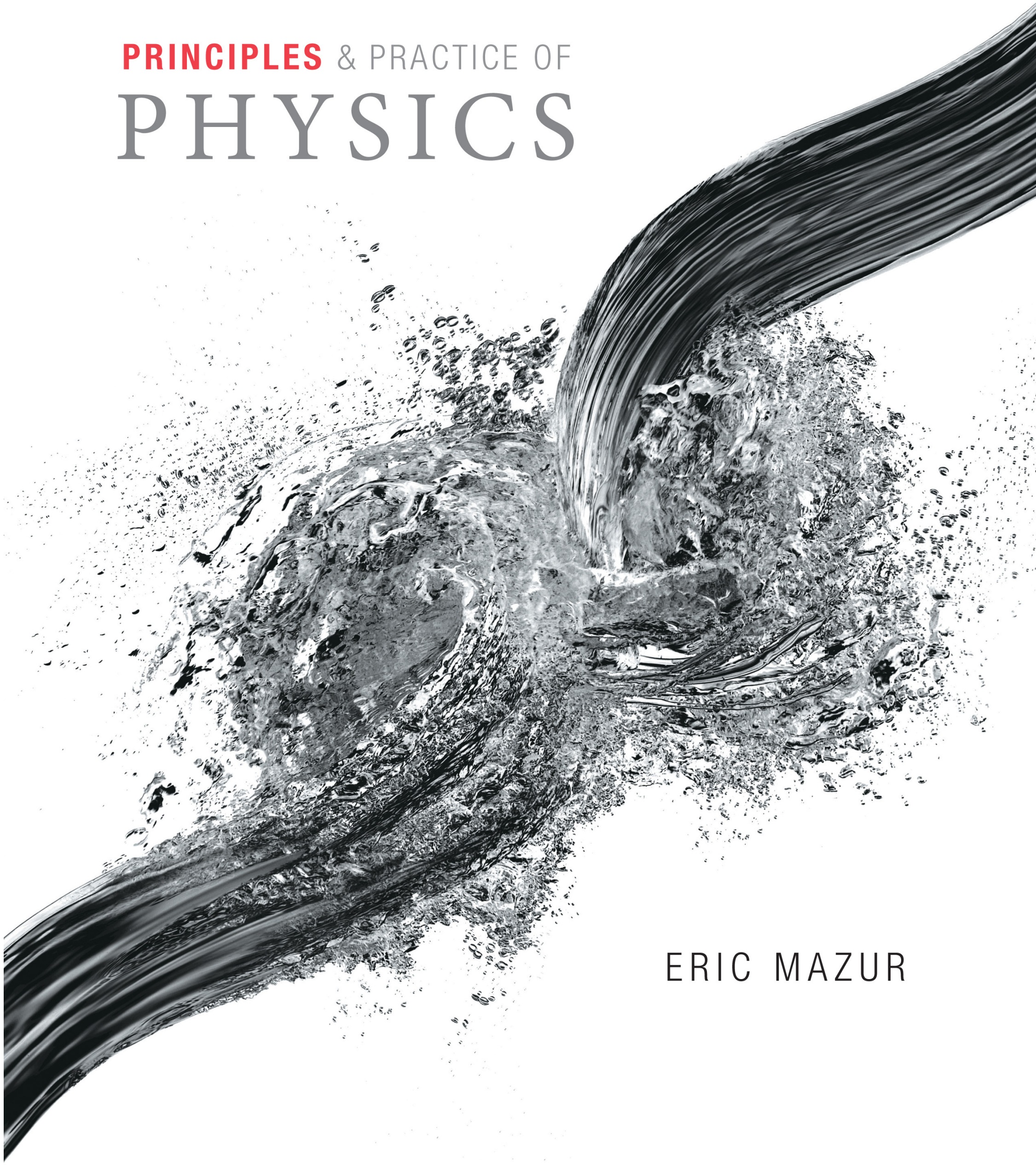
Chapter 19
Entropy



things

Final, previous exam solutions

- Exam 3 solution coming out this afternoon
- Exam 2 shortly as well

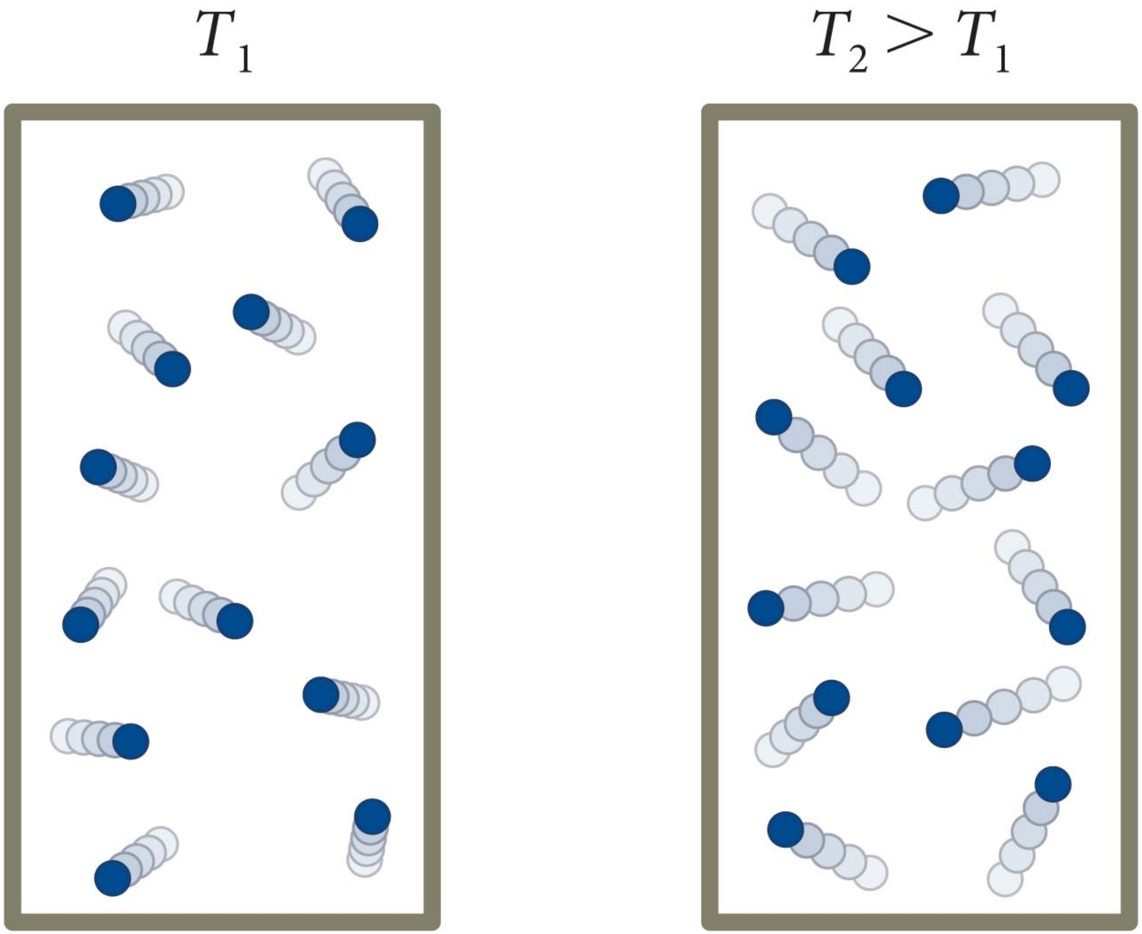
- Practice problems out

Section 19.7: Properties of a monatomic ideal gas

Section Goals

You will learn to

- Relate the **microscopic** and **macroscopic parameters** that describe an ideal gas.
- Derive the **ideal gas law** and the relationship between **thermal energy** and **temperature**.



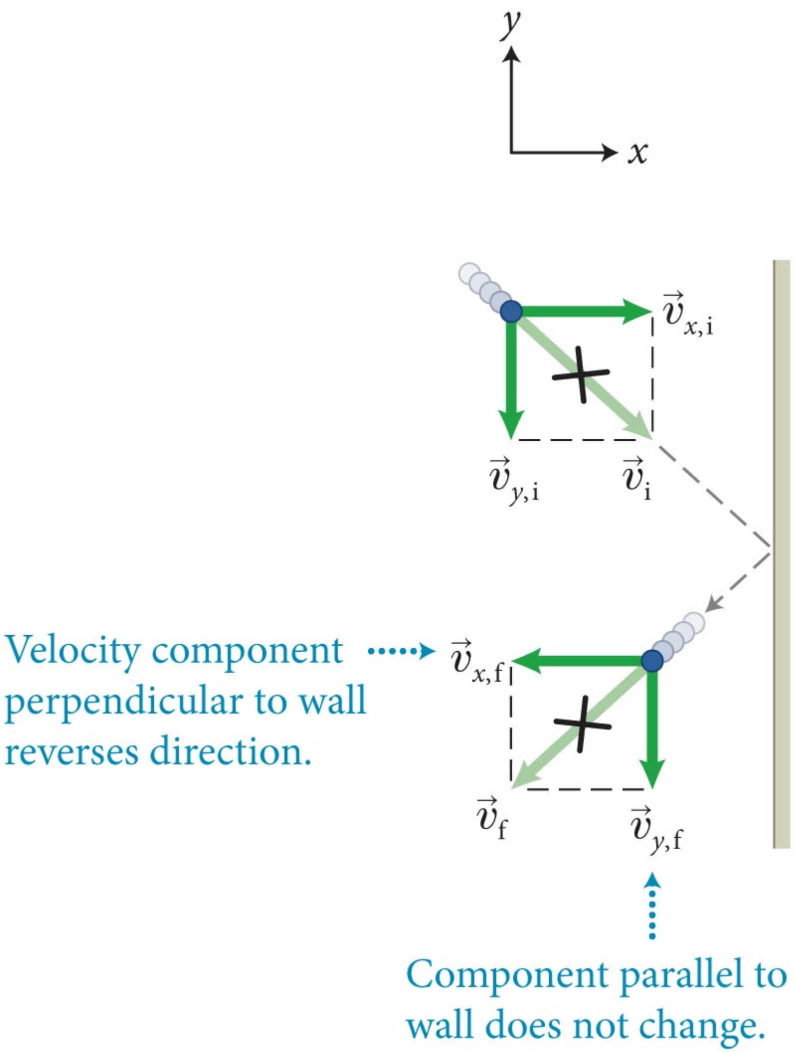
We assume you are aware of the ideal gas law

- $PV = NRT$

- Should have seen this in HS science (I hope)
- Crucial that T is in Kelvin

Section 19.7: Properties of a monatomic ideal gas

- We will now connect some of the atomic parameters that describe an ideal gas to its macroscopic parameters.
- Consider a monatomic gas that consists of a very large number N of indistinguishable atoms of mass m inside a closed rigid container of volume V.
- The figure shows one atom-wall collision.
- All collisions are elastic.



Velocity component perpendicular to wall reverses direction.

Component parallel to wall does not change.

Checkpoint 19.24

19.24 Let the system comprising the atoms and wall in Figure 19.25 be isolated.

(*a*) What is the direction of the atom's change in momentum?

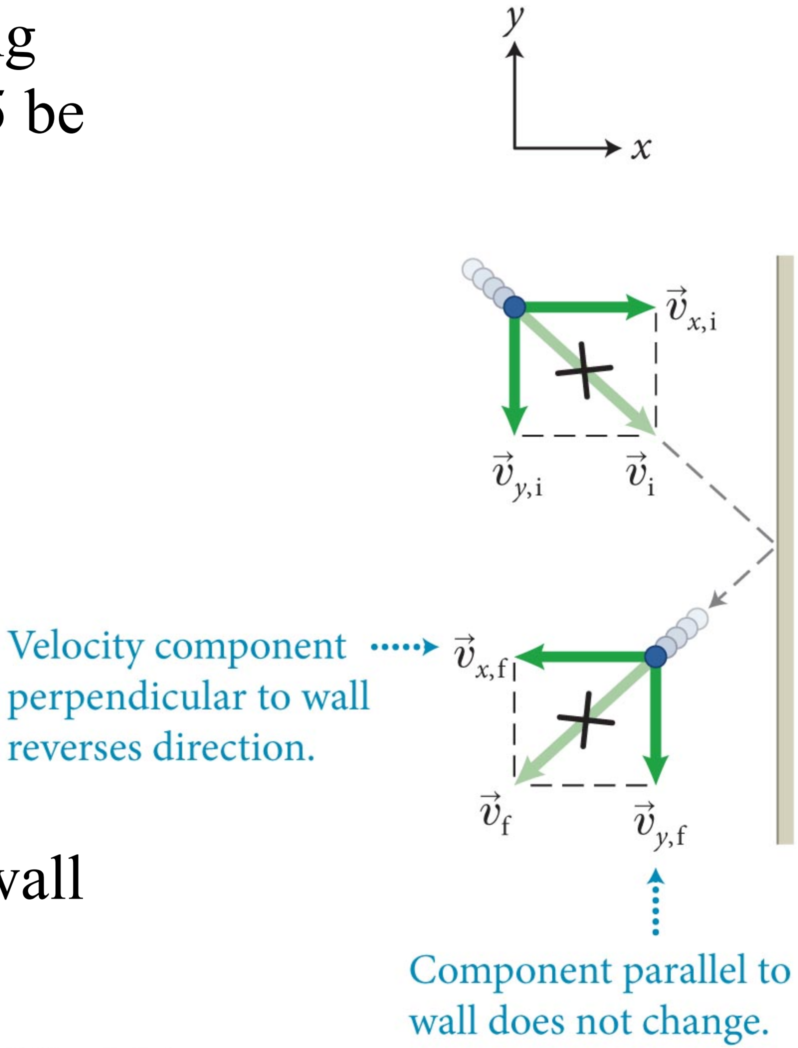
Perpendicular to wall, to left

(*b*) What is the direction of the change in momentum of the wall?

