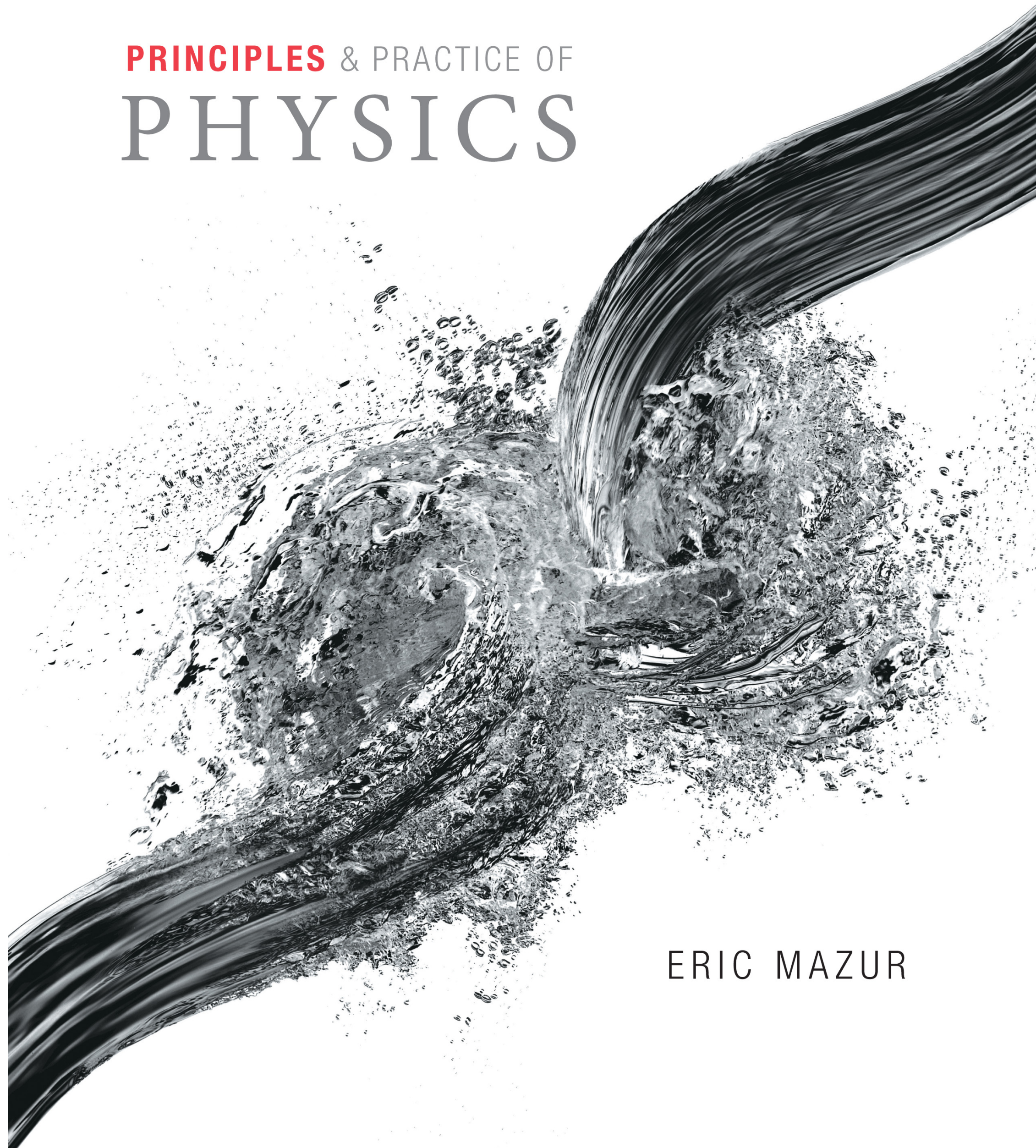
Chapter 9
Work



Homework

Let’s go over a few homework problems

Homework

8.35

- A 50-kg skier heads down a slope, reaching a speed of 21 km/h. She then slides across a horizontal snow field but hits a rough area. Assume the snow before the rough area is so slippery that you can ignore any friction between the skis and the snow.
- If the frictional force exerted by the snow in the rough area is 40 N, how far across the rough area does the skier travel before stopping?

Homework

8.35

- Watch units
- You know the acceleration is $a_x = -F/m$
- Know initial, final velocities and acceleration
- Don't know time
- Want displacement …

Homework

8.36

- A 2.19-kg cart on a long, level, low-friction track is heading for a small electric fan at 0.25 m/s . The fan, which was initially off, is turned on. As the fan speeds up, the magnitude of the force it exerts on the cart is given by at^2, where $a = 0.0200\ \mathrm{N/s^2}$.

- What is the speed of the cart 3.5 s after the fan is turned on?
- After how many seconds is the cart's velocity zero?

8.36

- You know the force, $a_x = -F/m$. (- b/c opposes motion)
- *Change* in velocity is

$$\Delta v = v_f - v_i = \int_0^{t_f} a \, dt$$

- For the second part? You want $v_f = 0$, solve for t

Homework

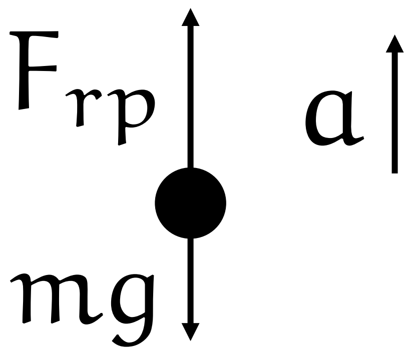
8.42

- You are climbing a rope straight up toward the ceiling.
- What is the magnitude of the force you must exert on the rope in order to accelerate upward at 1.1 m/s^2, assuming your inertia is 61 kg ?

Homework

8.42

- Start with a free body diagram:



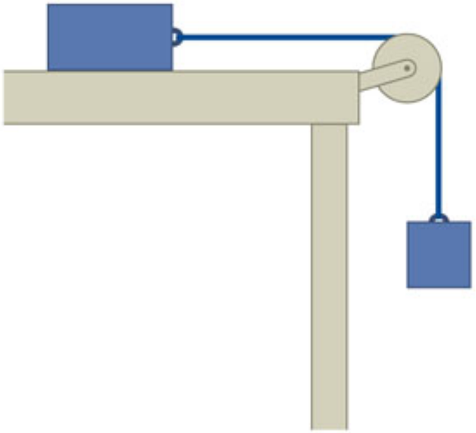
- This generates math

$$\sum F = F_{rp} - mg = ma$$

Homework

8.45

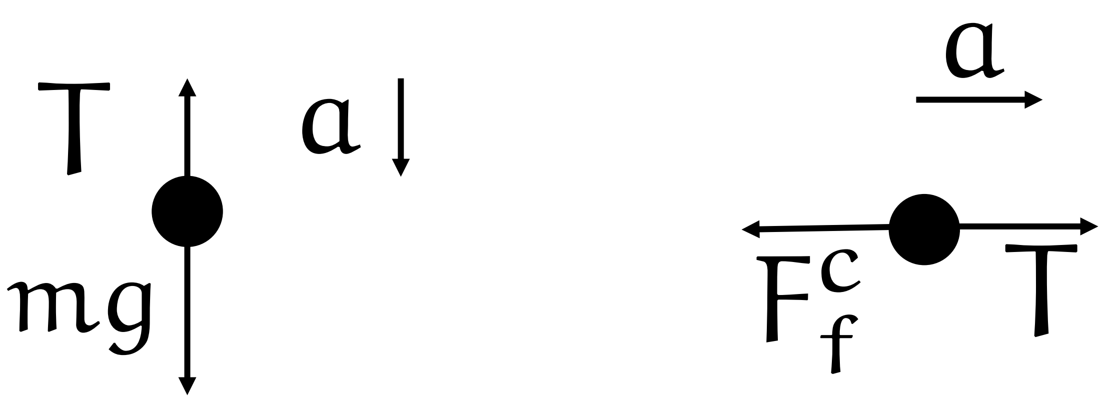
- Assume that the block on the table in (Figure 1) has half the inertia of the hanging block. You push the block on the table to the right so that it starts to move. The magnitude of the frictional force exerted by the table on the table block is half the magnitude of the gravitational force exerted on this block.



Homework

8.45

- Acceleration after removing hand?
- Start with two free body diagrams:



- This generates math – combine & solve for a.

$$\sum F = T - mg = -ma$$

$$\sum F = T - F_f^c = \frac{1}{2}ma$$

Homework

8.45

- Second part - now push the block to the left?
- The only thing that changes is that *friction now acts to the right.*
- Change the sign of the friction force and repeat

Homework

8.55

- When a 6.0-kg box is hung from a spring, the spring stretches to 57 mm beyond its relaxed length.

- In an elevator accelerating upward at 2.0 m/s^2, how far does the spring stretch with the same box attached?

Homework

8.55

- Free body diagrams: at rest, in motion

- At rest (left) gives you spring constant
- Use that in the equation generated from the moving diagram

Homework

8.55

- A horizontal force F_{slide} is exerted on a 9.0-kg box sliding on a polished floor. As the box moves, the magnitude of F_{slide} increases smoothly from 0 to 5.0 N in 5.0 s.
- What is the box's speed at $t = 5.0$ s if it starts from rest? Ignore any friction between the box and the floor.
- What is the box's speed at $t = 5.0$ s if at $t = 0$ it has a velocity of 3.5 m/s in the direction opposite the direction of F_{slide}? Ignore any friction between the box and the floor.

Homework

8.55

- Need an equation for force. *Increases smoothly* means linear: $F(t) = at$ (in this case $a = 1$)

- Then: impulse can be found:

$$J = \Delta p = \int_0^{t_f} F(t)\,dt = m(v_f - v_i)$$

- Difference in A, B? only initial velocity, force is *negative* in part B

Looking Ahead: Work done by a constant force

- In order for a force to do **work** on an object, the point of application of the force must undergo a **displacement**.
- Work is the ‘useful’ application of a force
- The SI unit of work is the **joule** (J).

Chapter 9: Work

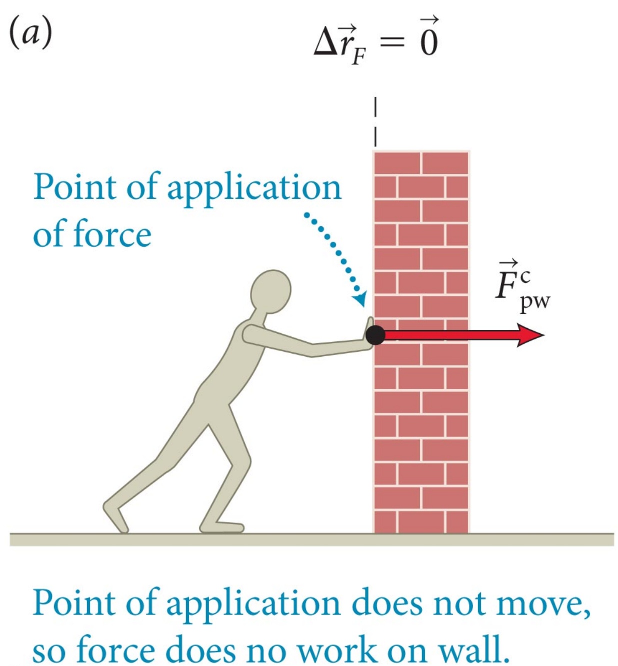
- Forces can change the physical state of an object (internal energy) as well as its state of motion (kinetic energy).
- To describe these changes in energy, physicists use the concept of **work:**
  - **Work is the change in the energy of a system due to external forces.**
- The SI unit of work is the joule (J).

Section 9.1: Force displacement

- **Work** amounts to a **mechanical transfer of energy**, either from a system to the environment or from the environment to a system.
- Do external force *always* cause a change in energy on a system?
  - To answer this question, it is helpful to consider an example

Checkpoint 9.1

9.1 Imagine pushing against a brick wall as shown in Figure 9.1*a*. (*a*) Considering **the wall as the system**, is the force you exert on it internal or external? (*b*) Does this force accelerate the wall? Change its shape? Raise its temperature? (*c*) Does the energy of the wall change as a result of the force you exert on it? (*d*) Does the force you exert on the wall do work on the wall?



Checkpoint 9.1

9.1 Imagine pushing against a brick wall.

(a) Considering the wall as the system, is the force you exert on it internal or external? **external**

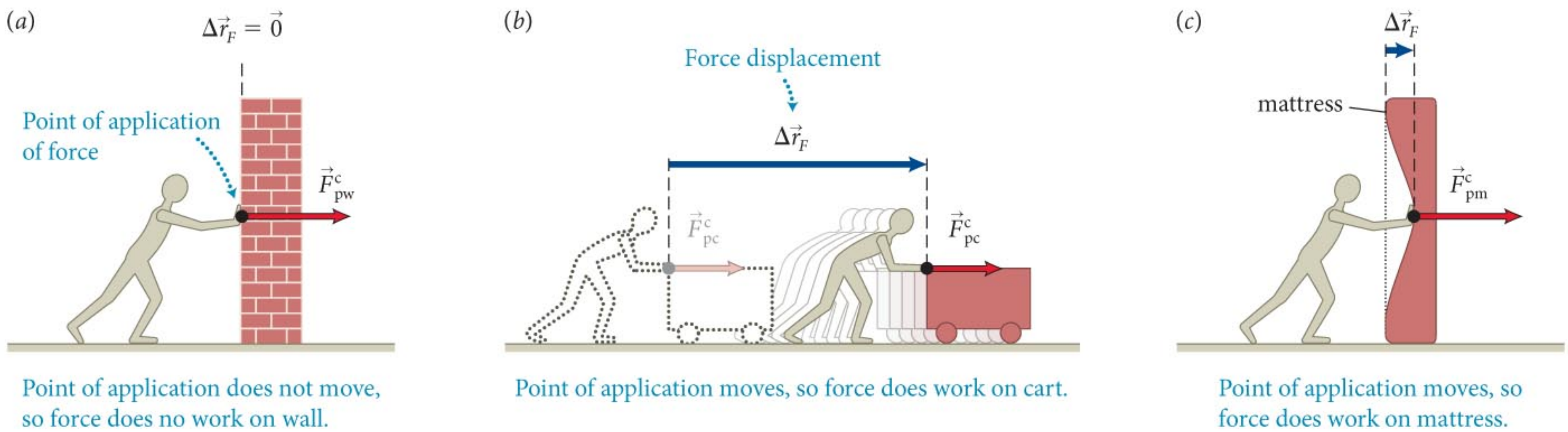
(b) Does this force accelerate the wall? Change its shape? Raise its temperature? **no, no, no**

(c) Does the energy of the wall change as a result of the force you exert on it? **no**

(d) Does the force you exert on the wall do work on the wall?

no

Section 9.1: Force displacement



- Even though the work is zero in (*a*), it is nonzero in (*b*) and (*c*).
- **In order for a force to do work, the point of application of the force must undergo a displacement.**
- The displacement of the point of application of the force is called the **force displacement**.

Exercise 9.1 Displaced forces

For which of the following forces is the force displacement nonzero:

(a) the force exerted by a hand compressing a spring

(b) the force exerted by Earth on a ball thrown upward,

(c) the force exerted by the ground on you at the instant you jump upward,

(d) the force exerted by the floor of an elevator on you as the elevator moves downward at constant speed?

Exercise 9.1 Displaced forces (cont.)

SOLUTION (*a*), (*b*), and (*d*).

(*a*) The point of application of the force is at the hand, which moves to compress the spring.

(*b*) The point of application of the force of gravity exerted by Earth on the ball is at the ball, which moves.

(*c*) The point of application is on the ground, which doesn't move.

(*d*) The point of application is on the floor of the elevator, which moves. ✔

Checkpoint 9.2

9.2 You throw a ball straight up in the air. Which of the following forces do work on the ball while you throw it? Consider the interval from the instant the ball is at rest in your hand to the instant it leaves your hand at speed v.

(a) The force of gravity exerted by Earth on the ball.

(b) The contact force exerted by your hand on the ball.

Checkpoint 9.2

9.2 You throw a ball straight up in the air. Which of the following forces do work on the ball while you throw it? Consider the interval from the instant the ball is at rest in your hand to the instant it leaves your hand at speed v. (*a*) The force of gravity exerted by Earth on the ball. (*b*) The contact force exerted by your hand on the ball.

both do work – for both, the point of application is the ball, and this point moves as you launch the ball

(your hand has to move to launch the ball)

Section 9.1
Question 1

A woman holds a bowling ball in a fixed position. The work she does on the ball

1. depends on the weight of the ball.
2. cannot be calculated without more information.
3. is equal to zero.

Section 9.1
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A woman holds a bowling ball in a fixed position. The work she does on the ball

1. depends on the weight of the ball.
2. cannot be calculated without more information.
3. ✔ is equal to zero.

Section 9.1
Question 2

A man pushes a very heavy load across a horizontal floor. The work done by gravity on the load

1. depends on the weight of the load.
2. cannot be calculated without more information.
3. is equal to zero.

Section 9.1
Question 2

A man pushes a very heavy load across a horizontal floor. The work done by gravity on the load

1. depends on the weight of the load.
2. cannot be calculated without more information.
3. ✔ is equal to zero.

gravity acts vertically, the displacement is horizontal.
the work is against the frictional force

Section 9.2: Positive and negative work

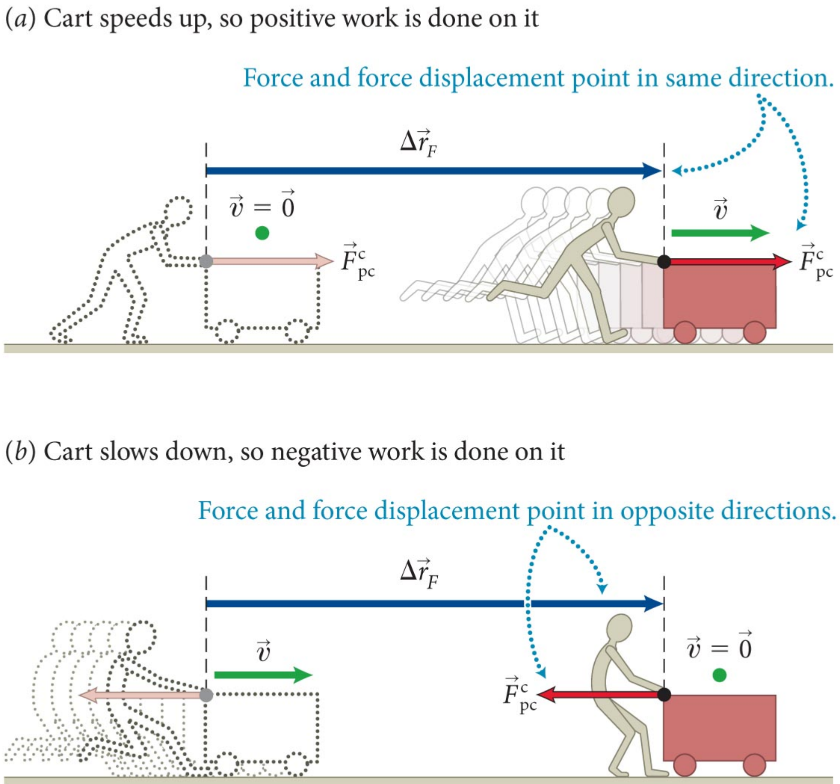
Section Goal

You will learn to

- Determine how the **sign** of the work done depends on the vector relationship between the force and the displacement.

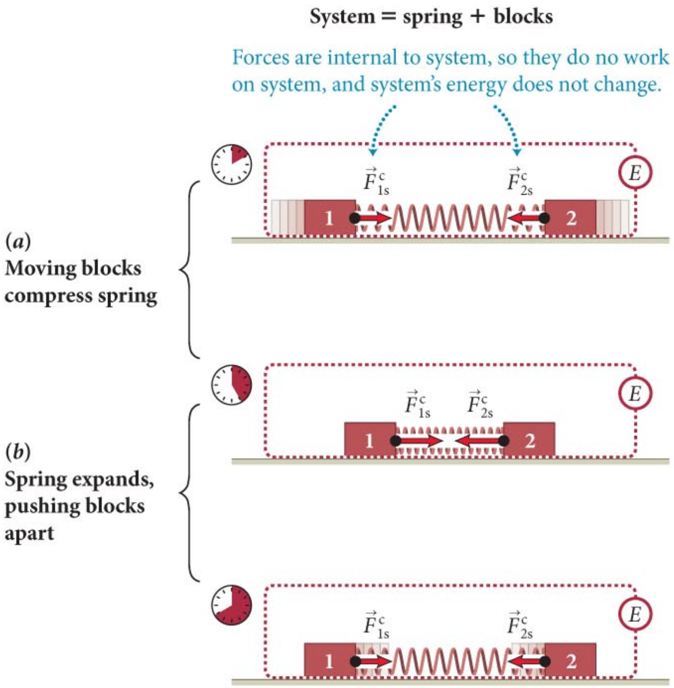
Section 9.2: Positive and negative work

- When the work done by an external force on a system is positive, the change in energy is positive, and when work is negative, the energy change is negative.
- *External force adds or subtracts energy from system*
- Examples of negative and positive work are illustrated in the figure
- **The work done by a force on a system is positive when the force and the force displacement point in the same direction and negative when they point in opposite directions.**



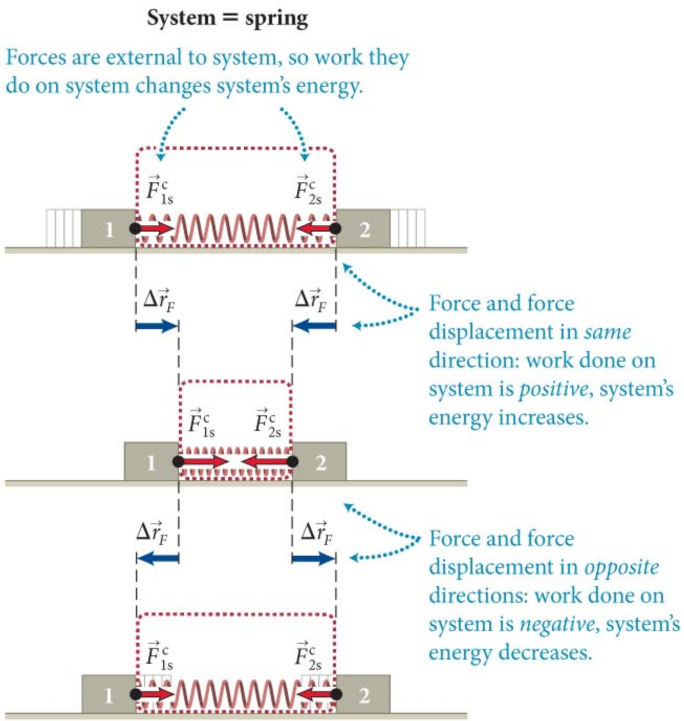
Section 9.2: Positive and negative work

- Let us now consider a situation involving potential energy.
- First, we consider the **spring + blocks to be a closed system.**
- In this case the change in potential energy will manifest as a change in the kinetic energy of the blocks, keeping the total energy constant.
- Because no external forces are exerted on the system, no work is involved.



Section 9.2: Positive and negative work

- Next, consider the **spring by itself as the system.**
- When the compressed spring is released, the decrease in the energy of the spring implies the work done by the block on the spring is negative.
- The force exerted on the spring by the block and the force displacement are in opposite directions, which confirms that the work is negative.



Checkpoint 9.3

9.3 A ball is thrown vertically upward.

(a) As it moves upward, it slows down under the influence of gravity. Considering the changes in energy of the ball, is the work done by Earth on the ball positive or negative?

(b) After reaching its highest position, the ball moves downward, gaining speed. Is the work done by the gravitational force exerted on the ball during this motion positive or negative?

Checkpoint 9.3

9.3 A ball is thrown vertically upward.

(a) As it moves upward, it slows down under the influence of gravity. Considering the changes in energy of the ball, is the work done by Earth on the ball positive or negative?

- As it moves upward, KE decreases, E_{int} is constant. The ball's energy ($K + E_{int}$) decreases, so work is negative (force & displacement opposite)

(b) After reaching its highest position, the ball moves downward, gaining speed. Is the work done by the gravitational force exerted on the ball during this motion positive or negative?

- Ball's energy now increases, so work is positive (force & displacement in same direction)

Exercise 9.2 Positive and negative work

Is the work done by the following forces positive, negative, or zero? In each case the **system is the object on which the force is exerted.**

(*a*) the force exerted by a hand compressing a spring,
(*b*) the force exerted by Earth on a ball thrown upward,
(*c*) the force exerted by the ground on you at the instant you jump upward

(*d*) the force exerted by the floor of an elevator on you as the elevator moves downward at constant speed.

Exercise 9.2 Positive and negative work (cont.)

SOLUTION

(*a*) Positive. To compress a spring, I must move my hand in the same direction as I push. ✔

(*b*) Negative. The force exerted by Earth points downward; the point of application moves upward. ✔

(*c*) Zero, because the point of application is on the ground, which doesn't move. ✔

(*d*) Negative. The force exerted by the elevator floor points upward; the point of application moves downward. ✔

Section 9.2
Question 3

You throw a ball up into the air and then catch it. How much work is done by gravity on the ball while it is in the air?

1. A positive amount
2. A negative amount
3. Cannot be determined from the given information
4. Zero

Section 9.2
Question 3

You throw a ball up into the air and then catch it. How much work is done by gravity on the ball while it is in the air?

1. A positive amount
2. A negative amount
3. Cannot be determined from the given information
4. Zero – comes back to where it started

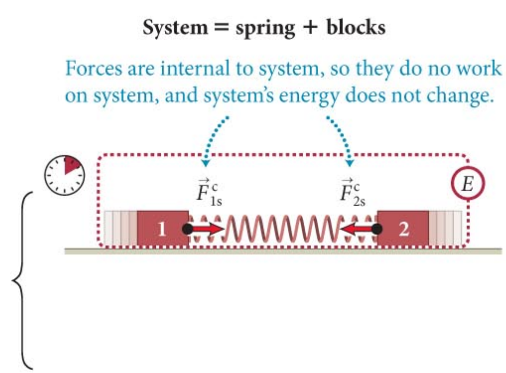
Checkpoint 9.5

9.5 Suppose that instead of the two moving blocks in Figure 9.3*a*, just one block is used to compress the spring while the other end of the spring is held against a wall.

(a) Is the system comprising the block and the spring closed?

(b) When the system is defined as being only the spring, is the work done by the block on the spring positive, negative, or zero? How can you tell?

(c) Is the work done by the wall on the spring positive, negative, or zero?



(*a*) Moving blocks compress spring

Checkpoint 9.5

9.5 Suppose that instead of the two moving blocks in Figure 9.3*a*, just one block is used to compress the spring while the other end of the spring is held against a wall.

(a) Is the system comprising the block and the spring closed?

yes – no changes in motion or state in environment

(b) When the system is defined as being only the spring, is the work done by the block on the spring positive, negative, or zero? How can you tell?

