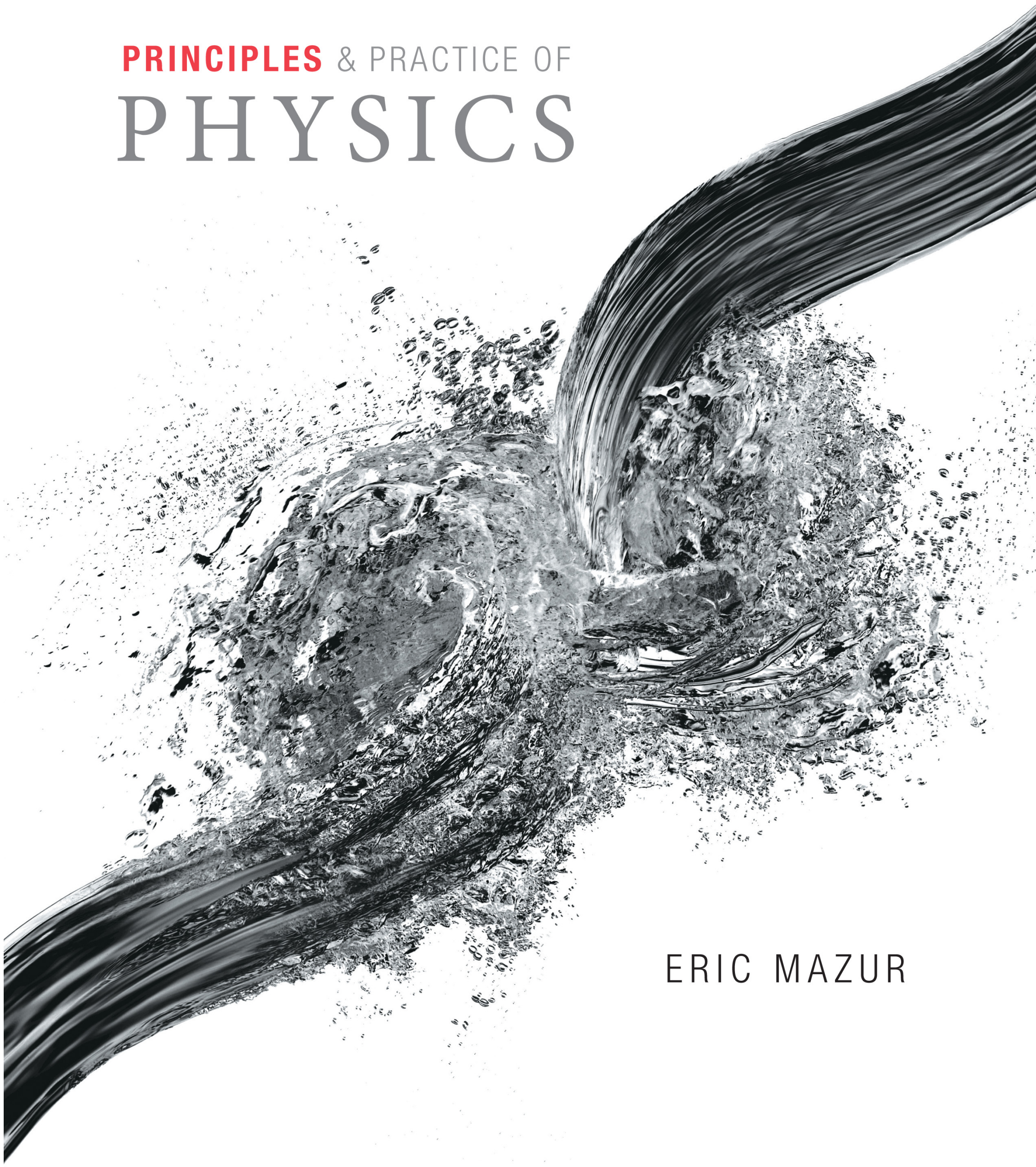
Chapter 3
Acceleration



PackBack

The first answer gives a good physical picture. The video was nice, and worth the second answer.

https://www.youtube.com/watch?v=m57cimnJ7fc

Asked by **Cody Phillips**
at The University of Alabama

How does physics apply to a knuckleball?

We all know that the spin and air resistance on a pitcher's curveball makes the ball move but how does this apply to a knuckleball?

7:35 PM, 1/22/2017 Options ▾

Add your own Response

Answered by **Alex Lindsey**
at The University of Alabama

Variations in airflow along the differences in the smooth and rougher surfaces of the ball are what cause the zigzag-like trajectory of a knuckleball. Basically, you are forcing the air flow to create an asymmetric drag to make the ball move up and down or side to side.

7:42 PM, 1/22/2017 Options ▾

Answered by **Alex Ambrose**
at The University of Alabama

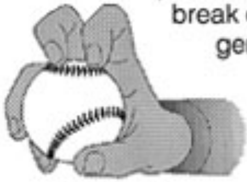
Here's a video I found that might be useful In helping you find your answer!

https://www.youtube.com/watch?v=m57cimnJ7fc

9:57 PM, 1/22/2017 Options ▾

The Knuckleball Hop

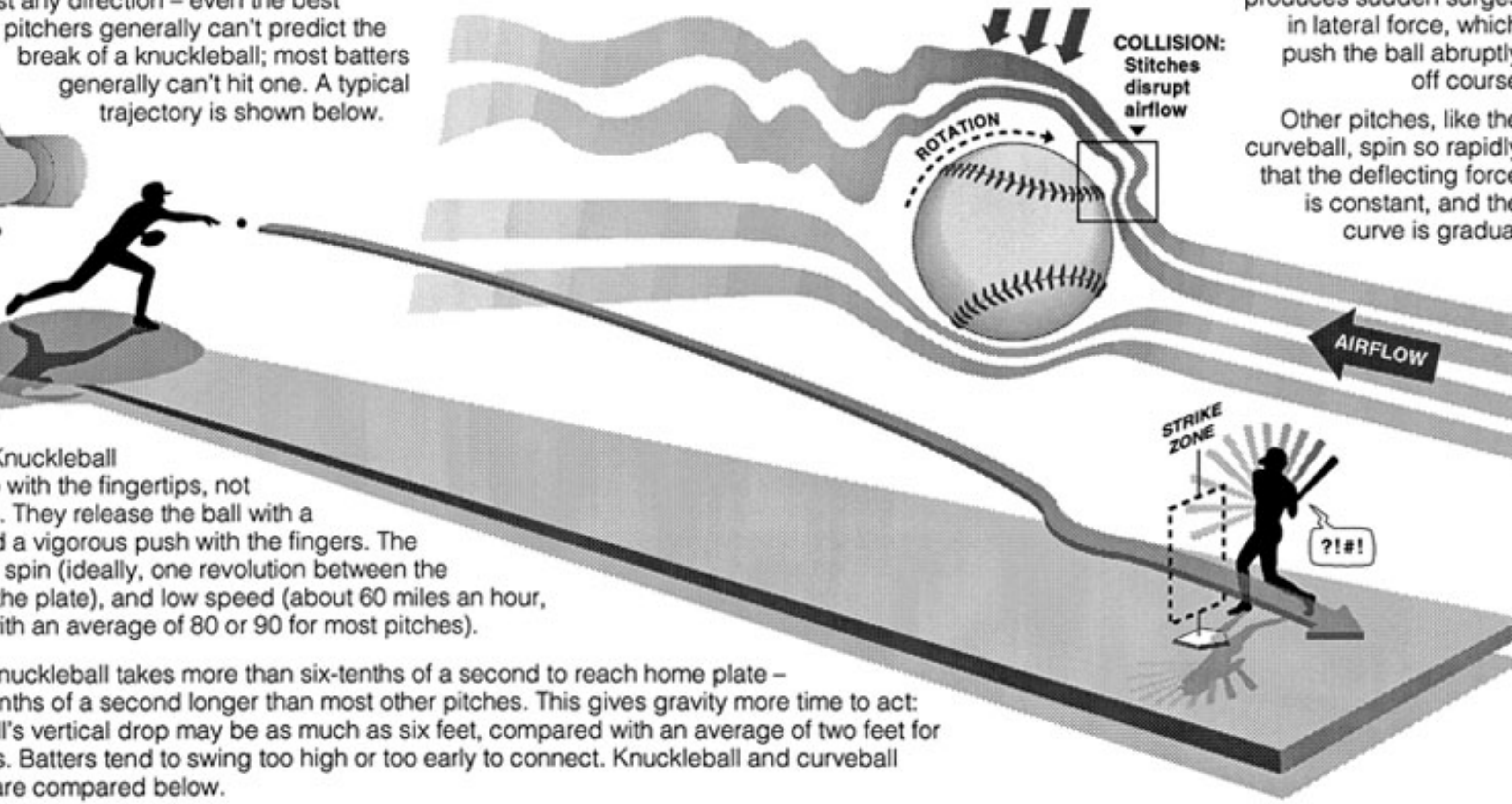
Physicists explain a knuckleball's hops and dips as collisions between the air and a baseball's stitches. They attribute the peculiar flight pattern to a combination of low speed and minimal spin. The pitch can veer in almost any direction – even the best pitchers generally can't predict the break of a knuckleball; most batters generally can't hit one. A typical trajectory is shown below.



THE GRIP

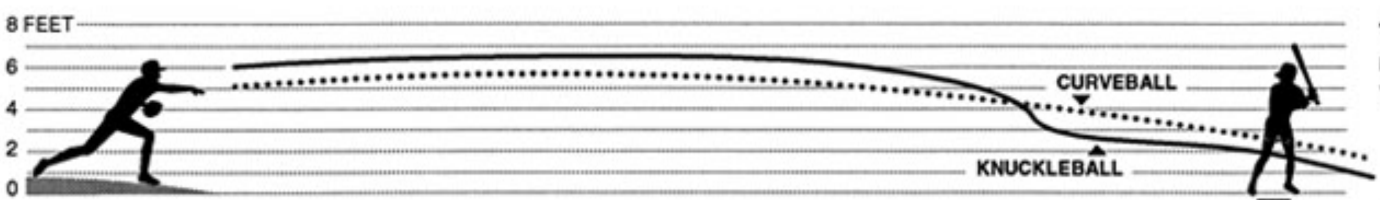
AERODYNAMICS: As the baseball slowly turns, its stitches rotate into the path of oncoming air and create increased air pressure near the "collision" point. As the ball keeps turning, the seam moves out of the airflow and pressure decreases on that side. Fluctuating air pressure produces sudden surges in lateral force, which push the ball abruptly off course.