Opposite the atom

(*c*) How are these momentum changes related to the forces that wall and atom exert on each other?

Force is proportional!



Velocity component perpendicular to wall reverses direction.

Component parallel to wall does not change.

Section 19.7: Properties of a monatomic ideal gas

- If $\vec{F}^{\text{c}}_{\text{g}A}$ is the force exerted by the gas on the wall of surface area A, then using the definition of pressure we can write

$$P \equiv \frac{F^{\text{c}}_{\text{g}A}}{A} = \frac{F^{\text{c}}_{A\text{g}}}{A}$$

- The force exerted by the wall on the gas $\vec{F}^{\text{c}}_{A\text{g}}$ is equal to the rate of change of momentum of the gas.
- From Figure 19.25, we can determine that the magnitude of the atom's change in momentum is $2m|v_x|$.
- We need to determine how many collisions occur in time Δt.

Section 19.7: Properties of a monatomic ideal gas

- For the atom to collide with the wall in time Δt, it has to be within a distance $|v_x|\Delta t$.
- On average, only one half of the atoms in the cylinder of volume $A|v_x|\Delta t$ will strike the wall in time Δt. (Half are going the wrong way!)
- The number of atoms in the cylinder is

$$N_{\text{cyl}} = (N/V)A|v_x|\Delta t.$$

- The change in momentum for all gas collisions in time Δt is

$$|\Delta p| = \tfrac{1}{2}\frac{N}{V}(A|\upsilon_x|\Delta t)(2m|\upsilon_x|) = \frac{NAm\upsilon_x^2}{V}\Delta t$$

Section 19.7: Properties of a monatomic ideal gas

- Now we can obtain for the pressure in the gas:

$$P = \frac{|\Delta p| / \Delta t}{A} = \frac{Nm\upsilon_x^2}{V}$$

- Using $\upsilon_{\text{rms}}^2 = 3(\upsilon_x^2)_{\text{av}}$, (same v in all 3 directions on average) we get

$$P = \frac{Nm\frac{1}{3}\upsilon_{\text{rms}}^2}{V} = \frac{2}{3}\frac{N(\frac{1}{2}m\upsilon_{\text{rms}}^2)}{V}$$

- $N(\frac{1}{2}mv^2)$ is the thermally-acquired kinetic energy
- Now substituting

$$E_{\text{th}} = N\left(\frac{1}{2}m\upsilon_{\text{rms}}^2\right),$$

$$P = \frac{2}{3}\frac{E_{\text{th}}}{V} \quad \text{(monatomic ideal gas)}$$

- Thermal energy?
  - Long story short: an object gets ½$k_B T$ worth of thermal energy from its surroundings on average *per degree of freedom*
  - Point particle: free to move in 3 directions
  - Molecule? Add possible rotations too
  - Aside from details, $E_{th} \sim T$ *defines* temperature

- So: for N single atoms in 3 dimensions, average thermal energy is known:

$$E_{th} = \tfrac{3}{2} N k_B T \quad \text{(monatomic ideal gas)}$$

Section 19.7: Properties of a monatomic ideal gas

- From equations 19.50 and 19.48 we obtain the **ideal gas law**:

$$P = \frac{N}{V} k_B T \quad \text{(ideal gas system at equilibrium)}$$

- Because thermal energy of a gas is the sum of the average kinetic energies of its atoms ($E_{th} = NK$av), we can write

$$K_{av} = \tfrac{1}{2} m\upsilon_{rms}^2 = \tfrac{3}{2} k_B T$$

- The equation allows us to express the rms speed of an atom in a monatomic gas:

$$\upsilon_{rms} = \sqrt{\frac{3k_B T}{m}}$$

- Helium at room temperature: ~1400 m/s

Checkpoint 19.26

19.26 A certain number of helium atoms are placed in one container, and an equal number of argon atoms are placed in a separate identical container. The pressure is the same in both containers. Helium atoms have a smaller mass than argon atoms. For which gas, if either, (*a*) is the thermal energy smaller, (*b*) is the average kinetic energy of an atom smaller, (*c*) is the rms speed of an atom smaller?

(a) Neither – P & V are the same, so T is the same, and then so is thermal energy.

(b) Same number of atoms, same thermal energy, so same per atom

(c) Argon – same KE, but it is heavier

Problem 19.6 (bag of chips)

A bag of potato chips contains 2.00 L of air when it is sealed at sea level at a pressure of 1.00 atm and a temperature of 20.0°C.

What will be the volume of the air in the bag if you take it with you, still sealed, to the mountains where the temperature is 7.00°C and atmospheric pressure is 70.0 kPa?

Assume that the bag behaves like a balloon and that the air in the bag is in thermal equilibrium with the outside air. (1 atm = 1.01×10^5 Pa)

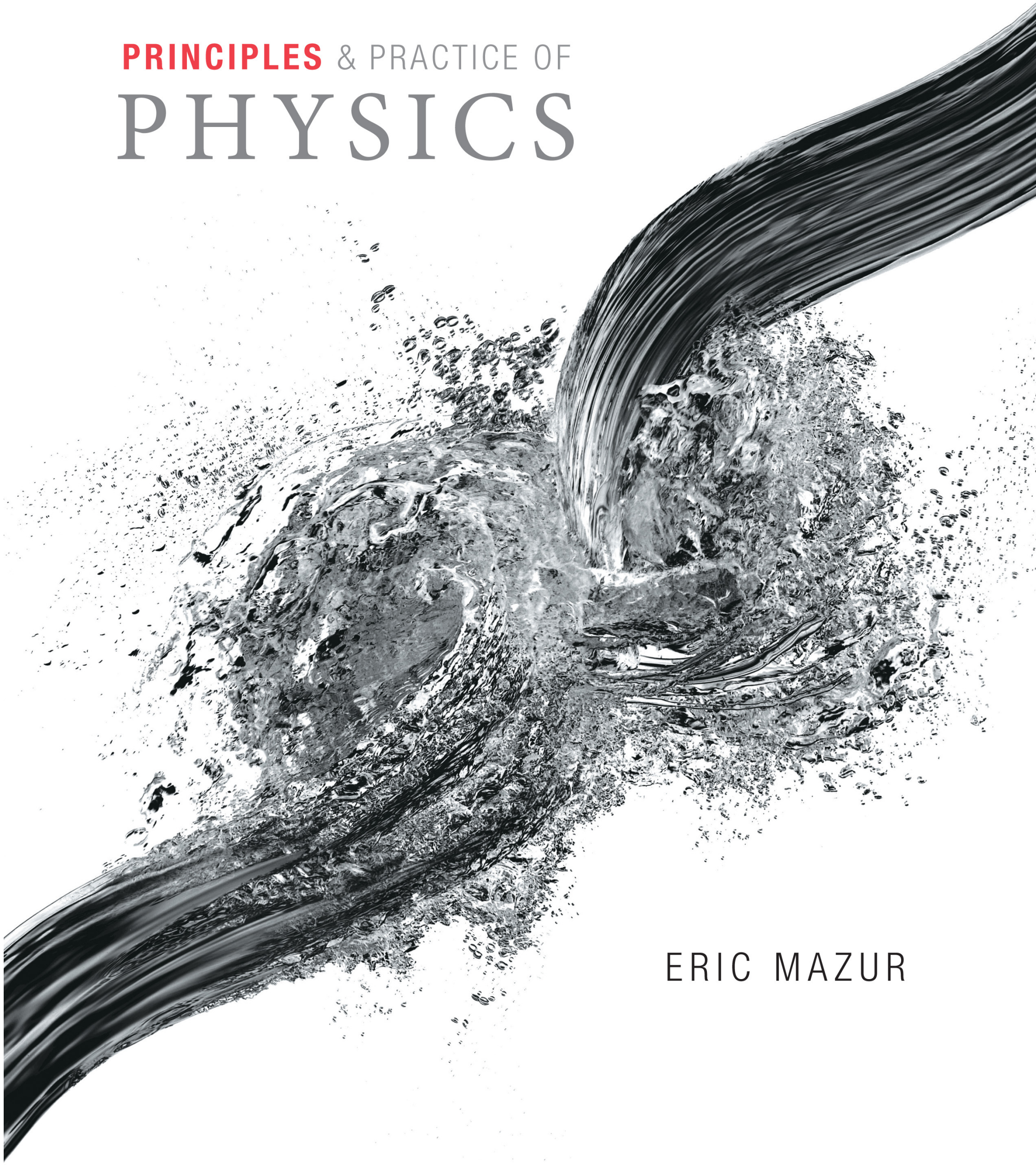
Problem 19.6

Number of atoms is constant, so solve idea gas equation for that. The resulting quantity is the same initially and finally. That's basically it.

$$nR = \text{const} = \frac{pV}{T} \qquad \Longrightarrow \qquad \frac{p_i V_i}{T_i} = \frac{p_f V_f}{T_f}$$

Watch the units. Always use K for temperature.

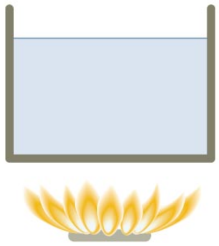
Chapter 20
Energy Transferred Thermally



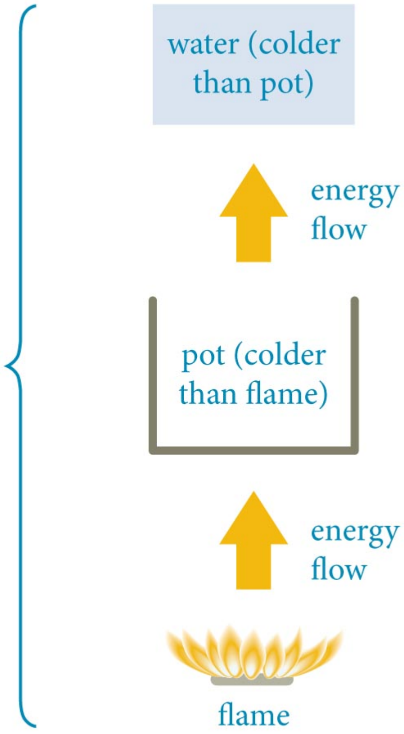
Section 20.1: Thermal interactions

- When two objects that are not at the same temperature are in contact with each other, energy is transferred from the higher-temperature (“hotter”) object to the lower-temperature (“cooler”) object.
- We attribute this transfer of energy to a **thermal interaction** between the objects.
- As the lower-temperature object gets hotter, its thermal energy increases.