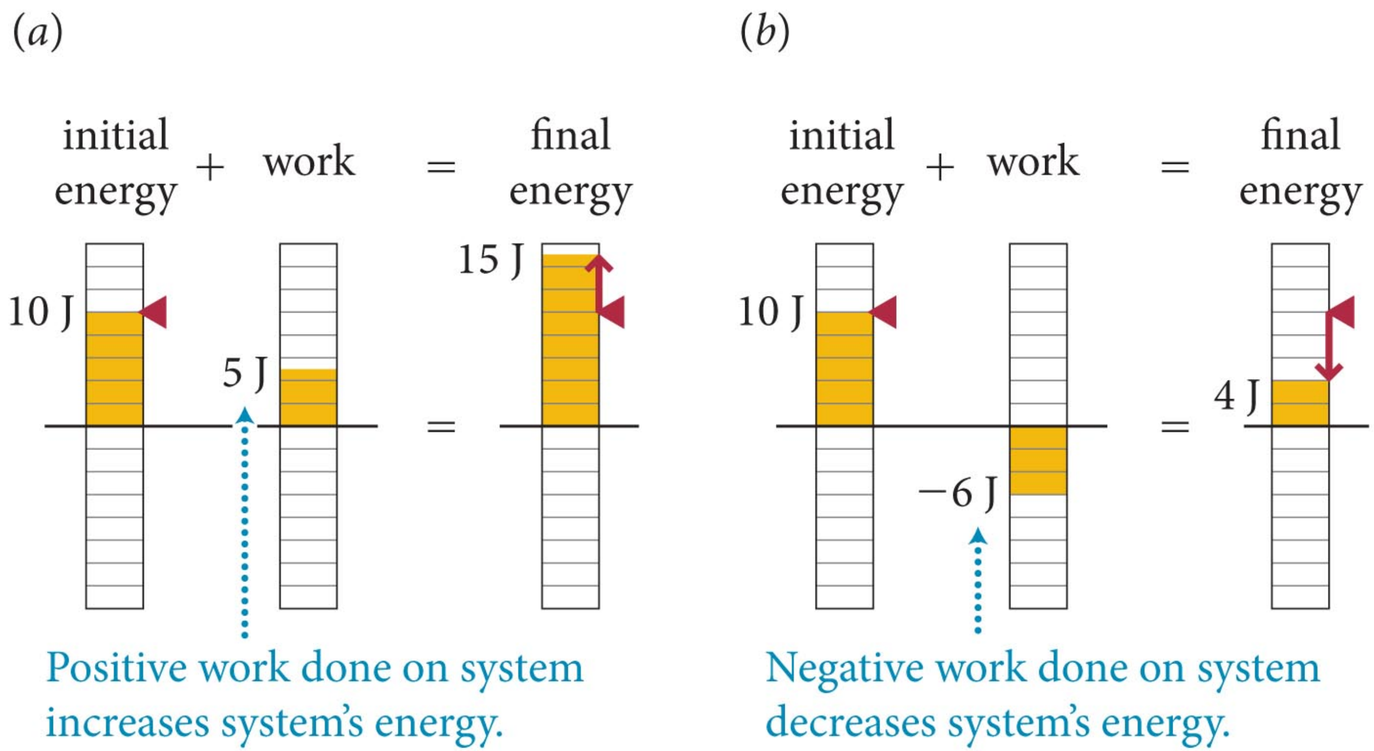
positive – force on spring is in direction point of contact moves

(c) Is the work done by the wall on the spring positive, negative, or zero?

zero – point of contact doesn't move

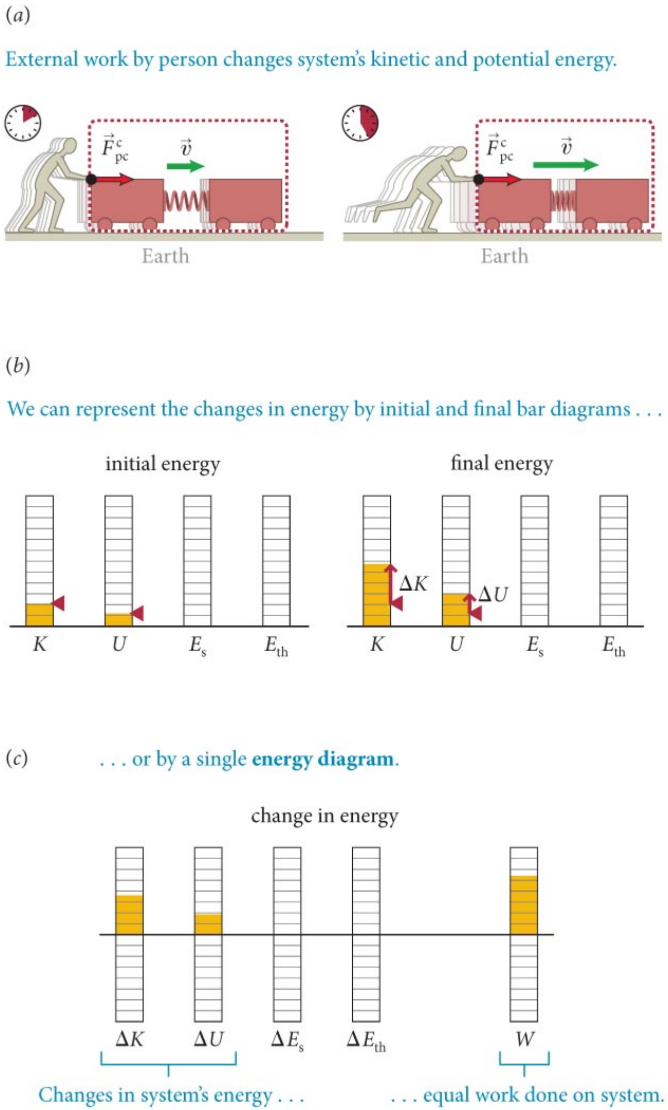
Section 9.3: Energy diagrams

- We can use energy bar charts to visually analyze situations involving work.
- This is why the sign is important



Section 9.3: Energy diagrams

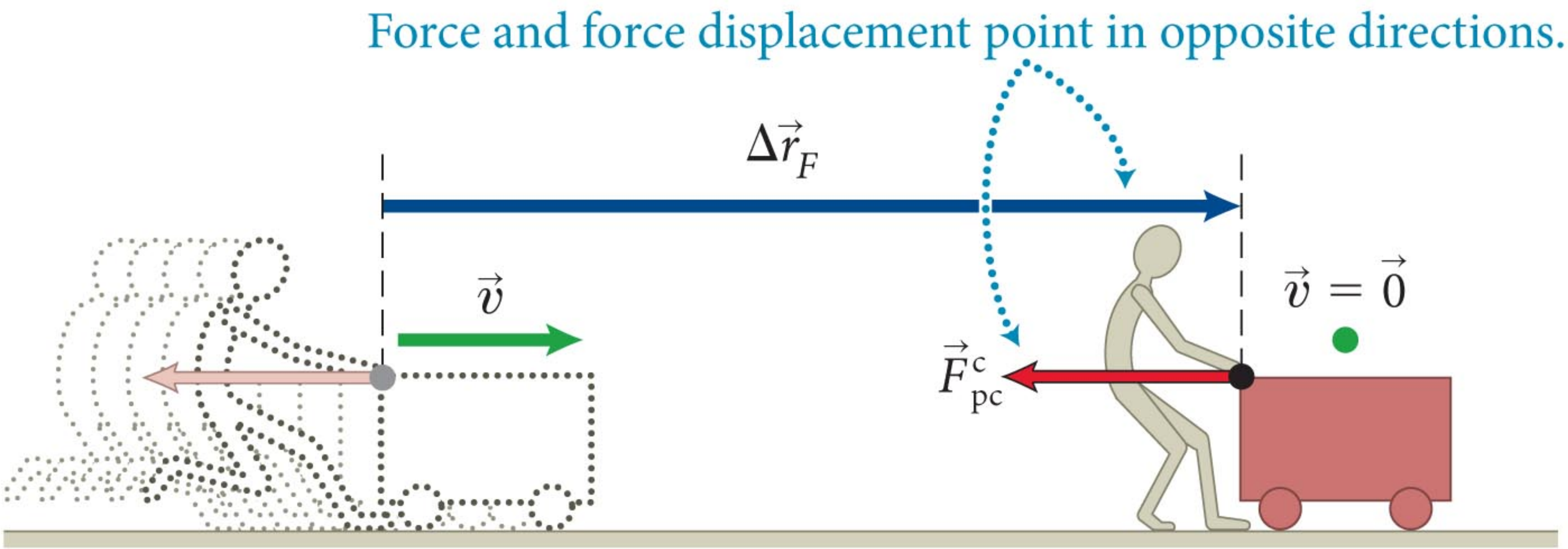
- Because any of the four kinds of energy can change in a given situation, we need more details in our energy bar charts [part (*b*)].
- As shown in part (*c*), we can also draw one set of bars for change in each category of energy, and a fifth bar to represent work done by external forces.
  - These are called **energy diagrams.**



Checkpoint 9.6

9.6 Draw an energy diagram for the cart in Figure 9.2*b*.

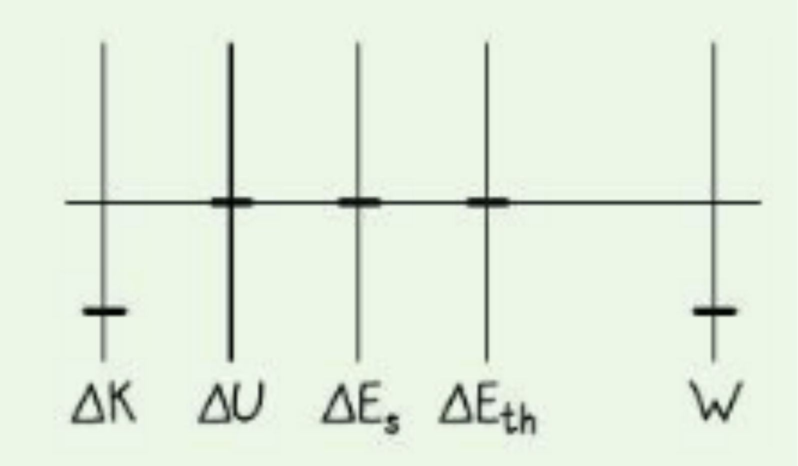
(*b*) Cart slows down, so negative work is done on it



Checkpoint 9.6

9.6

- The cart's KE decreases to zero, no changes in other forms of energy.
- Person's force is to the left, displacement to the right: work is negative.
- Change in KE should be same as work done in magnitude



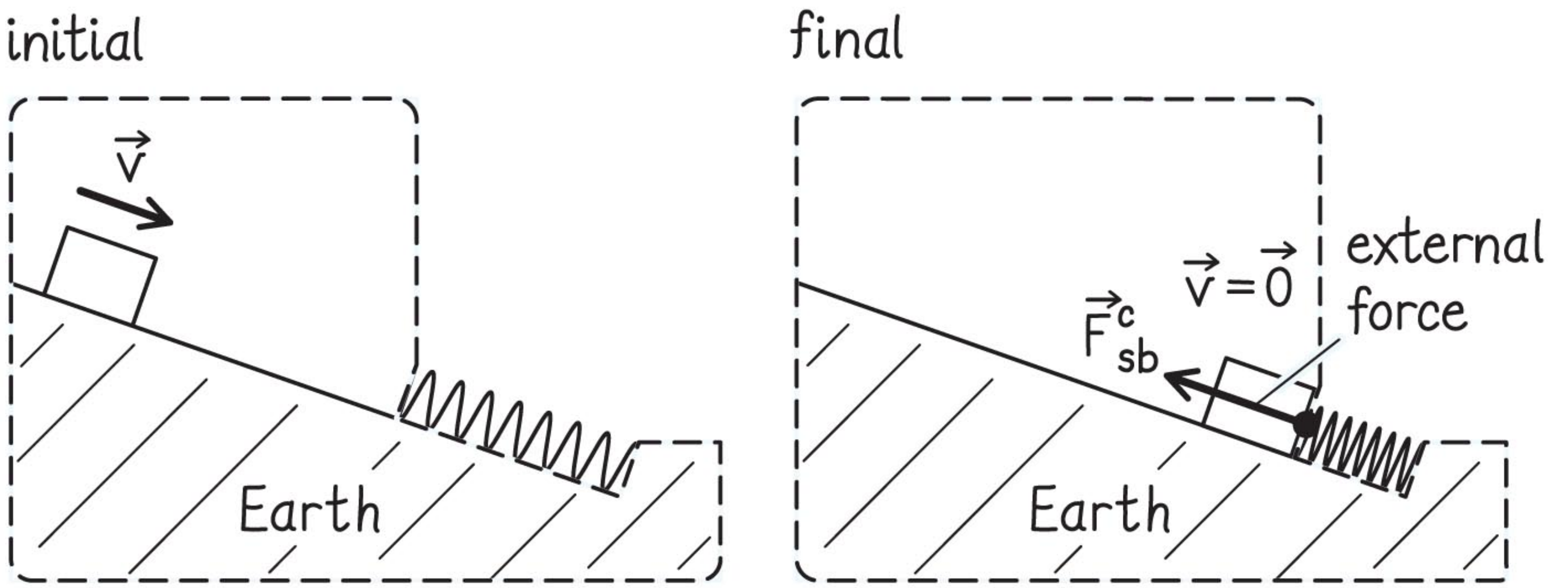
Exercise 9.4 Compressing a spring

A block initially at rest is released on an inclined surface. The block slides down, compressing a spring at the bottom of the incline; there is friction between the surface and the block.

Consider the time interval from a little after the release, when the block is moving at some initial speed v, until it comes to rest against the spring. Draw an energy diagram for the **system that comprises the block, surface, and Earth.**

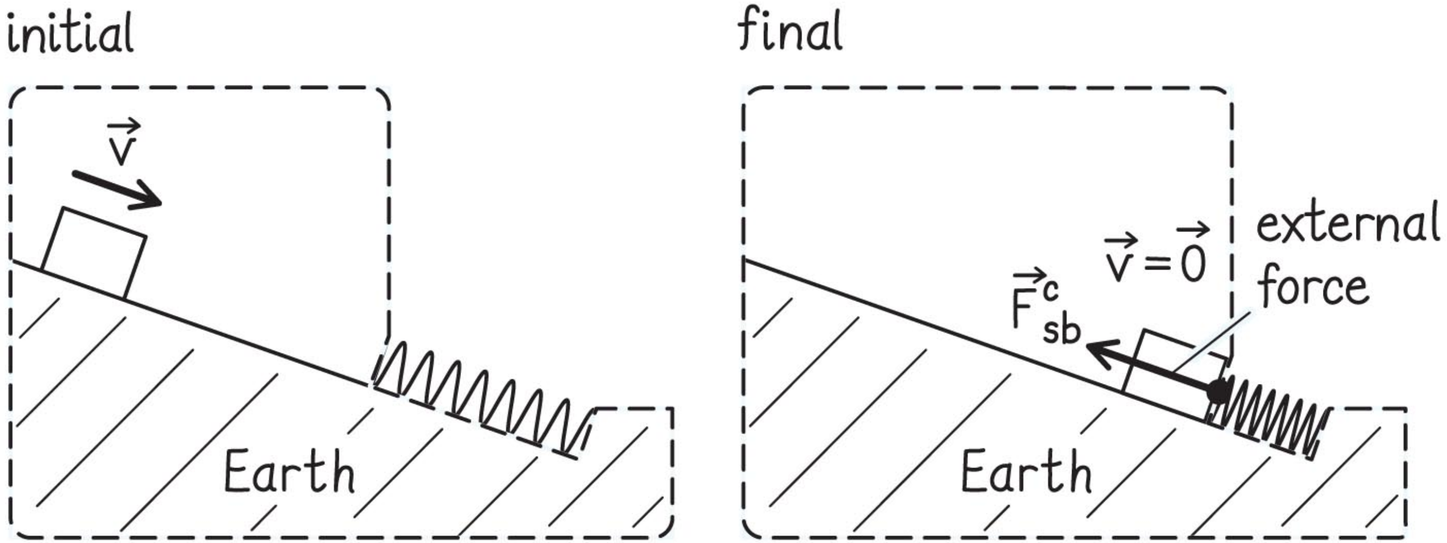
Exercise 9.4 Compressing a spring (cont.)

SOLUTION I begin by listing the objects that make up the system: block, surface, and Earth. Then I sketch the initial and final states of the system (Figure 9.8). The spring exerts external forces on the system.



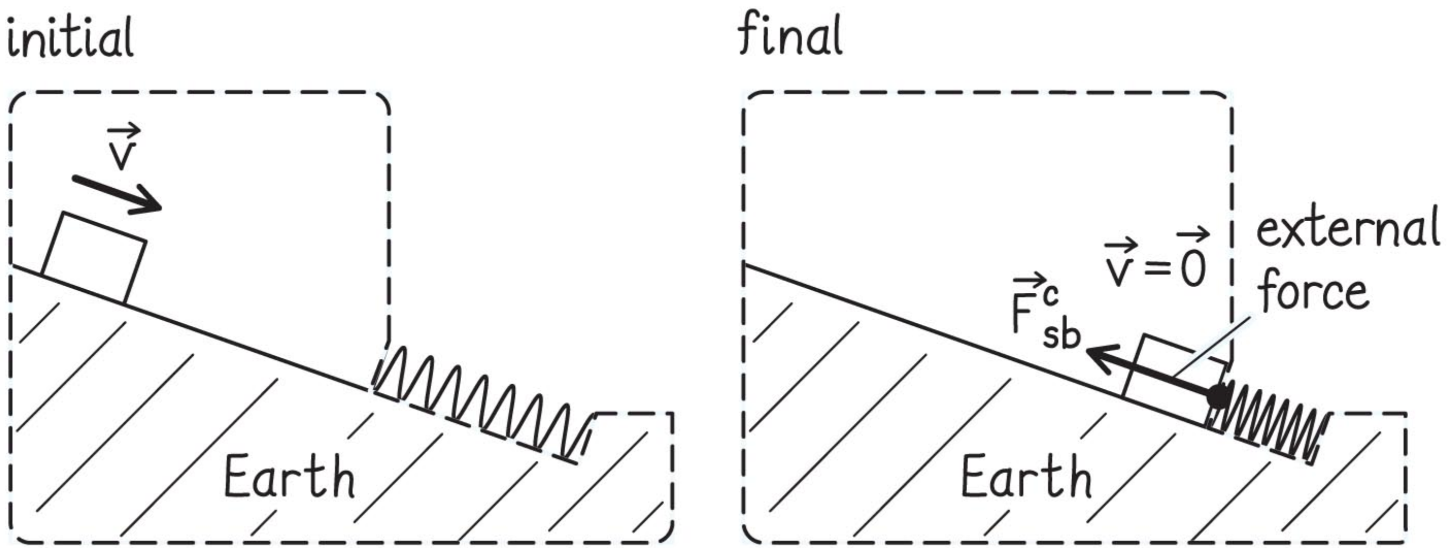
Exercise 9.4 Compressing a spring (cont.)

SOLUTION The bottom end of the spring exerts a force on the surface edge, but this force has a force displacement of zero. The top end of the spring exerts a force $\vec{F}^{c}_{sb}$ on the block. Because this force undergoes a nonzero force displacement, I include it in my diagram and show a dot at its point of application.



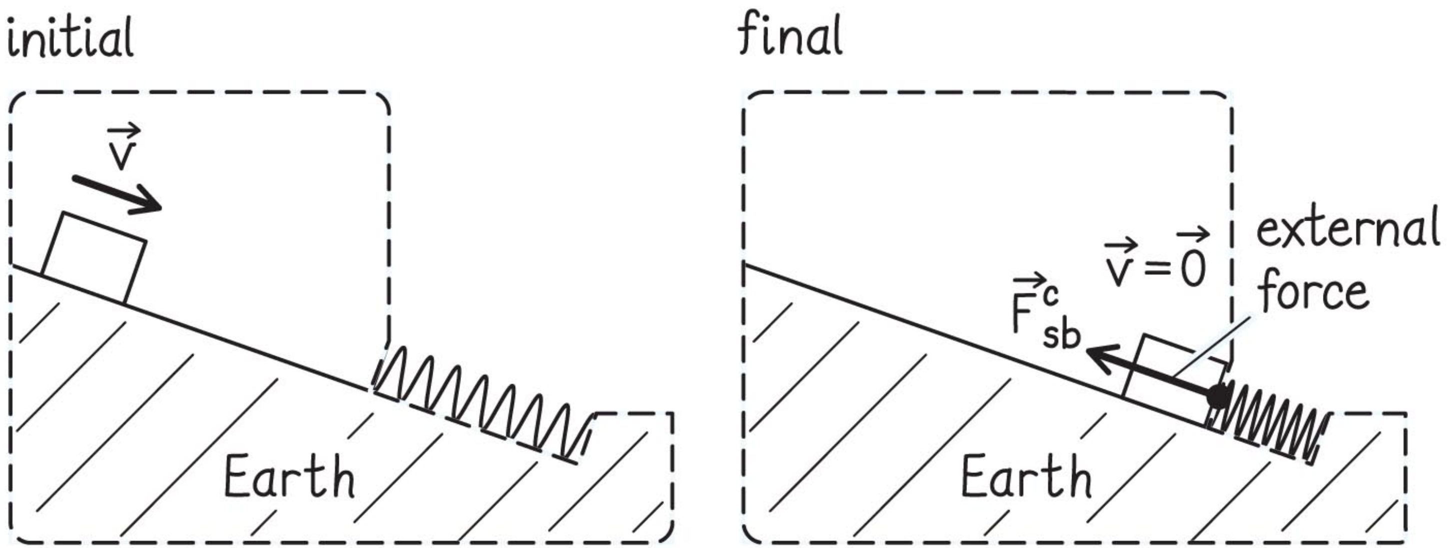
Exercise 9.4 Compressing a spring (cont.)

SOLUTION Next I determine whether there are any energy changes. Kinetic energy: The block's kinetic energy goes to zero, and the kinetic energies of the surface and Earth do not change. Thus the kinetic energy of the system decreases, and ΔK is negative.



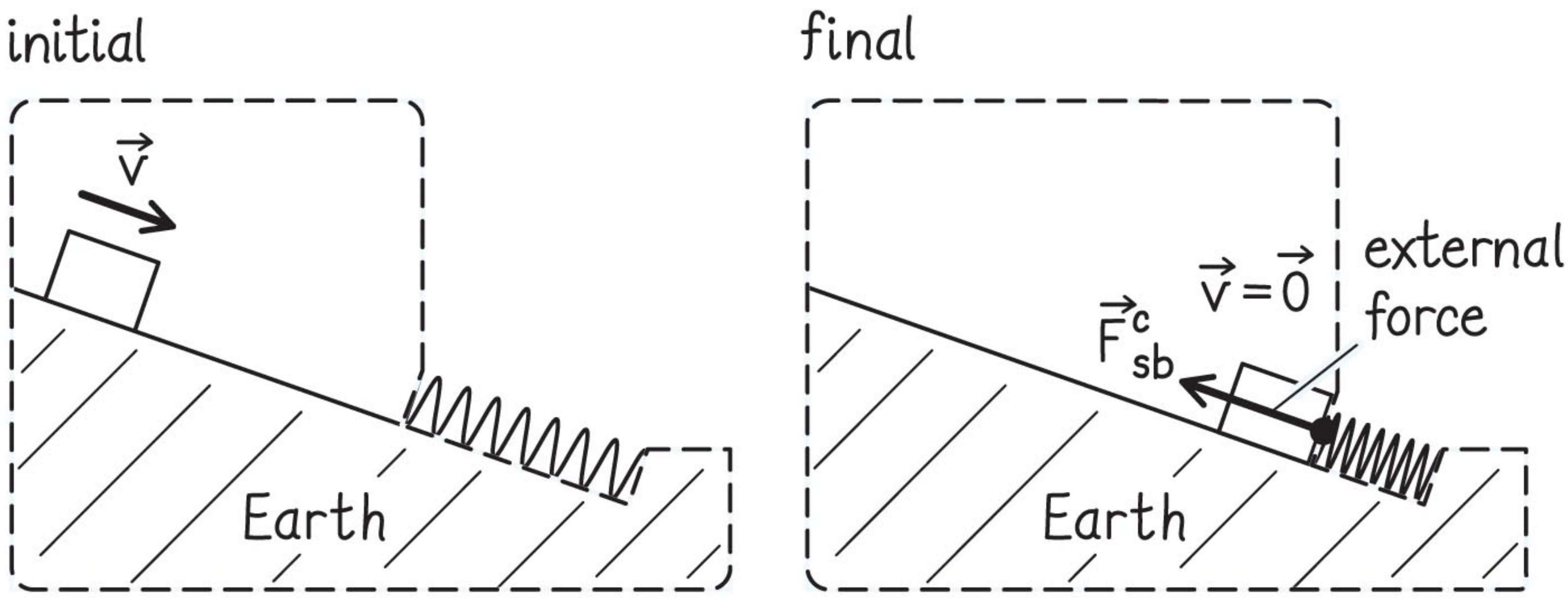
Exercise 9.4 Compressing a spring (cont.)

SOLUTION Potential energy: As the block moves downward, the gravitational potential energy of the block-Earth system decreases, and so ΔU is negative. (Because the spring gets compressed, its elastic potential energy changes, but the spring is not part of the system.)



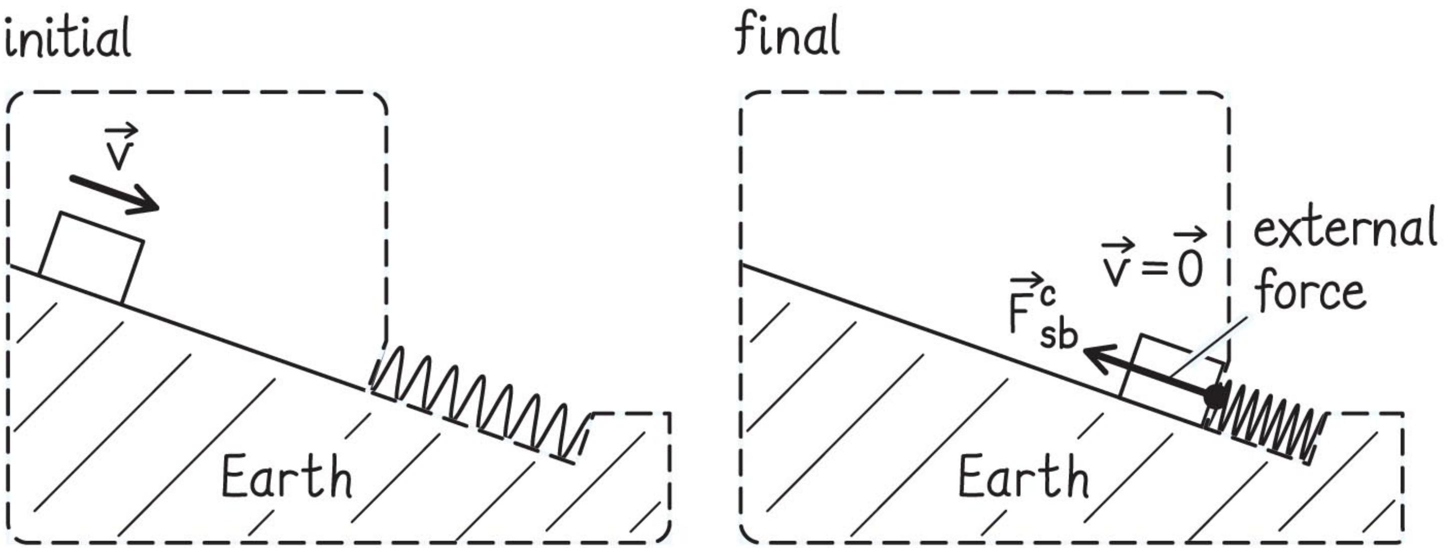
Exercise 9.4 Compressing a spring (cont.)

SOLUTION Source energy: none (no fuel, food, or other source of energy is converted in this problem). Thermal energy: As the block slides, energy is dissipated by the friction between the surface and the block, so ΔE_{th} is positive.



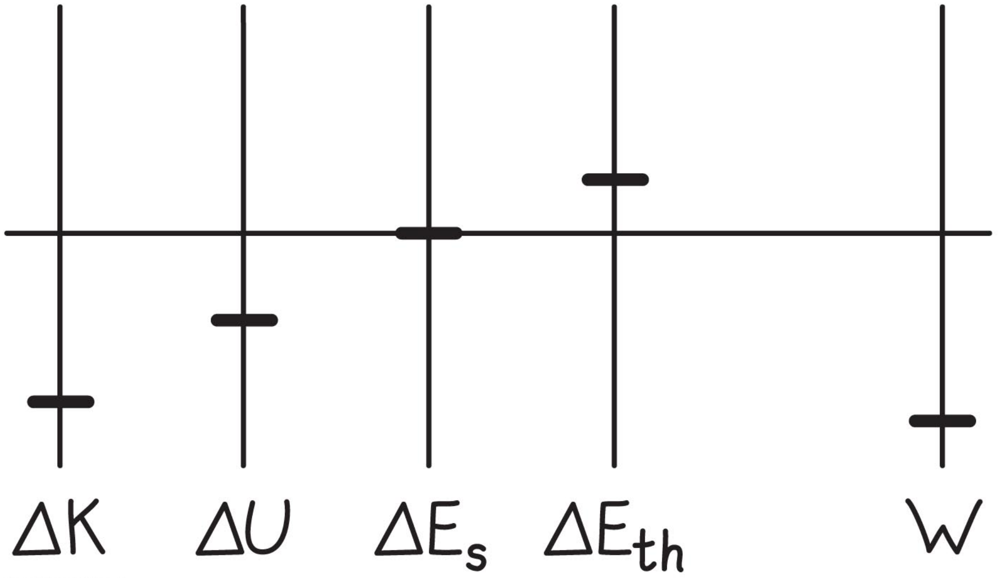
Exercise 9.4 Compressing a spring (cont.)

SOLUTION To determine the work done on the system, I look at the external forces exerted on it. The point of application of the external force $\vec{F}^{c}_{sb}$ exerted by the spring on the block undergoes a force displacement opposite the direction of the force, so that force does negative work on the system.



Exercise 9.4 Compressing a spring (cont.)

SOLUTION Thus the work done on the system by the external forces is negative, and the *W* bar extends below the baseline (Figure 9.9). I adjust the lengths of the bars so that the length of the *W* bar is equal to the sum of the lengths of the other three bars, yielding the energy diagram shown in Figure 9.9. ✔



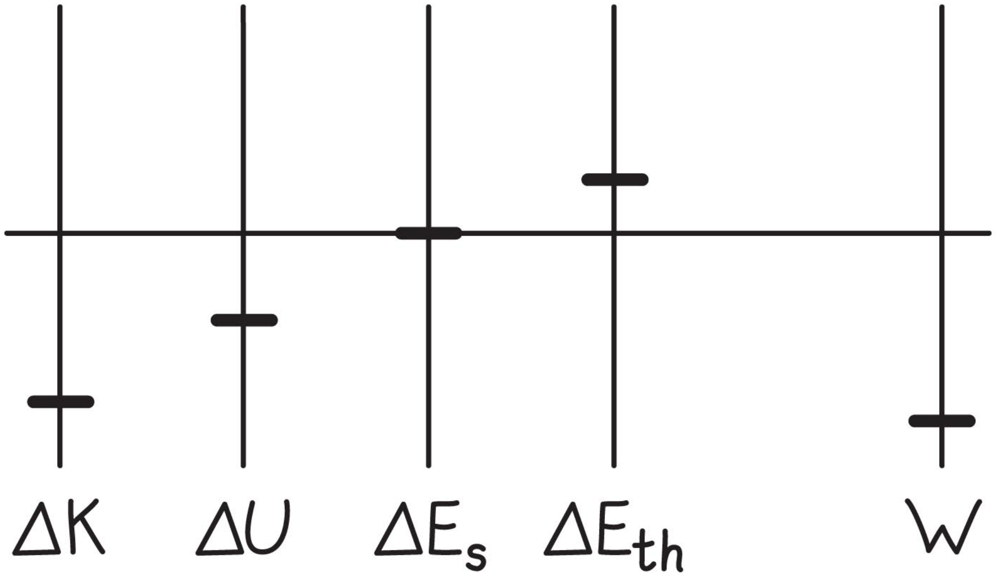
Exercise 9.4 Compressing a spring (cont.)