Other pitches, like the curveball, spin so rapidly that the deflecting force is constant, and the curve is gradual.



DELIVERY: Knuckleball pitchers grip with the fingertips, not the knuckles. They release the ball with a stiff wrist and a vigorous push with the fingers. The ball has little spin (ideally, one revolution between the mound and the plate), and low speed (about 60 miles an hour, compared with an average of 80 or 90 for most pitches).

TIMING: A knuckleball takes more than six-tenths of a second to reach home plate – about two-tenths of a second longer than most other pitches. This gives gravity more time to act: a knuckleball's vertical drop may be as much as six feet, compared with an average of two feet for other pitches. Batters tend to swing too high or too early to connect. Knuckleball and curveball trajectories are compared below.



THE CATCH: Bob Uecker, former major league catcher, described the best technique for catching a knuckleball: "Wait until the ball stops rolling and then pick it up."

Sources: Peter J. Brancazio, Brooklyn College Department of Physics; "Sport Science" (Brancazio); "Newton at the Bat," Eric W. Schrier and William F. Allman

Megan Jaegerman/The New York Times

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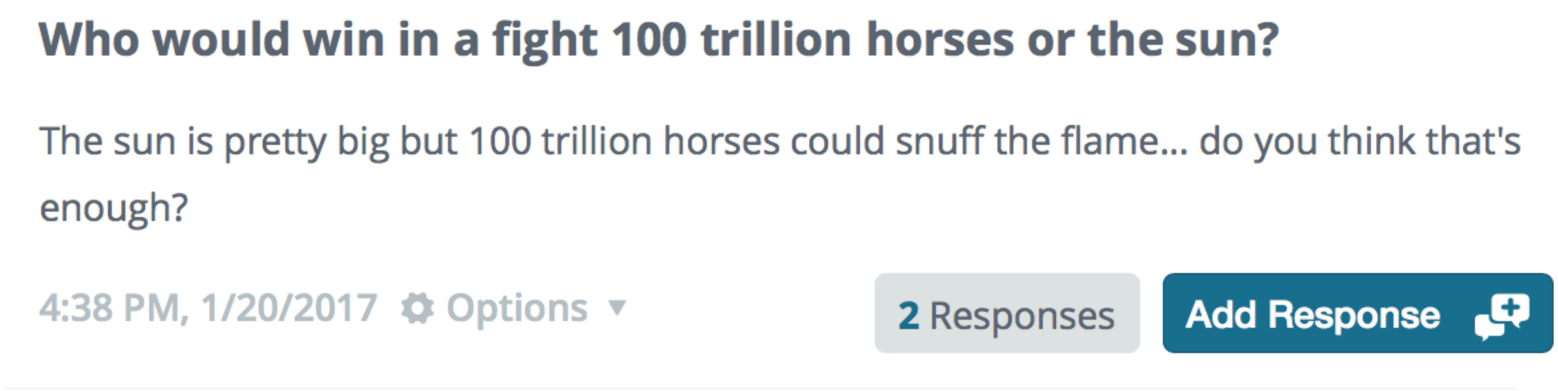


- It does not look good for the horses. Just saying. The sun is really big.

- But, serious point: you can answer weird questions calmly and logically, and that’s fine.

PackBack

- Lions or horses hardly matters
- There are technical difficulties
- It is fine to apply physics to ridiculous situations. That's how you know they are ridiculous.

Answered by **Max LaRocque**
at The University of Alabama

The average mass of a lion is 420 pounds, multiplying that by 100 trillion gets you 4.2*10^16 pounds. The mass of the sun is 4.385*10^30 pounds. The mass of the sun is 9.6*10^45 times greater than the mass of the lions. The lions don't stand a chance.

5:16 PM, 1/20/2017 Options ▾

Answered by **Gabriel Wood**
at The University of Alabama

I'm pretty sure the horses would lose in the event that they picked a fight with the sun. Mass aside, the horses would be unable to reach the sun in the first place. The sun is powered by nuclear fusion and burns at around 5778 Kelvin, so it would be rather difficult for the horses to make their way to the sun without being vaporized. Sadly, I think the sun wins this round.

11:08 PM, 1/21/2017 Options ▾

Obligatory reference

PH105: “Sadly, I think the sun wins this round.”

CMB: Ever since the beginning of time, man has yearned to destroy the sun

http://i.imgur.com/M3lcUSv.jpg

Reading quiz 3

question 1

Prelecture Concept Question 3.09

Part A

A car traveling due east at 20 m/s reverses its direction over a period of 10 seconds so that it is now traveling due west at 20 m/s. What is the direction of the car's average acceleration over this period?

Reading quiz 3

question 1

Prelecture Concept Question 3.09

Part A

A car traveling due east at 20 m/s reverses its direction over a period of 10 seconds so that it is now traveling due west at 20 m/s. What is the direction of the car's average acceleration over this period?

- Change in velocity $\Delta\vec{v}$ is to the west
- That makes $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$ to the west as well
- Magnitude? Change is *40 m/s*, over 10 s, so 4 m/s^2

Reading quiz 3

Question 2

Prelecture Concept Question 3.05

Part A

Suppose that you toss a rock upward so that it rises and then falls back to the earth. If the acceleration due to gravity is 9.8 m/sec^2, what is the rock's acceleration at the instant that it reaches the top of its trajectory (where its velocity is momentarily zero)? Assume that air resistance is negligible.

Reading quiz 3

Question 2

Prelecture Concept Question 3.05

Part A

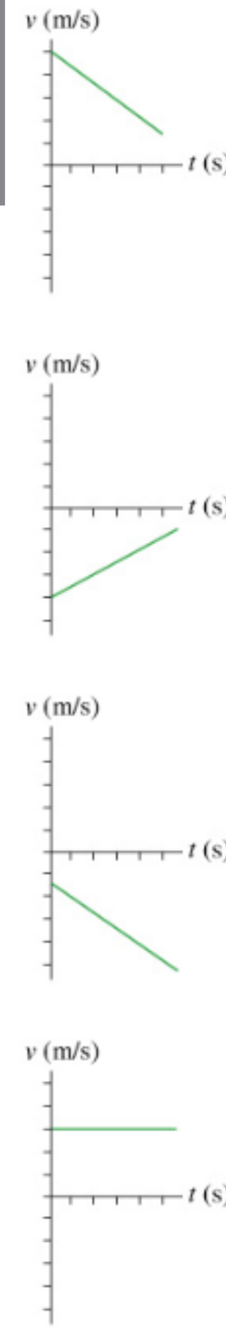
Suppose that you toss a rock upward so that it rises and then falls back to the earth. If the acceleration due to gravity is 9.8 m/sec^2, what is the rock's acceleration at the instant that it reaches the top of its trajectory (where its velocity is momentarily zero)? Assume that air resistance is negligible.

- Gravitational acceleration is constant – it is 9.8 m/s^2 at all times.
- Confusing *acceleration* and *velocity* is an easy thing to do – think carefully.

Reading quiz 3

Question 3

- Which figure could represent the velocity versus time graph of a motorcycle whose *speed* is increasing?

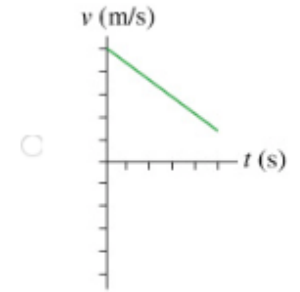


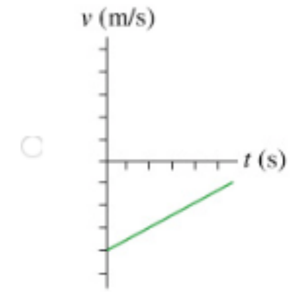
Reading quiz 3

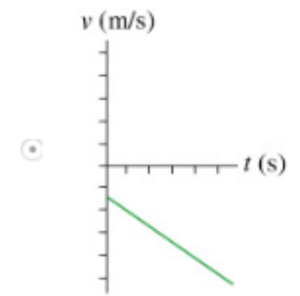
Question 3

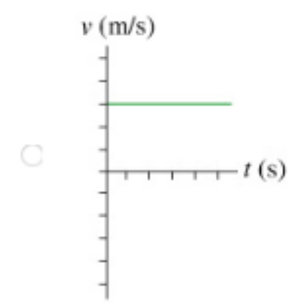
- Which figure could represent the velocity versus time graph of a motorcycle whose *speed* is increasing?

- Speed is the absolute value of velocity … magnitude of *v* increases
- Flip all curves to be in upper right quadrant and *then* compare









Last time

- Covered sections 3.1-2, the basics of acceleration.
- Finish Ch. 3 today, on to Ch. 4 on Thursday