(*a*) Flame heats pot of water

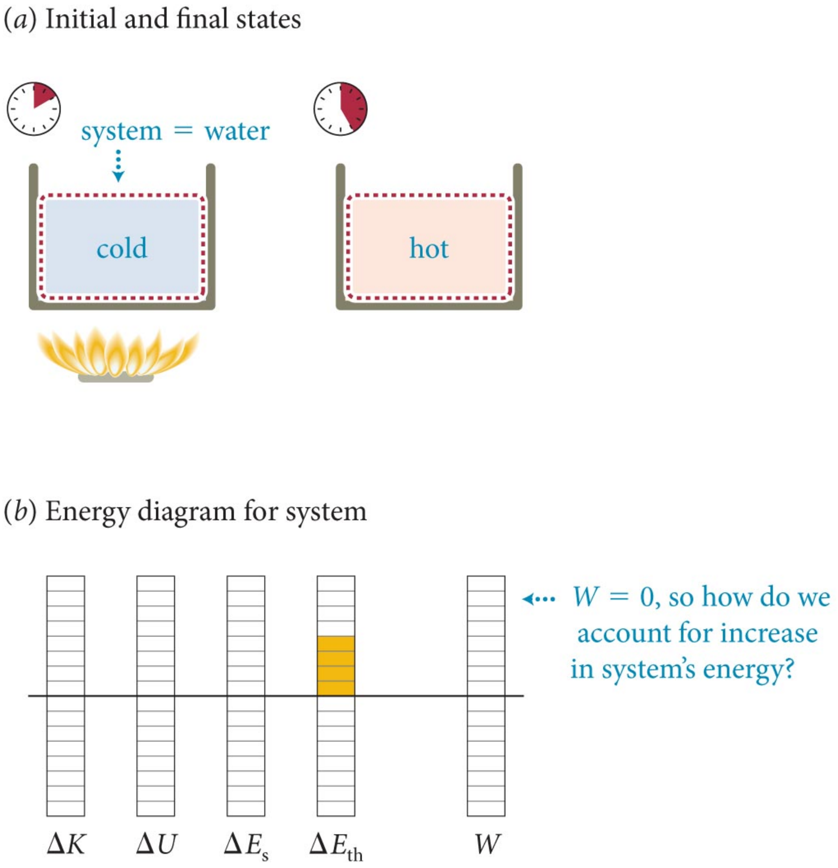


(*b*) Flow of energy from higher to lower temperature



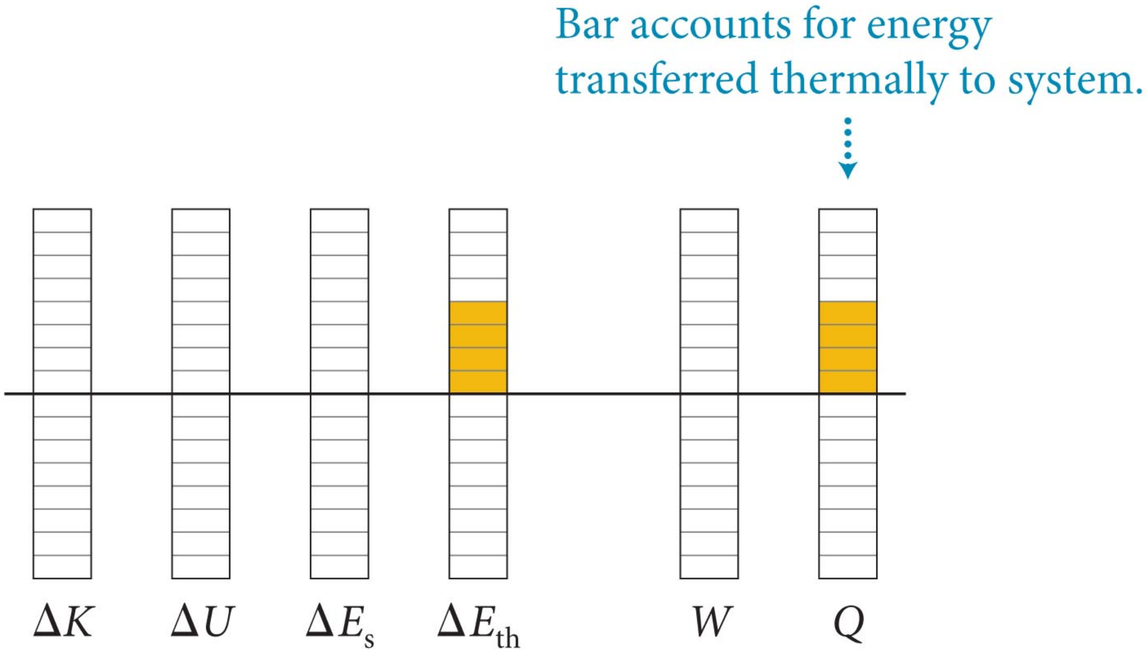
Section 20.1: Thermal interactions

- Consider the water in the figure as our system.
- Because there is no work done on the system, the energy diagram for the system is incomplete.
- To complete the energy diagram, we need to account for the energy transfer due to the thermal interaction.



Section 20.1: Thermal interactions

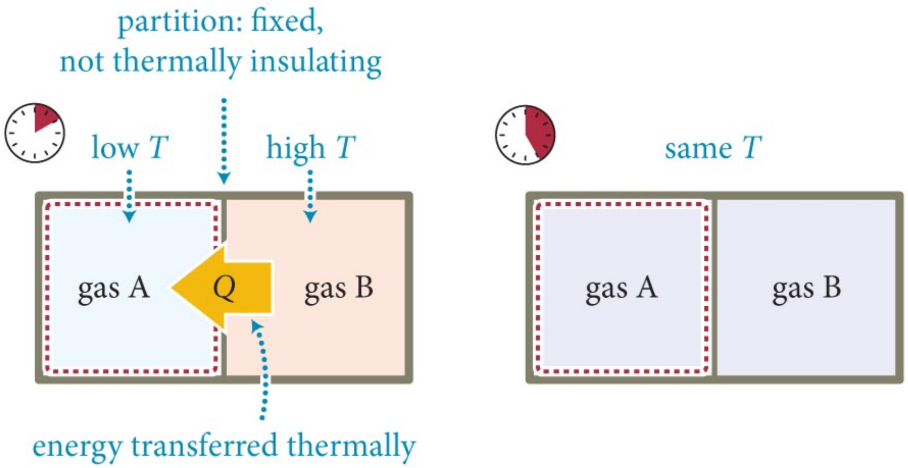
- The amount of energy transferred thermally is denoted by the letter Q.
- Q is generally referred to as *heat*.
- Calling it heat is misleading, it incorrectly suggests that heat is a property of an object. It is just another form of energy, ultimately kinetic.
- Better: use the term **energy transferred thermally** to refer to Q.


Bar accounts for energy transferred thermally to system.

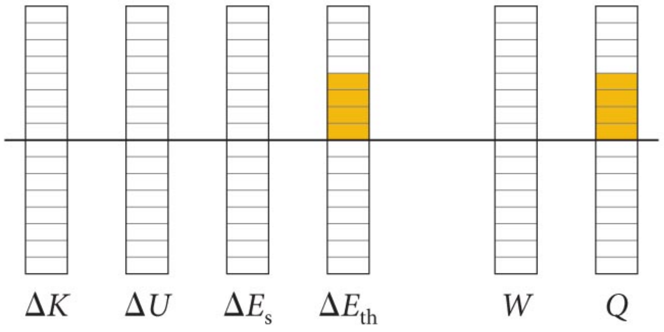
Section 20.1: Thermal interactions

- Consider the situation shown in the figure.
- The gasses interact thermally, and energy is transferred from B to A.
- If energy is transferred into the system, *Q* is positive.
- If energy is transferred out of the system, *Q* is negative.
- Energy will be transferred until an equilibrium *T* is reached
- Even then, energy back & forth, but at equal rates

(*a*) Gases at different temperatures are placed in thermal contact

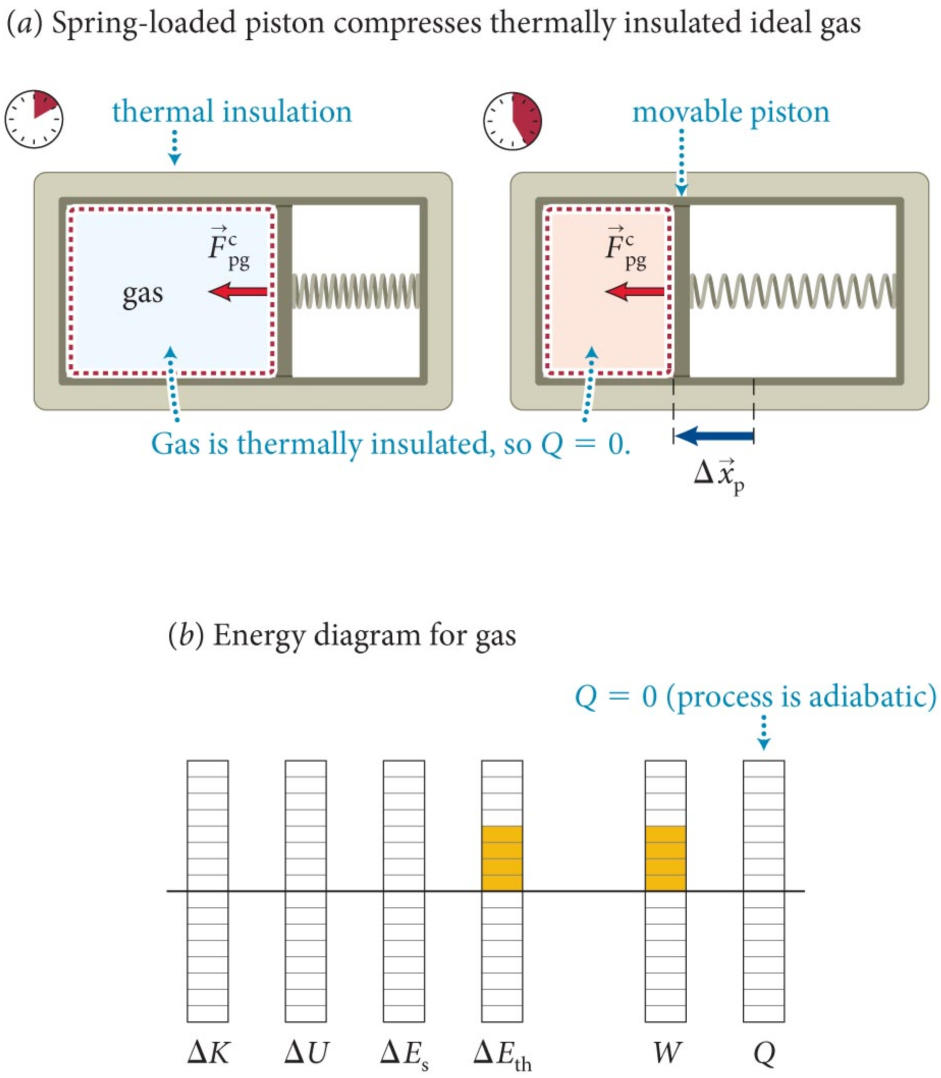


(*b*) Energy diagram for gas A



Section 20.1: Thermal interactions

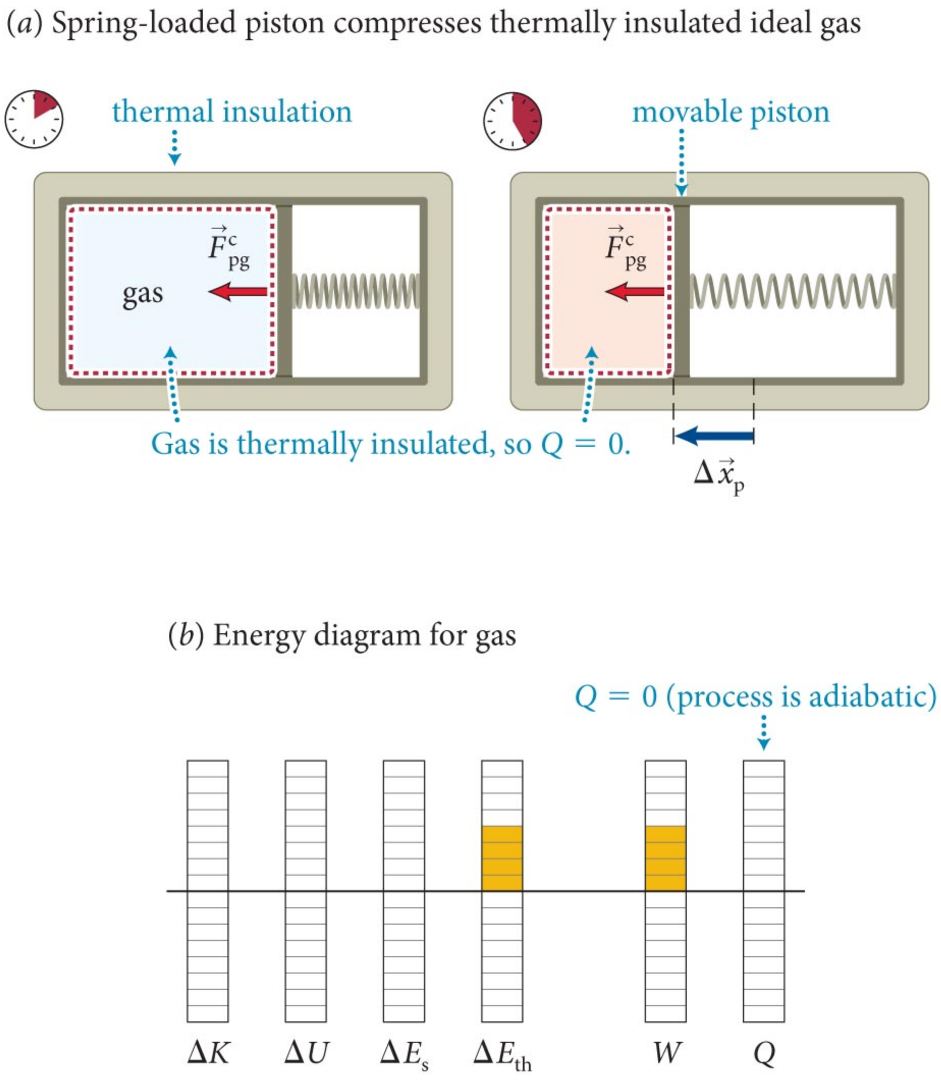
- The thermal energy of a system can be changed either by transferring energy thermally or by doing work on the system.
- A process that does not involve any thermal transfer of energy ($Q = 0$) is called an **adiabatic process**.
- This is mostly what we've worried about