SOLUTION

The change in kinetic energy is equal to the work done

The change in potential energy shows up as thermal energy (friction)

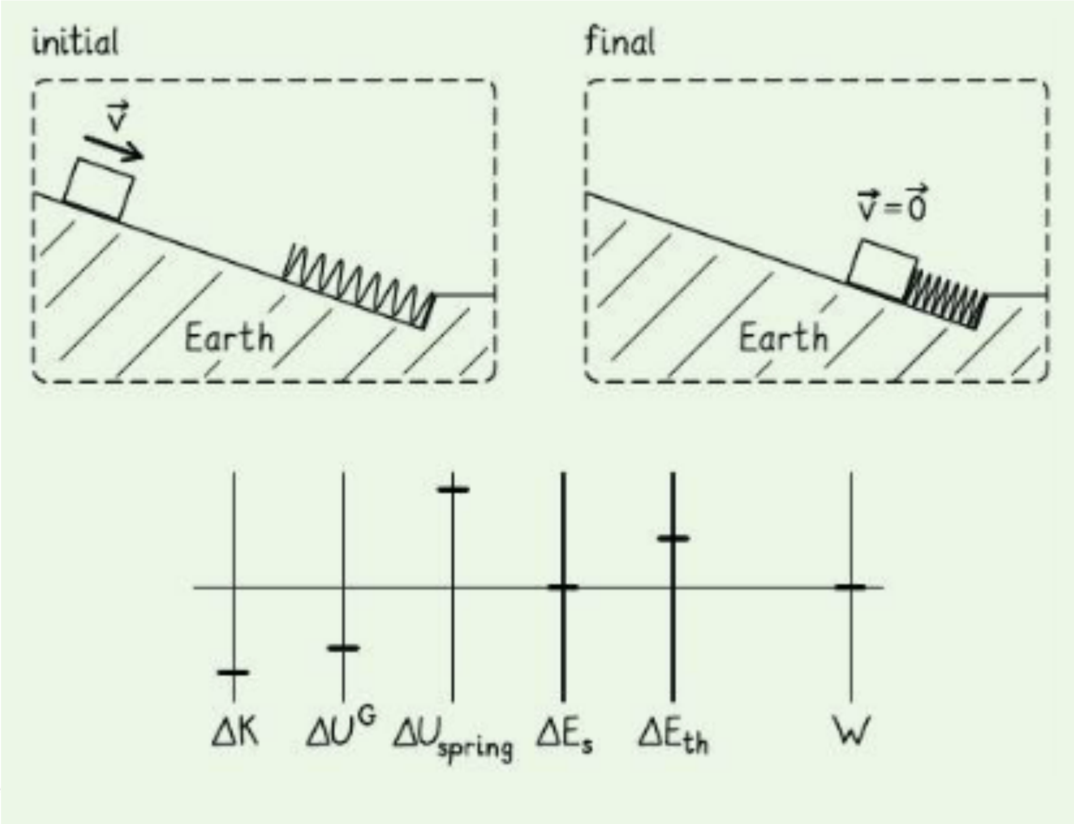
Now we could do math – all energy changes plus work sum to zero



Checkpoint 9.7

9.7 Draw an energy diagram for the situation presented in Exercise 9.4, but choose the system that comprises **block, spring, surface, and Earth. (i.e., include the spring now)**

Which changes should be equal? No external force now, no work.



What was work is now a change in PE

Distinction depends on choice of system!

Section 9.4: Choice of system

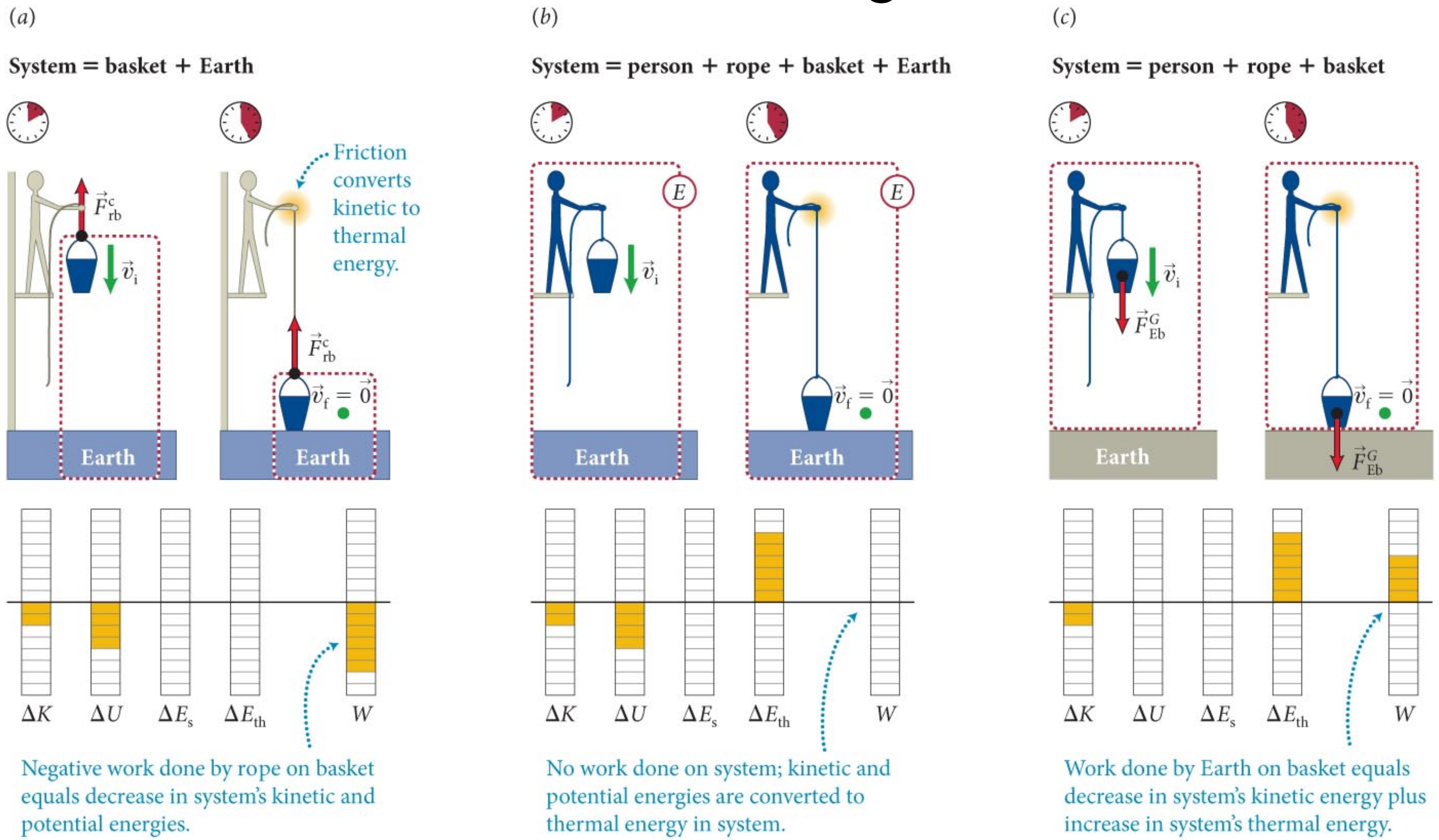
Section Goals

You will learn to

- Choose an **appropriate system** for a physical problem of interest in order to systematically account for the various energy changes.
- Recognize that a system chosen for which **friction** acts across the boundary is **difficult** to analyze. This is because in these situations thermal energy is generated in both the environment and the system, making energy accounting for the system problematic.

Section 9.4: Choice of system

- Different choices of systems lead to different energy diagrams.
- What is "work" in one context is energy conversion in another
- Work is involved when an *external* agent acts with a force

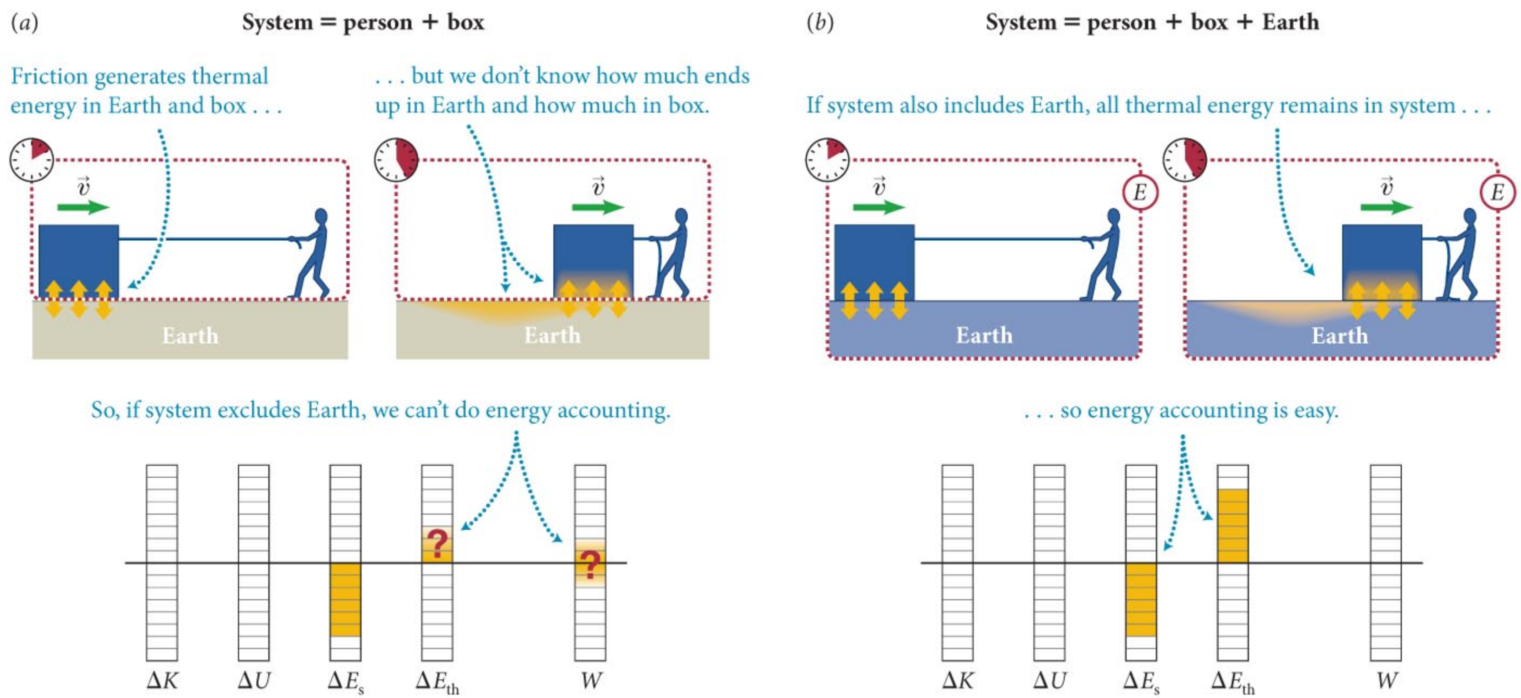


Section 9.4: Choice of system

- Need to be careful not to double count gravitational potential energy!
- It is important to remember the following point:
  - **Gravitational potential energy always refers to the relative position of various parts within a system, never to the relative positions of one component of the system and its environment.**
- In other words, depending on the choice of system, the gravitational interaction with the system can appear in energy diagrams as either a change in gravitational potential energy or work done by Earth, but not both.
- Earth outside system? Probably work (earth = external agent then) …

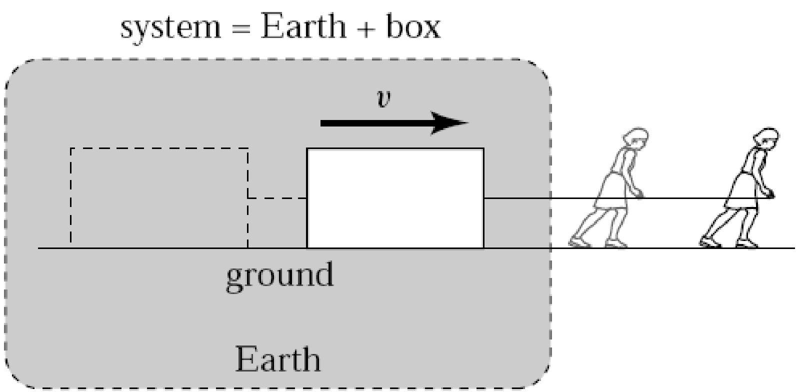
Section 9.4: Choice of system

- As seen in the figure, the thermal energy generated (in this case due to friction) ends up on both surfaces.
- As seen in part (*a*), certain choices of systems lead to complications:
  - **When drawing an energy diagram, do not choose a system for which friction occurs at the boundary of the system.**



Section 9.4
Question 5

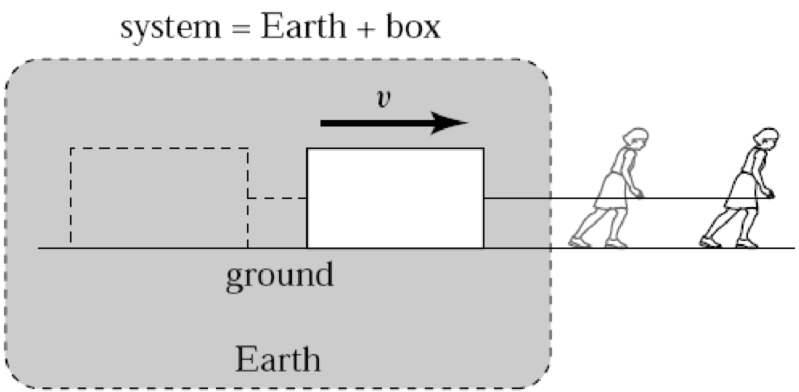
A person pulls a box along the ground at a constant speed. If we consider Earth and the box as our system, what can we say about the net external force on the system?



1. It is zero because the system is isolated.
2. It is nonzero because the system is not isolated.
3. It is zero even though the system is not isolated.
4. It is nonzero even though the system is isolated.
5. None of the above

Section 9.4
Question 5

A person pulls a box along the ground at a **constant speed**. If we consider Earth and the box as our system, what can we say about the net external force on the system?



1. It is zero because the system is isolated.
2. It is nonzero because the system is not isolated.
3. It is zero even though the system is not isolated. ✔
4. It is nonzero even though the system is isolated.
5. None of the above

Chapter 9: Self-Quiz #2

Consider a weightlifter holding a barbell motionless above his head.

(*a*) Is the sum of the forces exerted on the barbell zero?

(*b*) Is the weightlifter exerting a force on the barbell?

(*c*) If the weightlifter exerts a force, does this force do any work on the barbell?

(*d*) Does the energy of the barbell change?

(*e*) Are your answers to parts (*c*) and (*d*) consistent in light of the relationship between work and energy?

Chapter 9: Self-Quiz #2

Answer

(*a*) Yes, because the barbell remains motionless.

(*b*) Yes. He exerts an upward force to counter the downward gravitational force.

(*c*) No, because the point at which the lifter exerts a force on the barbell is not displaced.

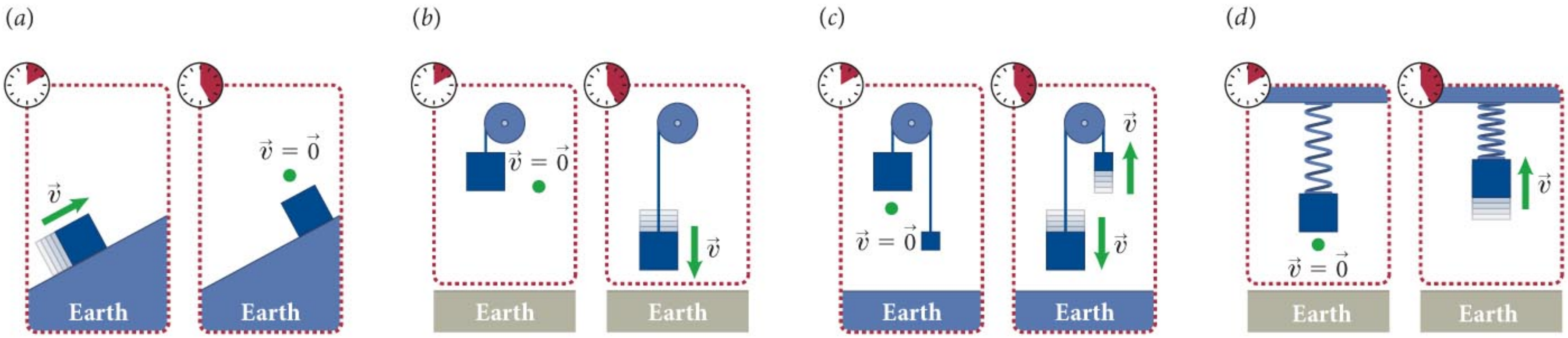
(*d*) No, it just sits there.

(*e*) They are consistent. If no work is done on a system, the energy of the system does not change.

Chapter 9: Self-Quiz #3

Do any of the systems in the figure undergo a change in potential energy?

If yes, is the change positive or negative? Ignore any friction.



Chapter 9: Self-Quiz #3

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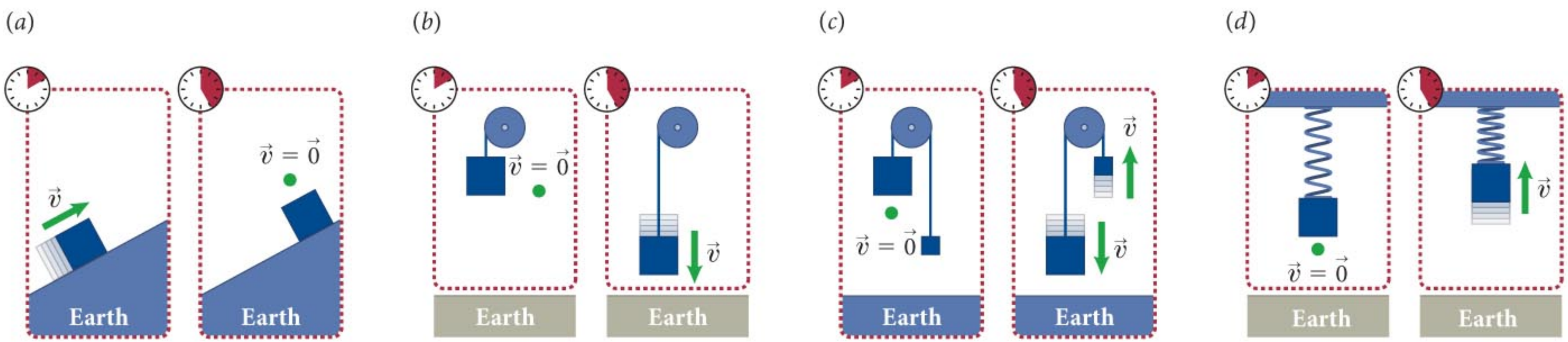
If yes, is the change positive or negative? Ignore any friction.



Yes: U^G changes

No: F^G is external

Yes: U^G changes

Yes: U^S changes

Quantitative Tools

Section 9.5: Work done on a single particle

- When work is done by external forces on a system, the energy change in the system is given by the **energy law:**

$$\Delta E = W$$

- To determine the work done by an external force, we will consider the simple case of a **particle:**
  - *Particle* refers to any object with an inertia m and no internal structure ($\Delta E_{\text{int}} = 0$).
- Only the kinetic energy of a particle can change, so

$$\Delta E = \Delta K \text{ (particle)}$$

- The constant force acting on the particle give is it an acceleration given by

$$a_x = \frac{\Sigma F_x}{m} = \frac{F_x}{m}$$

Section 9.5: Work done on a single particle

- Consider the motion of the particle in time interval $\Delta t = t_f - t_i$. From Equations 3.4 and 3.7, we can write

$$\upsilon_{x,f} = \upsilon_{x,i} + a_x \Delta t$$

$$\Delta x = \upsilon_{x,i} \Delta t + \tfrac{1}{2} a_x (\Delta t)^2$$

- The kinetic energy change of the particle is given by

$$\Delta K = K_f - K_i = \tfrac{1}{2} m(\upsilon_f^2 - \upsilon_i^2)$$

- Combining the above equations we get,

$$\Delta K = \tfrac{1}{2} m \left[\left(\upsilon_{x,i} + a_x \Delta t \right)^2 - \upsilon_{x,i}^2 \right]$$

$$= \tfrac{1}{2} m \left[2\upsilon_{x,i} a_x \Delta t + a_x^2 (\Delta t)^2 \right]$$

$$= m a_x \left[\upsilon_{x,i} \Delta t + \tfrac{1}{2} a_x (\Delta t)^2 \right]$$

$$= m a_x \Delta x_F = F_x \Delta x_F$$

where Δx_F is the force displacement.

Section 9.5: Work done on a single particle

- Since $\Delta E = W$, and for a particle $\Delta E = \Delta K$, we get

  $$W = F_x \Delta x_F \text{ (constant force exerted on particle, one dimension)}$$

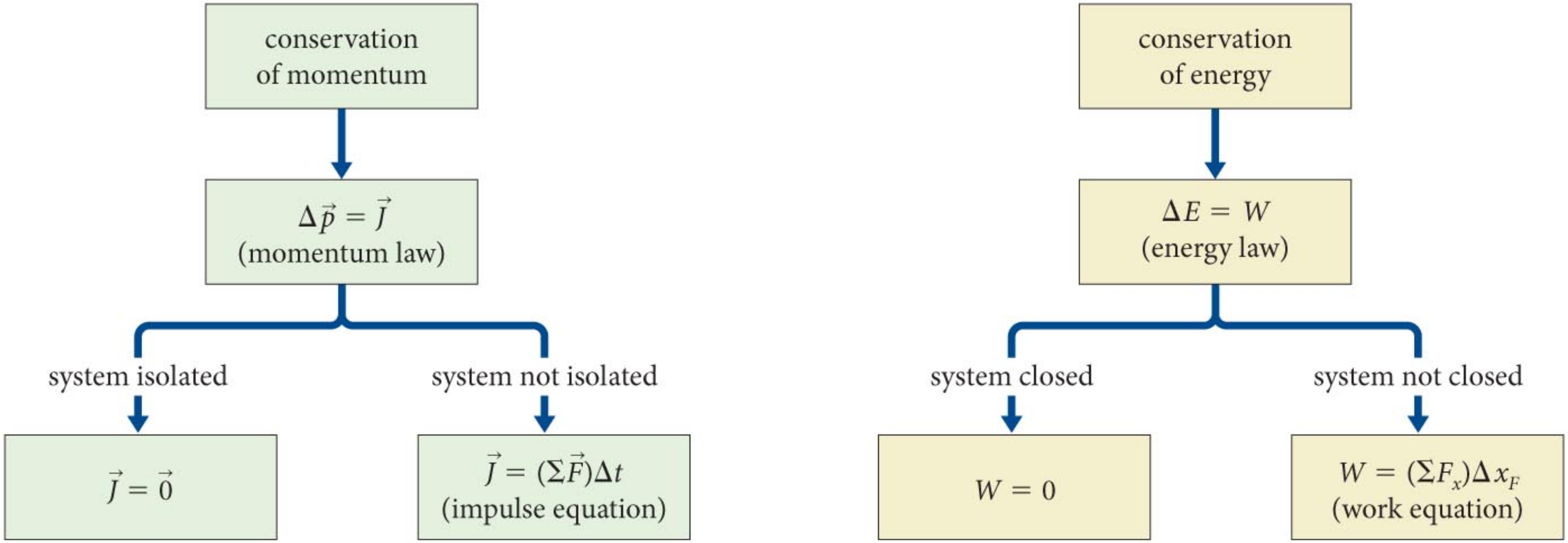
- The equation above in words:
  - **For motion in one dimension, the work done by a constant force exerted on a particle equals the product of the *x* component of the force and the force displacement.**
- If more than one force is exerted on the particle, we get

  $$W = (\Sigma F_x) \Delta x_F \text{ (constant forces exerted on particle, one dimension)}$$

- This is called the **work equation.**

Section 9.5: Work done on a single particle

- Notice the parallel between our treatment of momentum/impulse and energy/work, as illustrated in the figure.

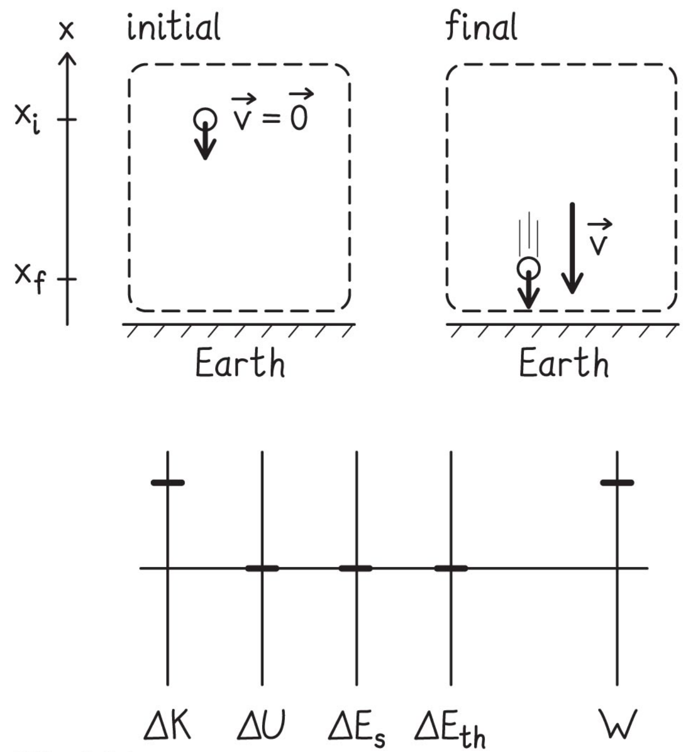


Example 9.6 Work done by gravity

A ball of inertia m_b is released from rest and falls vertically. What is the ball's final kinetic energy after a displacement $\Delta x = x_f - x_i$?

Example 9.6 Work done by gravity (cont.)

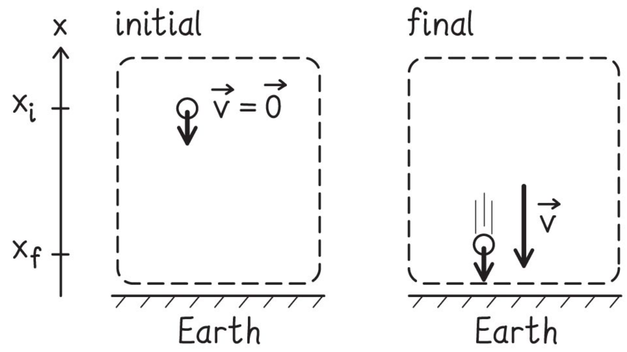
❶ GETTING STARTED I begin by making a sketch of the initial and final conditions and drawing an energy diagram for the ball (Figure 9.17). I choose an x axis pointing upward.

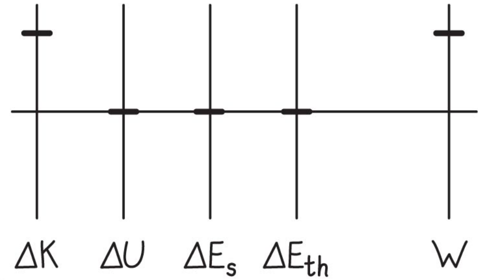


Section 9.5: Work done on a single particle

Example 9.6 Work done by gravity (cont.)

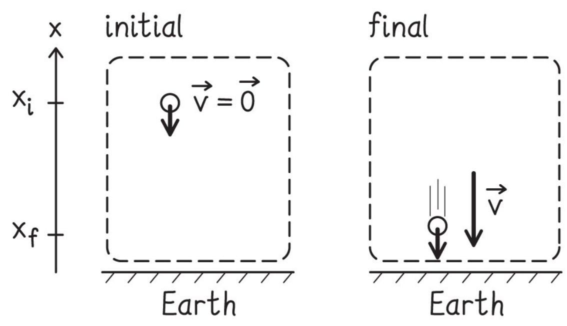
❶ GETTING STARTED Because the ball's internal energy doesn't change as it falls (its shape and temperature do not change), I can treat the ball as a particle. Therefore only its kinetic energy changes.

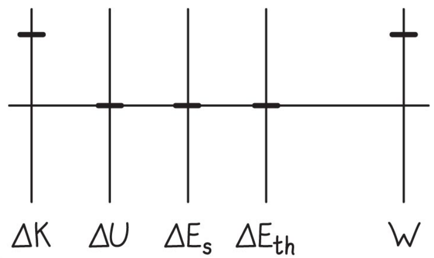




Example 9.6 Work done by gravity (cont.)