Section 3.3: Projectile motion

Section Goals

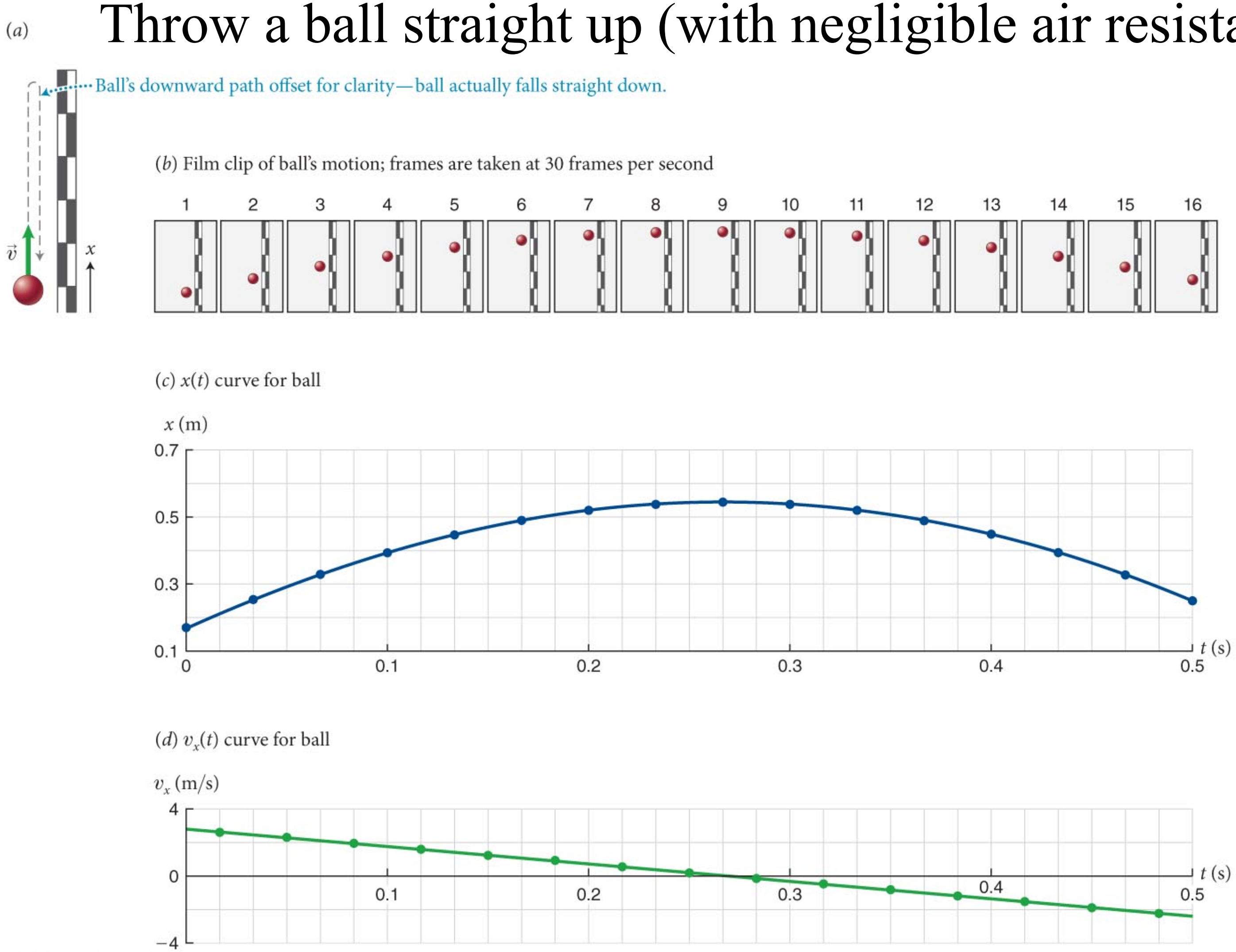
- Define the motion of objects that are launched but not self-propelled as projectile motion.
- Model the vertical trajectory of projectiles as objects that are in free fall.
- Represent projectile motion graphically using motion diagrams and motion graphs.

Section 3.3: Projectile motion

- An object that is launched but not self-propelled is called a **projectile**.
- Its motion is called **projectile motion**.
- The path the object follows is called its **trajectory**.

Section 3.3: Projectile motion

Throw a ball straight up (with negligible air resistance)

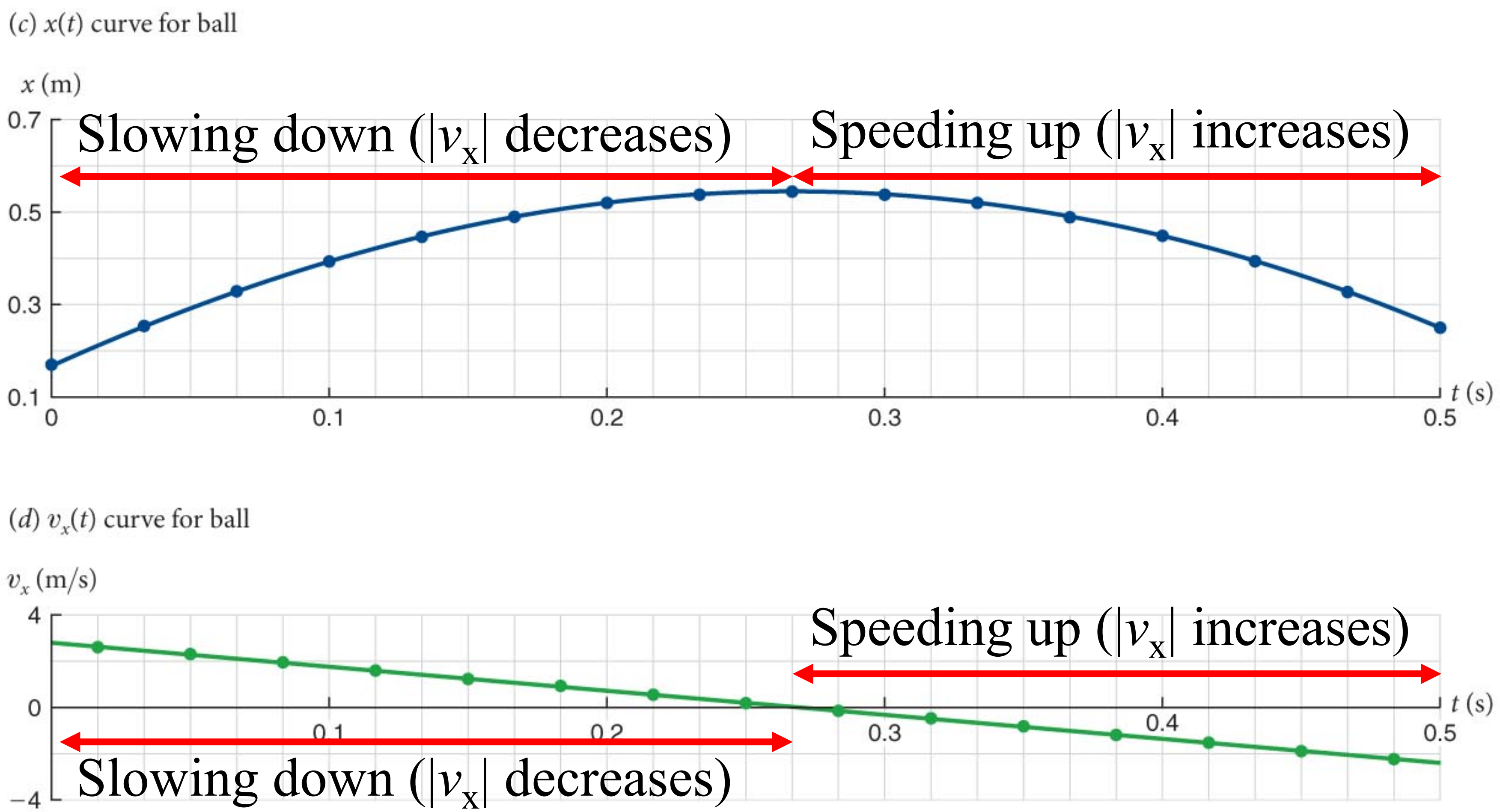


Section 3.3: Projectile motion

- Consider $x(t)$ and $v(t)$ curves:
  - **As the ball moves upward** it slows down
    - v and a are in opposite directions
    - since v is up, a must be down
  - **As the ball moves down** it speeds up
    - v and a must be in the same direction
    - since v is down, a is down
  - the **v (*t*) curve is a straight line** *for the whole motion*
    - slope approximately the *acceleration due to gravity*.

  - once the object is released, the rest of its motion is determined by gravity alone (free fall).

Section 3.3: Projectile motion



Checkpoint 3.8

3.8 Imagine throwing a ball downward so that it has an initial speed of 10 m/s.

What is its speed 1 s after you release it?

2 s after?

Constant acceleration: gain/lose same speed each second

- launched downward, so it speeds up
- $a \sim 10$ m/s^2, 1 second later: gain 10 m/s ➔ 20 m/s
- 2 seconds later: gain another 10 m/s ➔ 30 m/s

Section 3.3: Projectile motion

- What happens at the very top of the trajectory of a ball launched upward?
  - At the top, velocity changes from up to down, which means that acceleration must be nonzero.
  - At the very top, the instantaneous velocity is zero.
  - Acceleration, however, is nonzero.
  - *Acceleration is always ~9.8m/s*^{2}
  - *Remember: velocity can be zero while acceleration is not (and vice versa)*

Section 3.4: Motion diagrams

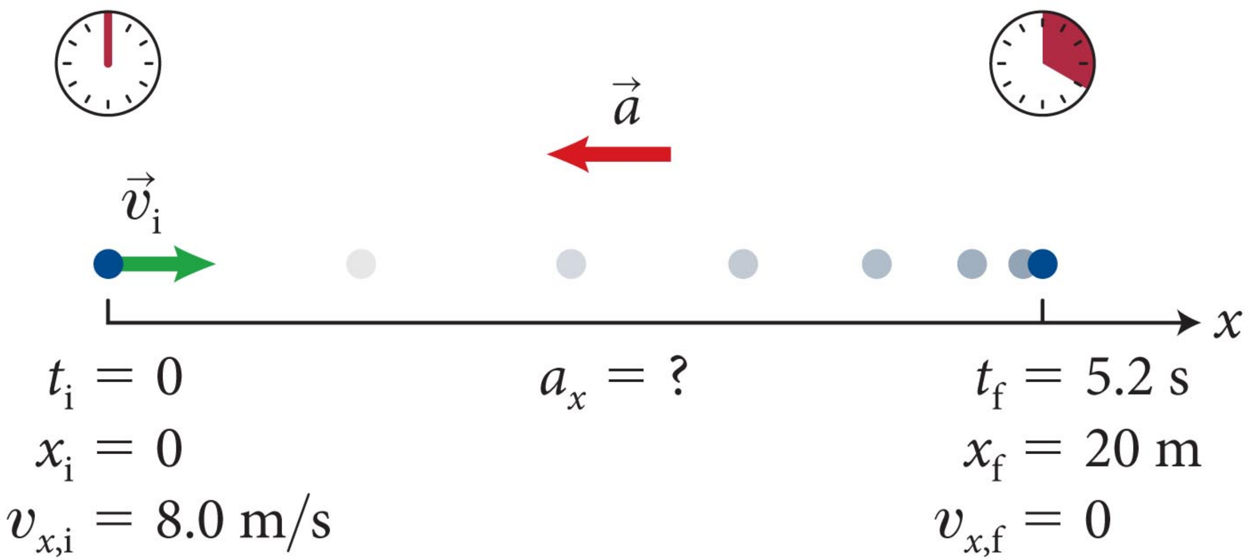
Section Goals

You will learn to

- Generalize the "frame sequence" diagram introduced in Chapter 2 to a new visual representation called a **motion diagram**.
- Represent and correlate the kinematic quantities, position, displacement, velocity, and acceleration on motion diagrams.

Section 3.4: Motion diagrams

- Motion diagrams are pictorial representations of objects in motion:
  - visualize the motion of an object described in a problem.
  - they show an object's *x*, *v*, and *a* at several equally spaced instances (including at the start and end).
  - it is basically a cartoon
- Below: a motion diagram for a bicycle with an initial velocity of 8.0 m/s slowing down to a stop.



