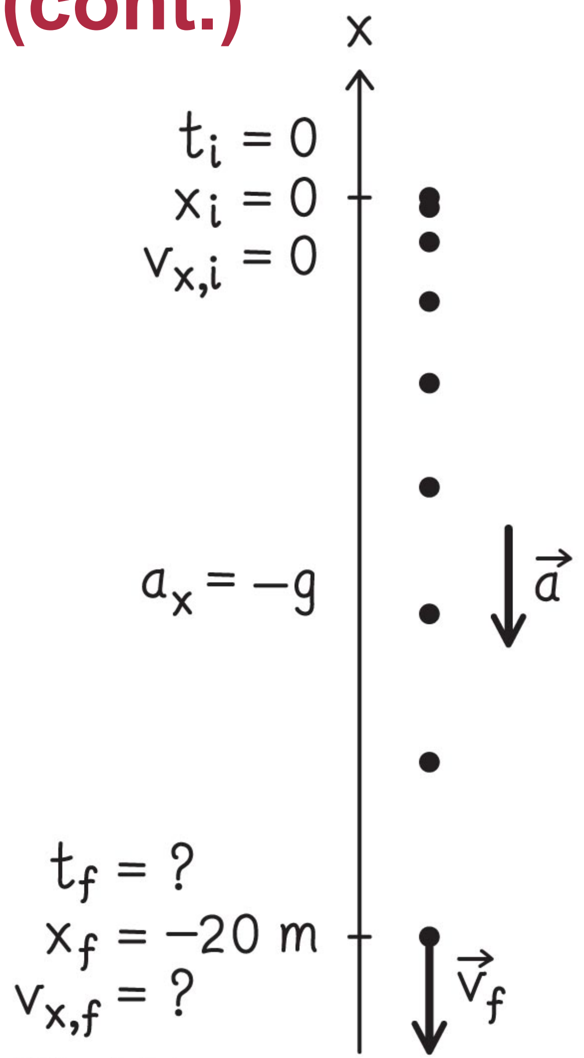
assumptions:

released from rest ($v_{x,\mathrm{i}} = 0$ at $t_\mathrm{i} = 0$)

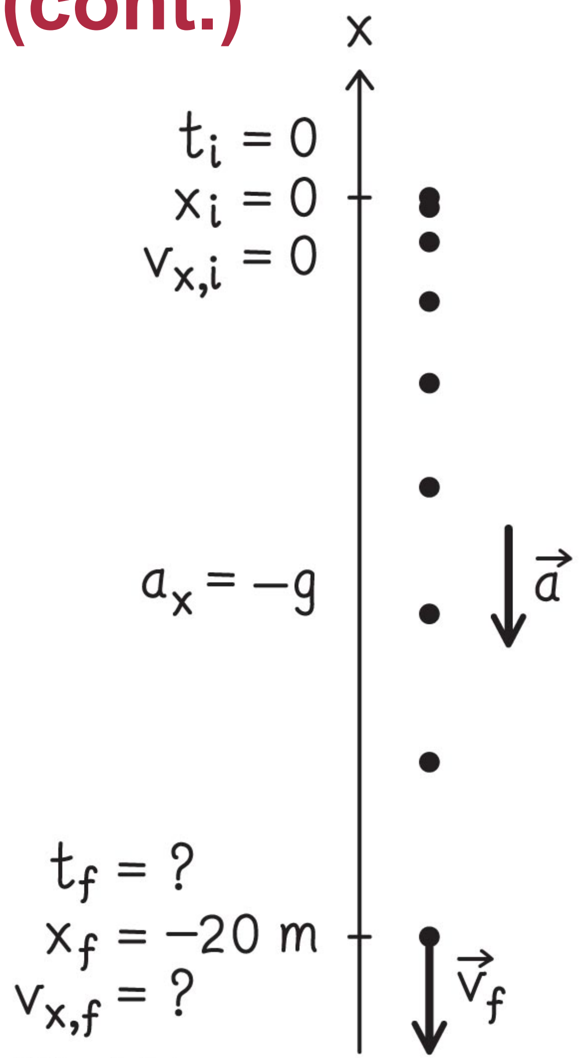
ignore air resistance

initial conditions are

$$t_\mathrm{i} = 0,\ x_\mathrm{i} = 0,\ v_{x,\mathrm{i}} = 0$$



Example 3.5 Dropping a ball (cont.)



❶ GETTING STARTED

final position x_f at instant t_f is a distance h below the initial position

just before impact at instant t_f, the final conditions are

$$t_f = ?,\ x_f = -h,\ v_{x,f} = ?$$

acceleration is negative, $a_x = -g$.

Example 3.5 Dropping a ball (cont.)

❷ DEVISE PLAN

Acceleration is constant, so our equations work.