(*a*) Spring-loaded piston compresses thermally insulated ideal gas

(*b*) Energy diagram for gas

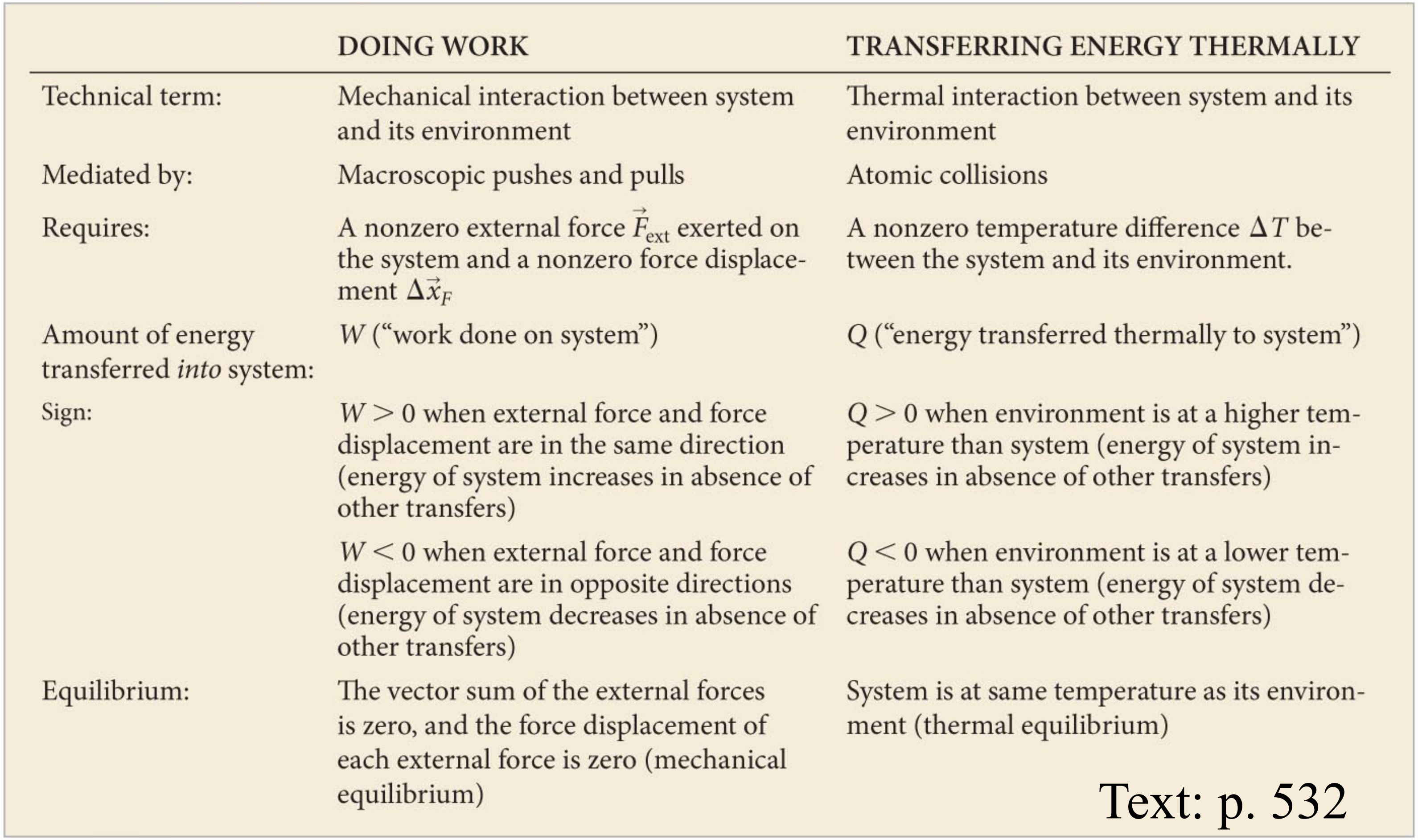
Section 20.1: Thermal interactions

- When the volume of a gas decreases, the work done on the gas is positive.
- When the volume increases, the work done on the gas in negative.
- When there is no change in volume, the work done on the gas is zero.



Section 20.1: Thermal interactions

Transferring energy to a system
Go back and read this carefully later.



	DOING WORK	TRANSFERRING ENERGY THERMALLY
Technical term:	Mechanical interaction between system and its environment	Thermal interaction between system and its environment
Mediated by:	Macroscopic pushes and pulls	Atomic collisions
Requires:	A nonzero external force $\vec{F}_{\text{ext}}$ exerted on the system and a nonzero force displacement $\Delta\vec{x}_F$	A nonzero temperature difference ΔT between the system and its environment.
Amount of energy transferred *into* system:	W ("work done on system")	Q ("energy transferred thermally to system")
Sign:	$W > 0$ when external force and force displacement are in the same direction (energy of system increases in absence of other transfers)	$Q > 0$ when environment is at a higher temperature than system (energy of system increases in absence of other transfers)
	$W < 0$ when external force and force displacement are in opposite directions (energy of system decreases in absence of other transfers)	$Q < 0$ when environment is at a lower temperature than system (energy of system decreases in absence of other transfers)
Equilibrium:	The vector sum of the external forces is zero, and the force displacement of each external force is zero (mechanical equilibrium)	System is at same temperature as its environment (thermal equilibrium)

Text: p. 532

Section 20.3: Heat capacity

- **Specific heat capacity**, denoted by letter *c*, is a measure of how much energy is required to raise the temperature of 1 kg of a certain material by 1 K.
- SI units of *c* are J/K - kg.
- To understand why the heat capacities of materials are so different from each other, let us first examine ideal gases.

Table 20.2 Specific heat capacities

Material	*c* (J/K · kg)
alcohol	2436
aluminum	897
brass	385
carbon graphite	685
copper	385
glass	837
gold	130
helium	3116
ice (268 K)	2090
iron	449
lead	130
marble	860
mercury	140
nickel	443
platinum	133
silver	235
steam (373 K)	1556
titanium	527
water	4181
wood	1700
zinc	388

All values are reported at a temperature of 298 K (unless otherwise noted) and atmospheric pressure and with the gases held at constant volume.

Section 20.3: Heat capacity

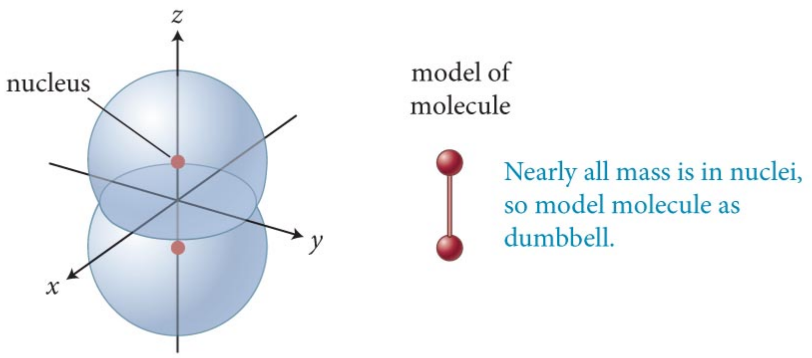
- As we saw in Chapter 19, each atom in a monatomic gas has an average thermal energy equal to $\frac{3}{2}k_B T$.
- $k_B = 1.38 \times 10^{-23}$ J/K
- Atoms in a monatomic gas are free to move along the x, y and z axes.
- These atoms carry energy exclusively as translational kinetic energy (3 degrees of freedom).

- Molecules in different gases may have additional modes of energy storage associated with vibration or rotational motion.
- We define the number of **degrees of freedom** as the number of independent ways of storing energy in a particle.

Section 20.3: Heat capacity

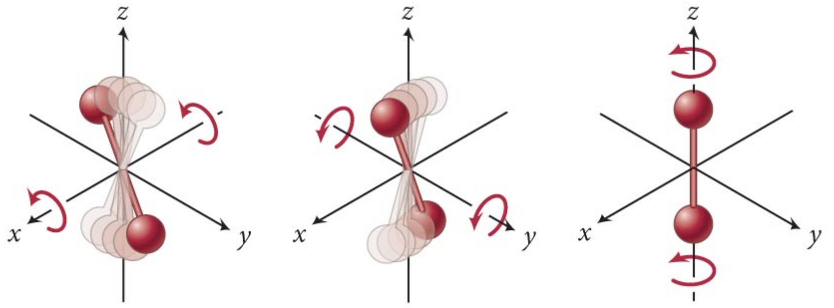
- In an ideal gas, each particle's **equipartition energy share** of the energy of the gas is ½ $dk_B T$, where d is the number of degrees of freedom.
- So, each atom of a monatomic gas has energy equal to $3(\frac{1}{2}k_B T)$.
- The dumbbell-shaped hydrogen molecule shown has two additional "rotational degrees of freedom" for a total of five.
- Hydrogen has thermal energy per molecule equal to $\frac{5}{2}k_B T$.

(*a*) Diatomic molecule



Nearly all mass is in nuclei, so model molecule as dumbbell.

(*b*) Rotational degrees of fredom for diatomic molecule



Rotation about *x* and *y* axes stores rotational energy.

Rotation about *z* axis stores negligible energy (because rotational inertia is negligible).

Section 20.3: Heat capacity

- If we raise the temperature of a monatomic ideal gas by 1 K, the thermal energy of each atom goes up by $\frac{3}{2}k_B T$.
- Therefore, we expect the heat capacity per atom to be $\frac{3}{2}k_B$.
- Heat capacities per particle (denoted by C) in units of k_B for a number of substances are listed in the table.

- Note water is weird. Hydrogen bonding.

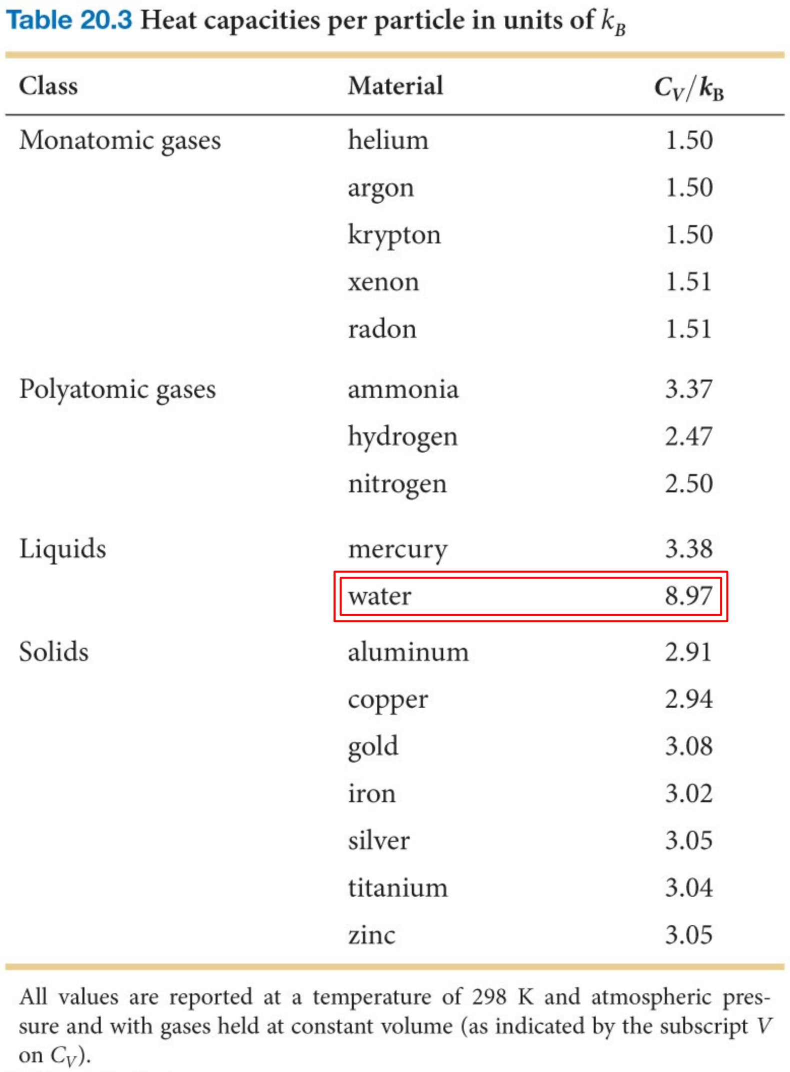


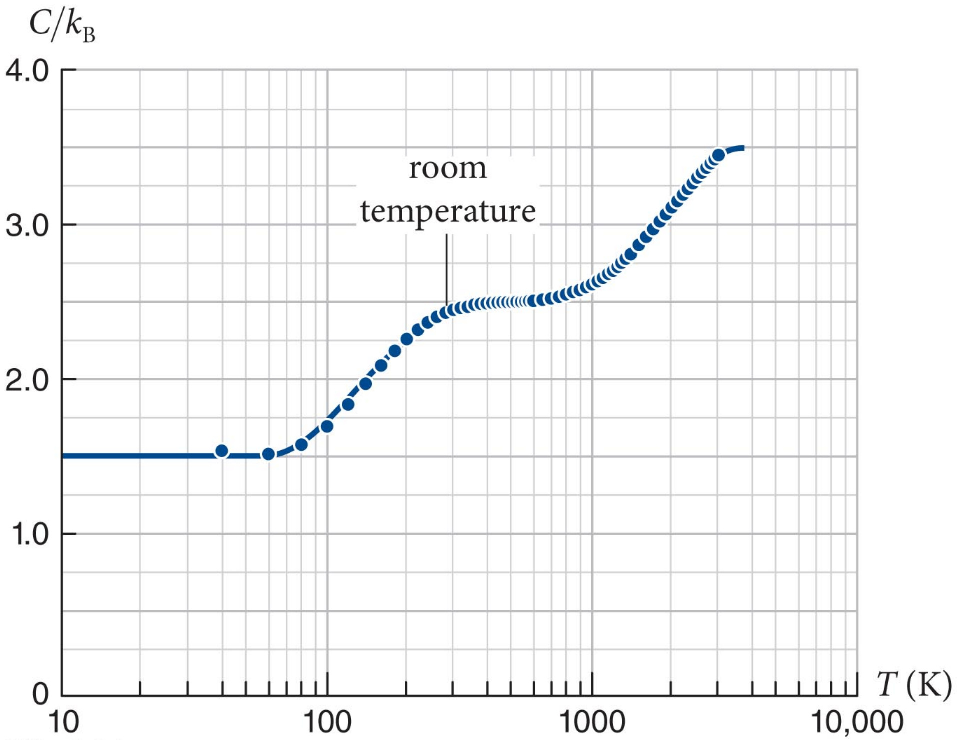
Table 20.3 Heat capacities per particle in units of k_B