❶ GETTING STARTED I can also assume air resistance is small enough to be ignored, so that the only external force exerted on the ball is a constant gravitational force. This force has a nonzero force displacement and so does work on the ball. I therefore include this force in my diagram.





Example 9.6 Work done by gravity (cont.)

❷ DEVISE PLAN If I treat the ball as a particle, the change in the ball's kinetic energy is equal to the work done on it by the constant force of gravity, the x component of which is:

$$F^G_{\text{Eb}\,x} = -m_\text{b} g.$$

(The minus sign means that the force points in the negative x direction.)

To calculate the work done by this force on the ball, I use Eq. 9.8.

Example 9.6 Work done by gravity (cont.)

❸ EXECUTE PLAN Substituting the x component of the gravitational force exerted on the ball and the force displacement $x_f - x_i$ into Eq. 9.8, I get

$$W = F^G_{\text{Eb}\,x}\Delta_{x_F} = -m_b g(x_f - x_i).$$

Example 9.6 Work done by gravity (cont.)

❸ EXECUTE PLAN Because the work is equal to the change in kinetic energy and the initial kinetic energy is zero, I have $W = \Delta K = K_f - 0 = K_f$, so

$$K_f = -m_b g(x_f - x_i). \checkmark$$

Example 9.6 Work done by gravity (cont.)

❹ EVALUATE RESULT Because the ball moves in the negative x direction, $\Delta x = x_f - x_i$ is negative and so the final kinetic energy is positive (as it should be).

Example 9.6 Work done by gravity (cont.)

❹ EVALUATE RESULT An alternative approach is to consider the **closed Earth-ball system**. For that system, the sum of the gravitational potential energy and kinetic energy does not change, and so, from Eq. 7.13,

$$\Delta K + \Delta U^G = \tfrac{1}{2} m_\text{b}(\upsilon_\text{f}^2 - \upsilon_\text{i}^2) + m_\text{b} g(x_\text{f} - x_\text{i}) = 0.$$

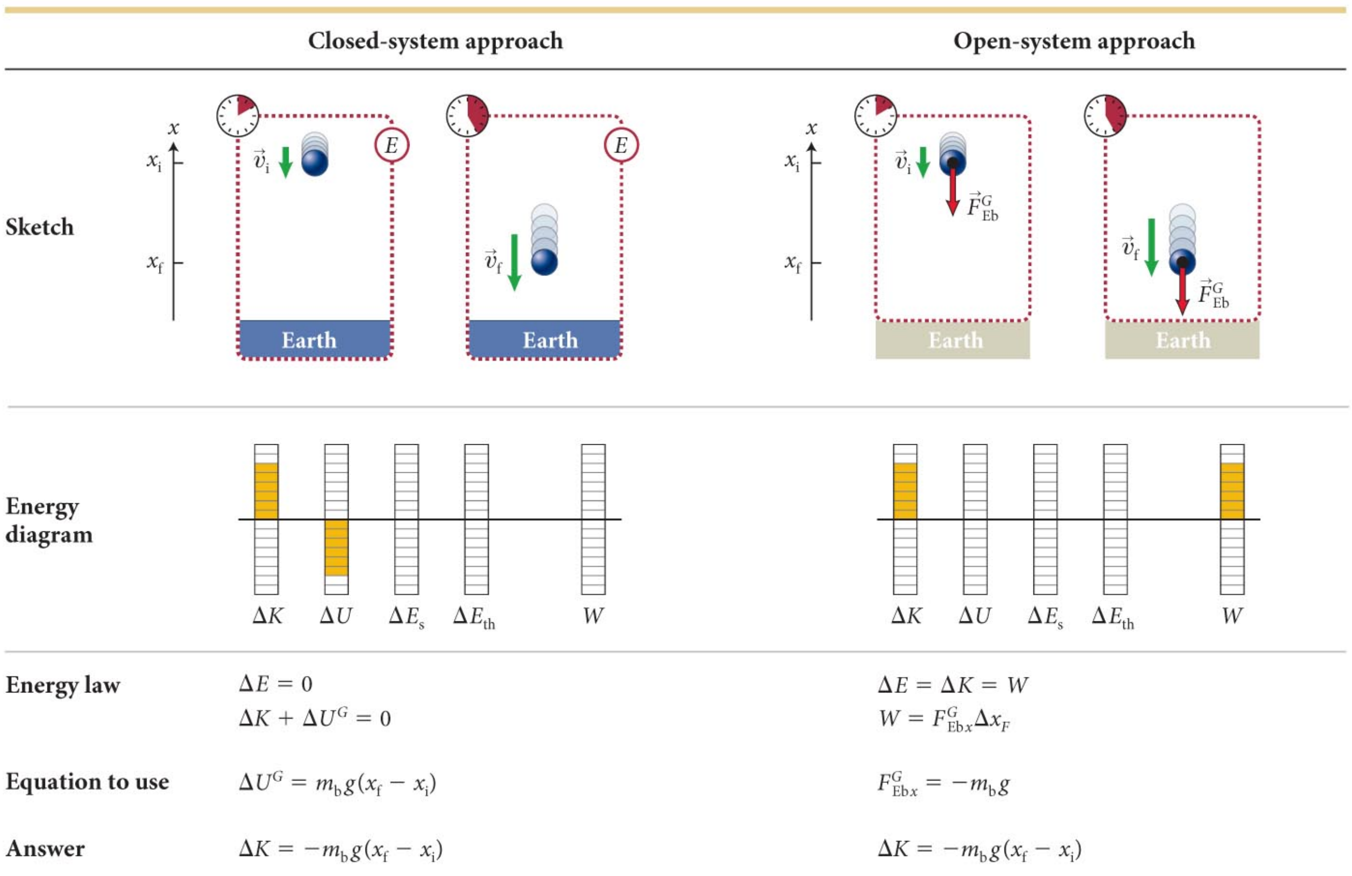
No work done here - the gravitational force is internal

Because the ball starts at rest, $\upsilon_\text{i} = 0$, and so I obtain the same result for the final kinetic energy:

$$\tfrac{1}{2} m_\text{b}\upsilon_\text{f}^2 = -m_\text{b} g(x_\text{f} - x_\text{i}).$$

Section 9.5: Work done on a single particle

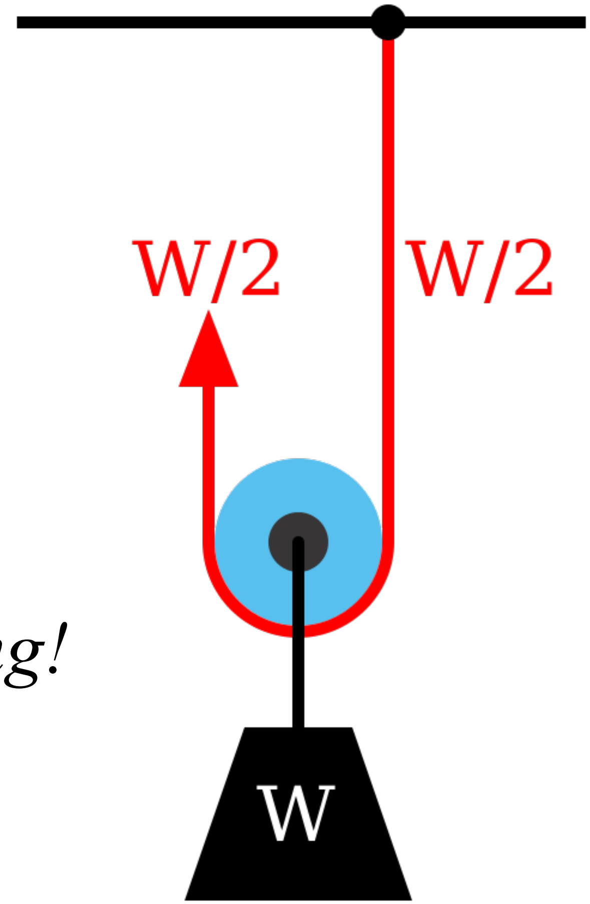
- The two approaches used in the previous example are shown schematically in the figure.



Interlude: what is a pulley good for?

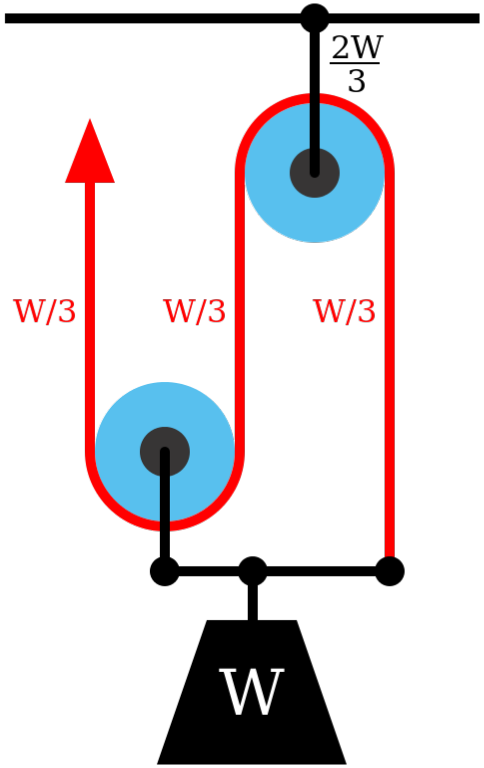
- Why use a pulley?
- Simple pulley redirects force
- Pull down to move object up
- But now you use two ropes
  - each has same tension
- Net downward force: W
- Net upward: W/2 + W/2

- *let the ceiling do half the pulling!*



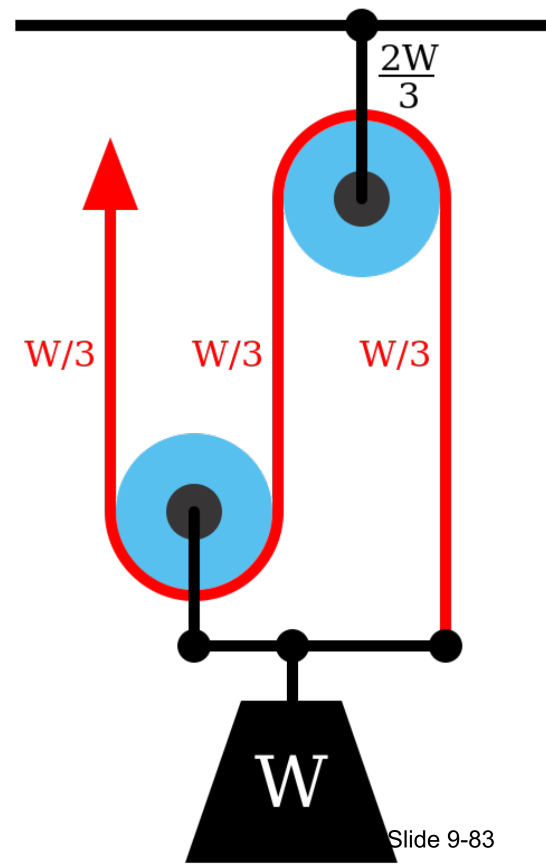
Pulley

- Compound pulley? Why not use more ropes!
- Same rope, same tension, but split it up
- 3 sections, pull with 1/3 weight
- tension same everywhere in the rope if it is light!



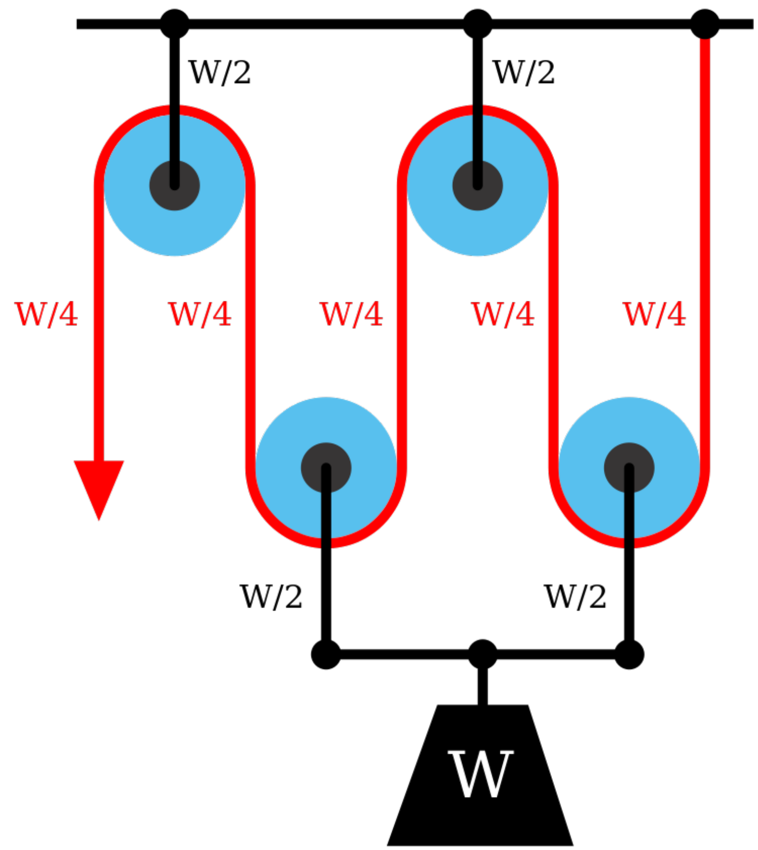
Pulley

- Work?
- if load moves by x, you have to pull L
- Work done by you = work by gravity
- $(W/3)L = Wx$ so $L = 3x$
- pull with 1/3 the force
- pay with 3x the distance



Pulley

- Pull with ¼ the force, but 4 times as far
- Mechanical advantage – trade force for distance



Drag forces

- Resistance in a fluid or gas
- Depends on:
  - Speed v (laminar) or v^2 (turbulent)
  - Shape (factor D, all speeds)
  - Cross sectional area (A)
  - Surface finish (D, esp. high speed "skin friction")
  - Density of fluid/gas (ρ high speed)
  - Viscosity of fluid (η low speed)

$$F_{\text{drag}} = \begin{cases} -\frac{1}{2}\rho v^2 DA & \text{high speed / turbulent} \\ -6\pi\eta r|v| & \text{low speed / laminar} \end{cases}$$

Low speed drag in 1D: solvable

- acceleration is $a_{drag} \sim -bv$
- b is the *drag coefficient*
- add to this acceleration of +g due to gravity

$$v(t) = \frac{g}{b}\left(1 - e^{-bt}\right) = v_{term}\left(1 - e^{-bt}\right)$$

- Qualitatively similar for high speeds

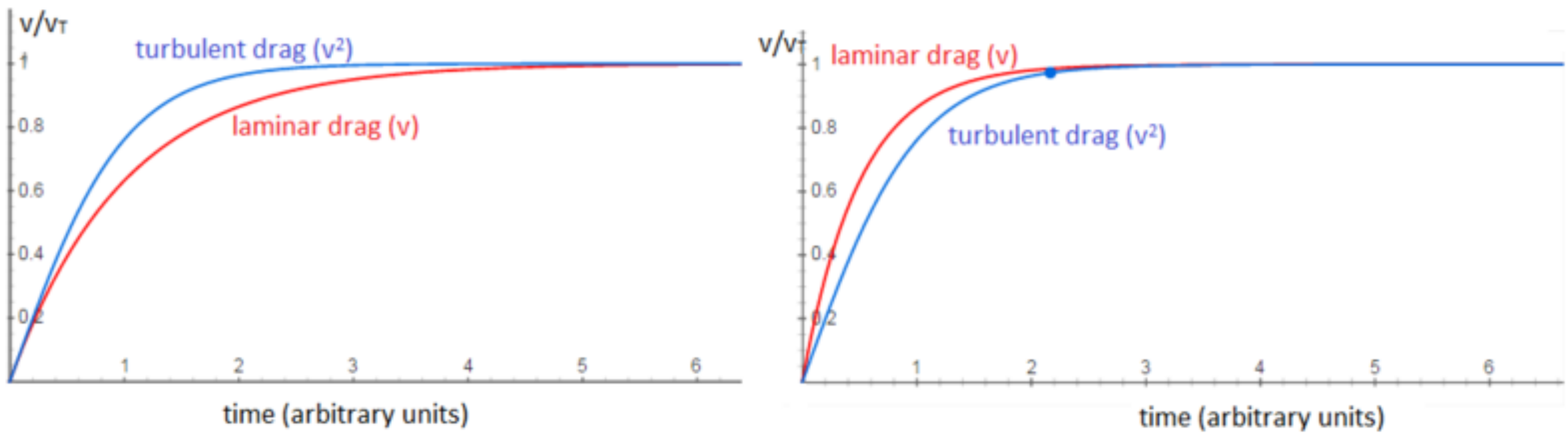


Image: wikipedia, "Drag (physics)

So what is "high speed"

- Depends on balance of inertial and viscous forces
- If the object has characteristic length *L*, characterized by *Reynolds number*

$$\mathrm{Re} = \frac{\text{intertial forces}}{\text{viscous forces}} = \frac{(\text{mass})\,(\text{acceleration})}{(\text{dynamic viscosity})\left(\frac{\text{velocity}}{\text{distance}}\right)(\text{area})} = \frac{\rho v L}{\eta}$$

- For Re << 1, nice laminar flow
  - Viscous forces dominate
- For Re ~ 10, turbulent
  - Inertial forces dominate
- Small viscosity = more easily turbulent

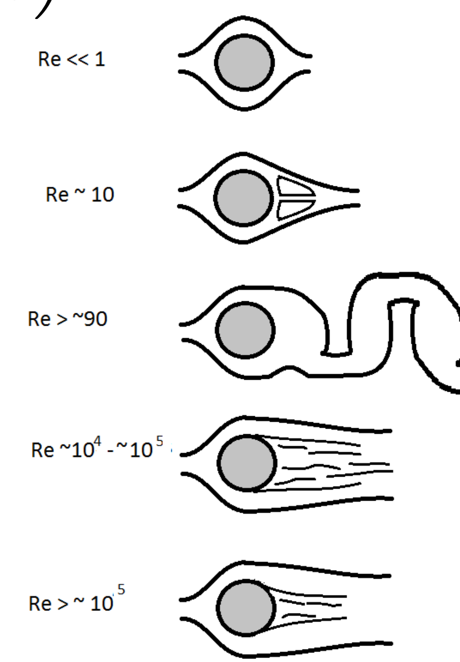


Image: wikipedia, "Reynolds number"

So?

Say we have a 1mm particle in air at 10 m/s (22 mph)

$$\text{Re} = \begin{cases} 355 & \text{water} \\ 18000 & \text{air} \end{cases}$$

Turbulent! How about a 1um particle at 10 m/s?

$$\text{Re} = \begin{cases} 11 & \text{water} \\ 588 & \text{air} \end{cases}$$

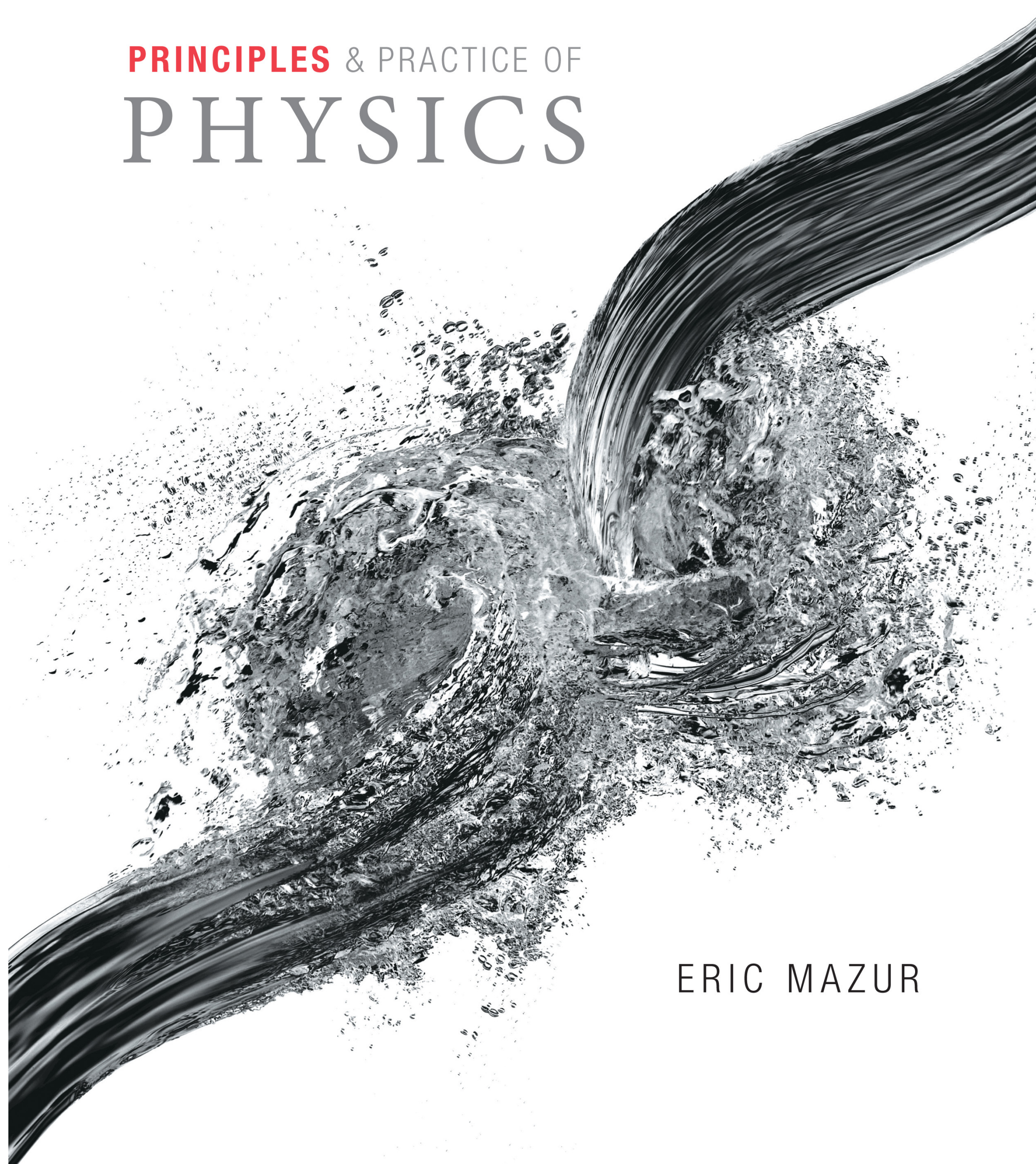
Only laminar below ~1m/s. Pollen is already 10-100um

Laminar? Very fine dust, bacteria swimming

most stuff is turbulent, and this is just terrible

PRINCIPLES & PRACTICE OF

PHYSICS



ERIC MAZUR

Reading quiz

9.06

A graph depicts force versus position. What represents the work done by the force over the given displacement?

- ○ The work done is equal to the slope of the curve.
- ○ The work done is equal to length of the curve.
- ○ Work cannot be determined from this type of graph.
- ○ The work done is equal to the area under the curve.
- ○ The work done is equal to the product of the maximum force times the maximum position.

Submit   My Answers   Give Up

Reading quiz

9.06

A graph depicts force versus position. What represents the work done by the force over the given displacement?

- ○ The work done is equal to the slope of the curve.
- ○ The work done is equal to length of the curve.
- ○ Work cannot be determined from this type of graph.
- ◉ The work done is equal to the area under the curve.
- ○ The work done is equal to the product of the maximum force times the maximum position.

Submit My Answers Give Up

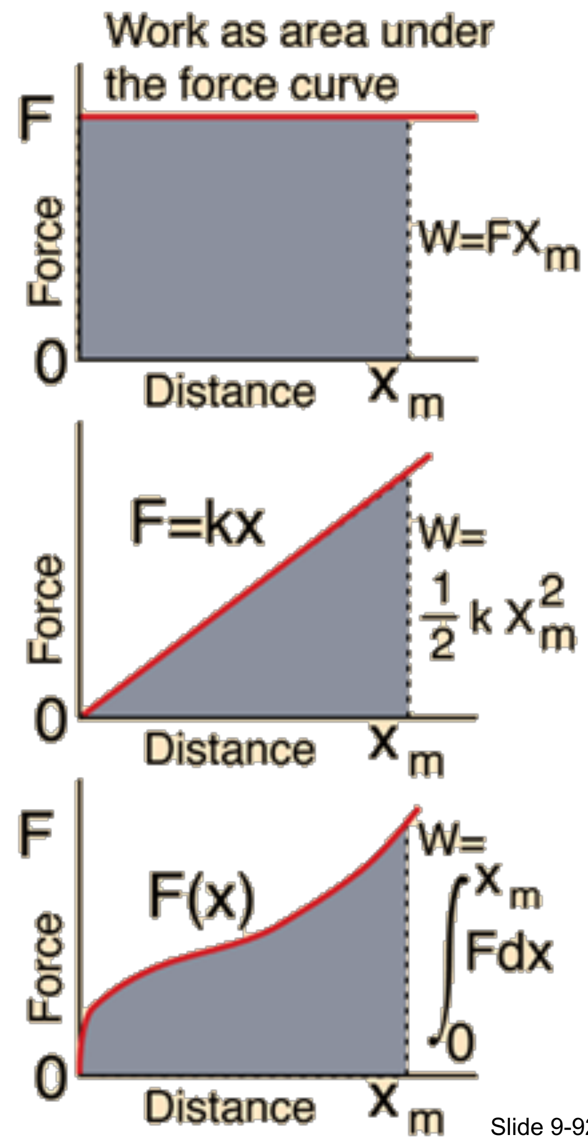
Correct

Reading quiz

9.06

$$\text{Work} = W = \int_0^{x_m} F(x)\,dx = \int_0^{x_m} kx\,dx = \frac{1}{2}kx_m^2 \qquad \text{spring}$$

$$\text{Work} = W = \int_{x_1}^{x_2} F(x)\,dx \qquad \text{general path}$$



Reading quiz

9.07

Which of the following statements is/are true?

Check all that apply.

- ☐ The SI unit of power is the watt.
- ☐ Power is the rate at which energy is transformed.
- ☐ Power is the rate at which work is done.
- ☐ A person is limited in the total work he or she can do by their power output.
- ☐ The SI unit of power is the horsepower.

Submit My Answers Give Up

Reading quiz

9.07

Which of the following statements is/are true?

Check all that apply.

- [x] The SI unit of power is the watt.
- [x] Power is the rate at which energy is transformed.
- [x] Power is the rate at which work is done.
- [ ] A person is limited in the total work he or she can do by their power output.
- [ ] The SI unit of power is the horsepower.

Submit   My Answers   Give Up

Correct

Reading quiz

9.08

Person B does twice the work of person A, and in one-half of the time . How does the power output of person B compare to person A?

- ◯ Person B has the same power output as person A.
- ◯ Person B has half the power output of person A.
- ◯ Person B has twice the power output of person A.
- ◯ Person B has four times the power output of person A.
- ◯ Person B has eight times the power output of person A.

Submit My Answers Give Up

Reading quiz

9.08

Person B does twice the work of person A, and in one-half of the time . How does the power output of person B compare to person A?

- Person B has the same power output as person A.
- Person B has half the power output of person A.
- Person B has twice the power output of person A.
- Person B has eight times the power output of person A.
- (selected) Person B has four times the power output of person A.

Submit   My Answers   Give Up

Correct

Reading quiz

9.05

If the net work done on an object is zero, what can you determine about the object's kinetic energy?

- ◯ The object's kinetic energy is increasing.
- ◯ The object's kinetic energy is zero.
- ◯ The object's kinetic energy is decreasing.
- ◯ The object's kinetic energy remains the same.

Submit My Answers Give Up

Reading quiz

9.05

If the net work done on an object is zero, what can you determine about the object's kinetic energy?

- ( ) The object's kinetic energy is increasing.
- ( ) The object's kinetic energy is zero.
- ( ) The object's kinetic energy is decreasing.
- (•) The object's kinetic energy remains the same.

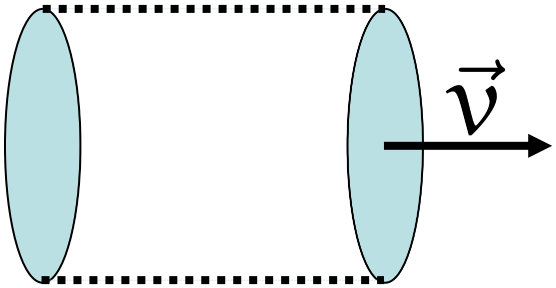
Submit   My Answers   Give Up

Correct

Where does high speed drag come from?

Have to move air out of the way.

Mass of air to move?



$$m = \rho V = \rho A \Delta x = \rho A v \Delta t$$

Change in momentum?

$$\Delta p = m\Delta v = (\rho A v \Delta t)(v - 0)$$

Force = time rate of change

$$F_{\text{drag}} = \frac{\Delta p}{\Delta t} = \rho A v^2$$

(add a drag coefficient 'fudge factor' D to handle shape/etc.)

How to calculate?

- Drag forces for turbulent flow don't often admit analytic solutions
- Make computers do the work. Let's say we start at rest at the origin at time t=0, with only gravity

$$a[0] = -g$$
$$v[0] = 0$$
$$x[0] = 0$$

- How about some infinitesimally small time later?