Procedure: Analyzing motion using motion diagrams

Solving motion problems: a diagram summarizing what you have & what you want may all but solve the problem

1. Use dots to represent the moving object at equally spaced time intervals. If the object **moves at constant speed**, the dots are evenly spaced; if the object **speeds up**, the spacing between the dots increases; if the object **slows down**, the spacing decreases.
2. Choose an x (position) axis that is convenient for the problem. Most often this is an axis that (*a*) has its origin at the initial or final position of the object and (*b*) is oriented in the direction of motion or acceleration.

Procedure: Analyzing motion using motion diagrams (cont.)

3. Specify x & v at all relevant instants. **Particularly, specify**
   - **the *initial conditions*** - position and velocity at the beginning of the time interval of interest
   - **the *final conditions*** - position and velocity at the end of that time interval.
   - also note where v reverses direction or a changes.
   - unknown parameters = question mark.
4. Indicate the acceleration of the object between all the instants specified

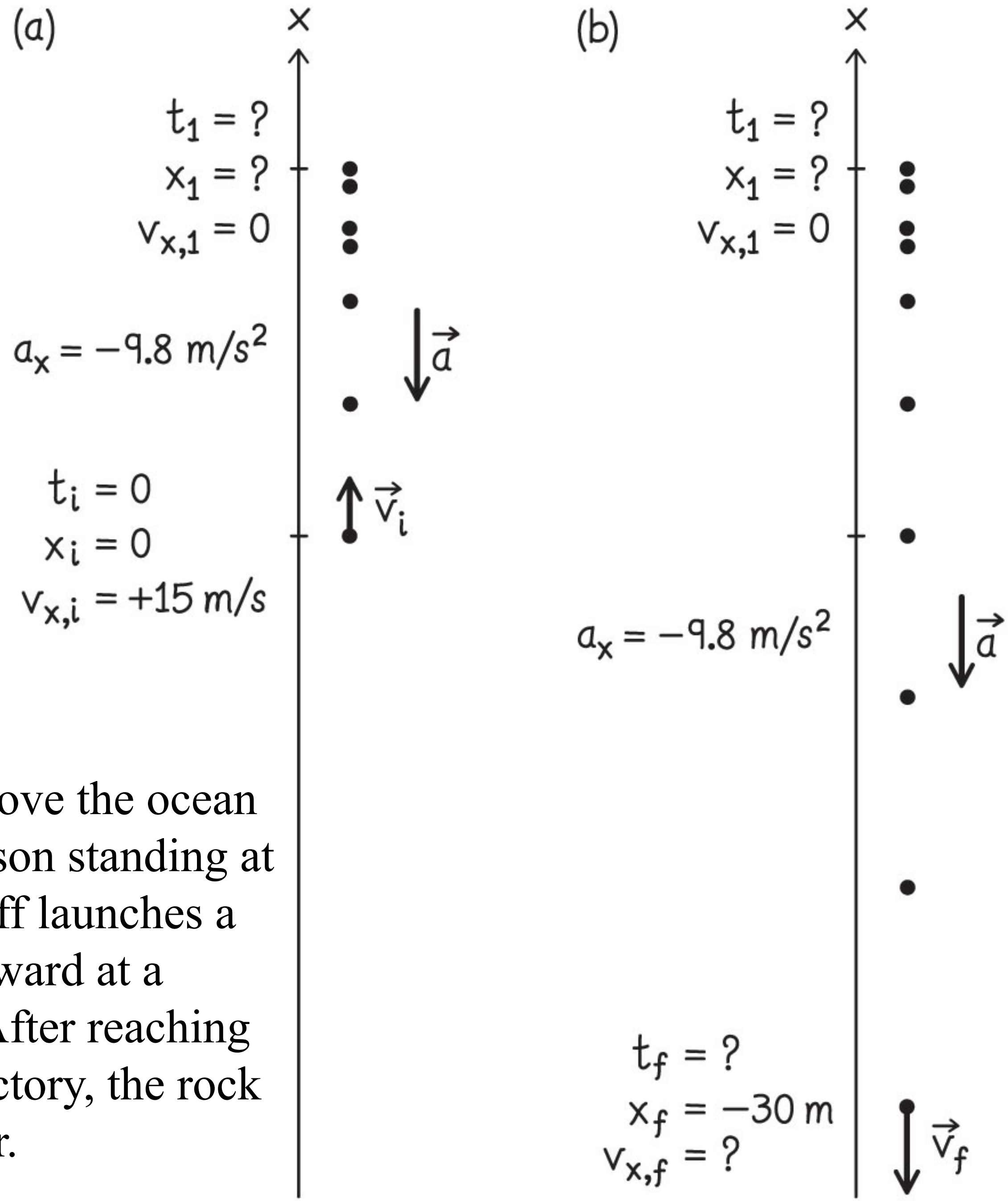
Procedure: Analyzing motion using motion diagrams (cont.)

5. With more than one object, draw separate diagrams side by side, using one common x axis.

6. If the object reverses direction, separate the motion diagram into two parts, one for each direction

Checkpoint 3.9

3.9 Make a motion diagram for the following situation: A seaside cliff rises 30 m above the ocean surface, and a person standing at the edge of the cliff launches a rock vertically upward at a speed of 15 m/s. After reaching the top of its trajectory, the rock falls into the water.



cliff rises 30 m above the ocean surface, and a person standing at the edge of the cliff launches a rock vertically upward at a speed of 15 m/s. After reaching the top of its trajectory, the rock falls into the water.

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Chapter 3: Self-Quiz #1

Two stones are released from rest at a certain height, one 1 s after the other.

(a) Once the second stone is released, does the difference in their speeds increase, decrease, or stay the same?

(b) Does their separation increase, decrease, or stay the same?

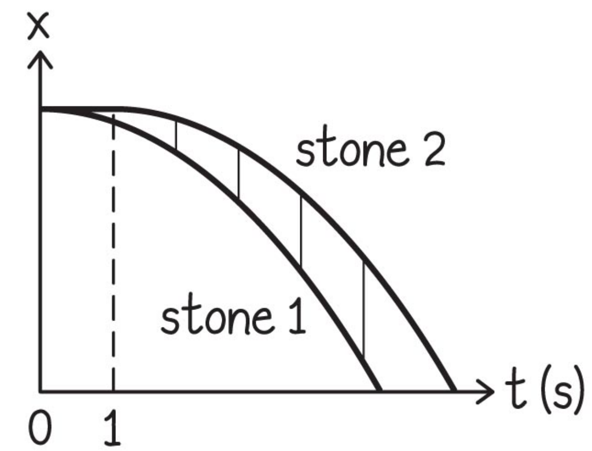
(c) Is the time interval between the instants at which they hit the ground less than, equal to, or greater than 1 s? (Use $x(t)$ curves to help you visualize this problem.)

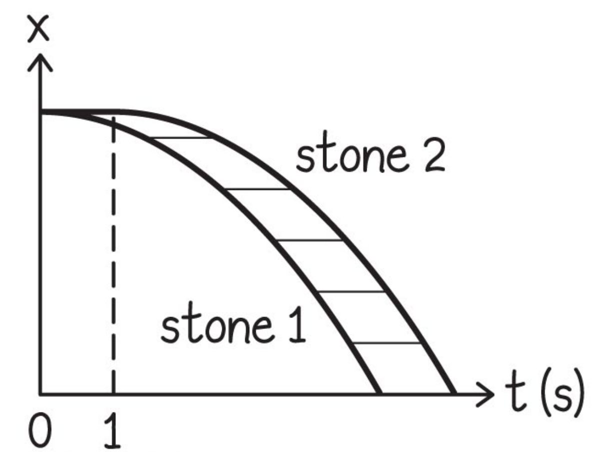
Answer

(a) Both stones accelerate at about 10 m/s^2, so the speeds increase at the same rate, thus the **difference in the speeds remains the same.**

(b) The separation increases because the speed of the first stone is always greater. As a result, for a given time interval the first stone always goes farther. (Position goes as *v* times *t*)

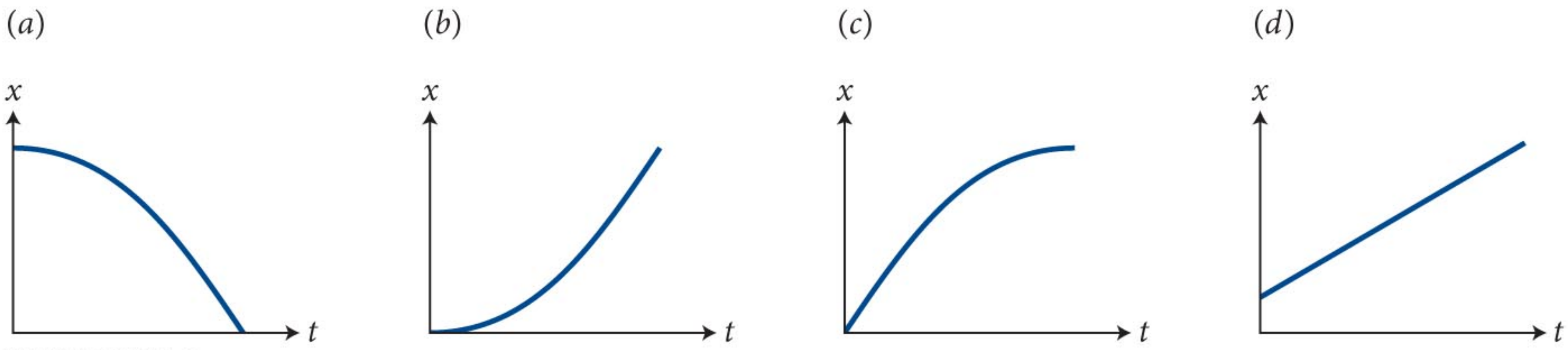
(c) the second stone always remains 1 s behind, this is how time works





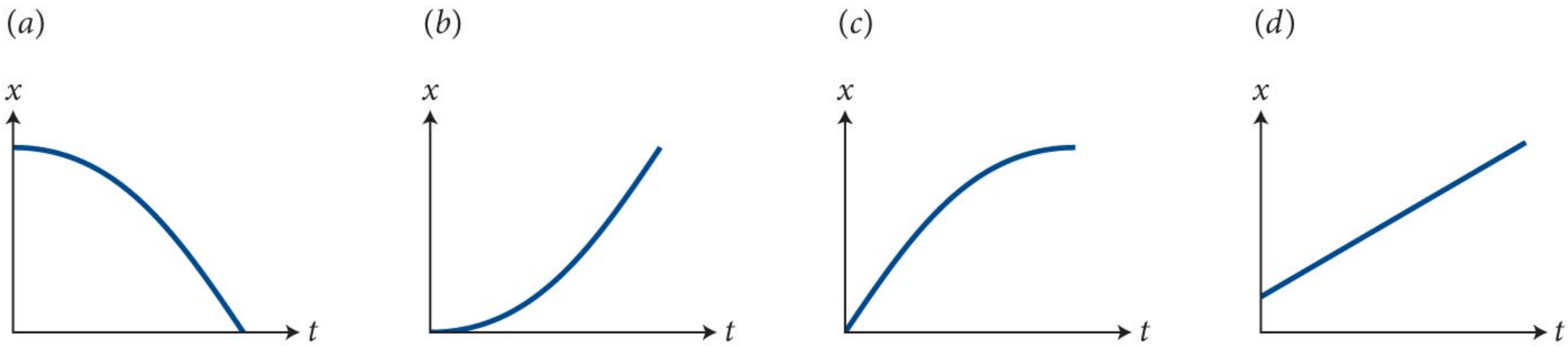
Chapter 3: Self-Quiz #2

Which of the graphs in Figure 3.12 depict(s) an object that starts from rest at the origin and then speeds up in the positive *x* direction?