Class	Material	C_V/k_B
Monatomic gases	helium	1.50
	argon	1.50
	krypton	1.50
	xenon	1.51
	radon	1.51
Polyatomic gases	ammonia	3.37
	hydrogen	2.47
	nitrogen	2.50
Liquids	mercury	3.38
	water	8.97
Solids	aluminum	2.91
	copper	2.94
	gold	3.08
	iron	3.02
	silver	3.05
	titanium	3.04
	zinc	3.05

All values are reported at a temperature of 298 K and atmospheric pressure and with gases held at constant volume (as indicated by the subscript V on C_V).

Section 20.3: Heat capacity

- The figure shows the temperature dependence of the heat capacity per particle for hydrogen gas molecules.

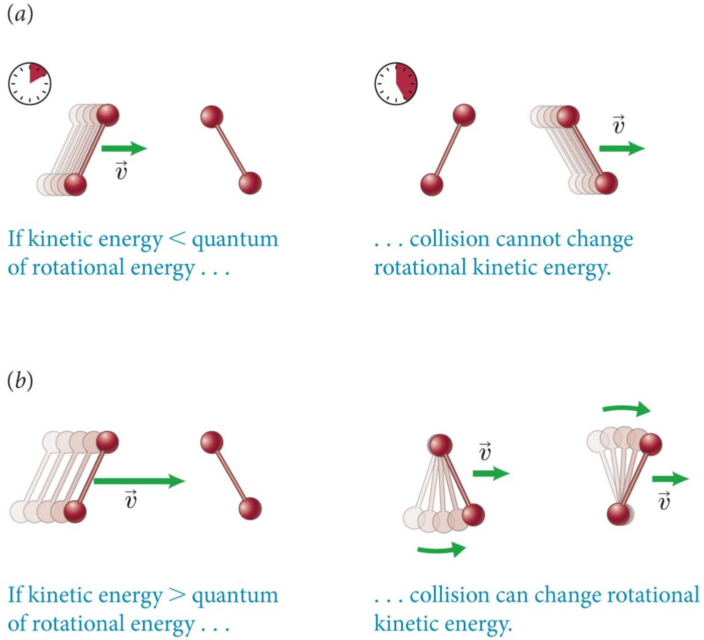


- The reason why *diatomic* hydrogen gas behaves like a *monatomic* gas at low temperatures can be explained using quantum mechanics.

Section 20.3: Heat capacity

- On small scales, energy is discrete (quantum). Molecules vibrate with only certain specific resonance frequencies, like standing waves.
- **Only degrees of freedom for which ½$k_B T$ is greater than the quantum of energy associated with that degree of freedom contribute to the heat capacity of a gas.**
- That means you can 'freeze out' some degrees of freedom at low temperature, since you don't have enough energy to excite them. This lowers the heat capacity

In thermal equilibrium, each contributing degree of freedom of a particle in an ideal gas stores ½$k_B T$ of thermal energy.



If kinetic energy < quantum of rotational energy . . .

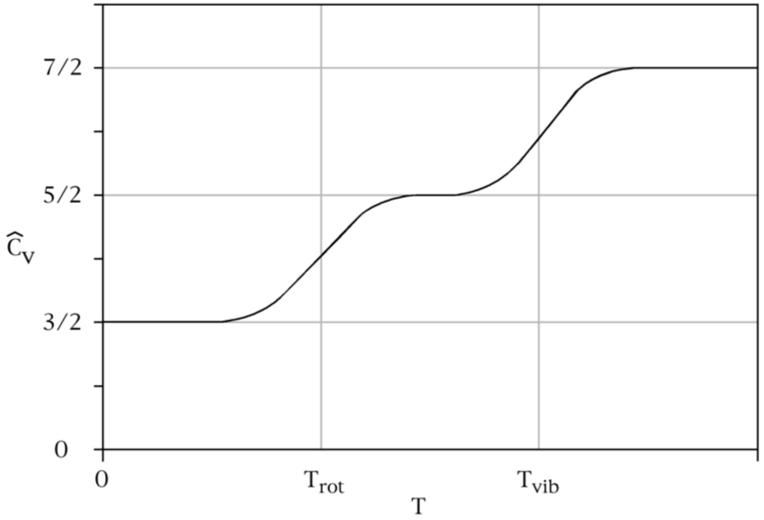
. . . collision cannot change rotational kinetic energy.

If kinetic energy > quantum of rotational energy . . .

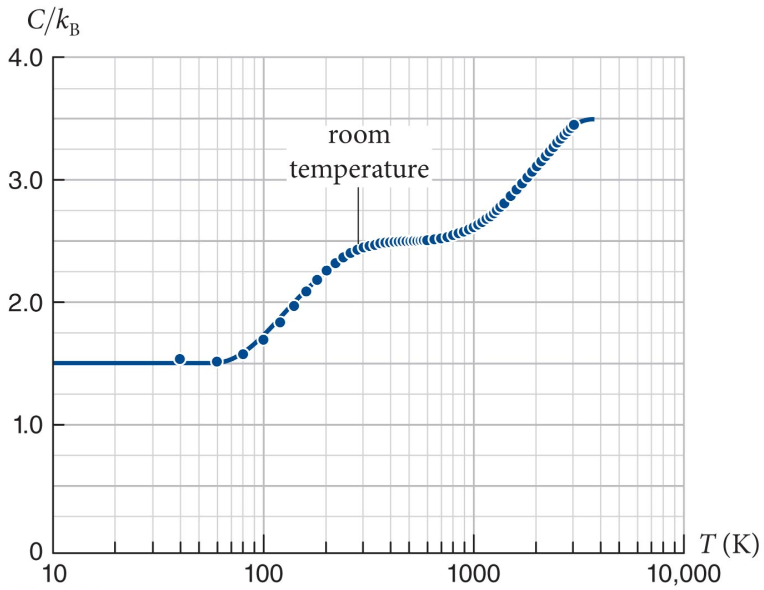
. . . collision can change rotational kinetic energy.

Ideal diatomic gas vs hydrogen

- High T: translation + rotation + vibration
- Middle T: freeze out vibration (highest energy, lose 2 d.f.)
- Low T: freeze out rotation (medium energy, lose 2 d.f.)
- Lowest T: only translation left, 3 directions = 3 d.f.



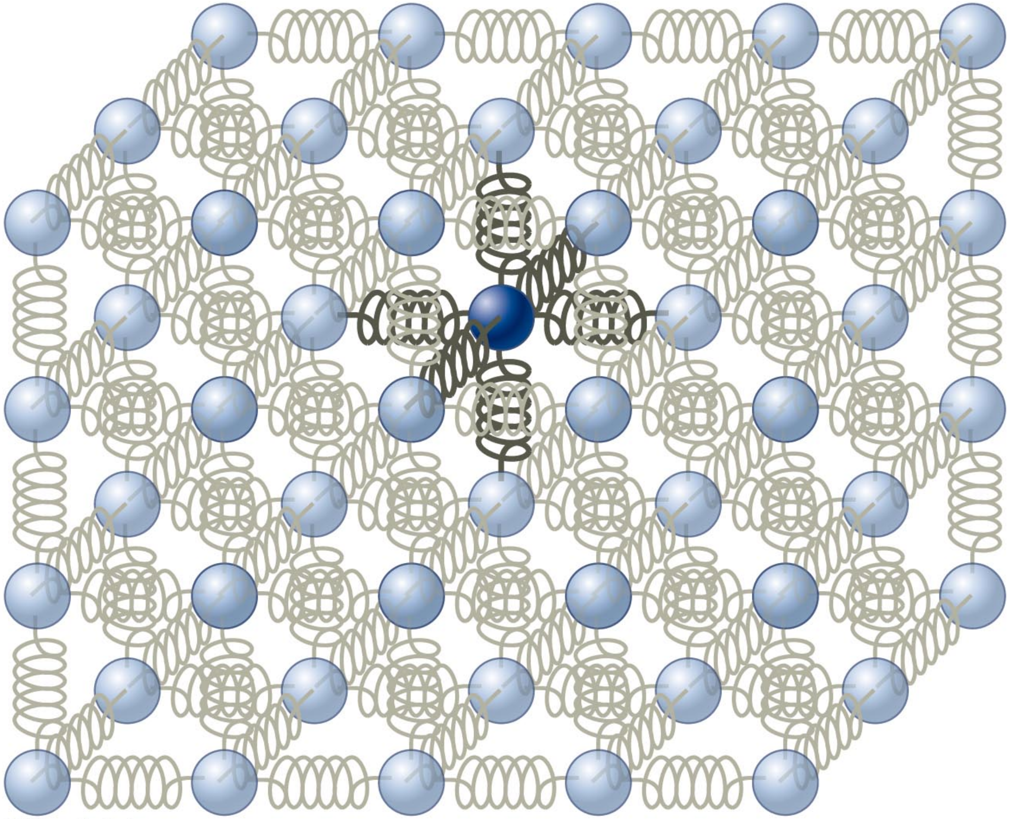
Ideal diatomic
https://en.wikipedia.org/wiki/Heat_capacity



Hydrogen (H_2)

Section 20.3: Heat capacity

- Consider the simple arrangement of atoms in a solid crystal.
- Each atom can vibrate along the three axes of the crystal.
- Vibration along each axis contributes 2 degrees of freedom, for a total of 6.
- Therefore, we expect the heat capacity per atom of a solid to be $3k_B$.
- This is in agreement with experimental measurements.



Checkpoint 20.8

20.8 Two identical volumes of gases—one monatomic and the other diatomic—are at the same pressure at room temperature. The temperature of each is raised by the same amount by thermally transferring energy to each of them. Is the energy transferred to the monatomic gas smaller than, equal to, or greater than the energy transferred to the diatomic gas?

Smaller than. More energy must be transferred to the diatomic gas because it has more degrees of freedom (and each needs some energy), so its heat capacity per particle is greater.

Diatomic = Bigger heat capacity = larger energy required for same temp rise

Chapter 20: Energy Transferred Thermally

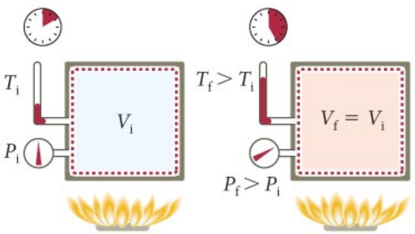
Quantitative Tools

Section 20.6: Isochoric an isoentropic ideal gas processes

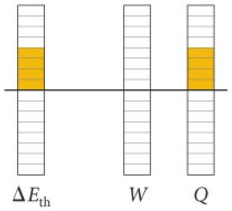
- An example of a **isochoric process** is shown in the figure.
- In an isochoric process, the volume does not change, and no work is done:

$$W = 0 \quad \text{(isochoric process)}$$

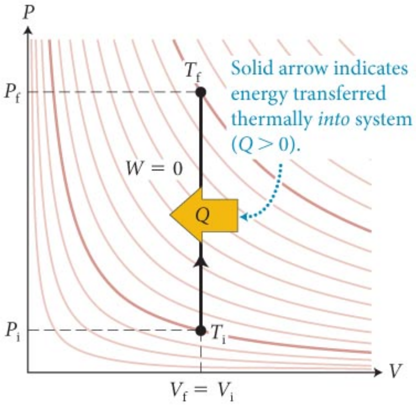
(*a*) Fixed volume of gas is heated, raising *T* and *P*



(*b*) Energy diagram for gas



(*c*) *PV* diagram showing the isochoric process



Solid arrow indicates energy transferred thermally *into* system ($Q > 0$).