A time *dt* later

$$a = -g$$

$$v[dt] = v[0] + g\,dt = g\,dt$$

$$x[dt] = v[dt]\,dt + \frac{1}{2}g(dt)^2$$

How about one step later?

$$a = -g$$

$$v[2dt] = v[dt] + g\,dt$$

$$x[2dt] = x[dt] + v[2dt]\,dt + \frac{1}{2}g(dt)^2$$

Notice a pattern

$$a = -g$$

$$v_{\text{now}} = v_{\text{prev}} + g\,dt$$

$$x_{\text{now}} = x_{\text{prev}} + v_{\text{now}}\,dt + \frac{1}{2}g(dt)^2$$

- We can use a known starting point and just increment one step *dt* at a time!
- Need stopping condition (e.g., x = -h to hit ground)
- Advantage? Can trivially add any acceleration

Example (python)

```
import math

#1D motion with drag for a dropped object

g=9.81          #grav accel defined as constant
v0=0.0          #starting velocity
h=100           #starting position
dt = 0.001      #time step; smaller=more accurate but slower
b = 0.1         #drag coefficient

def trajectory(v,h):
    y=h
    t=0.0
    a=-g

    while y>0:     #iterate until v change is tiny, terminal velocity reached
        a = -g + b*v**2     #drag is opposite gravity (in dir of v)
        v += a*dt
        y += v*dt+0.5*a*dt*dt
        t+=dt
    return (t,v)

time,speed=trajectory(v0,h)      #run with given starting speed and height

print "Time of flight (2)"
print "%.2f s" % (time)

print "Terminal velocity"
print "%.2f m/s" % (speed)
```

```
MacBook-Pro-82:python pleclair$ python ./1D-v2.py
Time of flight (2)
10.79 s
Terminal velocity
-9.90 m/s
MacBook-Pro-82:python pleclair$
```

So what?

- You can do this on your phone
- You are mostly science & engineering majors, this will totally come up again
- Don’t be afraid of code. Much better than labs, you can’t break anything.

- Some examples:
- http://faculty.mint.ua.edu/~pleclair/ph125/python/

Section 9.5
Question 6

When you do positive work on a particle, its kinetic energy

1. increases.
2. decreases.
3. remains the same.
4. We need more information about the way the work was done.

Section 9.5
Question 6

When you do positive work on a particle, its kinetic energy

✔ 1. increases.
2. decreases.
3. remains the same.
4. We need more information about the way the work was done.

Section 9.6: Work done on a many-particle system

Section Goals

You will learn to

- Extend the work-force-displacement relationship for single objects to **systems of interacting objects**.
- Recognize that **only** external forces contribute to the work done for many particle systems. Since the **internal** forces are members of an interaction pair the **work done** by that pair of forces always sums to **zero**.

Example 9.7 Landing on his feet

A 60-kg person jumps off a chair and lands on the floor at a speed of 1.2 m/s. Once his feet touch the floor surface, he slows down with constant acceleration by bending his knees. During the slowing down, his center of mass travels 0.25 m.

Determine the magnitude of the force exerted by the floor surface on the person and the work done by this force on him.

What?

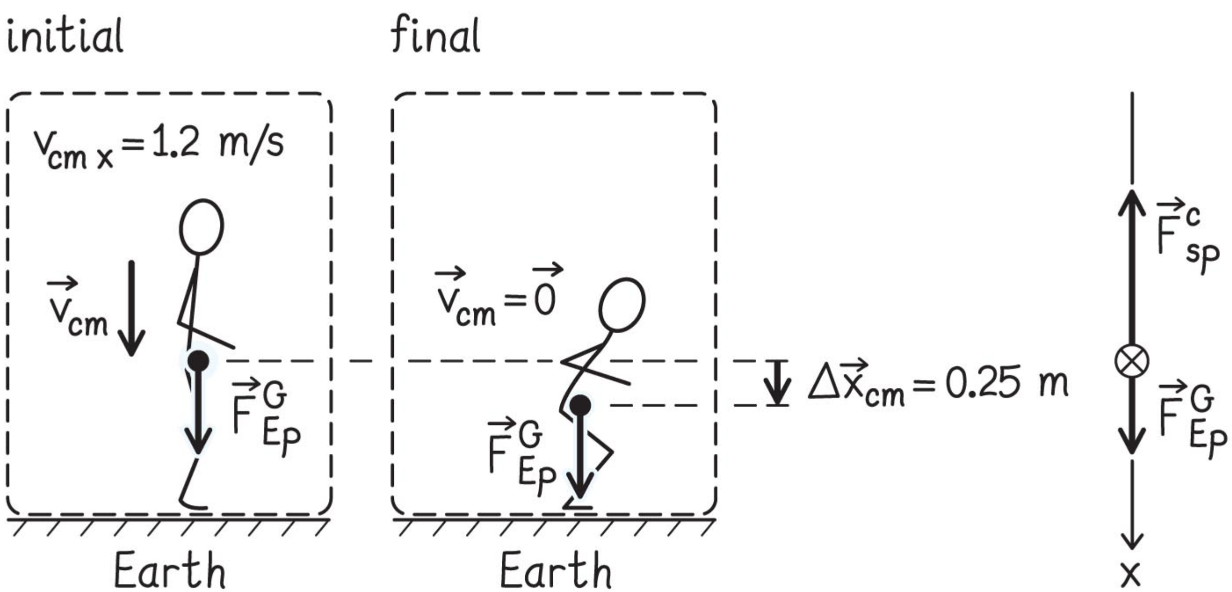
I have no idea what to do.

- What is the physics?
  - person's KE changes
  - external forces cause this change
  - the change in KE must be due to the work done by these forces

- Figure out the change in KE and the work done by gravity slowing down. From work you get force, knowing the displacement.

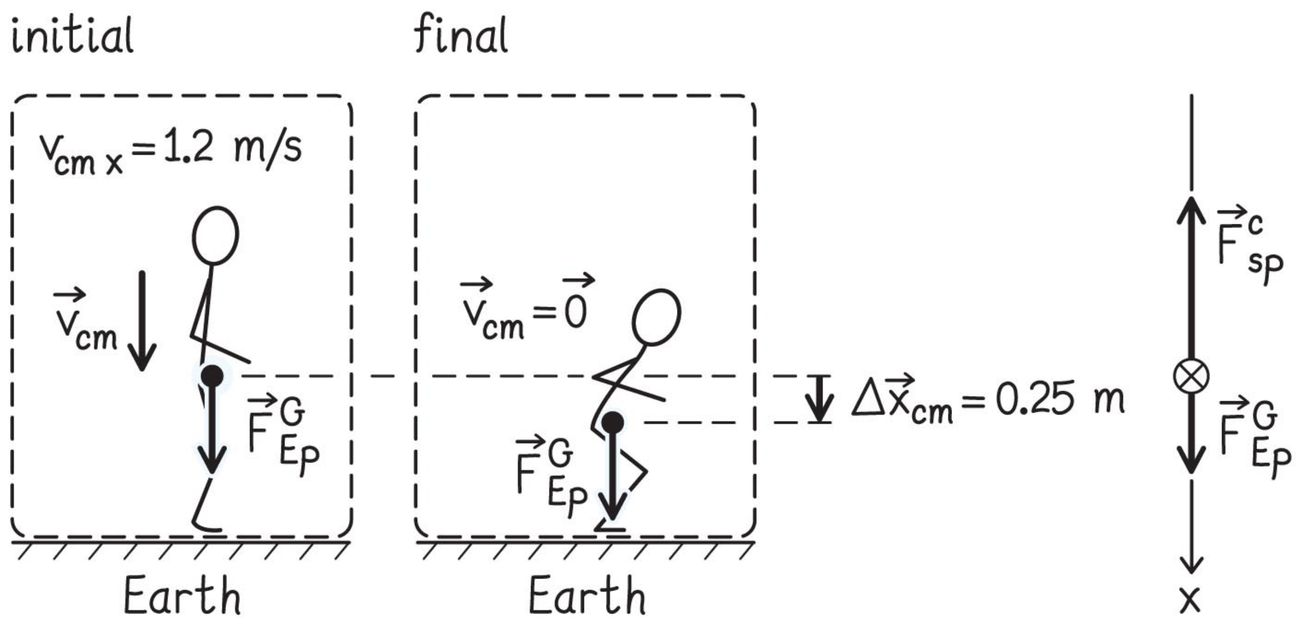
Example 9.7 Landing on his feet (cont.)

➊ GETTING STARTED I begin by making a sketch of the initial and final conditions, choosing my person as the system and assuming the motion to be entirely vertical (Figure 9.21).



Example 9.7 Landing on his feet (cont.)

❶ GETTING STARTED I point the x axis downward in the direction of motion, which means that the x components of both the displacement and the velocity of the center of mass are positive: $\Delta x_{cm} = +0.25$ m and $v_{cm\,x} = +1.2$ m/s.

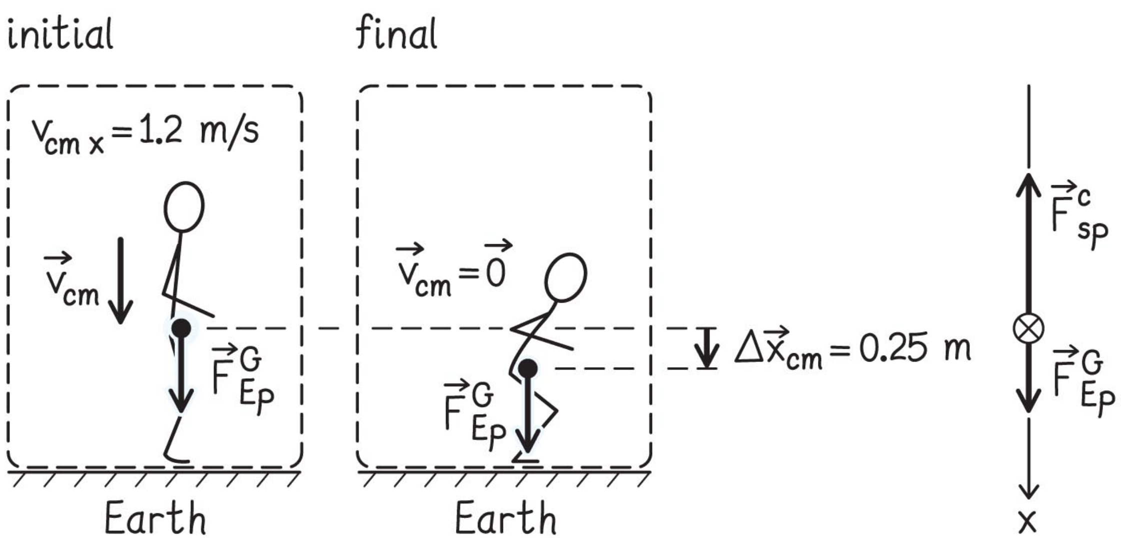


Section 9.6: Work done on a many-particle system

Example 9.7 Landing on his feet (cont.)

❶ GETTING STARTED Two external forces are exerted on the person: a downward force of gravity $\vec{F}^G_{Ep}$ exerted by Earth and an upward contact force $\vec{F}^c_{sp}$ exerted by the floor surface.

Only the point of application of the force of gravity undergoes a displacement, and so I need to include only that force in my sketch.



Example 9.7 Landing on his feet (cont.)

❷ DEVISE PLAN Knowing the initial center-of-mass velocity, I can use Eq. 9.13 to calculate the change in the person's translational kinetic energy ΔK_{cm}.

This change in kinetic energy must equal the work done by the net external force.

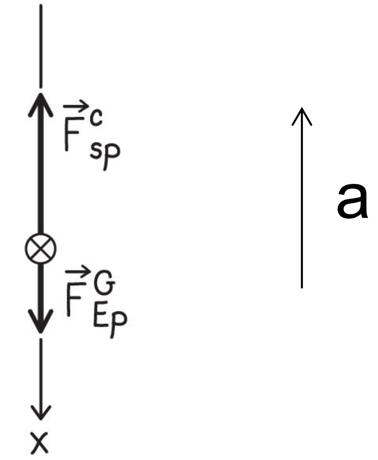
From that and the displacement $\Delta x_{cm} = +0.25$ m we can find the ***vector sum of the forces*** exerted on the person.

Example 9.7 Landing on his feet (cont.)

❷ DEVISE PLAN If I then subtract the force of gravity, I obtain the force exerted by the floor surface on the person. To determine the work done by this force on the person, I need to multiply it by the force displacement.

Example 9.7 Landing on his feet (cont.)

❷ DEVISE PLAN Because the person slows down as he travels downward, his acceleration is upward and so the vector sum of the forces is upward too. To remind myself of this, I draw a free-body diagram in which the arrow for the upward contact force is longer than the arrow for the downward force of gravity.



Section 9.6: Work done on a many-particle system

Example 9.7 Landing on his feet (cont.)

❸ EXECUTE PLAN Because the person ends at rest, his final translational kinetic energy is zero

$$\Delta K_{\text{cm}} = 0 - \tfrac{1}{2} m v_{\text{cm,i}}^2 = \tfrac{1}{2}(60\,kg)(1.2\ \text{m/s})^2 = -43\ \text{J}.$$

Example 9.7 Landing on his feet (cont.)

❸ EXECUTE PLAN Substituting this value and the displacement of the center of mass into Eq. 9.14 yields

$$\Sigma F_{\text{ext}\,x} = \frac{\Delta K_{\text{cm}}}{\Delta x_{\text{cm}}} = \frac{-43\ \text{J}}{0.25\ \text{m}} = -170\ \text{N}.$$

This is the *net* force. We need the free body diagram to disentangle the forces. It is negative, as our diagram indicated.

Example 9.7 Landing on his feet (cont.)

❸ EXECUTE PLAN To obtain the force exerted by the floor from this vector sum, look back to the free body diagram: just two forces.

$$\Sigma F_{\text{ext}\,x} = F^{G}_{\text{Ep}\,x} + F^{\text{c}}_{\text{sp}\,x}$$

and so $F^{\text{c}}_{\text{sp}\,x} = \Sigma F_{\text{ext}\,x} - F^{G}_{\text{Ep}\,x}$. The x component of the force of gravity is $F^{G}_{\text{Ep}\,x} = mg = (60\text{ kg})(9.8\text{ (m/s}^2)) = +590\text{ N}$ and so $F^{\text{c}}_{\text{sp}\,x} = -170\text{ N} - 590\text{ N} = -760\text{ N}$. ✔

Example 9.7 Landing on his feet (cont.)

❸ EXECUTE PLAN To determine the work done by this force on the person, I must multiply the x component of the force by the force displacement.

The point of application is the floor, which doesn't move. This means that the force displacement is zero, so the work done on the person by the surface is zero too: $W = 0$. ✔

Example 9.7 Landing on his feet (cont.)

❹ EVALUATE RESULT The contact force $F^{c}_{sp\,x}$ is negative because it is directed upward, as I expect. Its magnitude is larger than that of the force of gravity, as it should be in order to slow the person down.

There is work done, but it is by the force of gravity (since the earth is outside my system).

Section 9.7: Variable and distributed forces

Section Goals

You will learn to

- Derive the relationship for the **work** done by a **variable force**.
- Interpret the work done by a variable force **graphically**.
- Understand that distributed forces, like friction, have **no single point of application** on an object.

Section 9.7: Variable and distributed forces

- The acceleration of the center of mass of a system consisting of many interacting particles is given by

$$\vec{a}_{\text{cm}} = \frac{\Sigma\vec{F}_{\text{ext}}}{m}$$

- Following the same derivation as in the single-particle system, we can write

$$\Delta K_{\text{cm}} = ma_{\text{cm}\,x}\Delta x_{\text{cm}} = (\Sigma F_{\text{ext}\,x})\Delta x_{\text{cm}} \text{(constant forces, one dimension)}$$

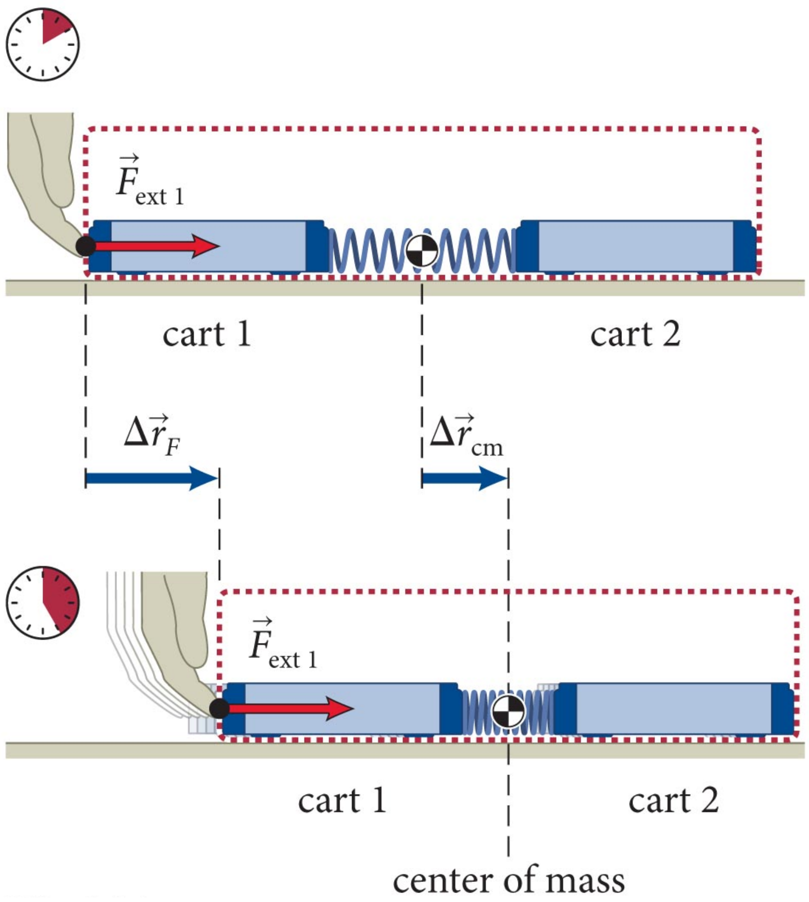
- This is *not* the work done by the external force on the system!
- The KE of the system is not *just* the CM KE – internal motion of constituents too!

Section 9.7: Variable and distributed forces

- For a system of many particles $K = K_{cm} + K_{conv}$ – there is some convertible KE due to internal motion of constituents
- Therefore, $\Delta K_{cm} \neq \Delta E$, and since $\Delta E = W$, we can see that

  $\Delta K_{cm} \neq W$ (many-particle system)

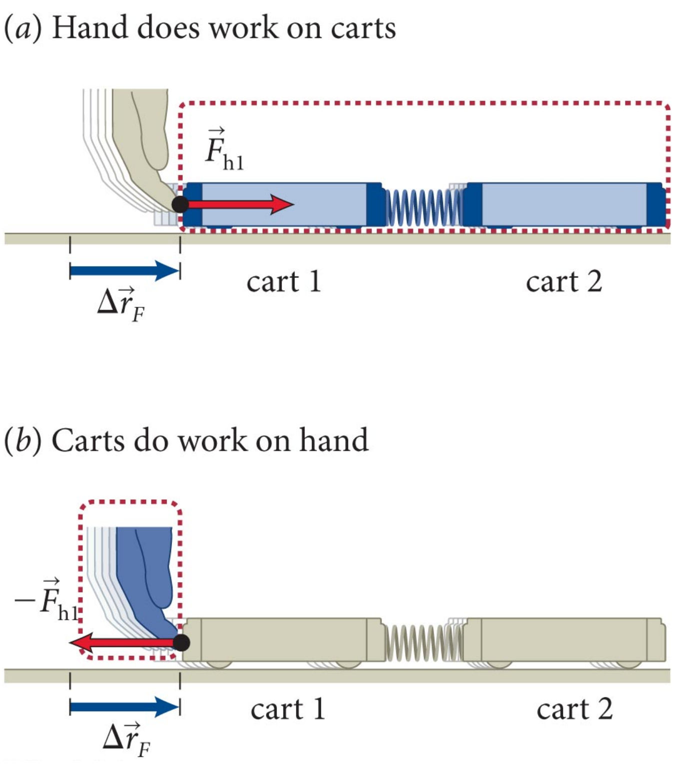
- This is explicitly illustrated in the example shown in the figure.
- The external force on cart 1 increases the kinetic energy **and** the internal energy of the system.



Section 9.7: Variable and distributed forces

- To determine the work done by external forces on a many particle system, we can use the fact that $W_{env} = -W_{sys}$.
- This makes sense – work is what crosses the boundary, and a loss to the environment is the same as a gain to the system (& vice versa)
- We can see from the figure that the work done by the two-cart system on the hand is $= -F_{h1x}\Delta x_F$.
- Then the work done by the external force on the two-cart system is

$$W = F_{ext\ 1x}\Delta x_F \text{ (constant nondissipative force, one dimension)}$$



Section 9.7: Variable and distributed forces

- Generalizing this work equation to many-particle systems subject to several constant forces, we get

$$W = W_1 + W_2 + \cdots = F_{\text{ext}\,1x}\Delta x_{F1} + F_{\text{ext}\,2x}\Delta x_{F2} + \cdots$$

or

$$W = \sum_n (F_{\text{ext}\,nx}\Delta x_{Fn}) \text{ (constant nondissipative forces, one dimension)}$$

Note the difference from

$$\Delta K_{\text{cm}} = ma_{\text{cm}\,x}\Delta x_{\text{cm}} = (\Sigma F_{\text{ext}\,x})\Delta x_{\text{cm}} \text{ (constant forces, one dimension)}$$

If we consider a varying force $F(x)$, we take infinitesimal displacements, and this becomes an integral

$$W = \int_{x_\text{i}}^{x_\text{f}} F_x(x)\,dx \text{ (nondissipative force, one dimension)}$$

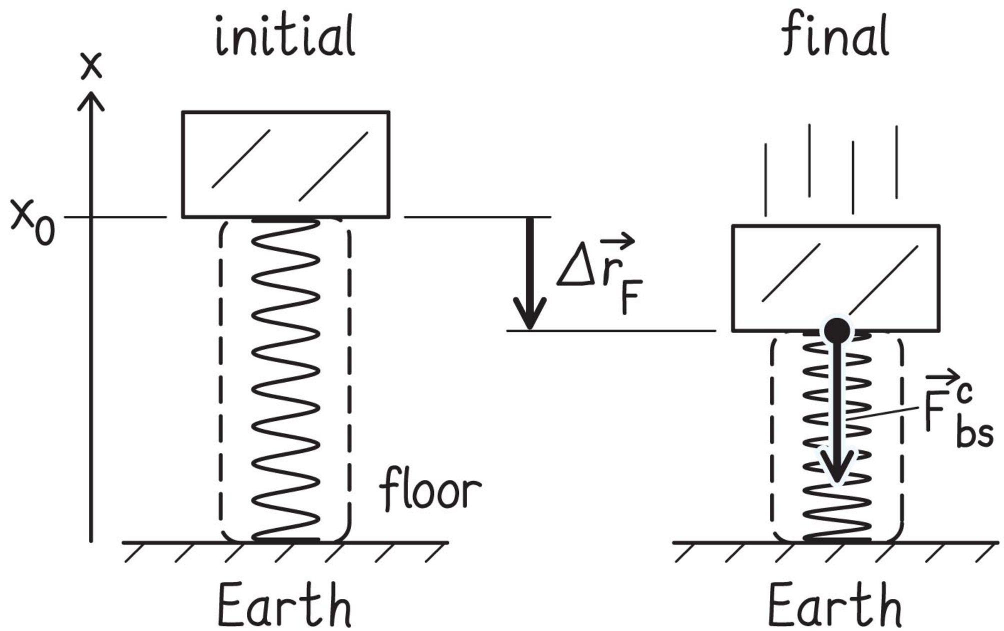
work is the area under the force-displacement curve!

Example 9.8 Spring work

A brick of inertia m compresses a spring of spring constant k so that the free end of the spring is displaced from its relaxed position. What is the work done by the brick on the spring during the compression?

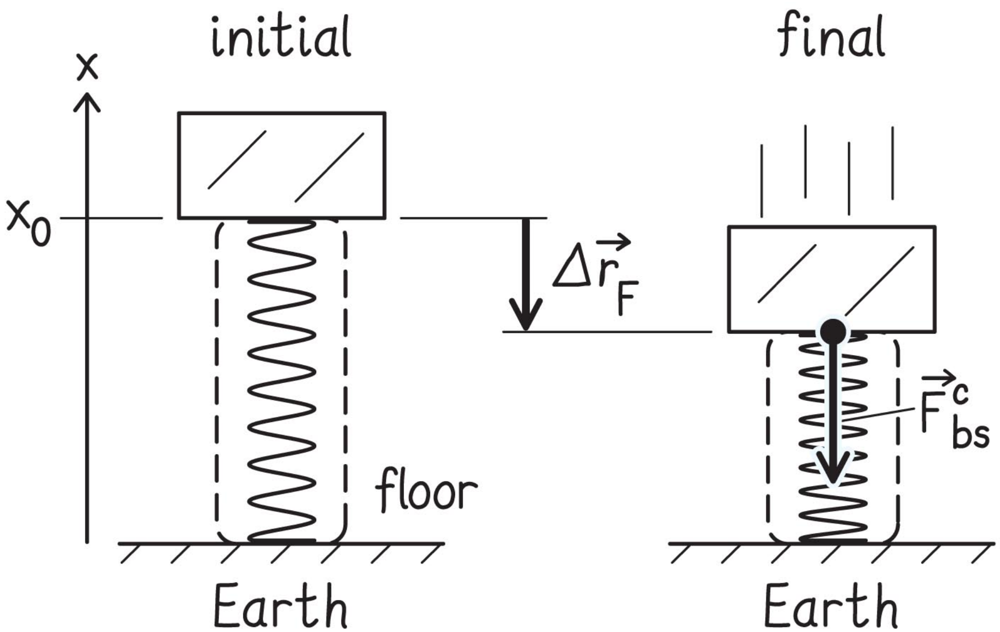
Example 9.8 Spring work (cont.)

❶ GETTING STARTED I begin by making a sketch of the situation as the free end of the spring is compressed from its relaxed position x_0 to a position x (Figure 9.23). Because I need to calculate the work done by the brick on the spring, I choose the spring as my system.



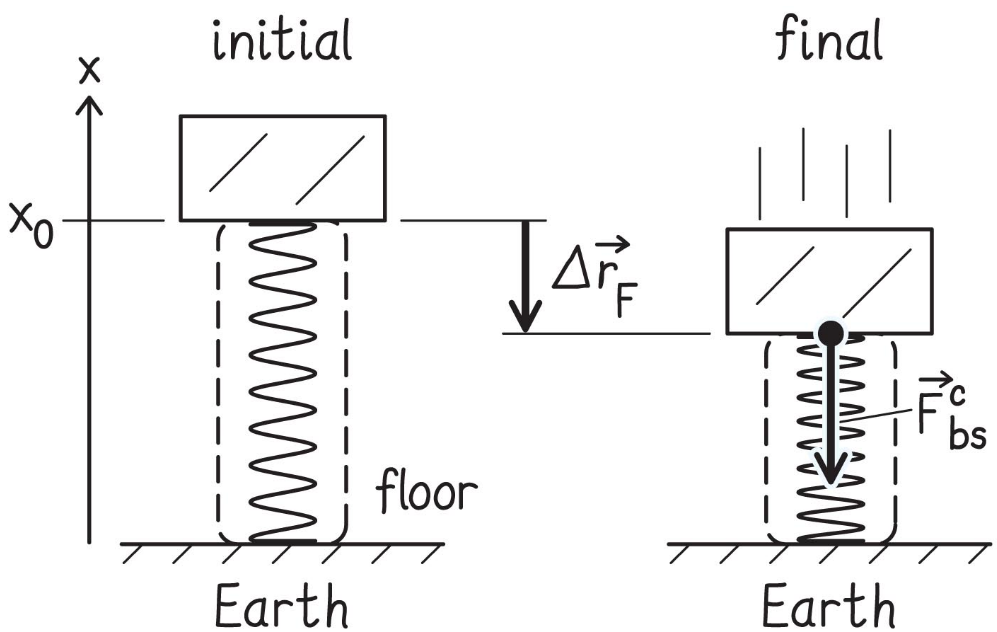
Example 9.8 Spring work (cont.)

❶ GETTING STARTED Three forces are exerted on the spring: contact forces exerted by the brick and by the floor, and the force of gravity. As usual when dealing with compressed springs, I ignore the force of gravity exerted on the spring (see Section 8.6).



Example 9.8 Spring work (cont.)

❶ GETTING STARTED Only the force $\vec{F}^{c}_{bs}$ exerted by the brick on the spring undergoes a nonzero force displacement, so I need to show only that force in my diagram. Because the brick and spring do not exert any forces on each other when the spring is in the relaxed position, I do not draw this force in the initial state.



Example 9.8 Spring work (cont.)

❷ DEVISE PLAN I need to calculate the work done by the brick on the spring. I could use Eq. 9.22 if I knew the force the brick exerts on the spring.

That force is not given, but the **force exerted by the brick on the spring and the force exerted by the spring on the brick form an interaction pair:** $\vec{F}^{c}_{bs} = -\vec{F}^{c}_{sb}$. I can use Eq. 8.20 to determine $\vec{F}^{c}_{sb}$ and then use it in Eq. 9.22.

Example 9.8 Spring work (cont.)

❸ EXECUTE PLAN Equation 8.20 tells me that the x component of the force exerted by the spring on the brick varies depending on how far the spring is compressed:

$$F_{\text{sb}\,x} = -k(x - x_0),$$

where x_0 is the coordinate of the relaxed position of the free end of the spring.