Answer

Choice *b* is the correct answer because its initial position is zero and the slope is initially zero but then increasing, indicating that the object speeds up.



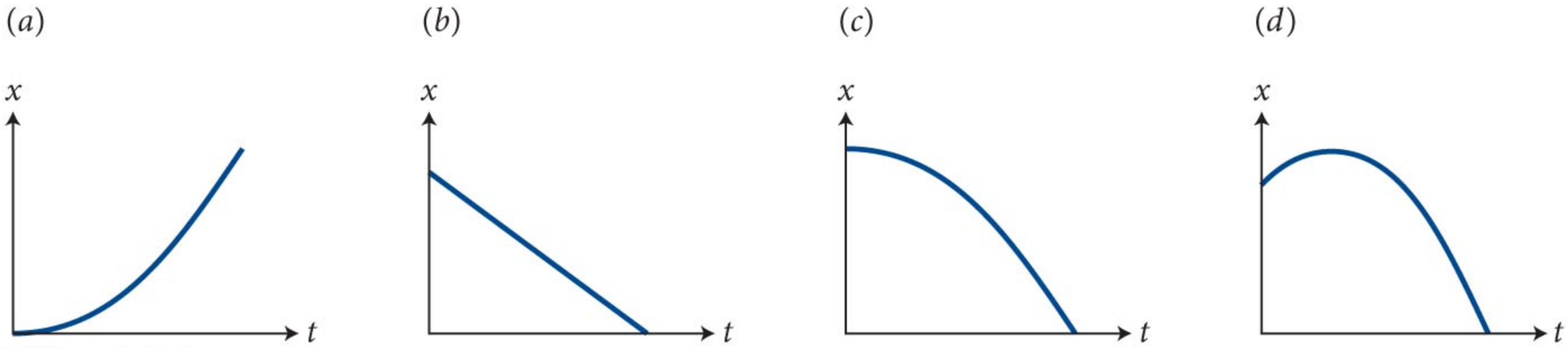
(a) speeds in –x
no start at origin

(c) slows

(d) no acceleration
no start at origin

Chapter 3: Self-Quiz #3

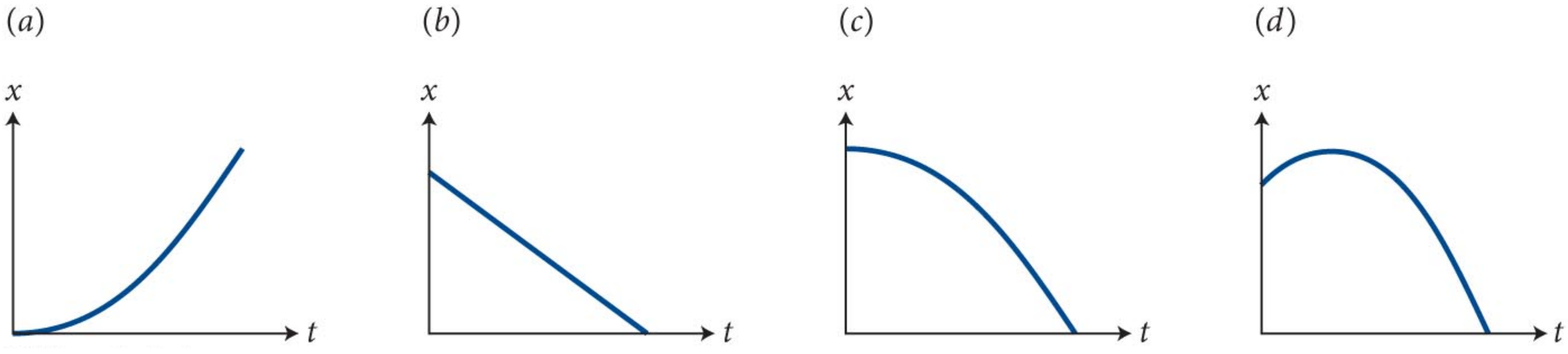
Which of the graphs in Figure 3.13 depict(s) an object that starts from a positive position with a positive x component of velocity and accelerates in the negative x direction?



Chapter 3: Self-Quiz #3

Answer

Choice *d*. Choice *a* does not have a positive initial position. Choice *b* represents zero acceleration. Choice *c* represents zero initial velocity.



Quantitative Tools

Section 3.5: Motion with constant acceleration

Section Goals

You will learn to

- Represent motion with constant acceleration using motion graphs and mathematics.
- Construct self-consistent position-versus-time, velocity-versus-time, and acceleration-versus-time graphs for specific motion situations.

Section 3.5: Motion with constant acceleration

- We can write down the definition for the x component of *average acceleration*:

$$a_{x,\text{av}} \equiv \frac{\Delta \upsilon_x}{\Delta t} = \frac{\upsilon_{x,\text{f}} - \upsilon_{x,\text{i}}}{t_\text{f} - t_\text{i}}$$

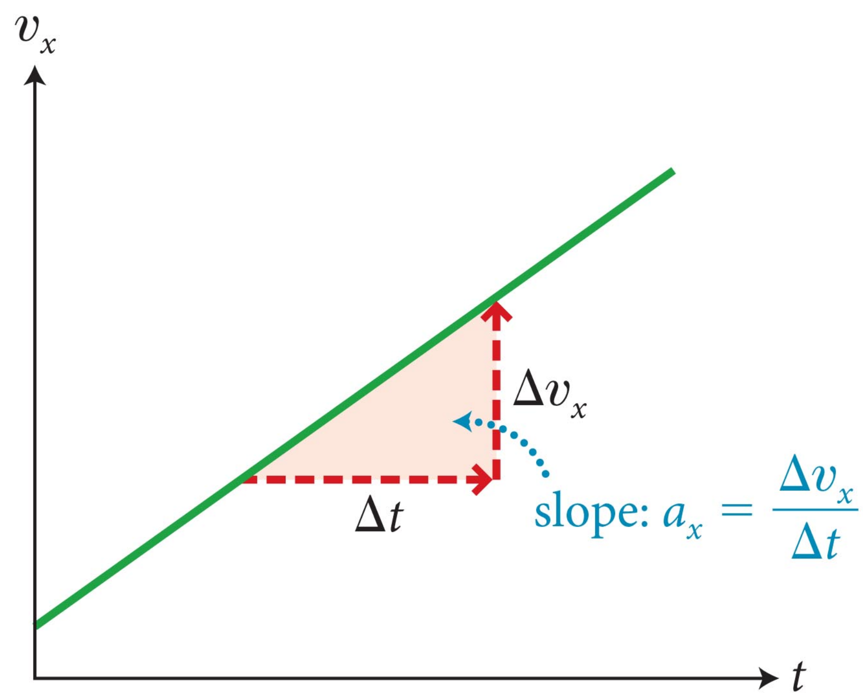
- Notice the similarity between this definition and the definition of average velocity in chapter 2:

$$\upsilon_{x,\text{av}} \equiv \frac{\Delta x}{\Delta t} = \frac{x_\text{f} - x_\text{i}}{t_\text{f} - t_\text{i}}$$

- *Rates of change* are our primary tools

Section 3.5: Motion with constant acceleration

- Now let us consider the motion of an object with **constant acceleration**:
  - For motion with constant acceleration, $a_{x,\mathrm{av}} = a_x$ and $v_x(t)$ curve is a straight line.
  - Rewriting our definition we can get the x-component of final velocity:



$$v_{x,\mathrm{f}} = v_{x,\mathrm{i}} + a_x \Delta t \quad \text{(constant acceleration)}$$

Section 3.5: Motion with constant acceleration

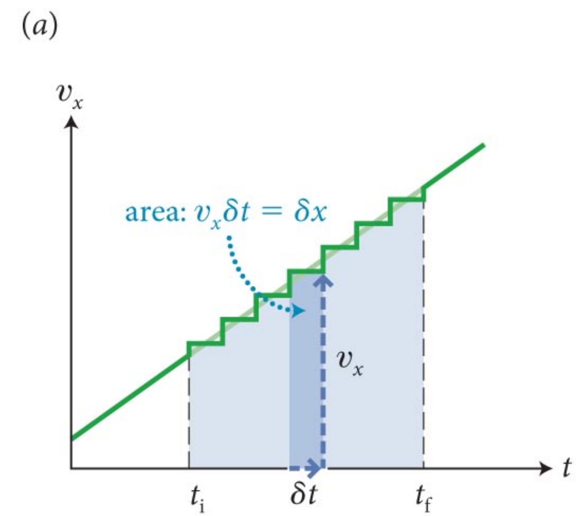
- displacement is the area under the $v_x(t)$ curve.