Section 20.6: Isochoric an isoentropic ideal gas processes

- For an isochoric process, $W = 0$
- The energy law including heat is $\Delta E = W + Q$, which reduces to

$$\Delta E_{\text{th}} = Q \quad \text{(isochoric process)}$$

- Substituting the expression for ΔE_{th} given in Equation 20.5:

$$Q = \frac{d}{2} N k_{\text{B}} \Delta T \quad \text{(isochoric process)}$$

- Therefore, at constant volume, the **heat capacity per particle** of a gas containing N particles, each having d degrees of freedom is

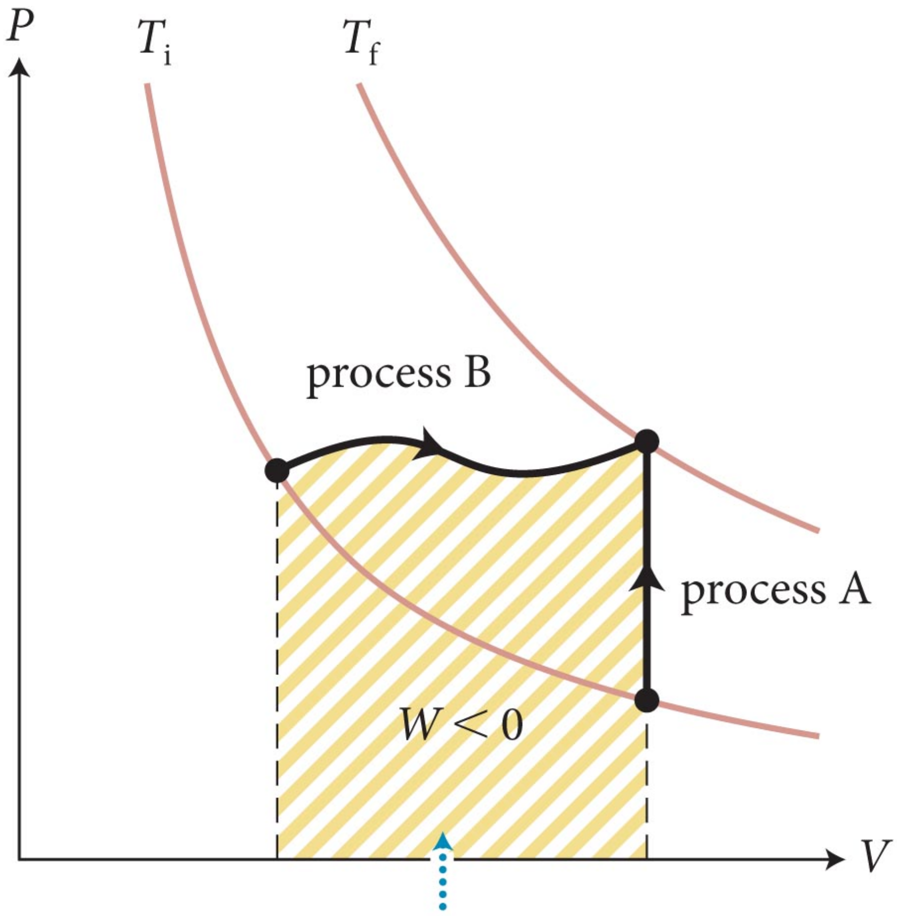
$$C_V = \frac{d}{2} k_B$$

- This equation allows us to express ΔE_{th} as $\Delta E_{\text{th}} = N C_V \Delta T$

Section 20.6: Isochoric an isoentropic ideal gas processes

- Processes A and B shown in the figure have the same ΔT.
- Process A is isochoric and B is some arbitrary process.
- Since processes A and B have the same ΔT they have the same ΔE_{th}.
- The total change in thermal energy for any process is given by the previous equation:

$$\Delta E_{th} = NC_V \Delta T \text{ (any ideal gas process)}$$



Hatched shading: Work done on gas in process B is *negative*.

Section 20.6: Isochoric an isoentropic ideal gas processes

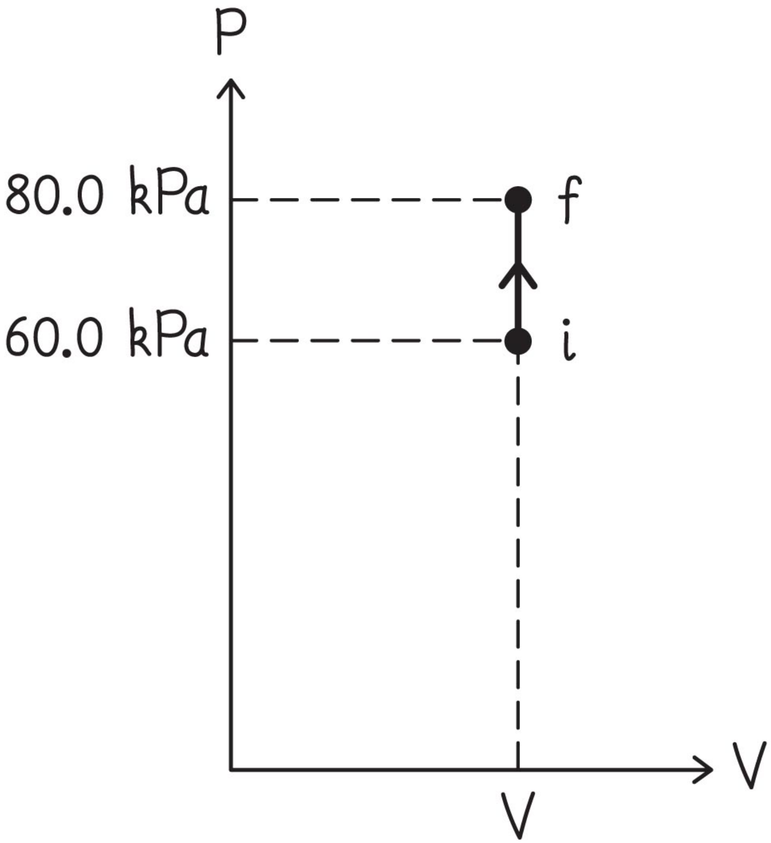
Example 20.6 Energy transferred thermally

A sample of hydrogen gas that contains 1.00 mol of H_2 molecules at a temperature of 300 K and a pressure of 60.0 kPa undergoes an isochoric process. The final pressure is 80.0 kPa. How much energy is transferred thermally to the gas?

Section 20.6: Isochoric an isoentropic ideal gas processes

Example 20.6 Energy transferred thermally (cont.)

❶ GETTING STARTED I begin by making a *PV* diagram of the process, treating the hydrogen as an ideal gas (Figure 20.32). Because the process is isochoric, I represent it by a vertical line. Because $P_f > P_i$, I indicate that the path representing the process points upward.



Section 20.6: Isochoric an isoentropic ideal gas processes

Example 20.6 Energy transferred thermally (cont.)

❷ DEVISE PLAN The quantity of energy transferred thermally during an isochoric process is given by Eq. 20.12. To use this equation, I need to know N, d, and ΔT. I know from Avogadro's number that my 1.00-mol sample contains $N = 6.02 \times 10^{23}$ molecules, and I can obtain d from Eq. 20.14 if I use Table 20.3 to get the value of C_V/k_B for H_2 molecules. For ΔT, I am given the initial temperature but not the final temperature. I know the initial and final pressures, however, and because the volume is fixed in this isochoric process, I can use the ideal gas law to determine the final temperature and so have a value for ΔT.

Section 20.6: Isochoric an isoentropic ideal gas processes

Example 20.6 Energy transferred thermally (cont.)

❸ EXECUTE PLAN From Table 20.3 I see that, for H_2 molecules, $C_V/k_B = 2.47$ at room temperature (about 300 K). From Figure 20.15 I see that C_V/k_B stays at that value through 700 K, and so, from Eq. 20.14, I know that $d/2 = C_V/k_B = 2.47$.

Section 20.6: Isochoric an isoentropic ideal gas processes

Example 20.6 Energy transferred thermally (cont.)

❸ EXECUTE PLAN Next I rewrite the ideal gas law (Eq. 19.51) in the form $V = Nk_BT/P$. Because V, N, and k_B are constant, I know that T/P is constant, and so $T_i/P_i = T_f/P_f$. Solving for T_f, I get

$$T_f = \frac{T_iP_f}{P_i} = \frac{(300\ \mathrm{K})(80.0\ \mathrm{kPa})}{60.0\ \mathrm{kPa}} = 400\ \mathrm{K},$$

so $\Delta T = 400\ \mathrm{K} - 300\ \mathrm{K} = 100\ \mathrm{K}$. Substituting my values for N, d, and ΔT in Eq. 20.12 gives me

$$Q = \frac{d}{2}Nk_B\Delta T = 2.47(6.02\times10^{23})(1.38\times10^{-23}\ \mathrm{J/K})(100\ \mathrm{K})$$

$$= 2.05\times10^{3}\ \mathrm{J}.$$ ✓

Section 20.6: Isochoric an isoentropic ideal gas processes

Example 20.6 Energy transferred thermally (cont.)

❹ EVALUATE RESULT Because the temperature of the gas increases, its thermal energy must increase. The process is isochoric, so no work is done on the gas. Therefore the value of the quantity of energy transferred thermally must be positive, as my result is.

Section 20.6: Isochoric an isoentropic ideal gas processes

Example 20.6 Energy transferred thermally (cont.)

❹ EVALUATE RESULT The quantity of energy transferred is significant—approximately equal to the kinetic energy of a 1-kg object moving at about 65 m/s—but I know that thermal energies tend to be large relative to kinetic energies. (See, for example, the box “Coherent versus incoherent energy” in Chapter 7.)

Section 20.7: Isobaric and isothermal ideal gas processes

Section Goals

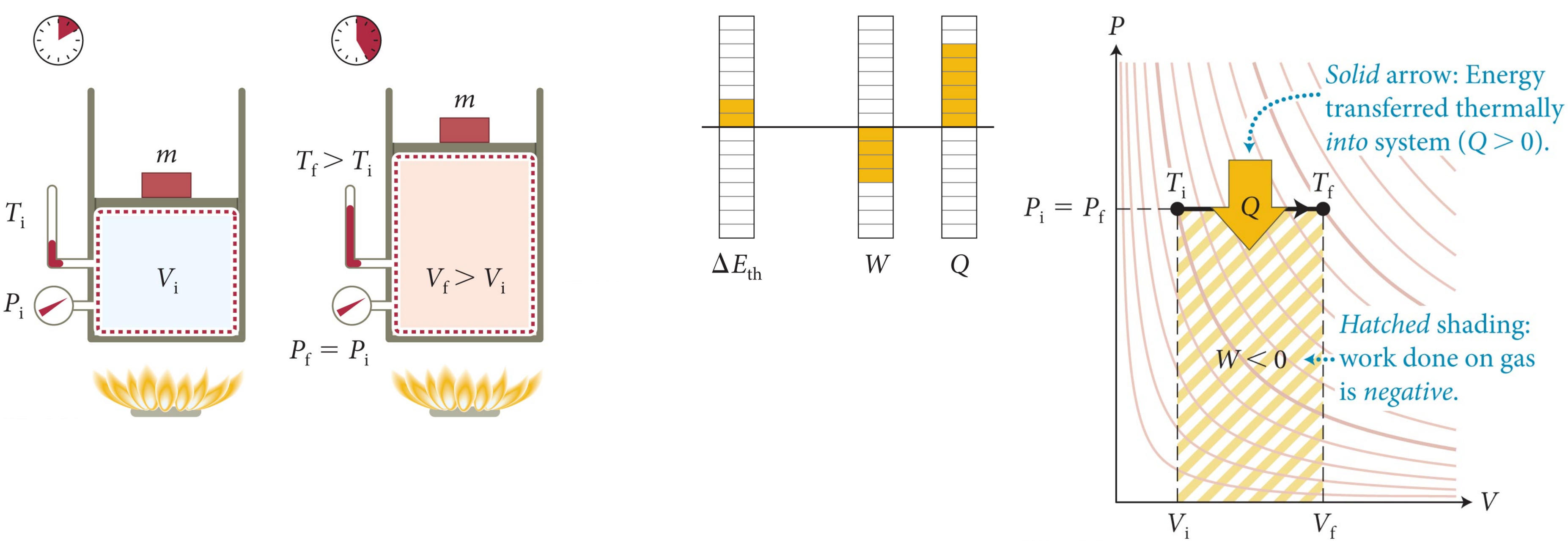
You will learn to

- Represent **isobaric** and **isothermal** processes for an ideal gas mathematically, diagrammatically, and using energy bar charts.
- Derive the equations for the **heat capacity per particle** for **isobaric** ideal gas processes.