Gives me two equations & two unknowns: t_f and $v_{x,f}$

$$x(t) = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_f = v_i + at$$

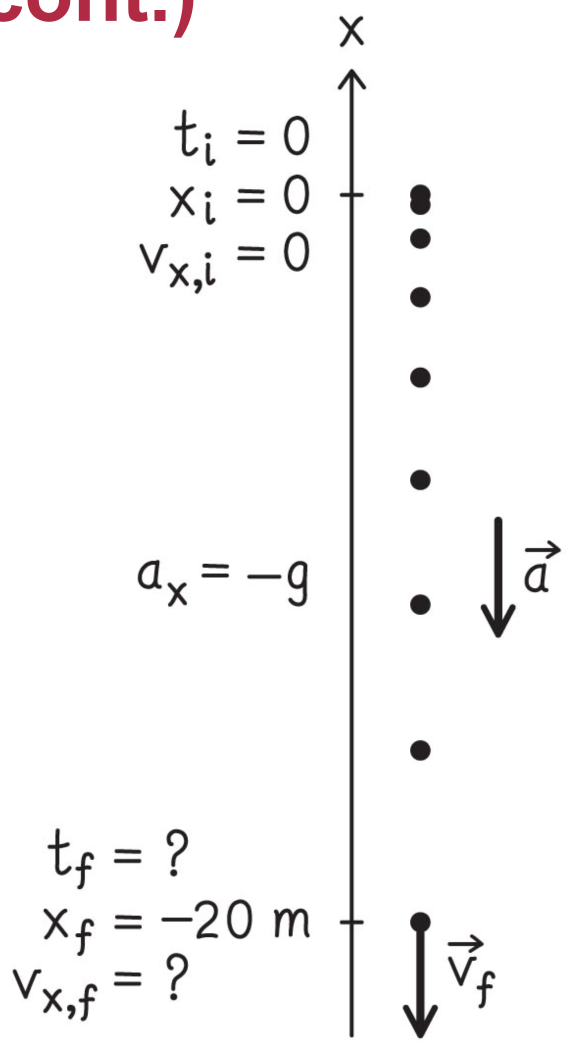
Example 3.5 Dropping a ball (cont.)

❸ EXECUTE PLAN Substituting the initial and final conditions

$$-h = 0 + 0 - \tfrac{1}{2} g t_{\mathrm{f}}^2 = -\tfrac{1}{2} g t_{\mathrm{f}}^2$$

and so

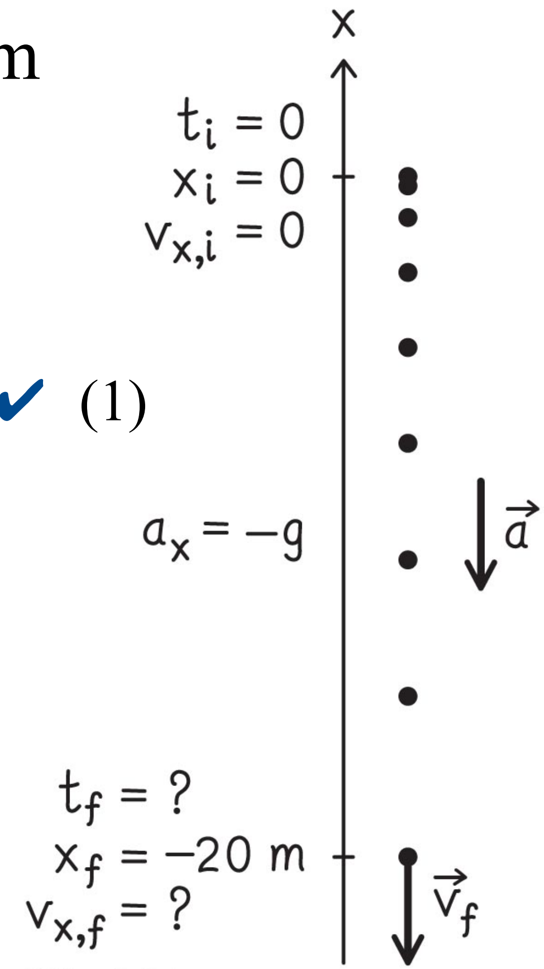
$$t_{\mathrm{f}} = \sqrt{\frac{2h}{g}}$$



Example 3.5 Dropping a ball (cont.)

❸ EXECUTE PLAN Substituting $h = 20$ m and $g = 9.8$ m/s^2

$$\Delta t = t_\mathrm{f} - t_\mathrm{i} = \sqrt{\frac{2h}{g}} - 0 = \sqrt{\frac{2(20\ \mathrm{m})}{9.8\ \mathrm{m/s^2}}} = \sqrt{4.0\ \mathrm{s}^2} = 2.0\ \mathrm{s}\ ✔ \quad (1)$$