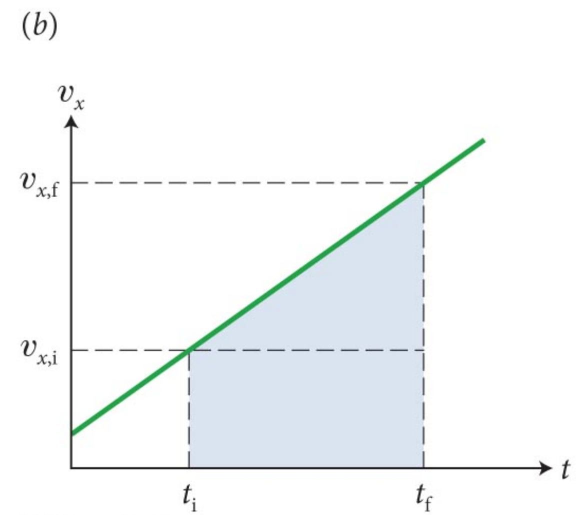
- for an object in motion with constant acceleration, the displacement ($\Delta x = x_f - x_i$) in time interval ($\Delta t = t_f - t_i$) is given by the area of the shaded trapezoid
- Setting $t_i = 0$, the object's final position can be written as

$$x_f = x_i + \upsilon_{x,i} t_f + \tfrac{1}{2} a_x t_f^2 \quad \text{(constant acceleration)}$$

- we can determine the object's final velocity

$$\upsilon_{x,f} = \upsilon_{x,i} + a_x t_f \quad \text{(constant acceleration)}$$





Section 3.5: Motion with constant acceleration

- Since t_f is an arbitrary instant in time in the object's motion, we can drop the subscript *f* and rewrite as

$$x(t) = x_i + \upsilon_{x,i} t + \tfrac{1}{2} a_x t^2 \quad \text{(constant acceleration)}$$

$$\upsilon_x(t) = \upsilon_{x,i} + a_x t \quad \text{(constant acceleration)}$$

- This is basically it for 1D motion!

- This is easier with calculus, assuming constant *a*

$$a = \frac{dv}{dt} \quad \Longrightarrow \quad v = \int a\,dt = at + C$$

- Noting $v(t = 0) = v_i$: $v(t) = v_i + at$

- Once more:

$$x = \frac{dv}{dt} \quad \Longrightarrow \quad x = \int v\,dt = v_i t + \frac{1}{2}at^2 + C'$$

- C' is $x(t = 0)$ or x_i

$$x(t) = x_i + v_i t + \frac{1}{2}at^2$$

In terms of displacement

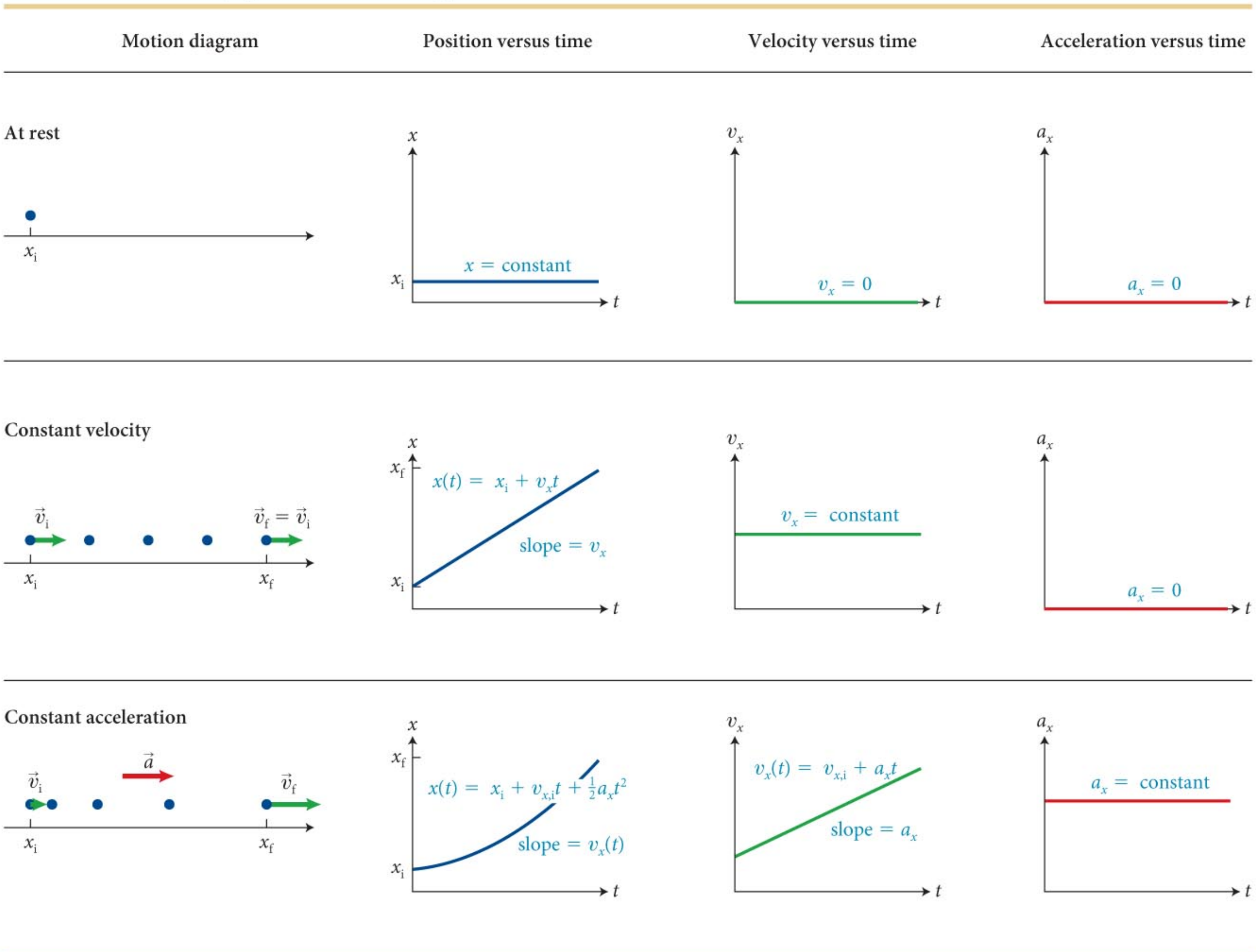
$$\Delta x = x_f - x_i = v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$

Sanity check: with no acceleration, distance = rate x time
set $a = 0$:

$$\Delta x = v_i \Delta t$$

Section 3.5: Motion with constant acceleration

Table 3.1 Kinematics graphs for three basic types of motion



Example 3.4 Collision or not?

You are bicycling at a steady 6.0 m/s when someone suddenly walks into your path 2.5 m ahead. You immediately apply the brakes, which slow you down at 6.0 m/s^2. Do you stop in time to avoid a collision?

Example 3.4 Collision or not? (cont.)

❶ GETTING STARTED

In order to avoid a collision, you must come to a stop in less than 2.5 m.

Need to calculate the distance traveled under the given conditions. Is it more or less than 2.5 m?

Example 3.4 Collision or not? (cont.)

❷ DEVISE PLAN I have equations for displacement, but I don't know the time interval Δt.

From the definition of acceleration: $a_{x,\text{av}} \equiv \dfrac{\Delta \upsilon_x}{\Delta t} = \dfrac{\upsilon_{x,\text{f}} - \upsilon_{x,\text{i}}}{t_\text{f} - t_\text{i}}$

$$\Delta t = (\upsilon_{x,\text{f}} - \upsilon_{x,\text{i}})/a_x$$

which contains no unknowns on the right side.

Example 3.4 Collision or not? (cont.)

❸ EXECUTE PLAN Substituting the expression for the time interval gives the *x* component of the displacement necessary to stop:

$$\Delta x = v_{x,i}\Delta t + \frac{1}{2}a_x(\Delta t)^2$$

$$\Delta x = \upsilon_{x,i}\underbrace{\frac{\upsilon_{x,f}-\upsilon_{x,i}}{a_x}}_{\Delta t} + \frac{1}{2}a_x\Bigg(\underbrace{\frac{\upsilon_{x,f}-\upsilon_{x,i}}{a_x}}_{\Delta t}\Bigg)^2 = \frac{\upsilon_{x,f}^2-\upsilon_{x,i}^2}{2a_x} \quad (1)$$

Example 3.4 Collision or not? (cont.)

❸ EXECUTE PLAN

With $+x$ along the direction of the motion

$$v_{x,\mathrm{i}} = +6.0 \text{ m/s}$$

$$v_{x,\mathrm{f}} = 0$$

$$a_x = -6.0 \text{ m/s}^2.$$

$$\Delta x = \frac{0 - (+6.0 \text{ m/s})^2}{2(-6.0 \text{ m/s}^2)} = +3.0 \text{ m}$$

more than the 2.5 m required. You will totally collide. ✔

Section 3.5: Motion with constant acceleration

- Notice: rearranging we can find the final velocity of an object under constant acceleration over a certain displacement (Δx):

$$\upsilon_{x,\mathrm{f}}^2 = \upsilon_{x,\mathrm{i}}^2 + 2a_x \Delta x \quad \text{(constant acceleration)}$$

- advantage: don't need to know time!
- In general: motion is *overdetermined* by x, v, a, t
  – only need 3 of 4
  – can eliminate 1 variable

Checkpoint 3.10

3.10 Determine the velocity of the stone dropped from the top of the Empire State Building in Example 3.2 just before the stone hits the ground.

(The empire state building is ~300m tall)

$$v_{x,\mathrm{f}}^2 = v_{x,\mathrm{i}}^2 + 2a_x \Delta x \quad \text{(constant acceleration)}$$

initial velocity is zero

displacement ~ 300 m

acceleration ~ 10 m/s^2

$$v_{x,f} = \sqrt{v_{x,i}^2 + 2a_x \Delta x} = \sqrt{0 + 2(10\,\mathrm{m/s^2})(300\,\mathrm{m})} \approx 80\,\mathrm{m/s}$$

Section 3.6: Free-fall equations

Section Goals

- Model free-fall motion using the concept of gravity and the definitions of velocity and acceleration.
- Manipulate the equations for free-fall into a form that allows the prediction of the future motion of an object from its present state of motion.

Section 3.6: Free-fall equations

- The magnitude of the **acceleration due to gravity** is designated by the letter *g*:

$$g \equiv \left|\vec{a}_{\text{free fall}}\right|$$

  - Near Earth's surface $g = 9.8\ \text{m/s}^2$.
  - The direction of the acceleration is downward, and **if we chose a positive axis pointing upward**, $a_x = -g$.
- If an object is dropped from a certain height with zero velocity along an **upward-pointing *x*-axis**, then

$$x_{\text{f}} = x_{\text{i}} - \tfrac{1}{2} g t_{\text{f}}^2$$

$$\upsilon_{x,\text{f}} = -g t_{\text{f}}$$

Example 3.5 Dropping the ball

Suppose a ball is dropped from height $h = 20$ m above the ground. How long does it take to hit the ground, and what is its velocity just before it hits?