Section 20.7: Isobaric and isothermal ideal gas processes

- In an **isobaric process**, the pressure is a constant, and the volume changes by $\Delta V = V_f - V_i$.
- The work done on the gas is $W = -P\Delta V$.
- Using the ideal gas law, we get

$$W = -P\Delta V = -Nk_B\Delta T \text{ (isobaric process)}$$



(*a*) Ideal gas heated at constant pressure

(*b*) Energy diagram for gas

(*c*) *PV* diagram showing the isobaric process

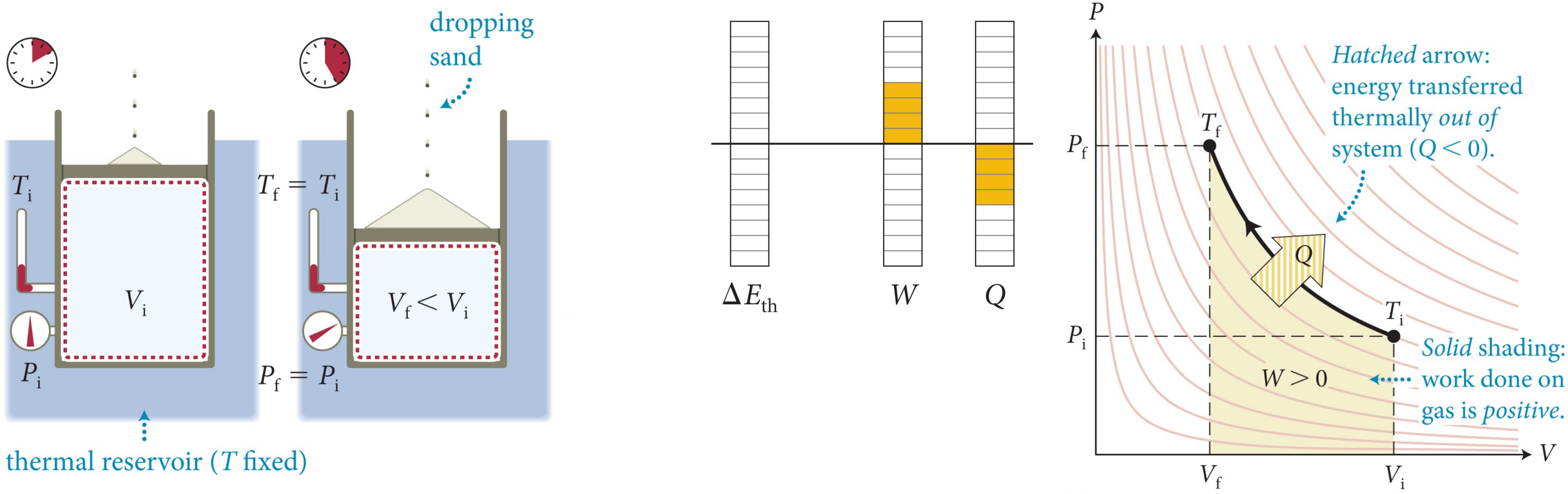
Solid arrow: Energy transferred thermally *into* system ($Q > 0$).

Hatched shading: work done on gas is *negative*.

Section 20.7: Isobaric and isothermal ideal gas processes

- An example of a quasistatic **isothermal process** is illustrated in the figure.
- Since the temperature of the gas remains constant,

$$\Delta E_{\text{th}} = 0 \quad \text{(isothermal process)}$$



(*a*) Ideal gas compressed slowly at constant temperature

(*b*) Energy diagram for gas

(*c*) *PV* diagram showing the isothermal process

Section 20.7: Isobaric and isothermal ideal gas processes

- For an isothermal process, the energy law reduces to

$$Q = -W \quad \text{(isothermal process)}$$

- Using the ideal gas law, we can determine the work done on a gas during an isothermal process:

$$W = -\int_{V_\mathrm{i}}^{V_\mathrm{f}} \frac{Nk_\mathrm{B}T}{V} dV = -Nk_\mathrm{B}T \int_{V_\mathrm{i}}^{V_\mathrm{f}} \frac{dV}{V} \quad \text{(isothermal process)}$$

$$W = -Nk_\mathrm{B}T(\ln V_\mathrm{f} - \ln V_\mathrm{i}) = -Nk_\mathrm{B}T \ln\left(\frac{V_\mathrm{f}}{V_\mathrm{i}}\right) \quad \text{(isothermal process)}$$

Section 20.7: Isobaric and isothermal ideal gas processes

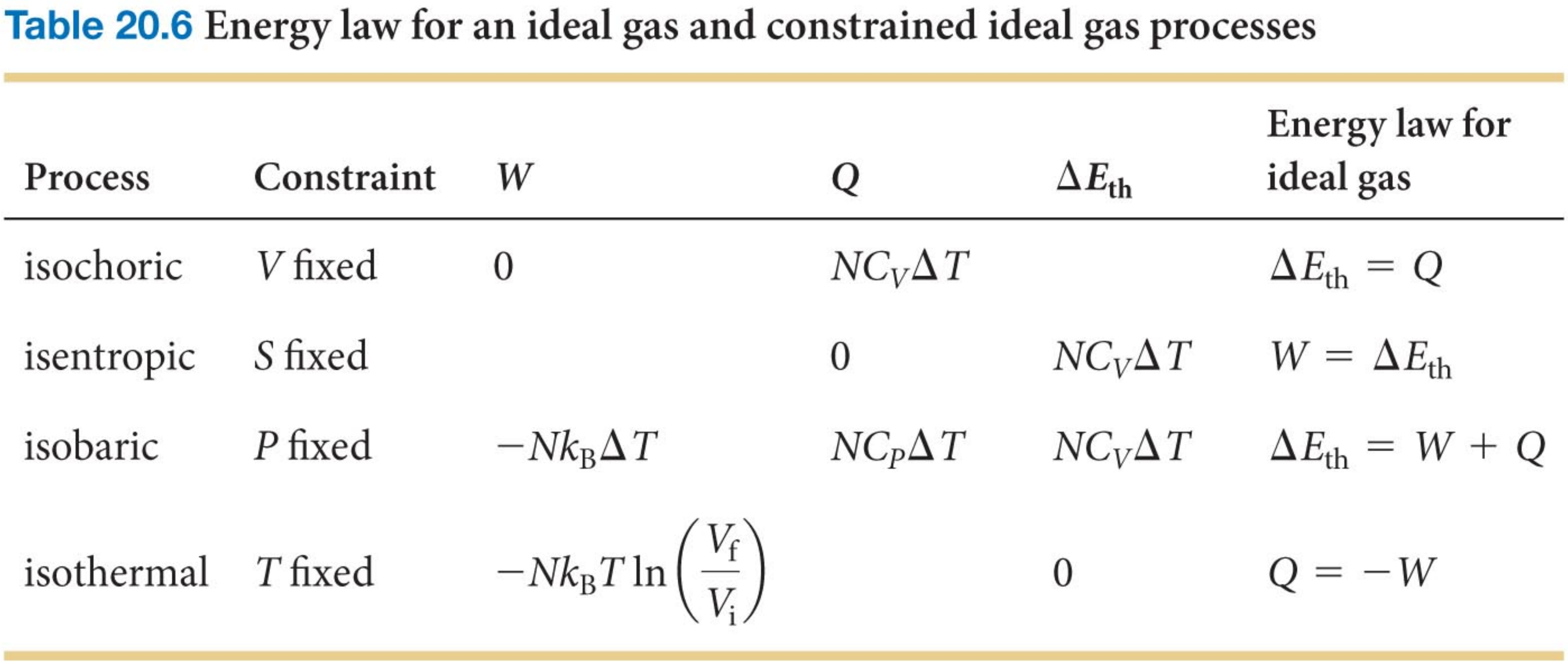


Table 20.6 Energy law for an ideal gas and constrained ideal gas processes

Process	Constraint	W	Q	ΔE_{th}	Energy law for ideal gas
isochoric	V fixed	0	$NC_V\Delta T$		$\Delta E_{\text{th}} = Q$
isentropic	S fixed		0	$NC_V\Delta T$	$W = \Delta E_{\text{th}}$
isobaric	P fixed	$-Nk_B\Delta T$	$NC_P\Delta T$	$NC_V\Delta T$	$\Delta E_{\text{th}} = W + Q$
isothermal	T fixed	$-Nk_BT\ln\left(\frac{V_f}{V_i}\right)$		0	$Q = -W$

Section 20.5: Change in energy and work

- From Chapter 9, the energy law can be written as

$$\Delta E = W + Q.$$

- If $\Delta K = \Delta U = \Delta E_s = 0$, then energy law reduces to

$$\Delta E_{\text{th}} = W + Q \quad \text{(ideal gas)}.$$

- For an ideal gas with d contributing degrees of freedom,

$$E_{\text{th}} = \frac{d}{2} N k_B T \quad \text{(ideal gas)}.$$

- If the gas contains a fixed number N of particles, then

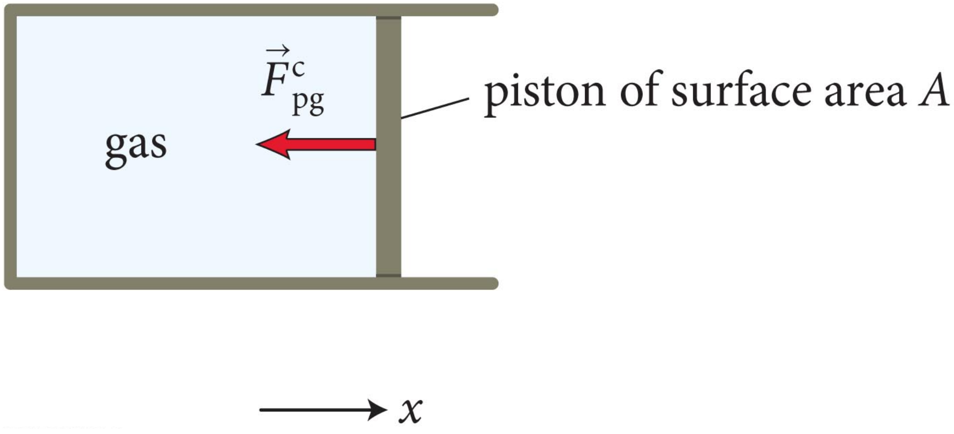
$$\Delta E_{\text{th}} = \frac{d}{2} N k_B \Delta T.$$

Section 20.5: Change in energy and work

- To calculate work done on a gas, consider the gas cylinder shown.
- The work done by the external force on the gas is $W = \int_{x_i}^{x_f} F^c_{\text{pg}\,x}\,dx$.
- Magnitude of F^c_{pg} is related to the gas pressure: $P = \frac{F^c_{\text{pg}}}{A}$.
- If we take the positive x-axis to the right, we have

$$W = -\int_{x_i}^{x_f} PA\,dx = -\int_{V_i}^{V_f} P\,dV$$

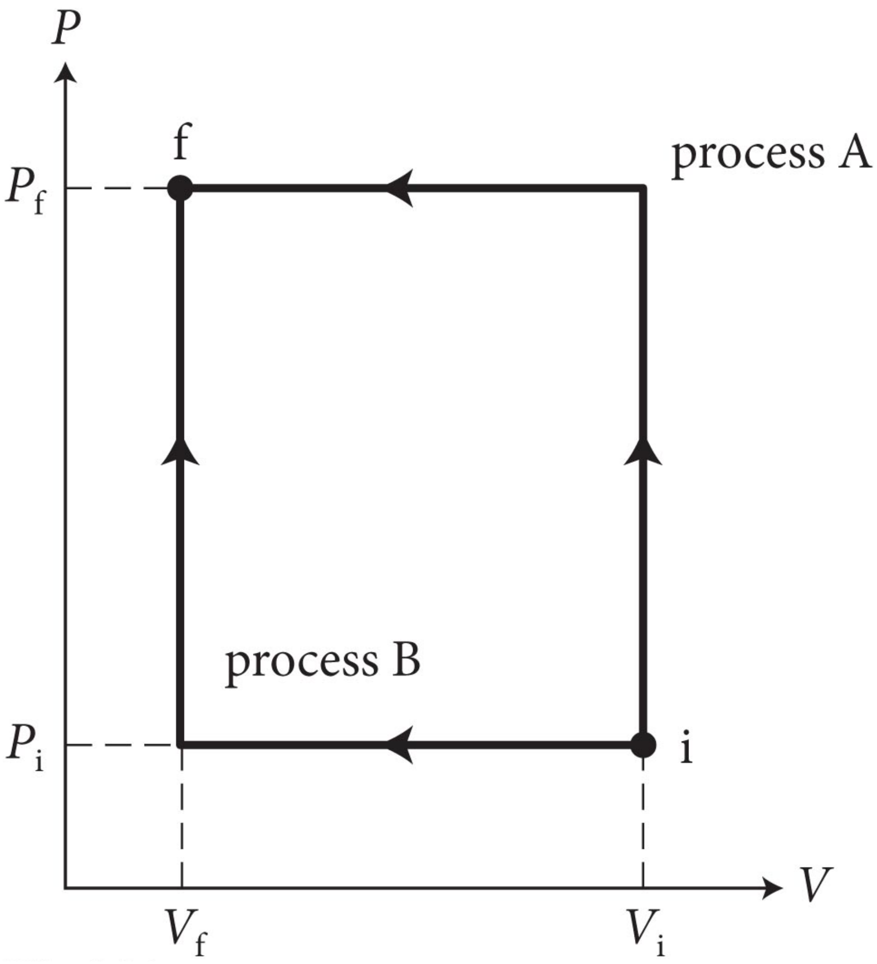
- When the pressure is constant, we get



$$W = -P\int_{V_i}^{V_f} dV = -P(V_f - V_i) = -P\Delta V \qquad \text{(quasistatic isobaric process)}$$

Example 20.5 Same change, different process

In Figure 20.28, an ideal gas is brought from an initial state i to a final state f by two processes. The initial volume and pressure are 0.50 m^3 and 100 kPa, and the final values for these variables are 0.10 m^3 and 500 kPa. How much energy is transferred thermally to the gas during (*a*) process A and (*b*) process B?