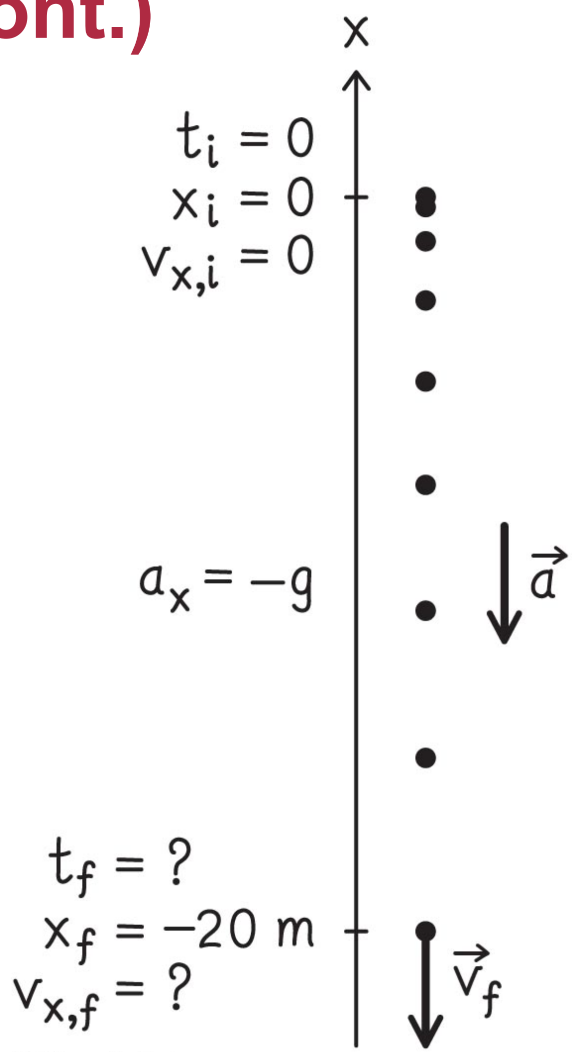
Section 3.6: Free-fall equations

Example 3.5 Dropping a ball (cont.)

❸ EXECUTE PLAN

Because the ball starts from rest:

$$v_{x,\mathrm{f}} = 0 - gt_\mathrm{f} = -gt_\mathrm{f} = -(9.8\ \mathrm{m/s^2})(2.0\ \mathrm{s})$$
$$= -20\ \mathrm{m/s}.\ ✔$$



Example 3.5 Dropping a ball (cont.)

❹ EVALUATE RESULT

Time is reasonable based on everyday experience

Final velocity $\Delta x_{,\mathrm{f}} = -20$ m/s also makes sense:

- negative because it points in the negative *x* direction
- if the ball was at a constant speed of 20 m/s, it would cover the 20-m distance in 1 s. It moves at that speed only at the end of the drop, so it takes longer to fall.

Section 3.7: Inclined planes

Section Goals

You will learn to

- Identify that one-dimensional motion along an incline plane can be related to free-fall motion along a non-vertical direction.
- Establish that purely horizontal and purely vertical motion are the special cases of motion along an incline plane.

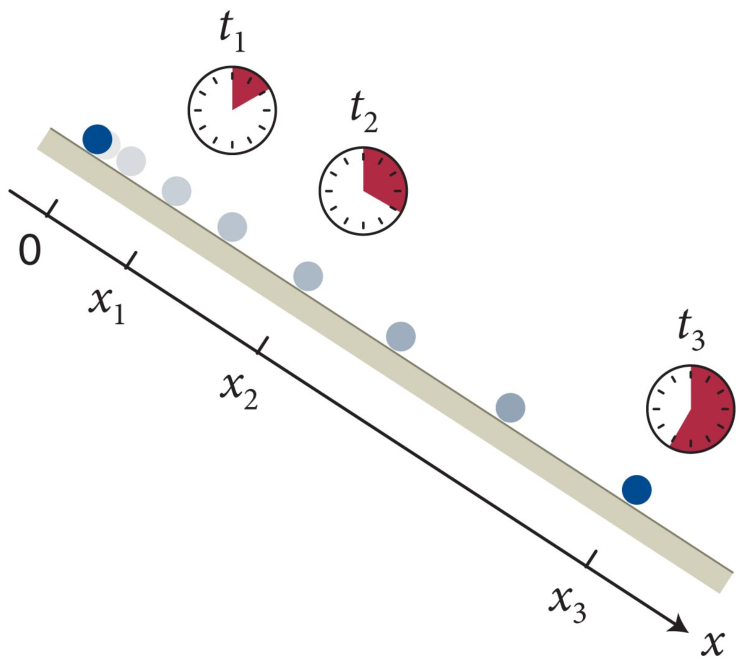
Section 3.7: Inclined planes

- Galileo used inclined planes to study motion of objects that are accelerated due to gravity:
  - He found that when a ball rolls down an incline starting at rest, the ratio of the distance traveled to the square of the amount of time needed to travel that distance is constant:

$$\frac{x_1}{t_1^2} = \frac{x_2}{t_2^2} = \frac{x_3}{t_3^2}$$

  - Using this and setting $x_i = 0$ and $t_i = 0$ we can show that this ratio is proportional to a_x:

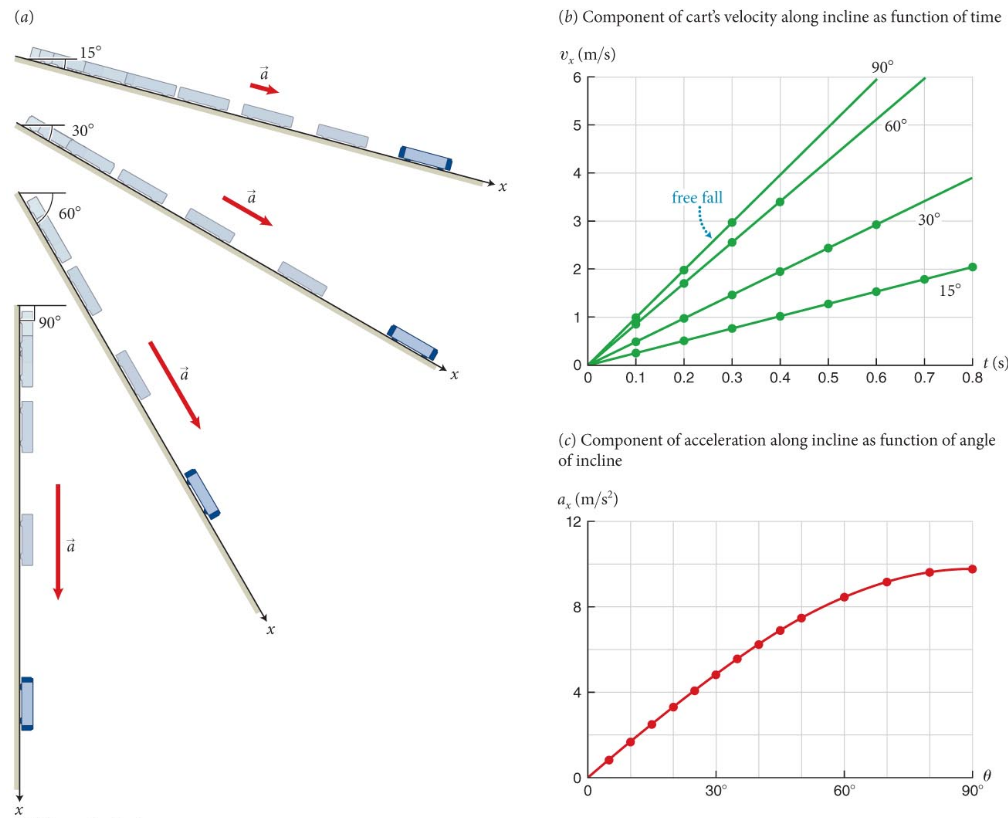
$$\frac{x_f}{t_f^2} = \tfrac{1}{2} a_x$$



Section 3.7: Inclined planes

- Galileo observed that
  - For each value of the angle θ, a_x along the incline is a constant.
  - a_x along the incline increases as θ increases.
  - Experimentally we can determine that the x component of the acceleration along the incline obey the relationship

$$a_x = +g \sin \theta$$



Inclined planes

- establishes gravity is vertical, constant acceleration
- it is a vector, and only the vertical component matters
- for inclined plane, the component along the plane is $a_x = +g \sin \theta$

Worked Problem 3.5 Inclined track

Your physics instructor prepares a laboratory exercise in which you will use a modern version of Galileo's inclined plane to determine acceleration due to gravity. In the experiment, an electronic timer records the time interval required for a cart initially at rest to descend 1.20 m along a low-friction track inclined at some angle θ with respect to the horizontal.

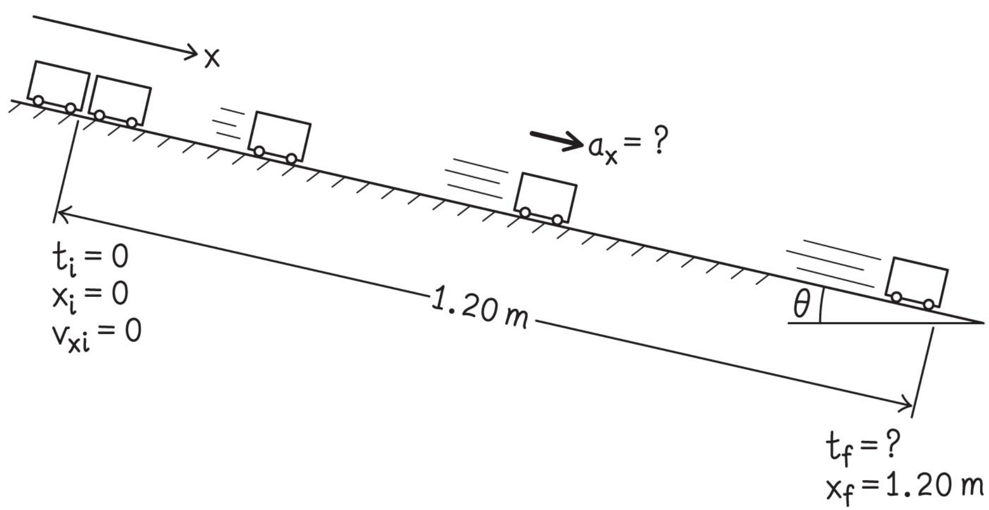
Worked Problem 3.5 Inclined track (cont.)