Example 9.8 Spring work (cont.)

❸ EXECUTE PLAN The x component of the force exerted by the brick on the spring is thus

$$F_{\text{bs}\,x} = +k(x - x_0). \tag{1}$$

Because $x_0 > x$, $F_{\text{bs}\,x}$ is negative, which means that $\vec{F}^{\text{c}}_{\text{bs}}$ points in the same direction as the force displacement. Thus the work done by the brick on the spring is positive.

Example 9.8 Spring work (cont.)

❸ EXECUTE PLAN Now I substitute Eq. 1 into Eq. 9.22 and work out the integral to determine the work done by the brick on the spring:

$$W_{\text{bs}} = \int_{x_0}^{x} F_{\text{bs}\,x}(x)\,dx = \int_{x_0}^{x} k(x - x_0)\,dx$$

$$= \left[\tfrac{1}{2}kx^2 - kx_0 x\right]_{x_0}^{x} = \tfrac{1}{2}k(x - x_0)^2. \checkmark \qquad (2)$$

limits of integration? start & end points of motion

Example 9.8 Spring work (cont.)

❹ Evaluate result Because the spring constant k is always positive (see Section 8.9 on Hooke's law), the work done by the brick on the spring is also positive. This is what I expect because the work done in compressing the spring is stored as potential energy in the spring.

Section 9.7
Question 7

When you plot the force exerted on a particle as a function of the particle's position, what feature of the graph represents the work done on the particle?

1. The maximum numerical value of the force
2. The area under the curve
3. The value of the displacement
4. You need more information about the way the work was done.

Question 7

When you plot the force exerted on a particle as a function of the particle's position, what feature of the graph represents the work done on the particle?

1. The maximum numerical value of the force
2. ✔ The area under the curve
3. The value of the displacement
4. You need more information about the way the work was done.

Section 9.8: Power

Section Goals

You will learn to

- Define power as the **time rate of change** of energy transferred or converted for a system.
- Calculate power for the special case of a **constant** external force applied to a system.

Concepts: Power

- **Power** is the *rate* at which energy is either converted from one form to another or transferred from one object to another.

- The SI unit of power is the **watt** (W), where 1 W = 1 J/s.

Quantitative Tools: Power

- The **instantaneous power** is

$$P = \frac{dE}{dt}$$

- If a constant external force $F_{\text{ext}\,x}$ is exerted on an object and the x component of the velocity at the point where the force is applied is υ_x, the power this force delivers to the object is

$$P = F_{\text{ext}\,x}\upsilon_x$$

Section 9.8: Power

- Short derivation:

$$P = \frac{dE}{dt} = \frac{dW}{dt} = \frac{d}{dt}\left(\int F(x)\,dx\right) \quad \text{general}$$

$$= \frac{d}{dt}(F\Delta x) = F\frac{d\Delta x}{dt} = Fv \quad \text{constant force}$$

- Single particle, $W = \Delta K$:

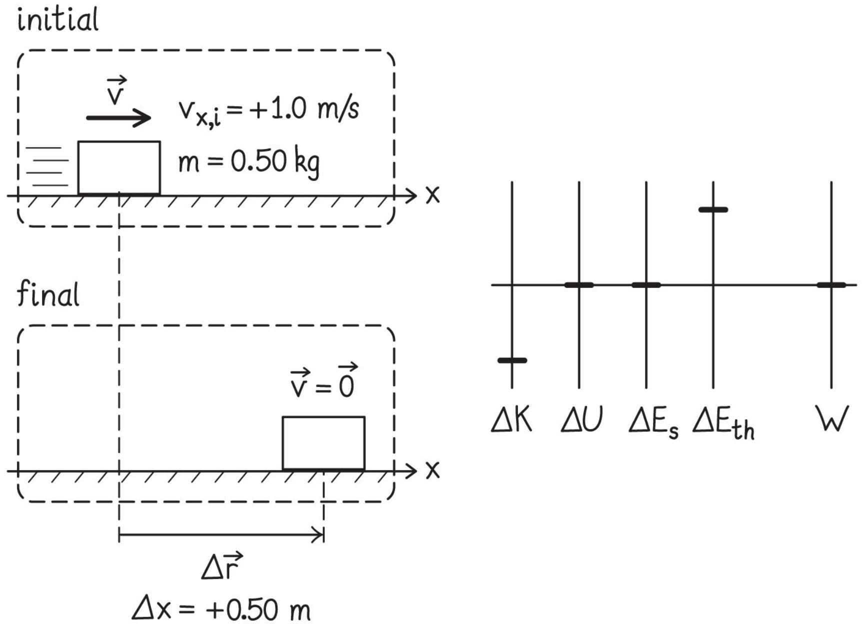
$$P = \frac{dE}{dt} = \frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = mv\frac{dv}{dt} = mva = Fv$$

Example 9.9 Frictional power

A 0.50-kg wood block initially traveling at 1.0 m/s slides 0.50 m on a horizontal floor before coming to rest. What is the average rate at which thermal energy is generated?

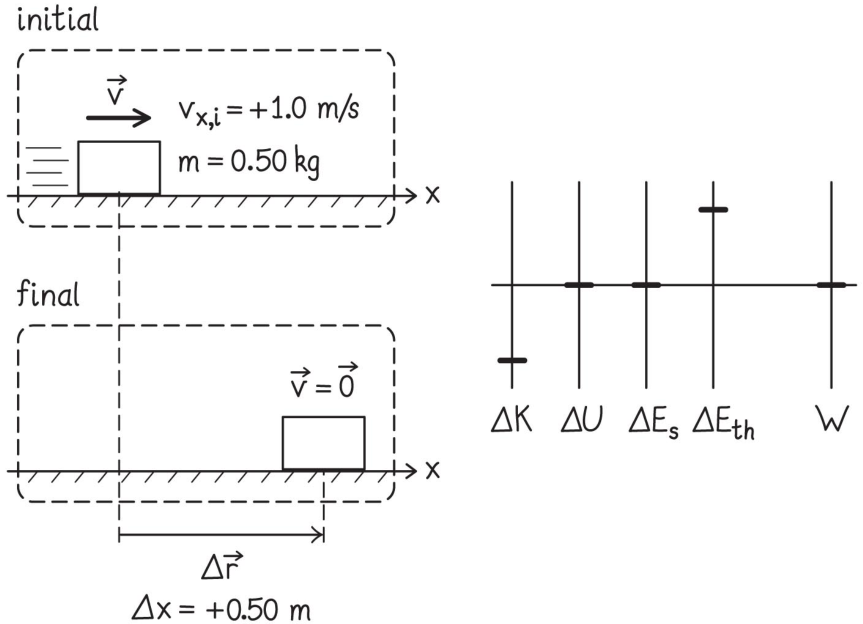
Example 9.9 Frictional power (cont.)

❶ GETTING STARTED I begin by making a sketch of the situation, listing all the given quantities and drawing an energy diagram for the closed block-floor system (Figure 9.25). I note that as the block slides to a stop, all of its kinetic energy is converted to thermal energy.



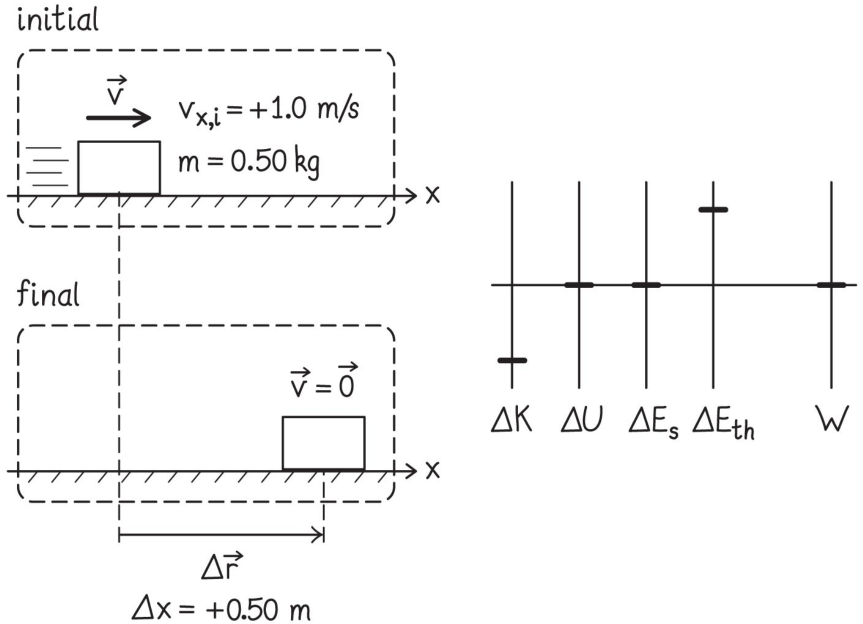
Example 9.9 Frictional power (cont.)

❷ DEVISE PLAN To calculate the average power at which thermal energy is generated, I need to know the amount ΔE_{th} of thermal energy generated and the time interval Δt over which it is generated. The ratio of these two quantities gives the average power (Eq. 9.29).



Example 9.9 Frictional power (cont.)

❷ DEVISE PLAN The amount of thermal energy is equal to the initial kinetic energy of the block. If I assume the block has constant acceleration as it comes to a stop, I can use equations from Chapter 3 to determine Δt from the block's initial speed and the distance over which it comes to a stop.



Example 9.9 Frictional power (cont.)

❸ EXECUTE PLAN Because the block ends at rest, I know from Eq. 3.4, $v_{x,\mathrm{f}} = v_{x,\mathrm{i}} + a_x \Delta t$, that

$$ax = -\frac{v_{x,\mathrm{i}}}{\Delta t}.$$

Example 9.9 Frictional power (cont.)

❸ EXECUTE PLAN Substituting this result into Eq. 3.8, $x_f = x_i + \upsilon_{x,i}\Delta t + \frac{1}{2}a_x(\Delta t)^2$, gives

$$\Delta x = \upsilon_{x,i}\Delta t - \frac{1}{2}\frac{\upsilon_{x,i}}{\Delta t}(\Delta t)^2 = \frac{1}{2}\upsilon_{x,i}\Delta t$$

or

$$\Delta t = \frac{2\Delta x}{\upsilon_{x,i}} = \frac{2(0.50\ \text{m})}{1.0\ \text{m/s}} = 1.0\ \text{s}.$$

Example 9.9 Frictional power (cont.)

❸ EXECUTE PLAN The block's initial kinetic energy is $\frac{1}{2}mv_{x,\mathrm{i}}^2 = \frac{1}{2}(0.50\ \mathrm{kg})(1.0\ \mathrm{m/s})^2 = 0.25\ \mathrm{J}$,

The thermal energy generated is thus $\Delta E_{\mathrm{th}} = 0.25\ \mathrm{J}$.

From Eq. 9.29, I then obtain for the average power at which this energy is generated: $P_{\mathrm{av}} = (0.25\ \mathrm{J})/(1.0\ \mathrm{s}) = 0.25\ \mathrm{W}$. ✔

Example 9.9 Frictional power (cont.)

❹ EVALUATE RESULT The value I obtain is rather small—as a gauge, consider that a 25-W light bulb consumes energy at 100 times this rate—but it's unlikely that the energy of a sliding block would be sufficient to generate any light.

Example 9.10 Car drag

A car requires 300 kJ of energy to overcome air resistance and maintain a constant speed of 20 m/s over a distance of 1.0 km. What is the force of air resistance exerted on the car?

Example 9.10 Car drag (cont.)

❶ GETTING STARTED The air resistance causes dissipation of the car's kinetic energy. The problem statement tells me that 300 kJ is dissipated over a distance of 1.0 km when the car moves at 20 m/s.

Example 9.10 Car drag (cont.)

❷ DEVISE PLAN From the speed and the distance traveled, I can calculate the time interval over which the 300 kJ is required. I can then use $P = \Delta E/\Delta t$ to calculate the rate at which the energy is converted.

Knowing the velocity, I can then obtain the force of air resistance from $P = Fv$.

Example 9.10 Car drag (cont.)

❸ EXECUTE PLAN At 20 m/s it takes the car 50 s to cover 1.0 km. The rate at which energy is converted is thus P = (300 kJ)/(50 s) = 6.0 kW. From Eq. 9.35, I obtain

$$F_x = \frac{P}{\upsilon_x} = \frac{6.0\ \text{kW}}{20\ \text{m/s}} = 300\ \text{N.}$$ ✔

Example 9.10 Car drag (cont.)

❹ EVALUATE RESULT The answer I obtain—300 N—is the magnitude of the gravitational force exerted on a 30-kg (65-lb) object and therefore equal in magnitude to the force required to hold up such an object.

Example 9.10 Car drag (cont.)

❹ EVALUATE RESULT I know, however, that if I stick my hand out a car window at highway speed, the force exerted by the air on my hand is not much larger than the force required to hold up a small object. The car, being much larger than my hand, intercepts a lot more air, but it is shaped more aerodynamically, and so it is reasonable that the force of air resistance is only 300 N.

Concepts: Work done by a constant force

- In order for a force to do work on an object, the point of application of the force must undergo a displacement.
- The SI unit of work is the **joule** (J).
- The work done by a force is positive when the force and the force displacement are in the same direction and negative when they are in opposite directions.

Quantitative Tools: Work done by a constant force

- When one or more constant forces cause a particle or a rigid object to undergo a displacement Δx in one dimension, the work done by the force or forces on the particle or object is given by the **work equation**:

$$W = \left(\sum F_x\right)\Delta x_F$$

- In one dimension, the work done by a set of constant nondissipative forces on a system of particles or on a deformable object is

$$W = \sum_n (F_{\text{ext}\,nx}\Delta x_{Fn})$$

Quantitative Tools: Work done by a constant force

- If an external force does work W on a system, the **energy law** says that the energy of the system changes by an amount

$$\Delta E = W$$

- For a closed system, $W = 0$ and so $\Delta E = 0$.
- For a particle or rigid object, $\Delta E_{\text{int}} = 0$ and so

$$\Delta E = \Delta K$$

- For a system of particles or a deformable object,

$$\Delta K_{\text{cm}} = \left(\sum F_{\text{ext}\,x}\right)\Delta x_{\text{cm}}$$

Concepts: Energy diagrams

- An **energy diagram** shows how the various types of energy in a system change because of work done on the system.
- In choosing a system for an energy diagram, avoid systems for which friction occurs at the boundary because then you cannot tell how much of the thermal energy generated by friction goes into the system.

Chapter 9: Summary

Concepts: Variable and distributed forces

- The force exerted by a spring is variable (its magnitude and/or direction changes) but nondissipative (no energy is converted to thermal energy).
- The frictional force is dissipative and so causes a change in thermal energy. This force is also a distributed force because there is no single point of application.

Quantitative Tools: Variable and distributed forces

- The work done by a variable nondissipative force on a particle or object is

$$W = \int_{x_i}^{x_f} F_x(x)\,dx$$

- If the free end of a spring is displaced from its relaxed position x_0 to position x, the change in its potential energy is

$$\Delta U_{\text{spring}} = \tfrac{1}{2}k(x - x_0)^2$$

- If a block travels a distance d_{path} over a surface for which the magnitude of the force of friction is a constant F^{f}_{sb}, the energy dissipated by friction (the thermal energy) is

$$\Delta E_{\text{th}} = F^{f}_{sb}\, d_{\text{path}}$$

Chapter 9: Summary

Concepts: Power

- **Power** is the *rate* at which energy is either converted from one form to another or transferred from one object to another.

- The SI unit of power is the **watt** (W), where $1 \text{ W} = 1 \text{ J/s}$.

Quantitative Tools: Power

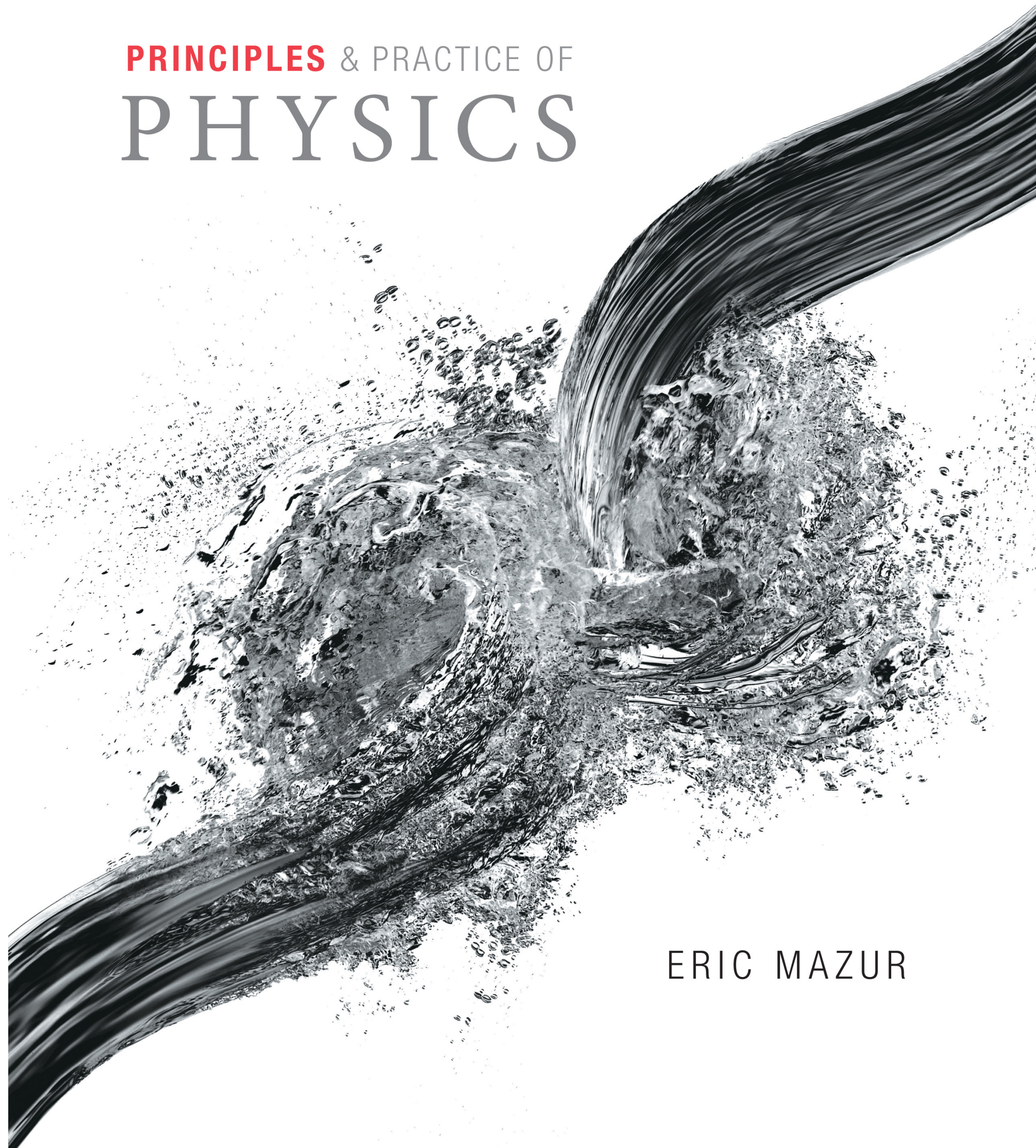
- The **instantaneous power** is

$$P = \frac{dE}{dt}$$

- If a constant external force $F_{\text{ext}\,x}$ is exerted on an object and the x component of the velocity at the point where the force is applied is υ_x, the power this force delivers to the object is

$$P = F_{\text{ext}\,x}\upsilon_x$$

Chapter 10 Motion in a Plane



Concepts

Section 10.1: Straight is a relative term

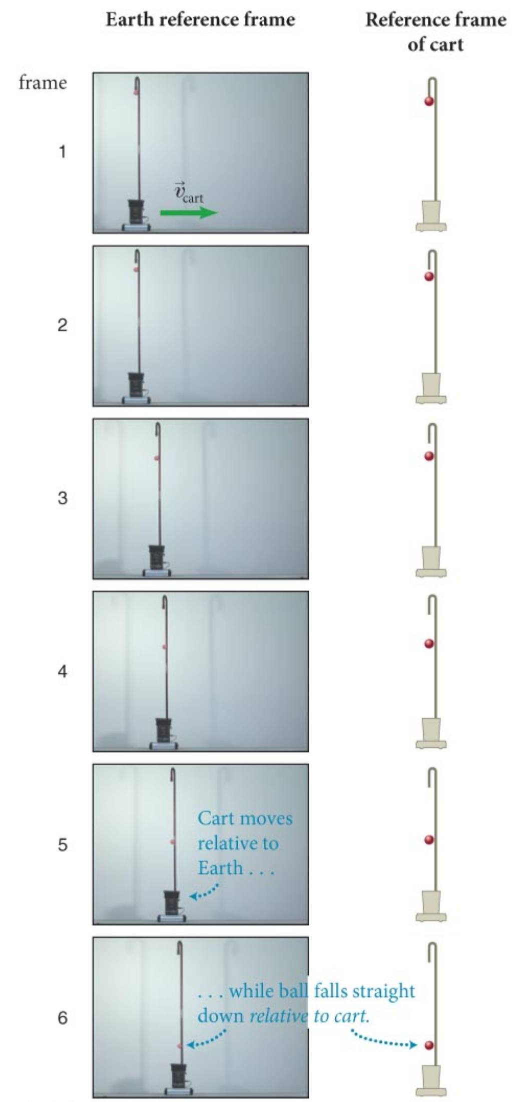
Section Goal

You will learn to

- Recognize that the trajectory followed by a free-falling object depends on the state of motion of the observer.

Section 10.1: Straight is a relative term

- To begin our discussion of motion in a plane, consider the film clip in the figure.
- The ball is dropped from a pole attached to a cart that is moving to the right at a constant speed.

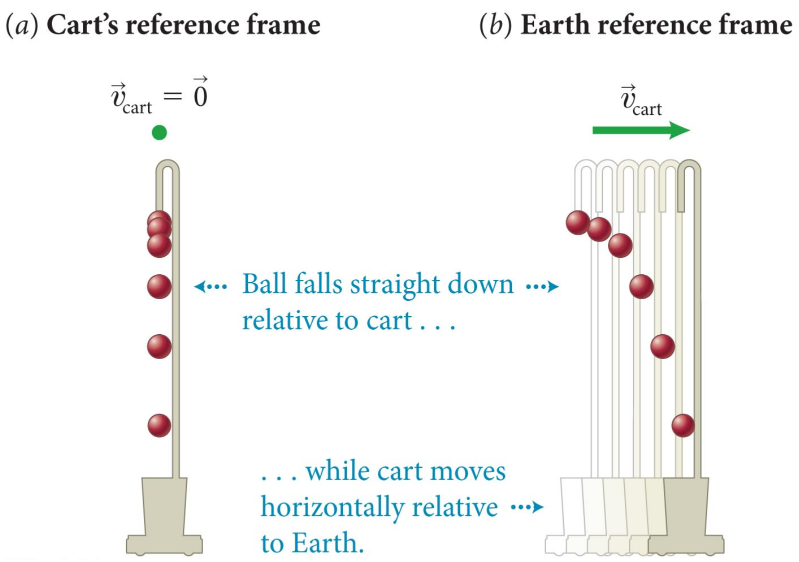


Video …

- https://www.youtube.com/watch?v=JCampZIwL5w

Section 10.1: Straight is a relative term

- The figure shows that
  - (*a*) The ball falls to the ground in a straight line if observed from the cart's reference frame.
  - (*b*) The ball has a horizontal displacement in addition to the straight downward motion when observed from Earth's reference frame.



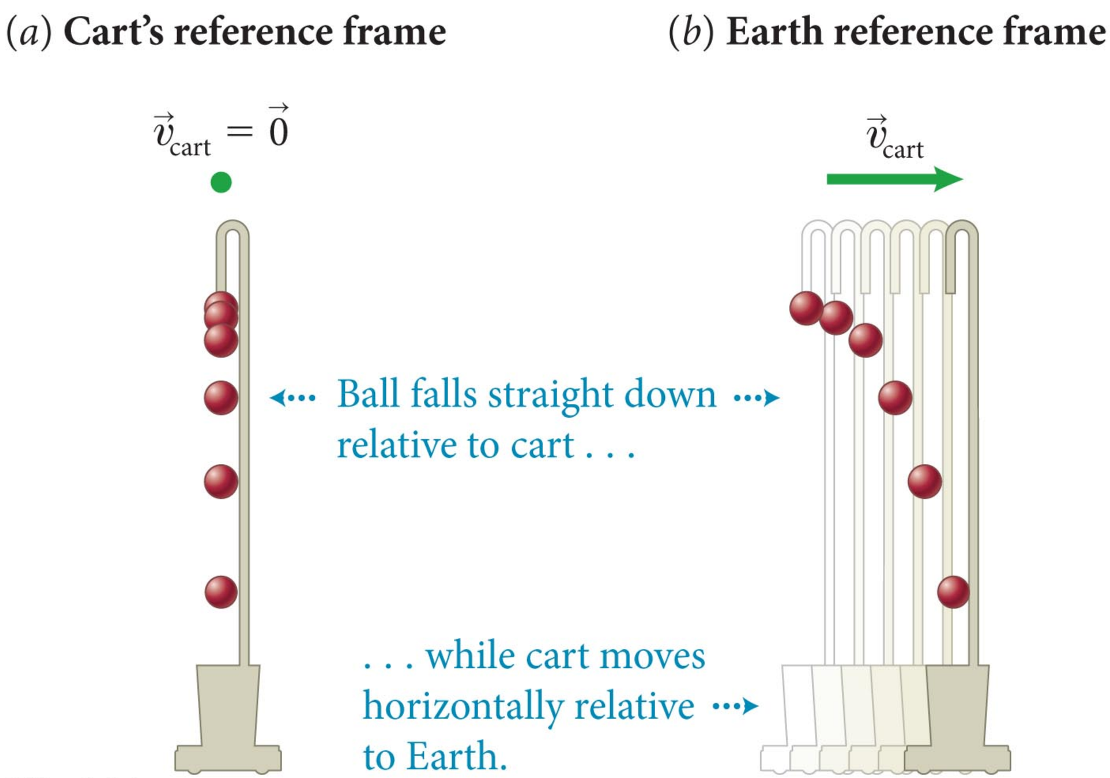
The figure shows us that the motion of the ball in Earth's reference frame can be broken down into two parts:

(a) Free fall in the vertical direction (only gravity acts)

(*b*) Constant velocity motion in the horizontal direction (follows cart traveling at constant velocity)

Checkpoint 10.1

10.1 (*a*) In Figure 10.2, what is the ball's velocity the instant before it is released? (*b*) Is the ball's speed in the reference frame of the cart greater than, equal to, or smaller than its speed in the Earth reference frame?



Checkpoint 10.1

10.1

In what reference frame? It depends …

Before release, $v = 0$ relative to cart (attached)

but in Earth's frame, velocity is v_{cart}

After release: ball now has a vertical component of velocity that the cart doesn't. Relative to the earth, its speed is higher.

Speed = vector magnitude of velocity

Section 10.1
Question 1

A passenger in a speeding train drops a peanut.

Which is greater?

1. The magnitude of the acceleration of the peanut as measured by the passenger.
2. The magnitude of the acceleration of the peanut as measured by a person standing next to the track.
3. Neither, the accelerations are the same to both observers.

Section 10.1
Question 1

A passenger in a speeding train drops a peanut.

Which is greater?

1. The magnitude of the acceleration of the peanut as measured by the passenger.
2. The magnitude of the acceleration of the peanut as measured by a person standing next to the track.
3. ✔ Neither, the accelerations are the same to both observers – same interaction

Section 10.2: Vectors in a plane

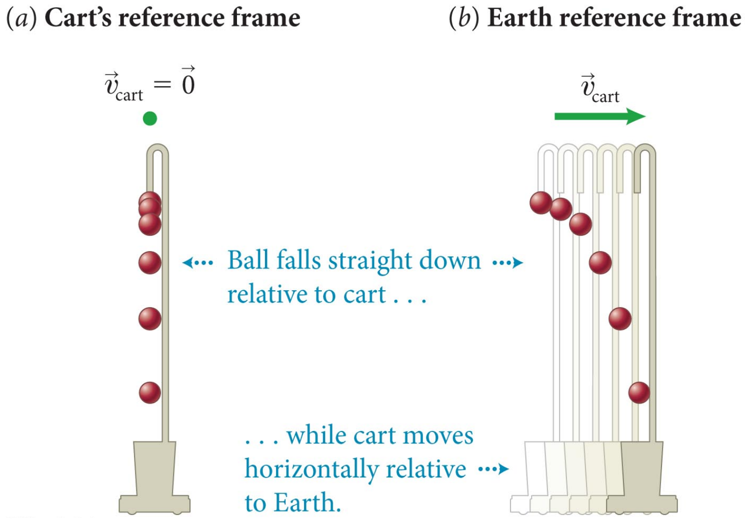
Section Goals

You will learn to

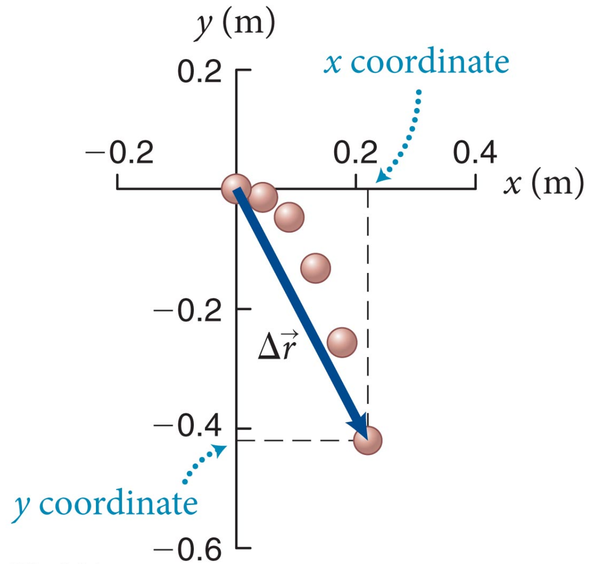
- Determine the sum and the difference of two vectors using a graphical method.
- Develop a procedure to compute the x and y components of a vector using a set of mutually perpendicular axes.
- Understand that the acceleration component parallel to the instantaneous velocity increases the speed of the object, and the perpendicular acceleration component changes the direction of the velocity.