Example 3.5 Dropping a ball (cont.)

❶ GETTING STARTED

x axis that points upward

origin at the initial position of the ball

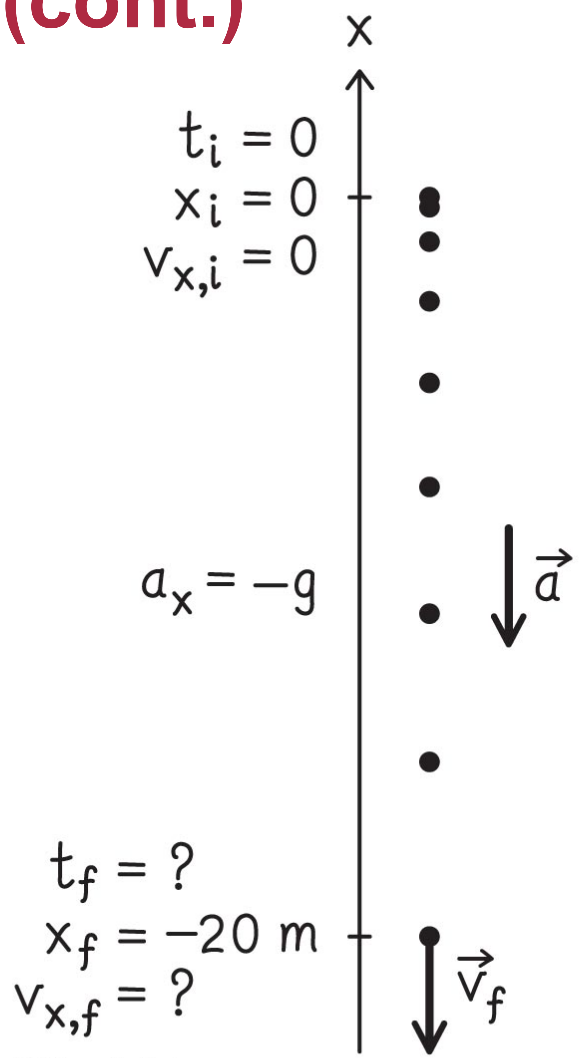
assumptions:

released from rest ($v_{x,\mathrm{i}} = 0$ at $t_\mathrm{i} = 0$)

ignore air resistance

initial conditions are

$$t_\mathrm{i} = 0,\ x_\mathrm{i} = 0,\ v_{x,\mathrm{i}} = 0$$



Example 3.5 Dropping a ball (cont.)

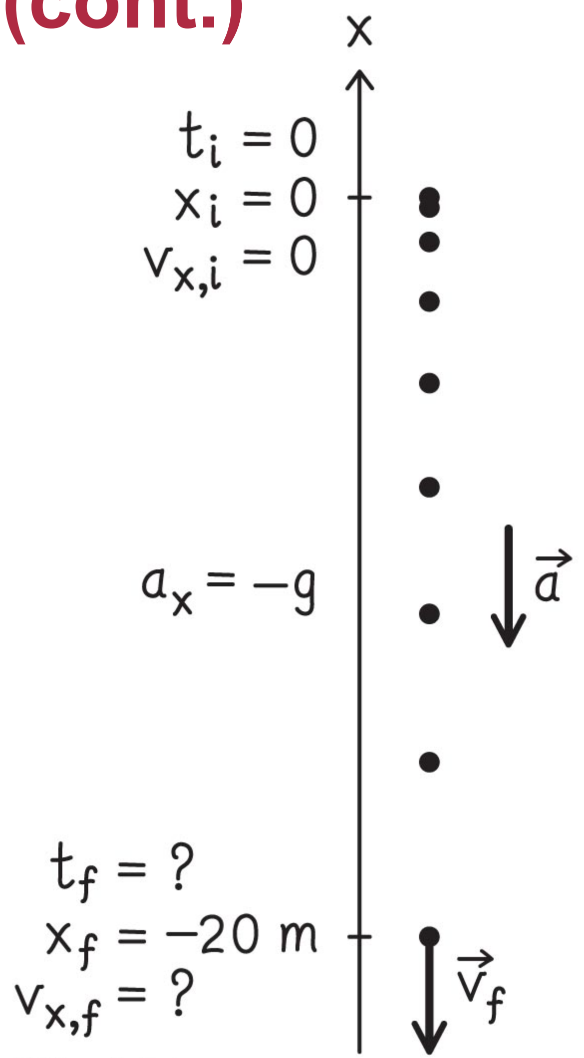
❶ GETTING STARTED

final position x_f at instant t_f is a distance h below the initial position

just before impact at instant t_f, the final conditions are

$$t_f = ?,\ x_f = -h,\ v_{x,f} = ?$$

acceleration is negative, $a_x = -g$.



Example 3.5 Dropping a ball (cont.)

❷ DEVISE PLAN

Acceleration is constant, so our equations are valid.

Gives us two equations & two unknowns: t_f and $v_{x,f}$

$$x(t) = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_f = v_i + at$$

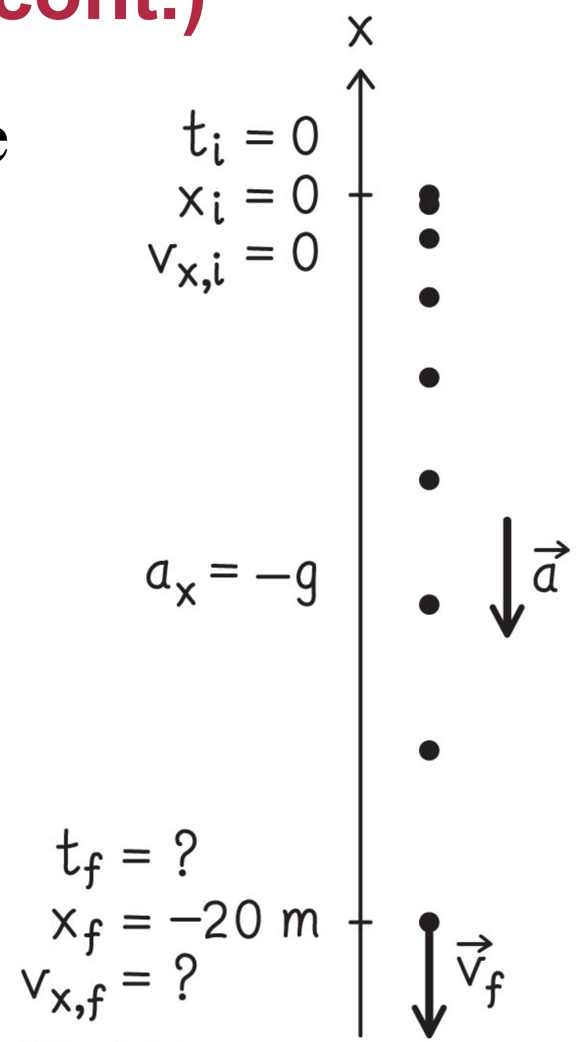
Example 3.5 Dropping a ball (cont.)

❸ EXECUTE PLAN Substituting the initial and final conditions into $x(t)$:

$$-h = 0 + 0 - \tfrac{1}{2} g t_f^2 = -\tfrac{1}{2} g t_f^2$$

and so

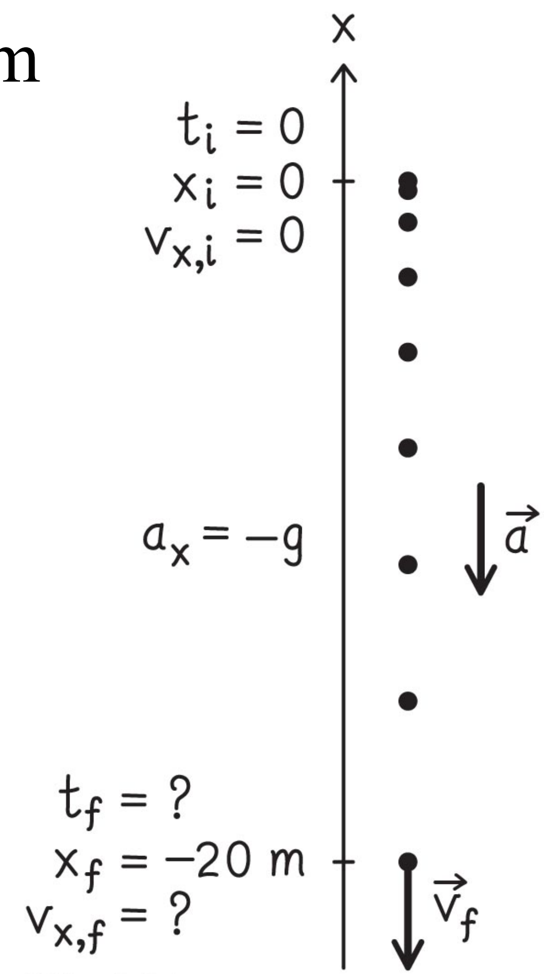
$$t_f = \sqrt{\frac{2h}{g}}$$



Example 3.5 Dropping a ball (cont.)

❸ EXECUTE PLAN Substituting $h = 20$ m and $g = 9.8$ m/s^2 into $v(t)$:

$$\Delta t = t_f - t_i = \sqrt{\frac{2h}{g}} - 0 = \sqrt{\frac{2(20\ \text{m})}{9.8\ \text{m/s}^2}} = \sqrt{4.0\ \text{s}^2} = 2.0\ \text{s}$$

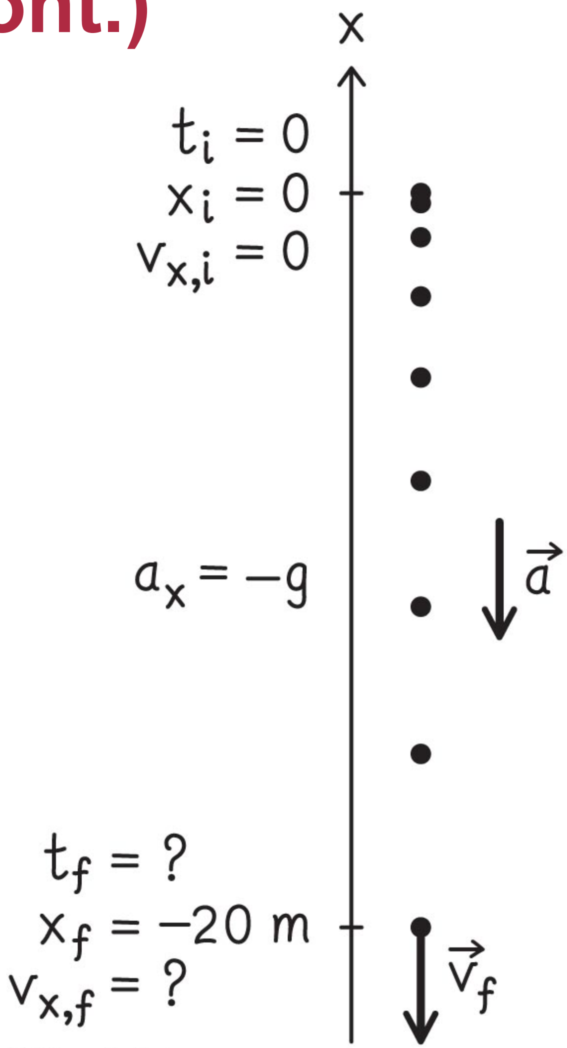


Example 3.5 Dropping a ball (cont.)