(*a*) In preparation for the experiment, you must obtain an equation from which you can calculate g on the basis of these measurements. What is that equation?

(*b*) To make it possible to check the students' measurements quickly, the instructor breaks the class into five groups and assigns one value of θ to each group. If no mistakes are made, these five θ values yield time intervals of 0.700, 0.800, 0.900, 1.00, and 1.20 s. What are the five θ values?

Worked Problem 3.5 Inclined track (cont.)

❶ GETTING STARTED The cart undergoes constant acceleration, from rest, on an inclined plane. We know how to analyze this type of motion, and we know how the acceleration at any given incline angle is related to the acceleration *g* due to gravity. We sketch a motion diagram, representing a cart moving down an inclined plane, and choose the positive *x* direction as pointing down the track.



Worked Problem 3.5 Inclined track (cont.)

❷ DEVISE PLAN

- We could use $a_x = +g \sin \theta$, but you will not be measuring a_x values directly.
- Measure displacements Δx and time intervals Δt
- We need an expression that gives acceleration in terms of these two variables. Use initial conditions & rearrange:

$$x(t) = x_i + v_i t + \frac{1}{2} a t^2$$

Worked Problem 3.5 Inclined track (cont.)

❸ EXECUTE PLAN

- With zero initial velocity, we have

  $a_x = 2(x_f - x_i)/t^2 = 2\Delta x/t^2$

- t_i was taken to be zero, the t^2 is actually $(\Delta t)^2$, so

$$a_x = \frac{2\Delta x}{(\Delta t)^2}$$

Worked Problem 3.5 Inclined track (cont.)

❸ EXECUTE PLAN Substituting this expression for a_x

$$\frac{2\Delta x}{(\Delta t)^2} = g \sin\theta$$

from which we obtain the expression for g to be used in the experiment:

$$g = \frac{2\Delta x}{(\Delta t)^2 \sin\theta} \quad ✔$$

Worked Problem 3.5 Inclined track (cont.)

❸ EXECUTE PLAN (*b*) Manipulation gives

$$g\sin\theta = \frac{2\Delta x}{g(\Delta t)^2}$$

$$\theta = \sin^{-1}\left(\frac{2\Delta x}{g(\Delta t)^2}\right)$$

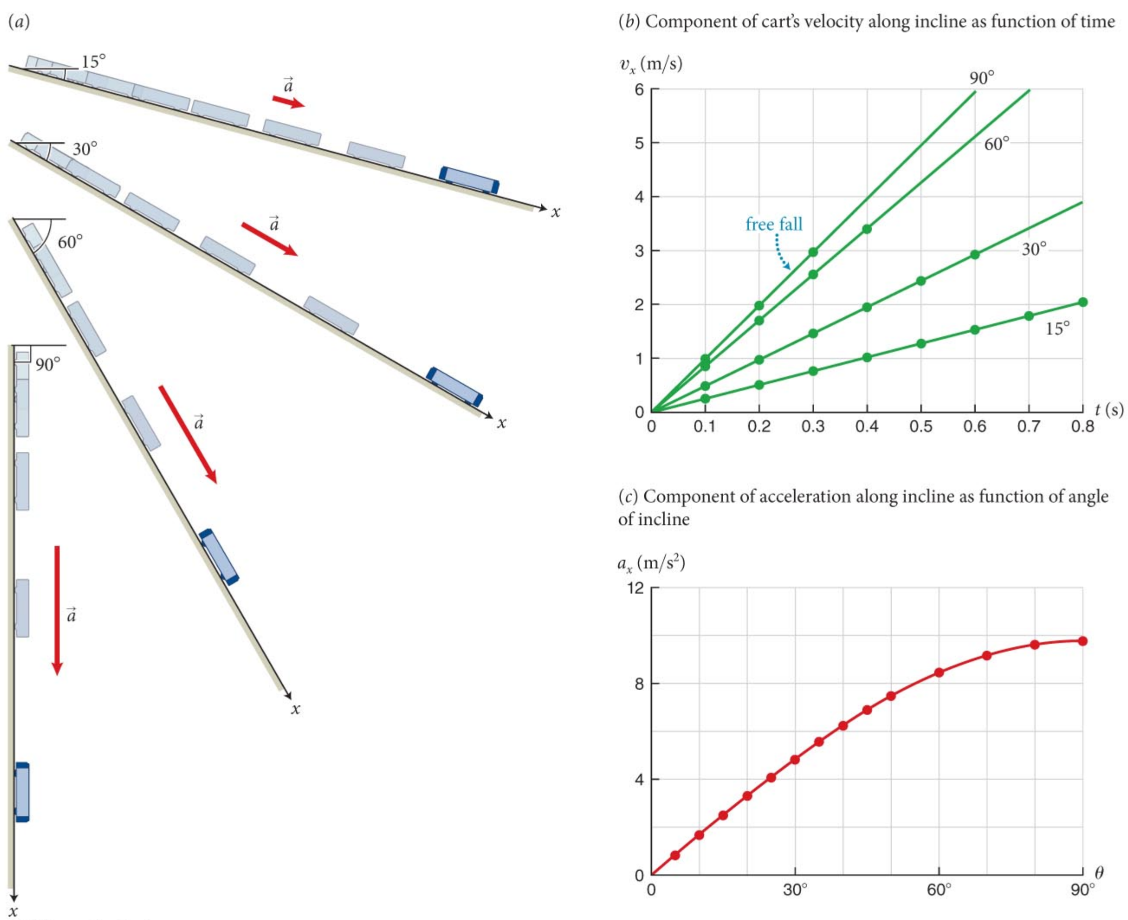
- First calculate the constant quantity $2\Delta x/g = 0.2449$ since we need it every time.
- Substitution of $\Delta t = 0.700, 0.800, 0.900, 1.00$, and 1.20 s into Eq. 2 yields the angles of incline she assigned to the five groups: 30.0°, 22.5°, 17.6°, 14.2°, and 9.79°. ✔