Section 10.2: Vectors in a plane

- To analyze the motion of an object moving in a plane, we need to define two reference axes, as shown on the right.

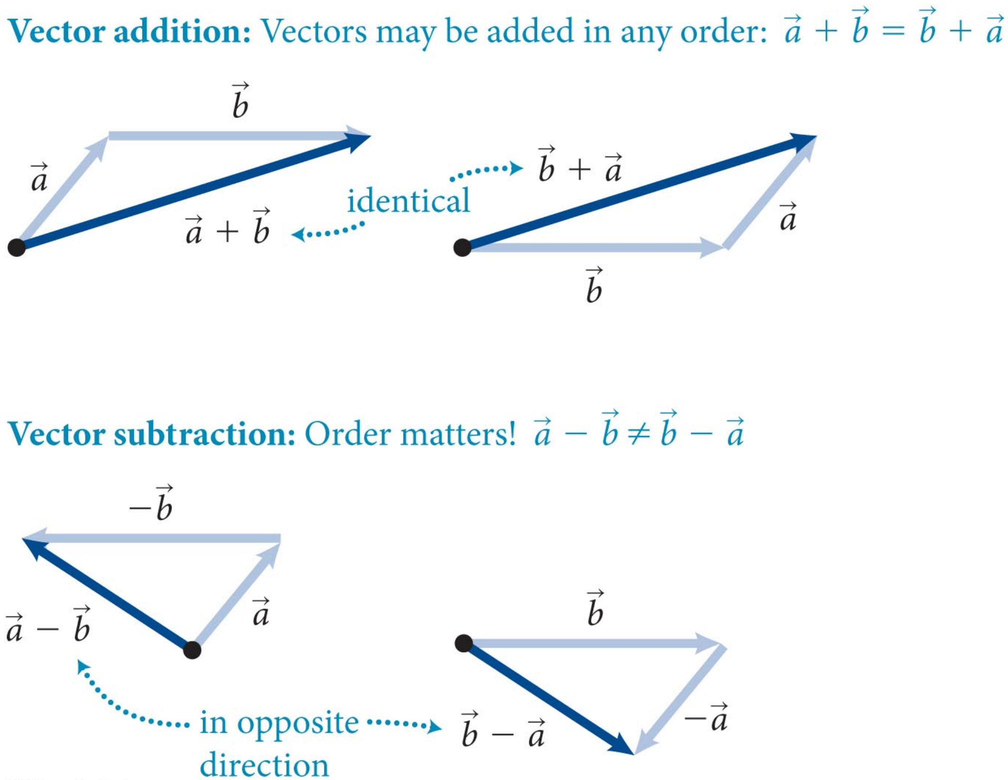


- We see from the figure at right that the ball's displacement in Earth's reference frame is the vector sum of the horizontal displacement $\Delta\vec{x}$ and the vertical displacement $\Delta\vec{y}$.



Section 10.2: Vectors in a plane

- The vector sum of two vectors in a plane is obtained by placing the tail of the second vector at the head of the first vector, as illustrated below.
- To subtract a vector $\vec{b}$ from a vector $\vec{a}$, reverse the direction $\vec{b}$ of and then add the reversed $\vec{b}$ to $\vec{a}$.



Section 10.2
Question 2

Is vector addition commutative? Is vector subtraction commutative?

1. Yes, yes
2. Yes, no
3. No, yes
4. No, no

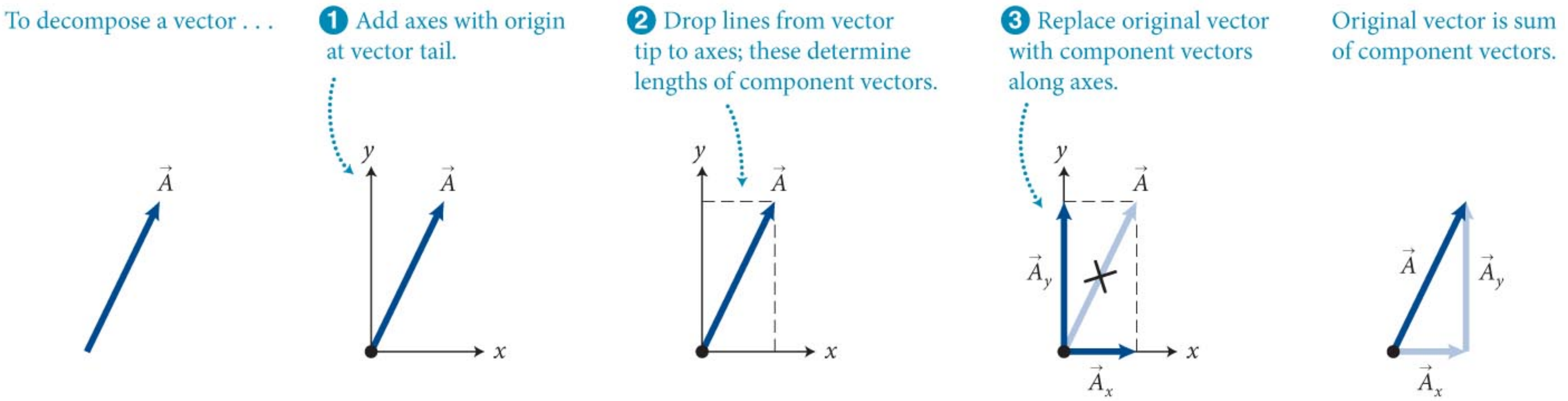
Section 10.2
Question 2

Is vector addition commutative? Is vector subtraction commutative?

1. Yes, yes
2. ✔ Yes, no – order matters for subtraction
3. No, yes
4. No, no

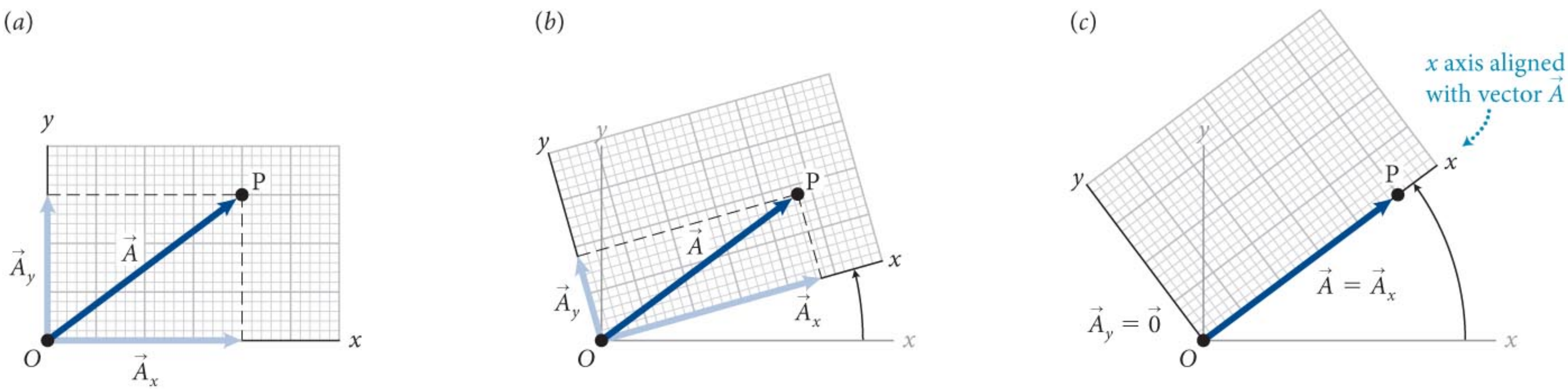
Section 10.2: Vectors in a plane

- Any vector $\vec{A}$ can be decomposed to **component vectors** $\vec{A}_x$ and $\vec{A}_y$ along the axes of some conveniently chosen set of mutually perpendicular axes, called a **rectangular coordinate system**.
- The procedure for decomposing a vector is shown below.



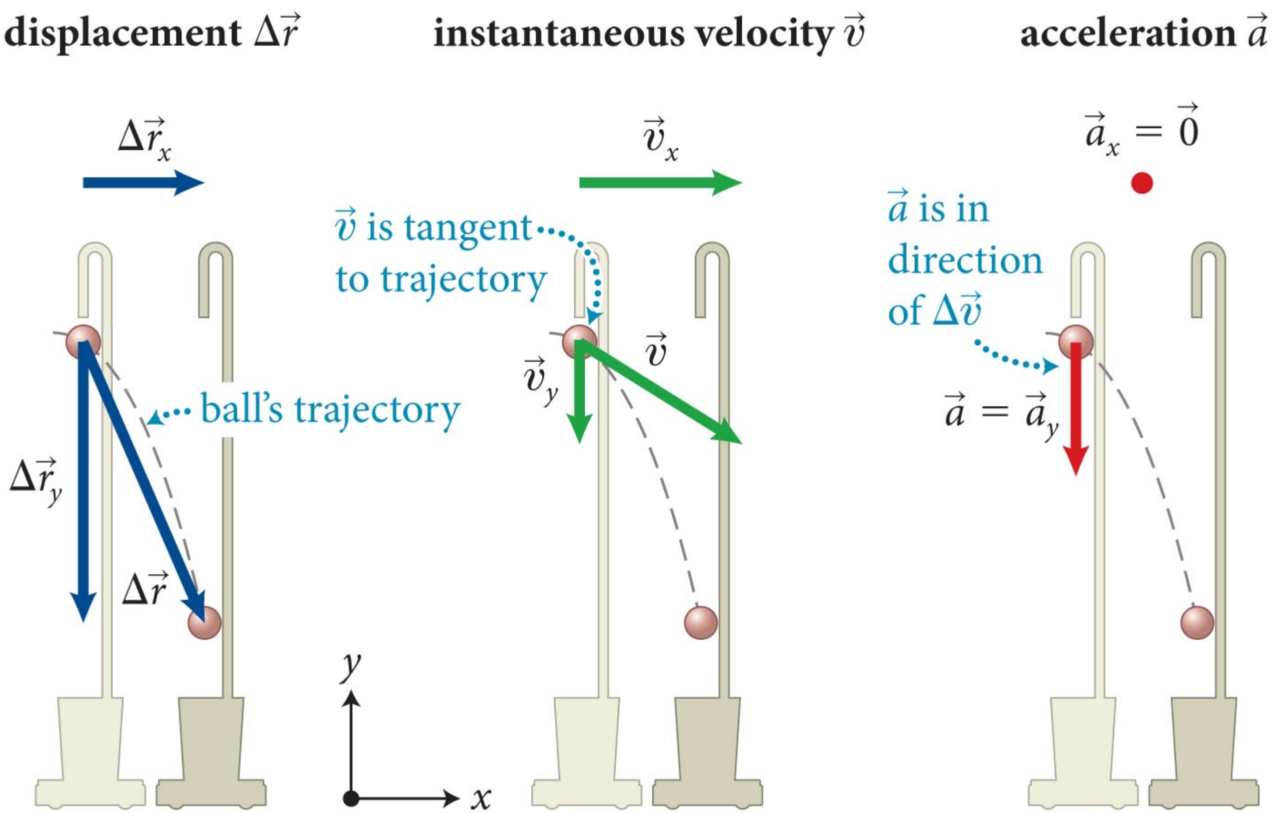
Section 10.2: Vectors in a plane

- The figure below shows the decomposition of a vector $\vec{A}$ in three coordinate systems.
- You need to choose the coordinate system that best suits the problem at hand.



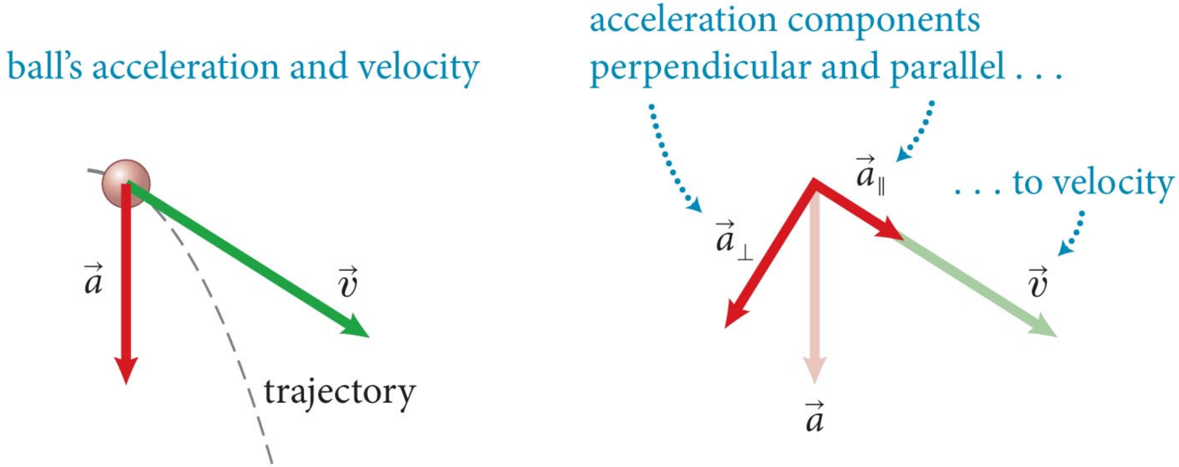
Section 10.2: Vectors in a plane

- The displacement, instantaneous velocity, and acceleration of the ball in the previous section is shown below.
- Notice that the instantaneous velocity and acceleration are not in the same direction. What does this mean?



Section 10.2: Vectors in a plane

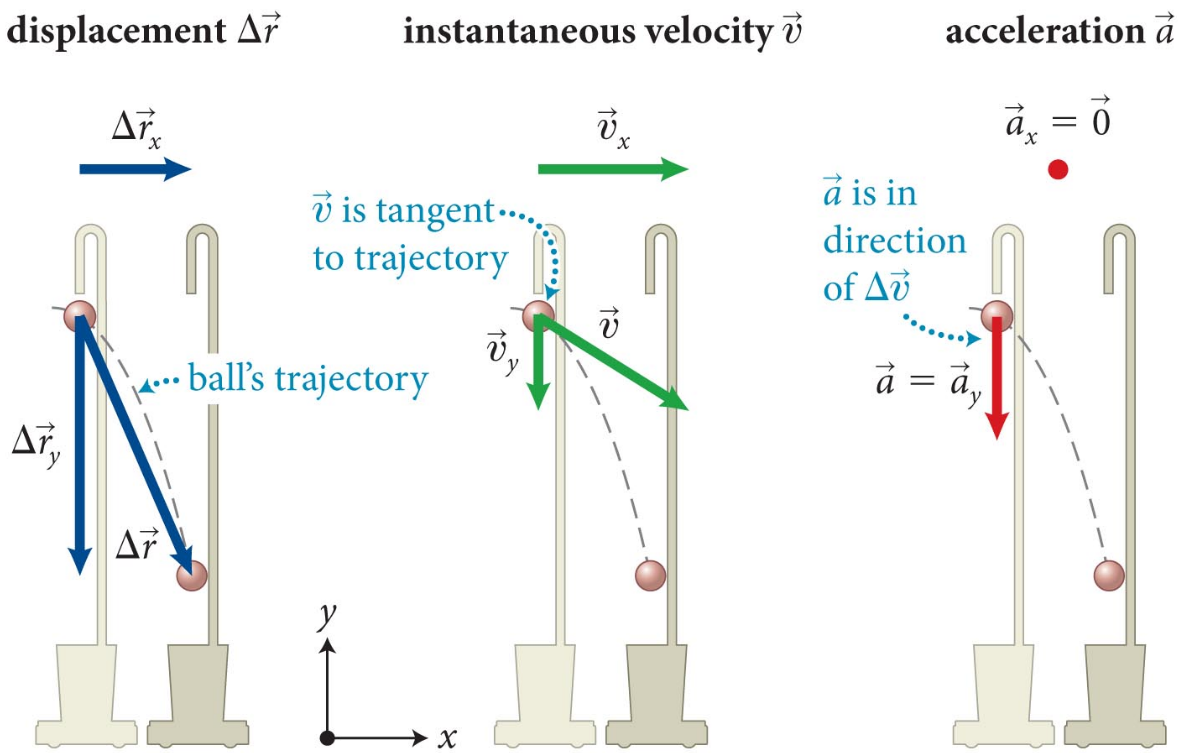
- To answer the question on the previous slide, we will decompose the acceleration vector into two components: one parallel to the instantaneous velocity, and one perpendicular to it.



- **In two-dimensional motion, the component of the acceleration parallel to the instantaneous velocity changes the speed; the component of acceleration perpendicular to the instantaneous velocity changes the direction of the velocity but not its magnitude.**

Checkpoint 10.2

10.2 In Figure 10.10, the ball's instantaneous velocity $\vec{v}$ does not point in the same direction as the displacement $\Delta\vec{r}$ (it points *above* the final position of the ball). Why?



Checkpoint 10.2

10.2

The ball's instantaneous velocity is the displacement over an *infinitesimal time interval*. It is tangential to the trajectory at any point.

The displacement vector is the *entire* net motion of the ball, pointing from initial to final positions.

(tangent vs secant)

Section 10.3: Decomposition of forces

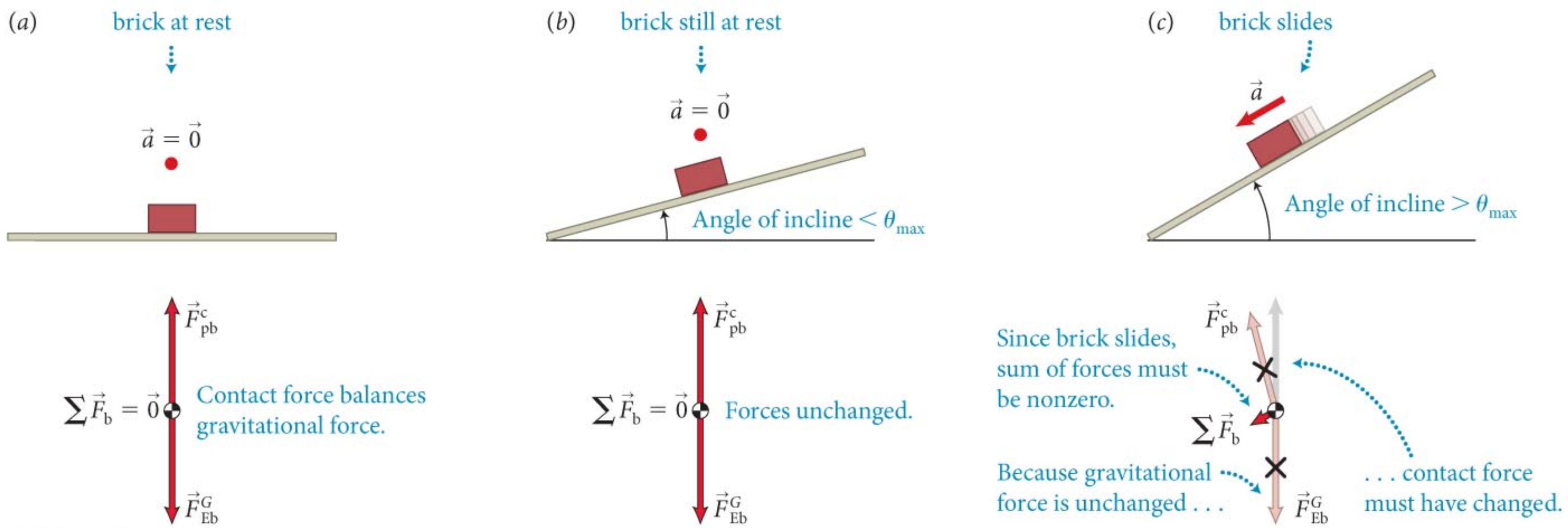
Section Goals

You will learn to

- Apply the vector decomposition technique to analyze the motion of a brick along an inclined surface.
- Realize that choosing a coordinate system such that one of the axes lies along the direction of acceleration of the object allows you to break the problem neatly into two parts.

Section 10.3: Decomposition of forces

- The figure shows a brick lying on a horizontal plank and then the plank is gently tilted.
- When the angle of incline exceeds a θ_{max} the brick accelerates down the incline.
- Then, the vector sum of the forces exerted on the brick must also point down the incline

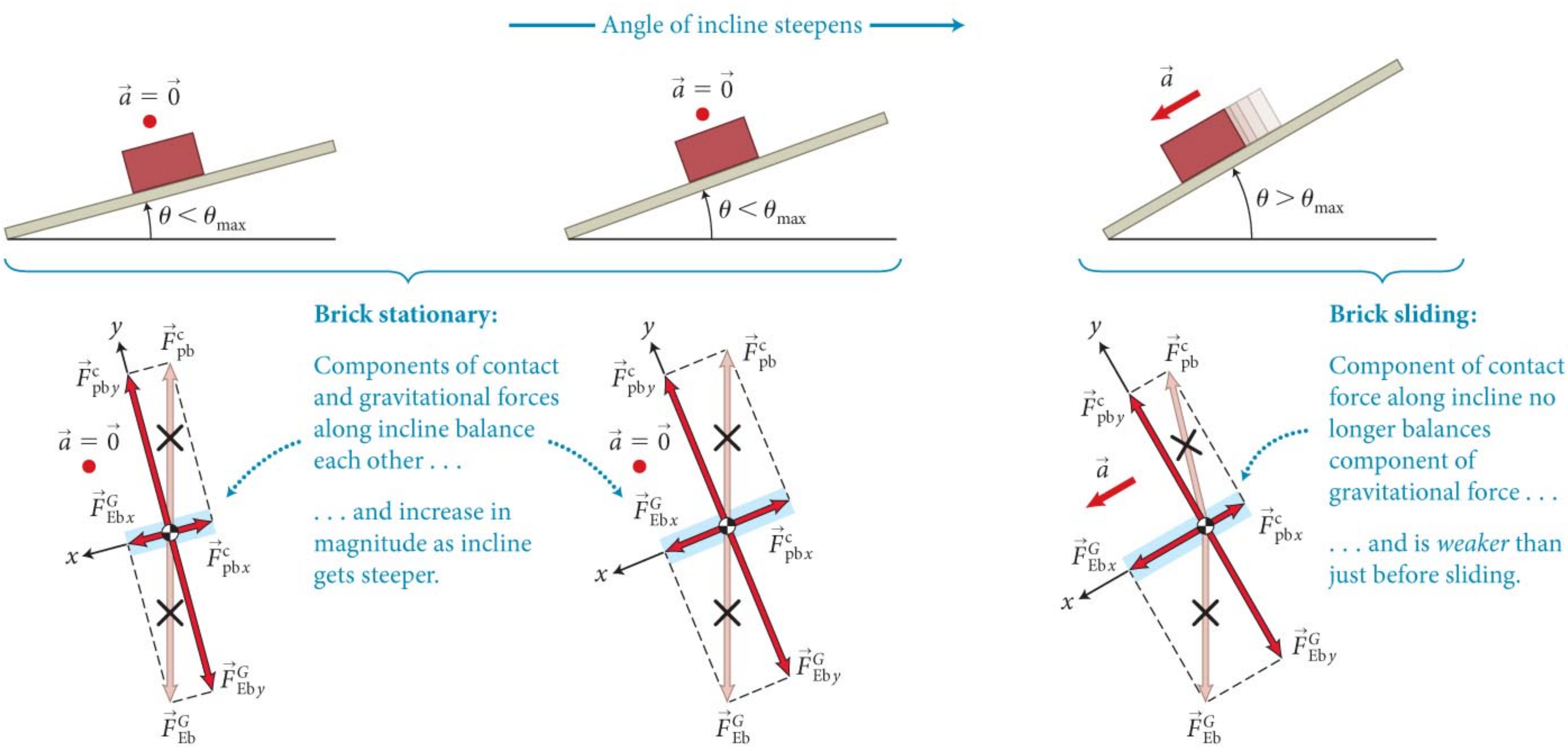


Section 10.3: Decomposition of forces

- Because the brick is constrained to move along the surface of the plank, it make sense to choose the x axis along surface
- The force components parallel to the surface are called **tangential components**.
- The force components perpendicular to the surface are called **normal components**. Normal components must cancel here!
- Contact force contains friction as tangential part

Section 10.3: Decomposition of forces

- Can resolve all forces into two components:
  - Component along the ramp
  - Component perpendicular to ramp

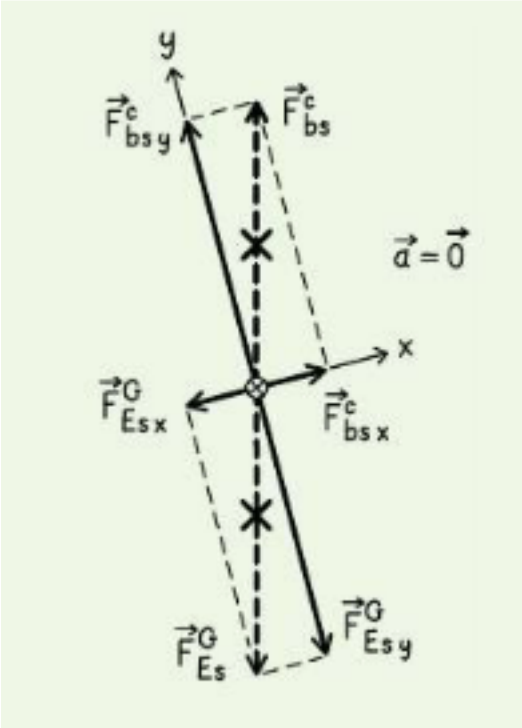


Checkpoint 10.3

10.3 A suitcase being loaded into an airplane moves at constant velocity on an inclined conveyor belt. Draw a free-body diagram for the suitcase as it moves up along with the belt. Show the normal and tangential components of the forces exerted on the suitcase.

Both vert & horz components

must sum to zero!