Section 20.5: Change in energy and work

Example 20.5 Same change, different process (cont.)

❶ GETTING STARTED Both processes are represented by a continuous path on a PV diagram and so are quasistatic. From the graph and the data given, I know that $V_i = 0.50\ \mathrm{m}^3$, $V_f = 0.10\ \mathrm{m}^3$, $P_i = 100$ kPa, and $P_f = 500$ kPa.

Example 20.5 Same change, different process (cont.)

❷ DEVISE PLAN To calculate the quantity of energy transferred thermally, I can use the energy law as given in Eq. 20.2, $\Delta E_{\mathrm{th}} = W + Q$, which gives me $Q = \Delta E_{\mathrm{th}} - W$.

From Checkpoint 20.13, I know that work is done on the gas only during the constant-pressure legs. To calculate W during these legs, I can use Eq. 20.9. The change in the thermal energy ΔE_{th} is given by Eq. 20.5, but I do not know d, N, or ΔT.

Section 20.5: Change in energy and work

Example 20.5 Same change, different process (cont.)

❷ DEVISE PLAN The ideal gas law, however, relates pressure and volume to temperature: $PV = Nk_BT$ (Eq. 19.51). I note that the product PV is the same for the initial and final states: $P_iV_i = (100\ \text{kPa})(0.50\ \text{m}^3) = 50\ \text{kPa - m}^3$ and $P_fV_f = (500\ \text{kPa})(0.10\ \text{m}^3) = 50\ \text{kPa - m}^3$.

This tells me that the initial and final states lie on an isotherm, so $\Delta T = 0$. Therefore Eq. 20.5 becomes $\Delta E_{\text{th}} = 0$, so Eq. 20.2 yields $Q = -W$.

Section 20.5: Change in energy and work

Example 20.5 Same change, different process (cont.)

❸ EXECUTE PLAN The change in volume for both processes is $\Delta V = V_f - V_i = (0.10\ \text{m}^3) - (0.50\ \text{m}^3) = -0.40\ \text{m}^3$.

(a) From Eq. 20.9, the work done on the gas for process A is $W = -P\Delta V = -(500\ \text{kPa})(-0.40\ \text{m}^3) = 2.0 \times 10^2$ kJ. Therefore $Q_A = -2.0 \times 10^2$ kJ. ✔

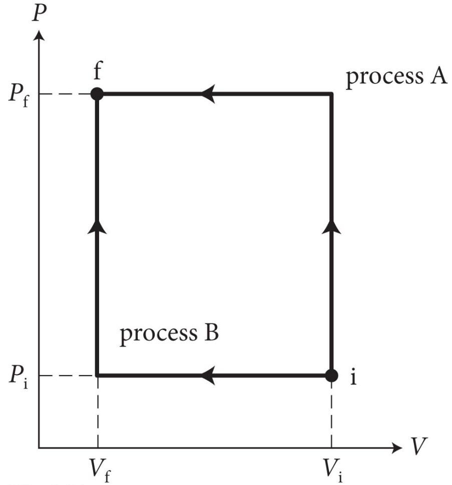
(*b*) $W = -P\Delta V = -(100\ \text{kPa})(-0.40\ \text{m}^3) = 40$ kJ, which leads to $Q_B = -40$ kJ. ✔

Section 20.5: Change in energy and work

Example 20.5 Same change, different process (cont.)

➍ EVALUATE RESULT The work done on the gas during process A is five times greater than the work done during process B. This is not surprising because in A the compression from V_i to V_f is done at a pressure that is five times greater than the pressure at which the compression is done in B.

Work = area under P(V) curve



Section 20.5: Change in energy and work

Example 20.5 Same change, different process (cont.)

❹ EVALUATE RESULT Because the thermal energy of the gas does not change, any energy acquired in the form of work done on the gas has to leave as energy transferred thermally out of the gas. Both Q_A and Q_B should therefore be negative, and Q_A should be greater than Q_B, which is what I found.

Section 20.7: Isobaric and isothermal ideal gas processes

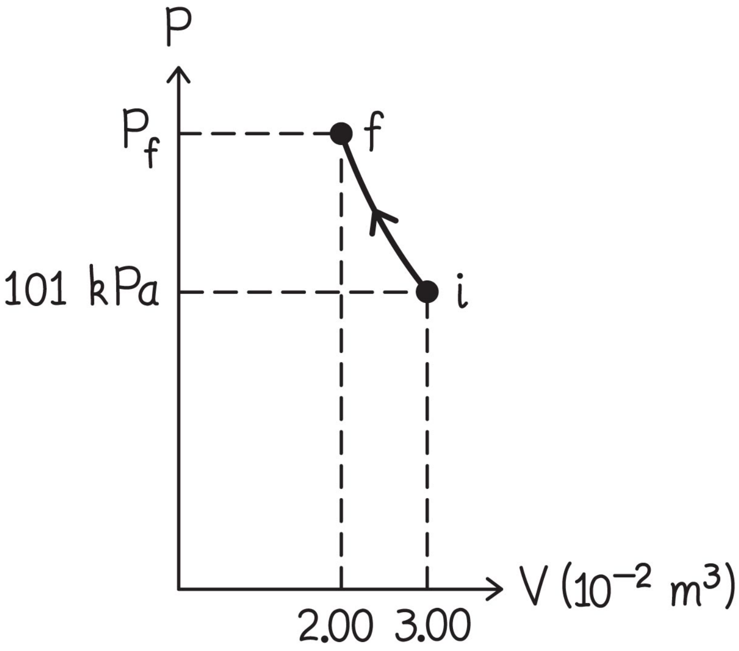
Example 20.8 Isothermal compression

An ideal gas sample is compressed quasistatically at constant temperature. The initial pressure and volume are $P_i = 1.01 \text{ x } 10^5$ Pa and $V_i = 3.00 \text{ x} 10^{-2} \text{ m}^3$, and the final volume is $V_f = 2.00 \text{ x } 10^{-2} \text{ m}^3$. How much work is done on the gas during the compression?

Section 20.7: Isobaric and isothermal ideal gas processes

Example 20.8 Isothermal compression (cont.)

➊ GETTING STARTED I begin by making a *PV* diagram of the process (Figure 20.36). The process is an isothermal compression, so it is represented by a path going up along an isotherm toward the left.



Example 20.8 Isothermal compression (cont.)

❷ DEVISE PLAN The work done on an ideal gas in an isothermal process is given by Eq. 20.30. I know the initial and final volumes, but not the factor Nk_BT. I can determine this factor from the ideal gas law, $PV = Nk_BT$; it does not matter whether I use initial values or final values in my calculation because the temperature is the same in both cases.

Because I do not know the final pressure, however, I should use initial values.

Section 20.7: Isobaric and isothermal ideal gas processes

Example 20.8 Isothermal compression (cont.)

❸ EXECUTE PLAN From the ideal gas law, I have $Nk_BT = P_iV_i$. From Eq. 20.30, the work done on the gas is

$$W = -Nk_BT\ln\left(\frac{V_f}{V_i}\right) = -P_iV_i\ln\left(\frac{V_f}{V_i}\right).$$

Substituting values for the variables, I get

$$W = -(1.01\times10^5\,\text{Pa})(3.00\times10^{-2}\,\text{m}^3)\ln\left(\frac{2.00\times10^{-2}\,\text{m}^3}{3.00\times10^{-2}\,\text{m}^3}\right)$$

$$= 1.23\times10^3\,\text{J}. ✓$$

Section 20.7: Isobaric and isothermal ideal gas processes

Example 20.8 Isothermal compression (cont.)

❹ EVALUATE RESULT My answer is positive, as I expect for the work done on a gas in compressing it.

To see whether the magnitude of my answer is in the right ballpark, I assume that the volume change is achieved by moving a piston of surface area 1×10^{-2} m^2 over a distance of 1 m. This gives the right volume change.

At a pressure of 1.01×10^5 Pa, the gas exerts on the piston a force of magnitude $(1 \times 10^{-2}\ \text{m}^2)(1.01 \times 10^5\ \text{Pa}) \approx 10^3$ N.

Section 20.7: Isobaric and isothermal ideal gas processes

Example 20.8 Isothermal compression (cont.)

❹ EVALUATE RESULT The piston exerts a force of equal magnitude on the gas. If this force were constant, the work done on the gas during a force displacement of 1 m would be $W = (10^3\ \text{N})(1\ \text{m}) = 10^3\ \text{J}$, which is of the same order of magnitude as the answer I obtained. (This estimate doesn’t take into account the increase in pressure and so is a bit lower than the actual value.)

Calculating how hot stuff gets

- Heat energy is $Q = mcT$, where c is heat capacity
- c is how much heat energy can flow per unit temperature difference per unit mass, J/kg-K
- So long as P and V do not change (no work done), heat is the only energy change.

- That means in the absence of phase transitions (e.g., melting/freezing or boiling/condensing), the energy input required for a given temperature rise is

$$E = Q = mc\Delta T$$

How hot stuff gets

- If you have 2 objects in contact? Thermal energy (heat) is transferred from hot to cold until equilibrium is reached.
- If the two are in contact but isolated from surroundings, the net change in energy is zero.
- Say we have a hot thing (H) and a cold thing (C) in contact. Energy conservation gives

$$\Delta Q_H + \Delta Q_C = 0$$

$$m_h c_H (T_H - T_{eq}) + m_C c_C (T_C - T_{eq}) = 0$$

$$\Longrightarrow \quad T_{eq} = \frac{m_H c_H T_H + m_C c_C T_C}{m_H c_H + m_C c_C}$$

What if stuff melts or boils?

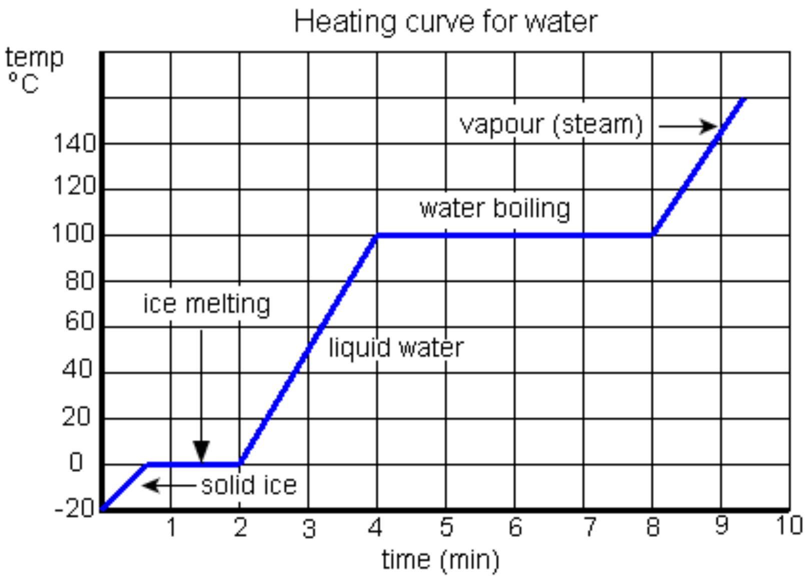
- At the phase change, energy added doesn't change T at all, it goes into breaking bonds and changing phase
- Before the phase change, you pay energy to raise temperature.
- At the phase change, you pay a toll: you have to give a certain amount of energy to accomplish the phase change before you can heat the new phase
- How much?

$$Q_{\text{transform}} = Lm$$

- L = latent heat. Vaporize/condense or fuse/melt

Ice to water

- Heat ice from T_i to melting point at 273K:
  - $Q = c_i m(T_i\text{-}273\text{K})$
- Melt ice. No T change, pay toll:
  - $Q = L_f m$
- Heat water up to boiling point (373K):
  - $Q = c_w m(373\text{K-}273\text{K})$
- Boil water. No T change:
  - $Q = L_v m$
- Heat steam
  - $Q = c_s m(T_f - 373\text{K})$



- http://www.spiritsd.ca/curr_content/physics20/heat/latent_heat.htm

Things

- Work problems for the final.
- For Ch. 13 & 20, probably just 1 problem for each!
- Look through the end of chapter formulas to make sure you know they are good for.

- Sleep before the final. Being sharp is better than cramming, this stuff is not about memorization.

- The final is right here, 8:00am on Wednesday. Bring **pencils** and calculators. If you are running late, still show up.