Worked Problem 3.5 Inclined track (cont.)

❹ EVALUATE RESULT The numerical values for the angles are reasonable: Larger angles are associated with smaller time intervals. Even the shortest interval is considerably longer than the time interval needed for an object to fall freely from a height of 1.2 m, as expected.

Checkpoint 3.13

3.13 As the angle θ of the incline used to collect the data of Figure 3.22 is increased beyond 90°, what happens to the acceleration? Does this result make sense (provided you always put the cart on the top side of the track)?



Section 3.8: Instantaneous acceleration

Section Goals

You will learn to

- Generalize the mathematical definition of the average acceleration of a moving object to instantaneous acceleration by use of a limiting process.
- Represent motion with continuous changes in velocity using motion graphs and mathematics.
- Relate the concept of a tangent line on a velocity-versus-time graph with the instantaneous acceleration.

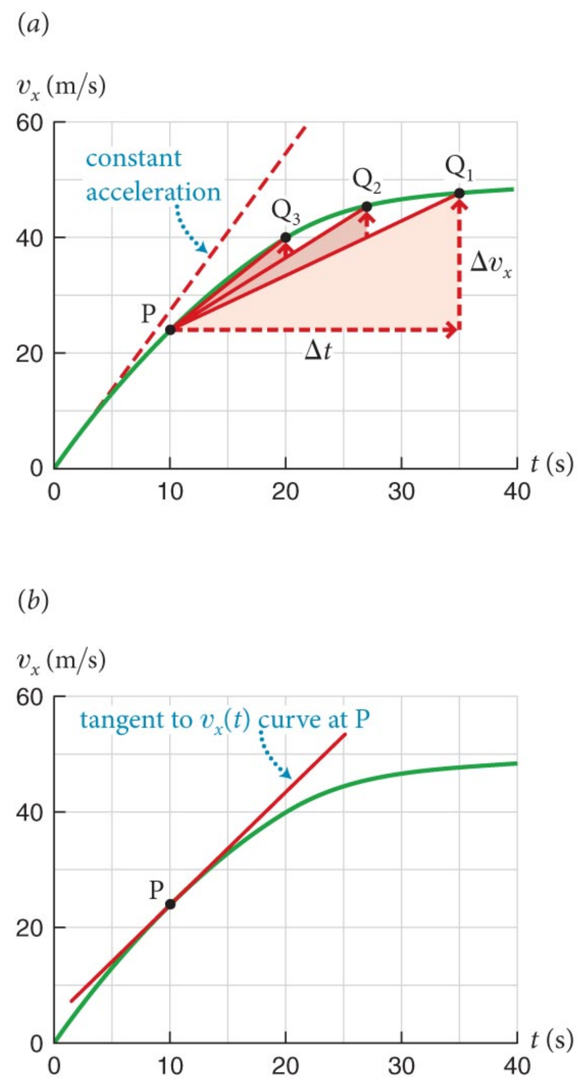
Section 3.8: Instantaneous acceleration

- What if acceleration is not constant?
  - The figure shows the $v_x(t)$ curve for a motion where the acceleration is not constant.
  - The instantaneous acceleration a_x is the slope of the tangent of the $v_x(t)$ curve at time t:

$$a_x = \frac{d\upsilon_x}{dt}$$

  - Or

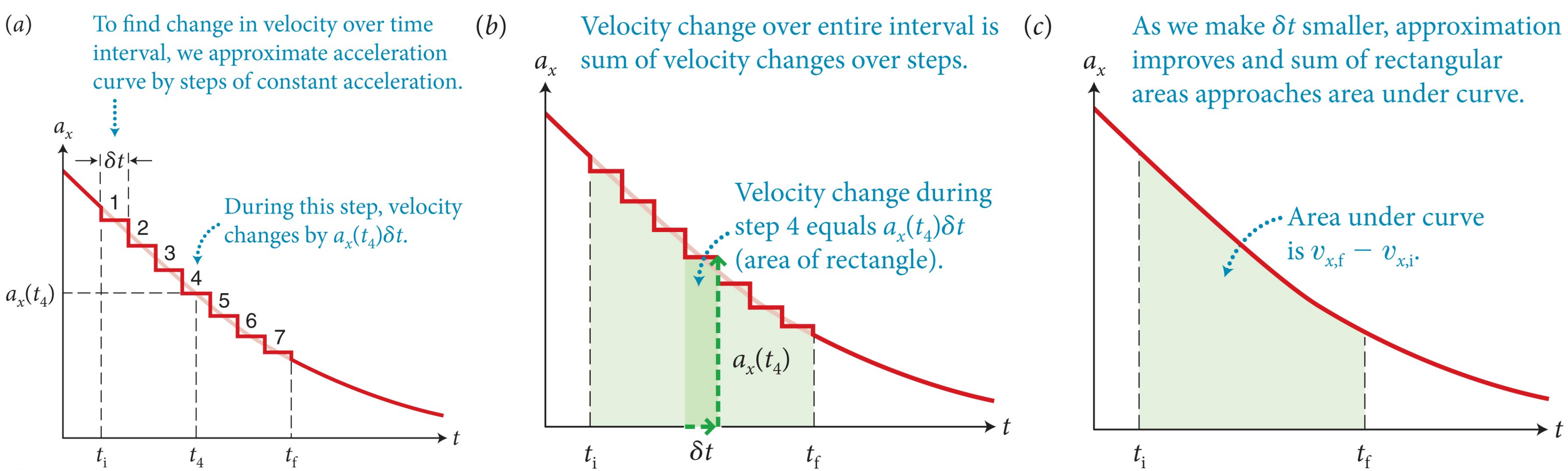
$$a_x = \frac{d\upsilon_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) \equiv \frac{d^2x}{dt^2}$$



Section 3.8: Instantaneous acceleration

- To find the change in velocity during the time interval (Δt), we can use the area under the $a_x(t)$ curve in the figure.
- Although, acceleration is not constant, we can divide motion into small intervals of Δt in which it is constant.
- In the limit $\Delta t \longrightarrow 0$, we can find

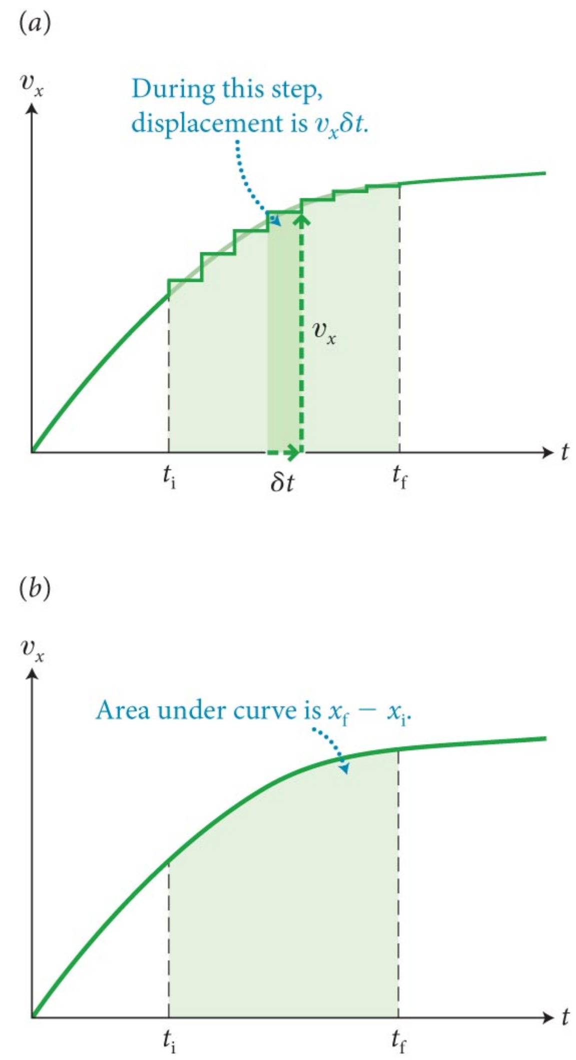
$$\Delta v_x = \int_{t_i}^{t_f} a_x(t)\,dt$$



Section 3.8: Instantaneous acceleration

- Once we know the velocity, we can use the same approach to obtain displacement:

$$\Delta x = \int_{t_i}^{t_f} v_x(t)\,dt$$



Substitute 3.11
Clicker Question 11

Exercise 3.7 Using calculus to determine displacement

- Suppose an object initially at x_i at $t_i = 0$ has a constant acceleration whose x component is a_x. Use calculus to show that the x component of the velocity and the x coordinate at some final instant t_f are given by Eqs. 3.10 and Eq. 3.9, respectively.

Exercise 3.7 Using calculus to determine displacement (cont.)