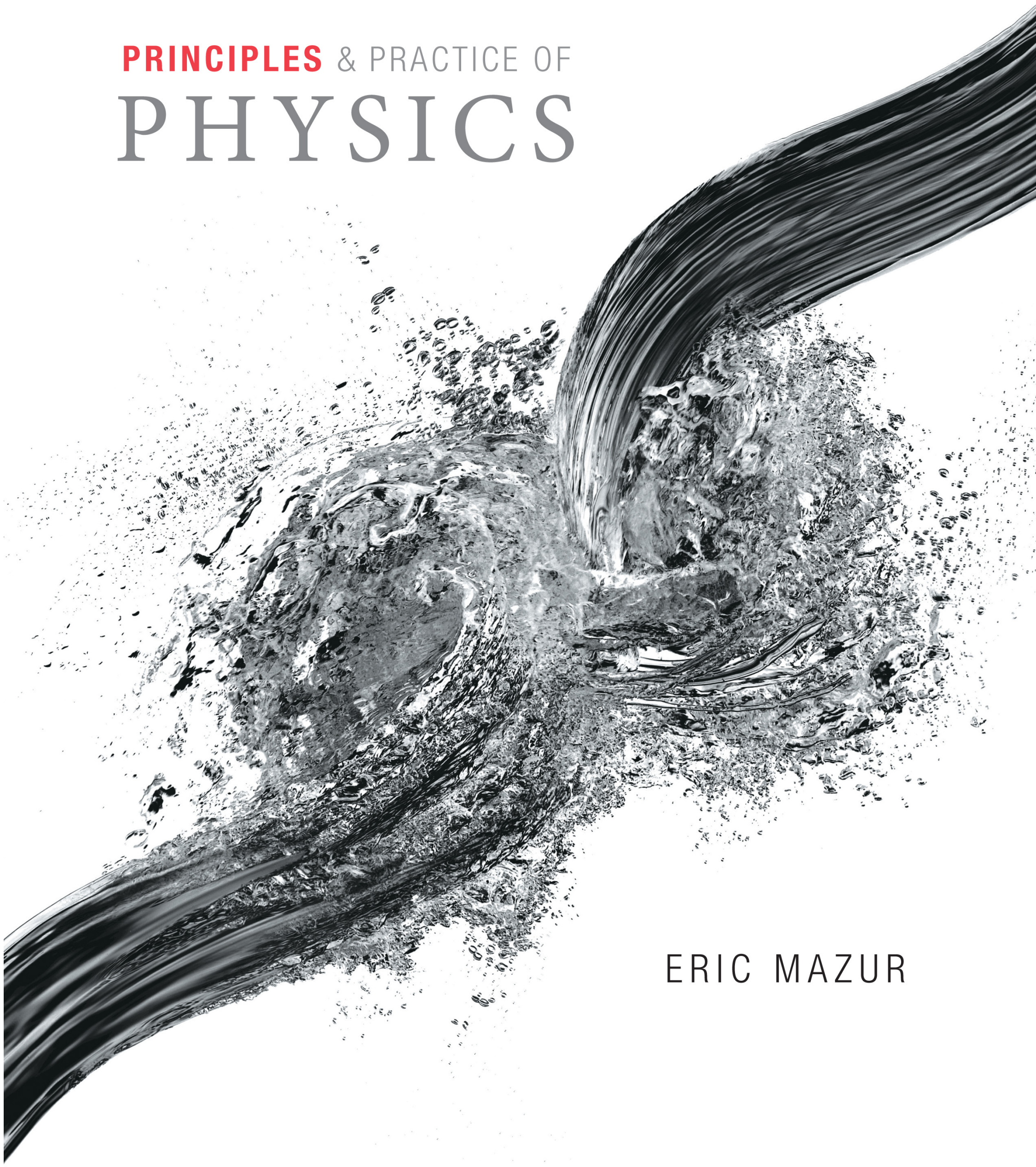
Section 10.3: Decomposition of forces

- The brick problem suggests which axes we should choose in a given problem:
  - **If possible, choose a coordinate system such that one of the axes lies along the direction of the acceleration of the object under consideration.**

- **Will it make a difference in the final answer?**
  - **No, but it can make things easier**

PRINCIPLES & PRACTICE OF

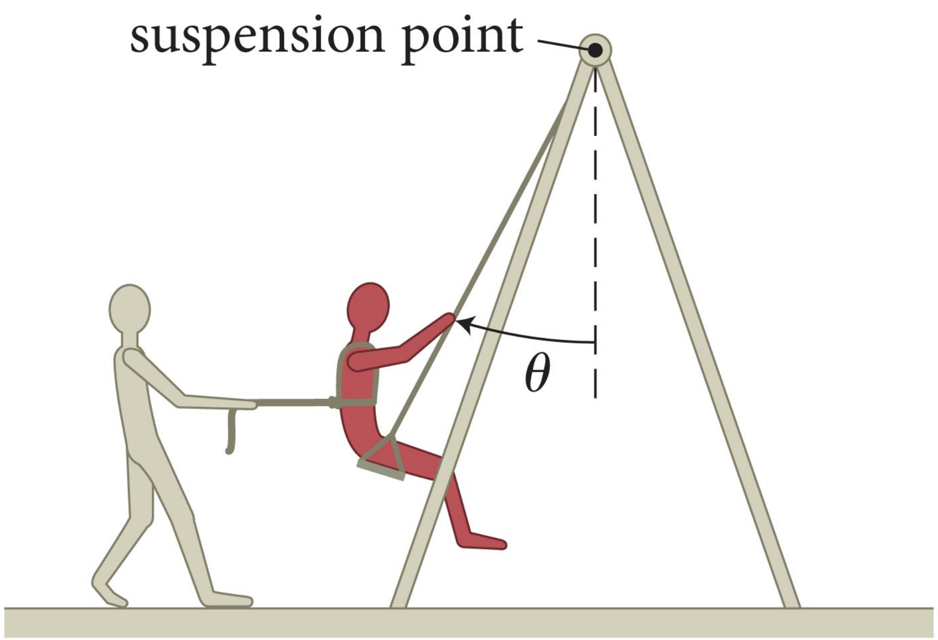
PHYSICS



ERIC MAZUR

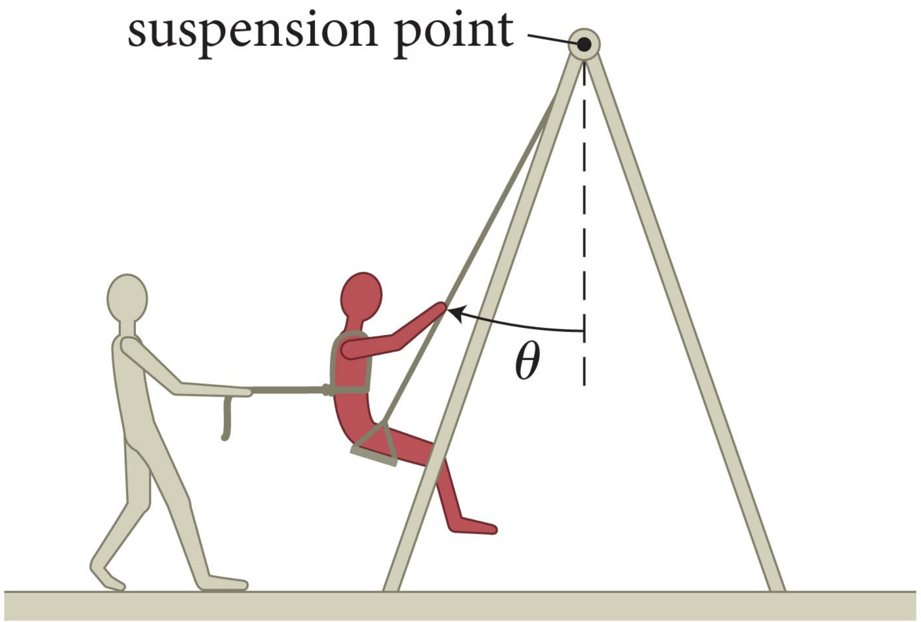
Example 10.2 Pulling a friend on a swing

Using a rope, you pull a friend sitting on a swing (Figure 10.16). (*a*) As you increase the angle θ, does the magnitude of the force $\vec{F}^{c}_{rp}$ required to hold your friend in place increase or decrease?



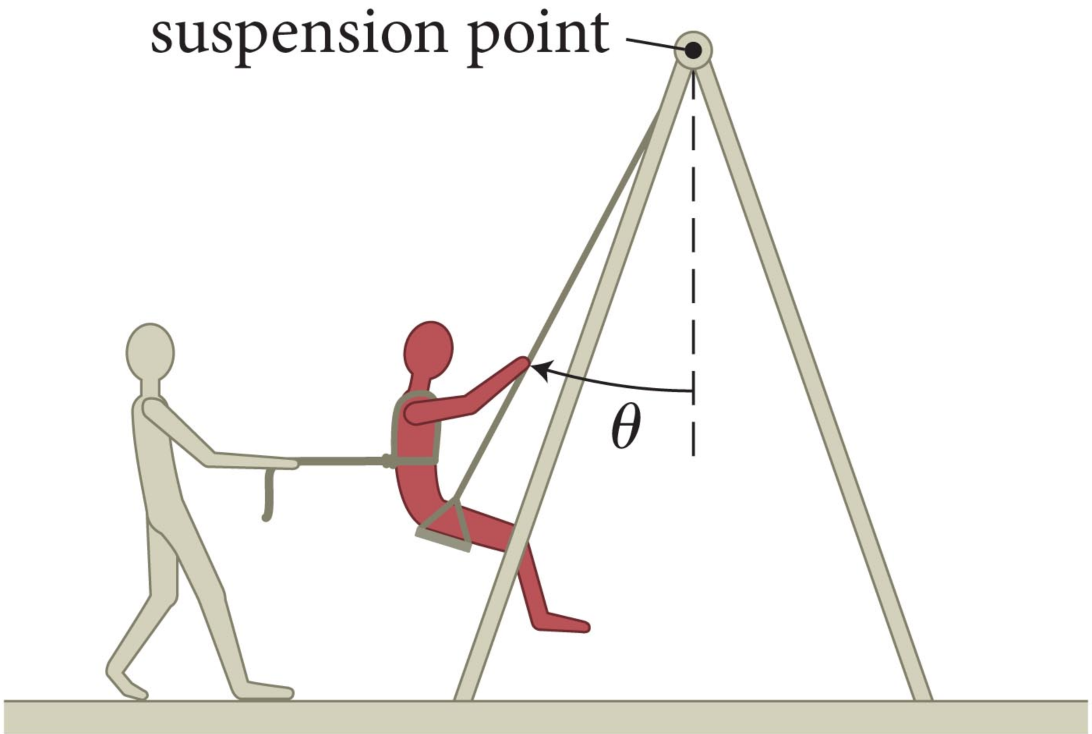
Example 10.2 Pulling a friend on a swing (cont.)

(*b*) Is the magnitude of that force larger than, equal to, or smaller than the magnitude of the gravitational force $\vec{F}^{G}_{\text{Ep}}$ exerted by Earth on your friend? (Consider the situation for both small and large values of θ.)



Example 10.2 Pulling a friend on a swing (cont.)

(*c*) Is the magnitude of the force $\vec{F}^{c}_{sp}$ exerted by the swing on your friend larger than, equal to, or smaller than F^{G}_{Ep}?



Example 10.2 Pulling a friend on a swing (cont.)

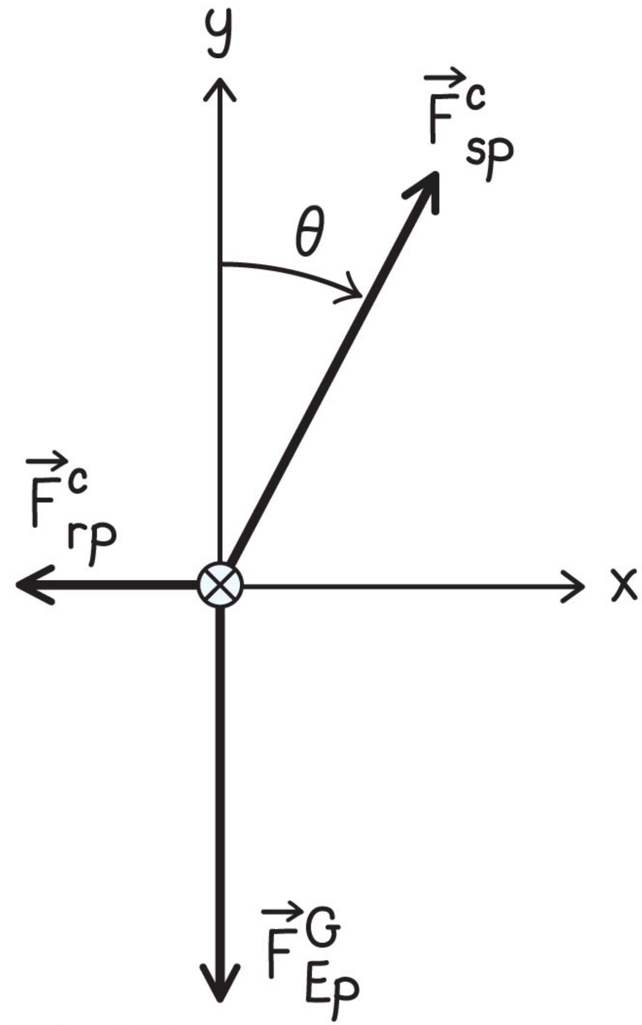
(*a*) As you increase the angle θ, does the magnitude of the force $\vec{F}^{c}_{rp}$ required to hold your friend in place increase or decrease?

(*b*) Is the magnitude of that force larger than, equal to, or smaller than the magnitude of the gravitational force $\vec{F}^{G}_{Ep}$ exerted by Earth on your friend?

(*c*) Is the magnitude of the force $\vec{F}^{c}_{sp}$ exerted by the swing on your friend larger than, equal to, or smaller than F^{G}_{Ep}?

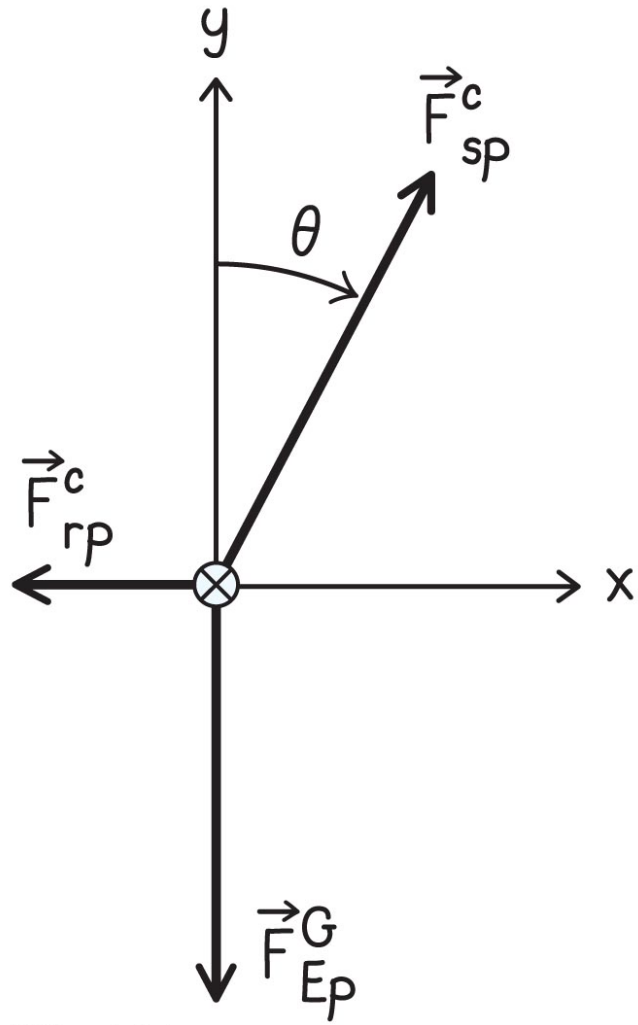
Example 10.2 Pulling a friend on a swing (cont.)

❶ GETTING STARTED I begin by drawing a free-body diagram of your friend (Figure 10.17). Three forces are exerted on him: $\vec{F}^{G}_{\text{Ep}}$, the force of gravity directed vertically downward, the horizontal force $\vec{F}^{c}_{\text{rp}}$ exerted by the rope, and a force $\vec{F}^{c}_{\text{sp}}$ exerted by the swing seat.



Example 10.2 Pulling a friend on a swing (cont.)

❶ GETTING STARTED This latter force $\vec{F}^{c}_{sp}$ is exerted by the suspension point via the chains of the swing and is thus directed along the chains. I therefore choose a horizontal *x* axis and a vertical *y* axis, so that two of the three forces lie along axes. Because your friend's acceleration is zero, the vector sum of the forces must be zero.

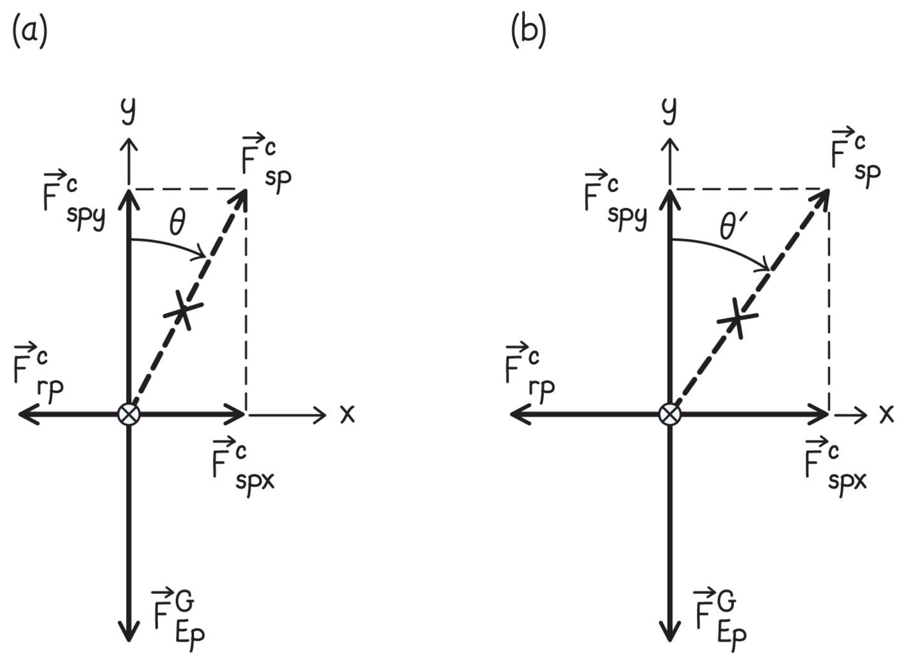


Example 10.2 Pulling a friend on a swing (cont.)

❷ DEVISE PLAN Because your friend is at rest, the vectors along the two axes must add up to zero. The best way to see how the magnitude of the force $\vec{F}^{c}_{rp}$ exerted by the rope must change as θ is increased is to draw free-body diagrams showing different values of θ. To answer parts *b* and *c*, I can compare the various forces in my free-body diagrams.

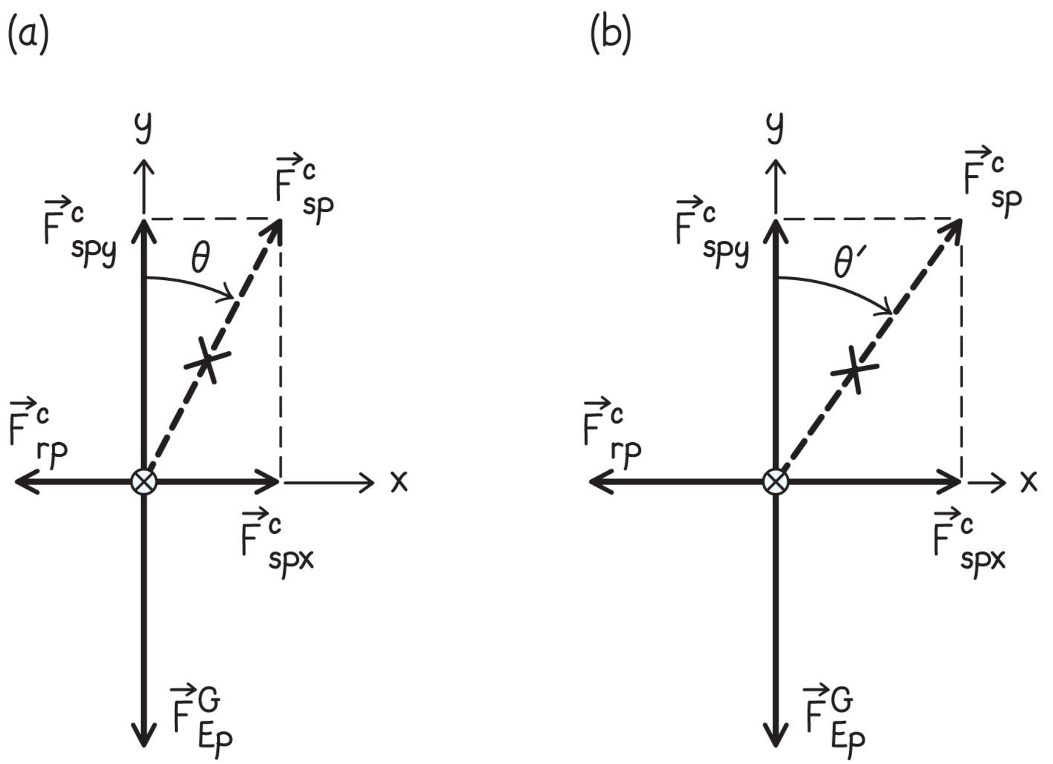
Example 10.2 Pulling a friend on a swing (cont.)

❸ EXECUTE PLAN (*a*) I begin by decomposing $\vec{F}^{c}_{sp}$ into *x* and *y* components (Figure 10.18*a*). Because the forces must add to zero along both axes, I conclude from my diagram that $\vec{F}^{c}_{sp\,y}$ must be equal in magnitude to $\vec{F}^{G}_{Ep}$, the downward force of gravity. Likewise, $\vec{F}^{c}_{sp\,x}$ must be equal in magnitude to $\vec{F}^{c}_{rp}$, the horizontal force the rope exerts on your friend.



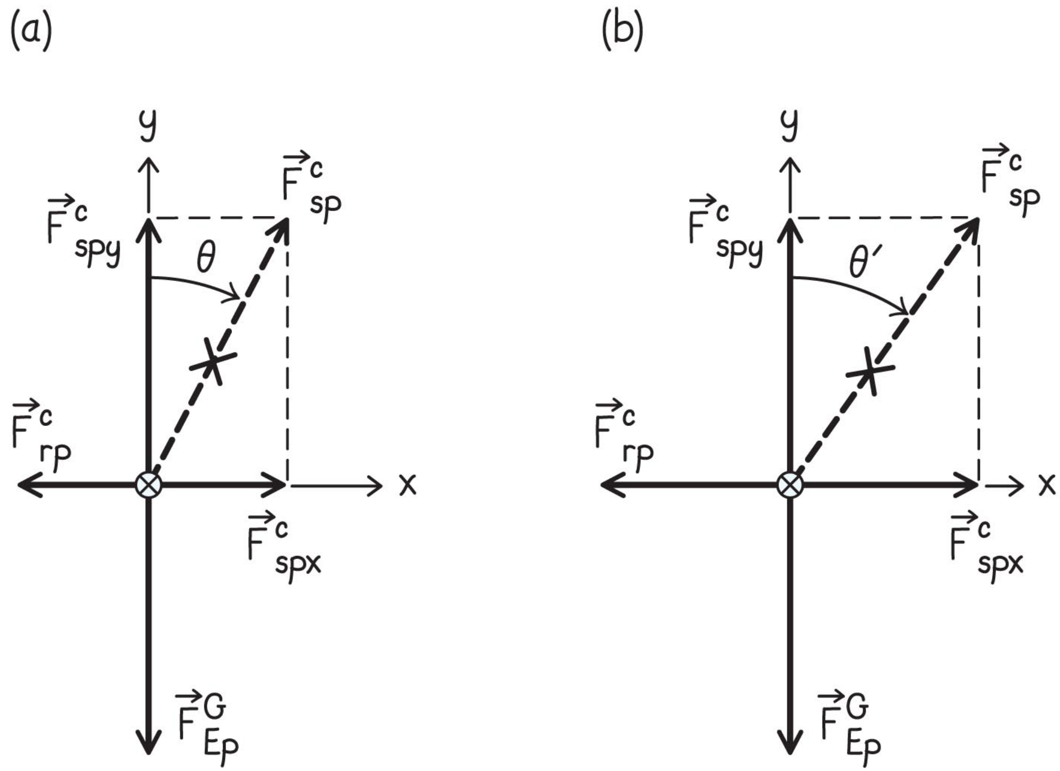
Example 10.2 Pulling a friend on a swing (cont.)

❸ EXECUTE PLAN Next, I draw a second free-body diagram for a larger angle θ (Figure 10.18*b*). As θ increases, $\vec{F}^{c}_{\text{sp}\,y}$ must remain equal in magnitude to $\vec{F}^{G}_{\text{Ep}}$ (otherwise your friend would accelerate vertically). As Figure 10.18*b* shows, increasing θ while keeping $\vec{F}^{c}_{\text{sp}\,y}$ constant requires the magnitude of $\vec{F}^{c}_{\text{sp}\,x}$ to increase.



Example 10.2 Pulling a friend on a swing (cont.)

❸ EXECUTE PLAN Because your friend is at rest, the forces in the horizontal direction must add up to zero and so $F^{c}_{rp} = \left| F^{c}_{sp\,x} \right|$. So if the magnitude of $\vec{F}^{c}_{sp\,x}$ increases, the magnitude of $\vec{F}^{c}_{rp}$ must increase, too. ✔



Example 10.2 Pulling a friend on a swing (cont.)

from the figure: $\tan\theta = \dfrac{|F^c_{sp,x}|}{|F^c_{sp,y}|}$

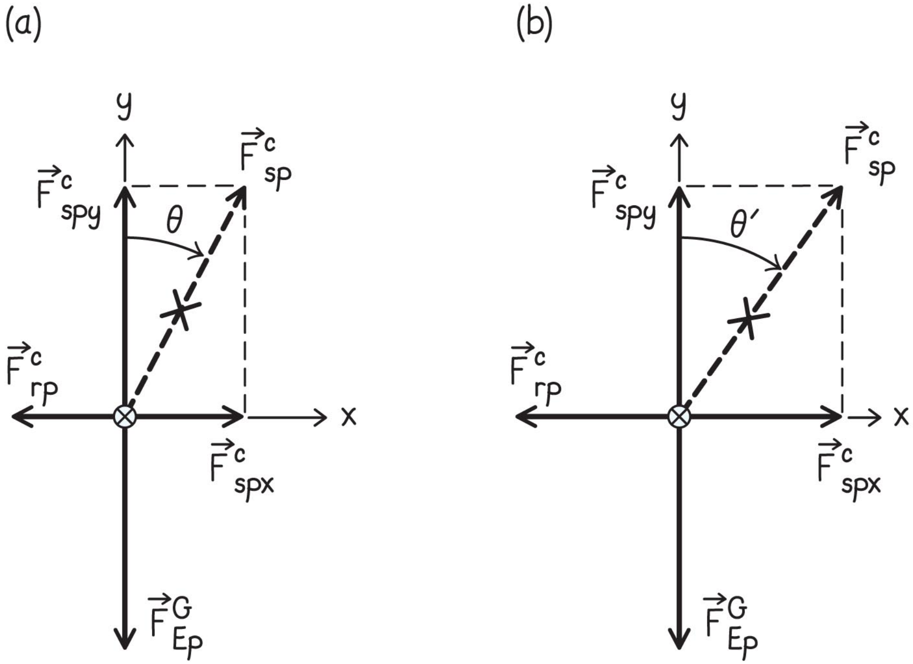
for $\theta < 45°$, $\tan\theta < 1$, so $|F^c_{sp,x}| < |F^c_{sp,y}|$

because $|F^c_{sp,y}| = F^G_{Ep}$ and $|F^c_{sp,x}| = F^c_{rp}$

that means for $\theta < 45°$ $F^c_{rp} < F^G_{Ep}$

when $\theta > 45°$, $\tan\theta > 1$ and thus

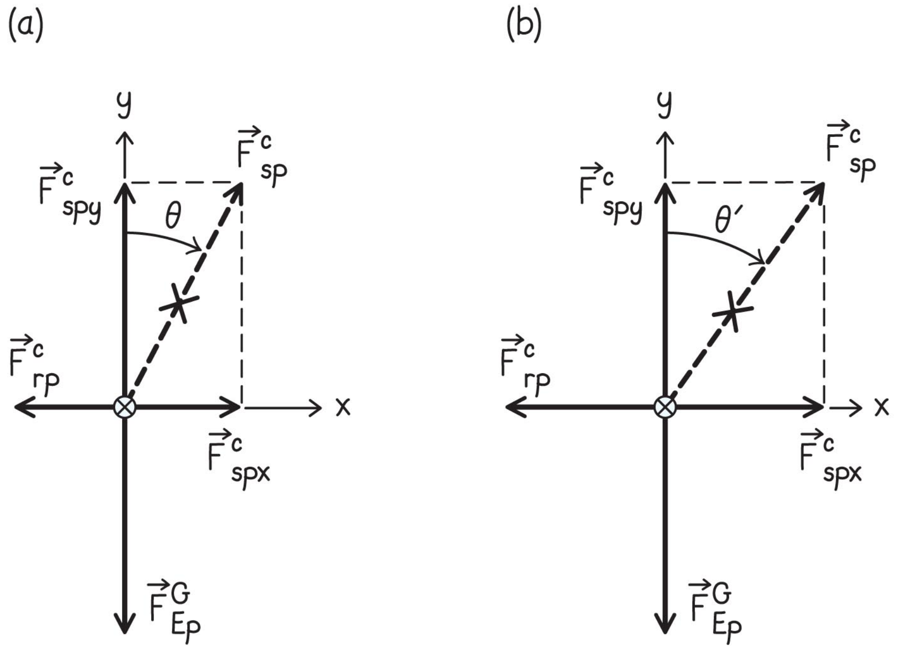
$|F^c_{sp,x}| > |F^c_{sp,y}|$ and $|F^c_{rp}| > |F^G_{Ep}|$



Example 10.2 Pulling a friend on a swing (cont.)

❸ EXECUTE PLAN (*c*) $\left|\vec{F}^{c}_{\text{sp}\,y}\right| = F^{G}_{\text{Ep}}$ and $F^{c}_{\text{sp}} = \sqrt{\left(F^{c}_{\text{sp}\,x}\right)^2 + \left(F^{c}_{\text{sp}\,y}\right)^2}$.

Therefore, F^{c}_{sp} must always be larger than F^{G}_{Ep} when $\theta \neq 0$. ✔



Since the y component of the force of the seat already equals the force of gravity, the *total* force of the seat must be always greater

Example 10.2 Pulling a friend on a swing (cont.)

❹ EVALUATE RESULT I know from experience that you have to pull harder to move a swing farther from its equilibrium position, and so my answer to part *a* makes sense. With regard to part *b*, when the swing is at rest at 45°, the forces $\vec{F}^{c}_{rp}$ and $\vec{F}^{G}_{Ep}$ on your friend make the same angle with the force $\vec{F}^{c}_{sp}$, and so $\vec{F}^{c}_{rp}$ and $\vec{F}^{G}_{Ep}$ should be equal in magnitude.

Section 10.3: Decomposition of forces

Example 10.2 Pulling a friend on a swing (cont.)

❹ EVALUATE RESULT The force of gravity is independent of the angle, but the force exerted by the rope increases with increasing angle, and so it makes sense that for angles larger than 45°, $\vec{F}^{c}_{rp}$ is larger than $\vec{F}^{G}_{Ep}$. In part *c*, because the vertical component of the force $\vec{F}^{c}_{sp}$ exerted by the seat on your friend always has to be equal to the force of gravity, adding a horizontal component makes $\vec{F}^{c}_{sp}$ larger than $\vec{F}^{G}_{Ep}$, as I found.