SOLUTION Because the acceleration is constant, I can pull a_x out of the integration in Eq. 3.27:

$$\Delta \upsilon_x = \upsilon_{x,\mathrm{f}} - \upsilon_{x,\mathrm{i}} = \int_{t_\mathrm{i}}^{t_\mathrm{f}} a_x \, dt = a_x \int_{t_\mathrm{i}}^{t_\mathrm{f}} dt = a_x (t_\mathrm{f} - t_\mathrm{i})$$

Substituting $t_\mathrm{i} = 0$ and rearranging terms, I obtain Eq. 3.10:

$$\upsilon_{x,\mathrm{f}} = \upsilon_{x,\mathrm{i}} + a_x t_\mathrm{f} \checkmark$$

Exercise 3.7 Using calculus to determine displacement (cont.)

SOLUTION For an arbitrary final instant t, I can drop the subscript f. Substituting this expression into Eq. 3.28, I get

$$\Delta x = \int_{t_\mathrm{i}}^{t_\mathrm{f}} (\upsilon_{x,\mathrm{i}} + a_x t)\, dt$$

or, pulling constant terms out of the integration and then carrying out the integration,

$$\Delta x = x_\mathrm{f} - x_\mathrm{i} = \upsilon_{x,\mathrm{i}} \int_{t_\mathrm{i}}^{t_\mathrm{f}} dt + a_x \int_{t_\mathrm{i}}^{t_\mathrm{f}} t\, dt = \upsilon_{x,\mathrm{i}} t_\mathrm{f} + a_x \left[\tfrac{1}{2} t^2 \right]_{t_\mathrm{i}}^{t_\mathrm{f}}$$

which yields Eq. 3.9:

$$x_\mathrm{f} = x_\mathrm{i} + \upsilon_{x,\mathrm{i}} t_\mathrm{f} + \tfrac{1}{2} a_x t_\mathrm{f}^2 \checkmark$$

Checkpoint 3.14

3.14 Take the first and second time derivatives of x_f in Eq. 3.9. What do you notice?

$$x_f = x_i + \upsilon_{x,i} t_f + \tfrac{1}{2} a_x t_f^2 \quad \text{(constant acceleration)}$$

Concepts: Accelerated motion

- If the velocity of an object is changing, the object is **accelerating**. The x component of an object's **average acceleration** is the change in the x component of its velocity divided by the time interval during which this change takes place.
- The x component of the object's **instantaneous acceleration** is the x component of its acceleration at any given instant.
- A **motion diagram** shows the positions of a moving object at equally spaced time intervals.

Quantitative Tools: Accelerated motion

- The x component of the **average acceleration** is
$$a_{x,\text{av}} \equiv \frac{\Delta \upsilon_x}{\Delta t} = \frac{\upsilon_{x,\text{f}} - \upsilon_{x,\text{i}}}{t_\text{f} - t_\text{i}}$$
- The x component of the **instantaneous acceleration** is
$$a_x \equiv \frac{d\upsilon_x}{dt} = \frac{d^2x}{dt^2}$$
- The x component of the change in velocity over a time interval is given by
$$\Delta \upsilon_x = \int_{t_\text{i}}^{t_\text{f}} a_x(t)\,dt$$
- The x component of the displacement over a time interval is given by
$$\Delta x = \int_{t_\text{i}}^{t_\text{f}} \upsilon_x(t)\,dt$$

Chapter 3: Summary

Concepts: Motion with constant acceleration

- If an object has constant acceleration, the $v_x(t)$ curve is a straight line that has a nonzero slope and the $a_x(t)$ curve is a horizontal line.

Quantitative Tools: Motion with constant acceleration

- If an object moves in the x direction with constant acceleration a_x starting at $t = 0$, with initial velocity $\upsilon_{x,\text{i}}$ at initial position x_i, its x coordinate at any instant t is given by

$$x(t) = x_\text{i} + \upsilon_{x,\text{i}} t + \tfrac{1}{2} a_x t^2$$

- The x component of its instantaneous velocity is given by

$$\upsilon_x(t) = \upsilon_{x,\text{i}} + a_x t$$

- And the x component of its final velocity is given by

$$\upsilon_{x,\text{f}}^2 = \upsilon_{x,\text{i}}^2 + 2a_x \Delta x$$

Concepts: Free fall and projectile motion

- An object subject only to gravity is in **free fall.** All objects in free fall near the surface of Earth have the same acceleration, which is directed downward. We call this acceleration the **acceleration due to gravity** and denote its magnitude by the letter g.
- An object that is launched but not self-propelled is in **projectile motion.** Once it is launched, it is in free fall. The it follows is called its **trajectory.**

Chapter 3: Summary

Quantitative Tools: Free fall and projectile motion

- The magnitude g of the downward acceleration due to gravity is

$$g = \left|\vec{a}_{\text{free fall}}\right| = 9.8\ \text{m/s}^2 \ \ \text{(near Earth's surface)}$$

Chapter 3: Summary

Concepts: Motion along an inclined plane

- An object moving up or down an inclined plane on which friction is negligible has a constant acceleration that is directed parallel to the surface of the plane and points downward along the surface.

Quantitative Tools: Free fall and projectile motion

- When friction is negligible, the x component of acceleration a_x for an object moving on an inclined plane that rises at an angle θ above the horizontal is

$$a_x = +g \sin \theta$$

  when the x axis is directed downward along the plane.