❸ EXECUTE PLAN

Because the ball starts from rest:

$$v_{x,\mathrm{f}} = 0 - gt_\mathrm{f} = -gt_\mathrm{f} = -(9.8\ \mathrm{m/s^2})(2.0\ \mathrm{s})$$
$$= -20\ \mathrm{m/s}. ✔$$



Example 3.5 Dropping a ball (cont.)

❹ EVALUATE RESULT

Time is reasonable based on everyday experience

Final velocity $\Delta x_f = -20$ m/s also makes sense:

- negative because it points in the negative x direction
- if the ball was at a constant speed of 20 m/s, it would cover the 20-m distance in 1 s. It moves at that speed only at the end of the drop, so it takes longer to fall.

- Know initial $v_{x,i}$ and a, so could also have used

$$\upsilon_{x,f}^2 = \upsilon_{x,i}^2 + 2a_x\Delta x \quad \text{(constant acceleration)}$$

Section 3.7: Inclined planes

Section Goals

You will learn to

- Identify that one-dimensional motion along an incline plane can be related to free-fall motion along a non-vertical direction.
- Establish that purely horizontal and purely vertical motion are the special cases of motion along an incline plane.

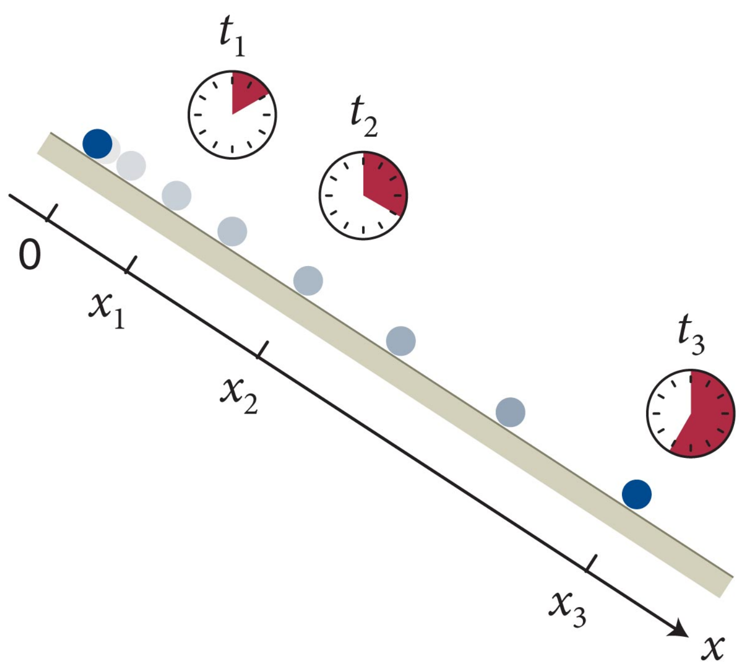
Section 3.7: Inclined planes

- Galileo used inclined planes to study motion of objects that are accelerated due to gravity:
  - He found that when a ball rolls down an incline starting at rest, the ratio of the distance traveled to the square of the amount of time needed to travel that distance is constant:

$$\frac{x_1}{t_1^2} = \frac{x_2}{t_2^2} = \frac{x_3}{t_3^2}$$

  - Using this and setting $x_i = 0$ and $t_i = 0$ we can show that this ratio is proportional to a_x:

$$\frac{x_f}{t_f^2} = \tfrac{1}{2} a_x$$



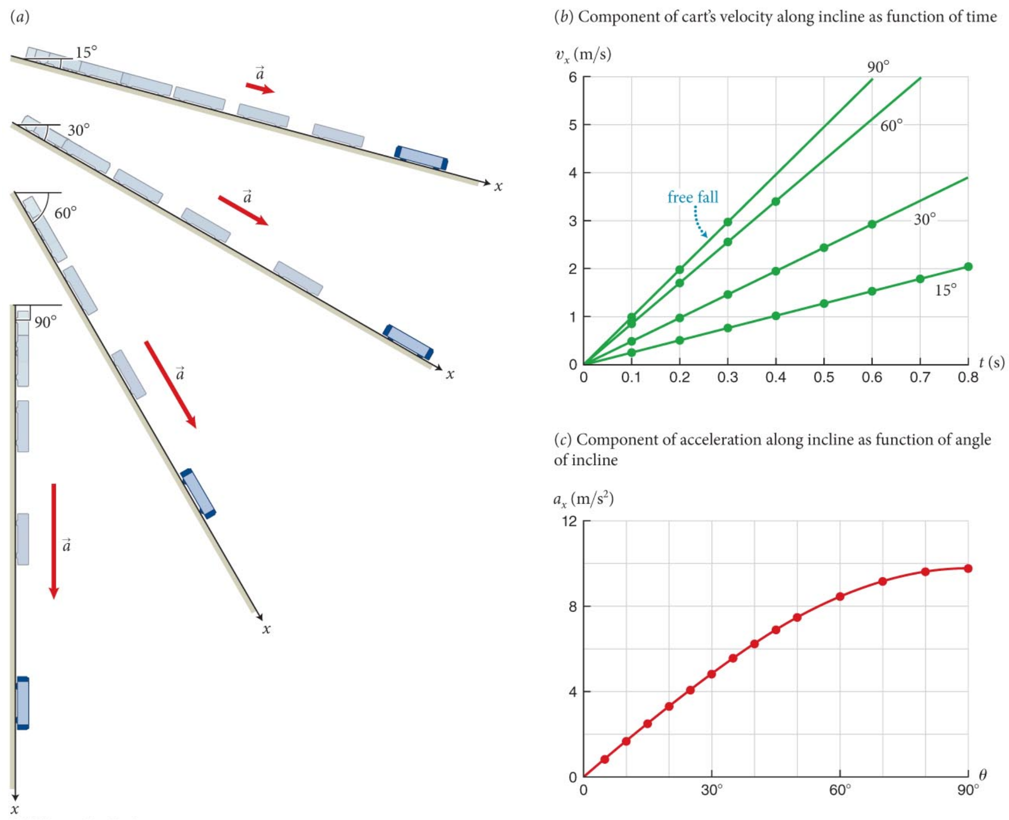
(from)

$$x(t) = x_i + v_i t + \frac{1}{2} a t^2$$

Section 3.7: Inclined planes

- Galileo observed that
  - For each value of the angle θ, a_x along the incline is a constant.
  - a_x along the incline increases as θ increases.
  - Experimentally we can determine that the x component of the acceleration along the incline obey the relationship

$$a_x = +g \sin \theta$$



Inclined planes

- establishes gravity is vertical, constant acceleration
- it is a vector, and only the *vertical component* matters
- for inclined plane, the component along the plane is $a_x = +g \sin \theta$
- (but sign depends on whether you go up or down the incline)

Section 3.8: Instantaneous acceleration

Section Goals

You will learn to

- Generalize the mathematical definition of the average acceleration of a moving object to instantaneous acceleration by use of a limiting process.
- Represent motion with continuous changes in velocity using motion graphs and mathematics.
- Relate the concept of a tangent line on a velocity-versus-time graph with the instantaneous acceleration.

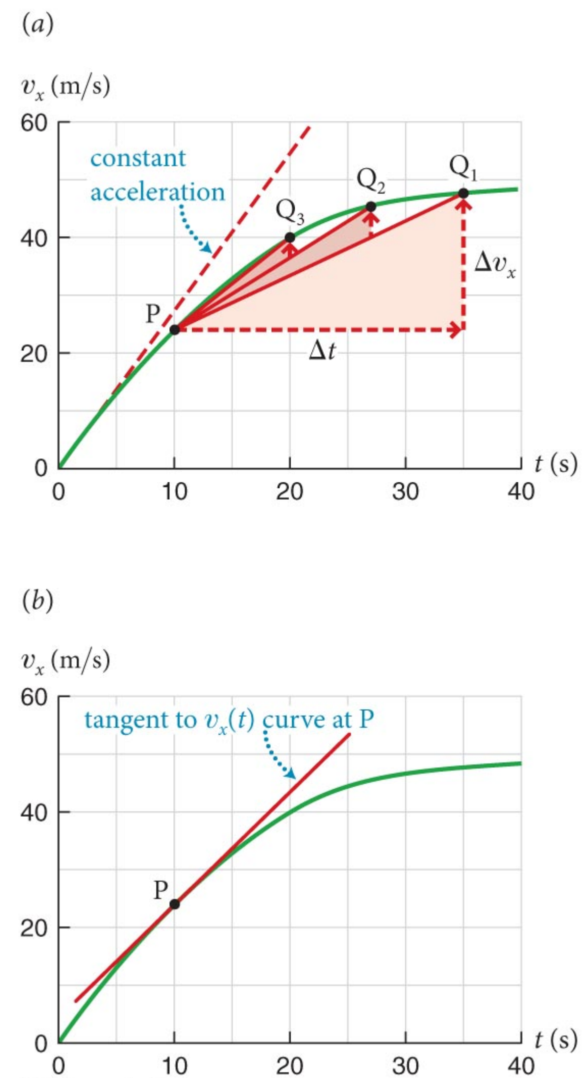
Section 3.8: Instantaneous acceleration

- What if acceleration is not constant? Use the calculus we did.
  - The figure shows the $v_x(t)$ curve for a motion where the acceleration is not constant.
  - The instantaneous acceleration a_x is the slope of the tangent of the $v_x(t)$ curve at time t:

$$a_x = \frac{dv_x}{dt}$$

  - Or

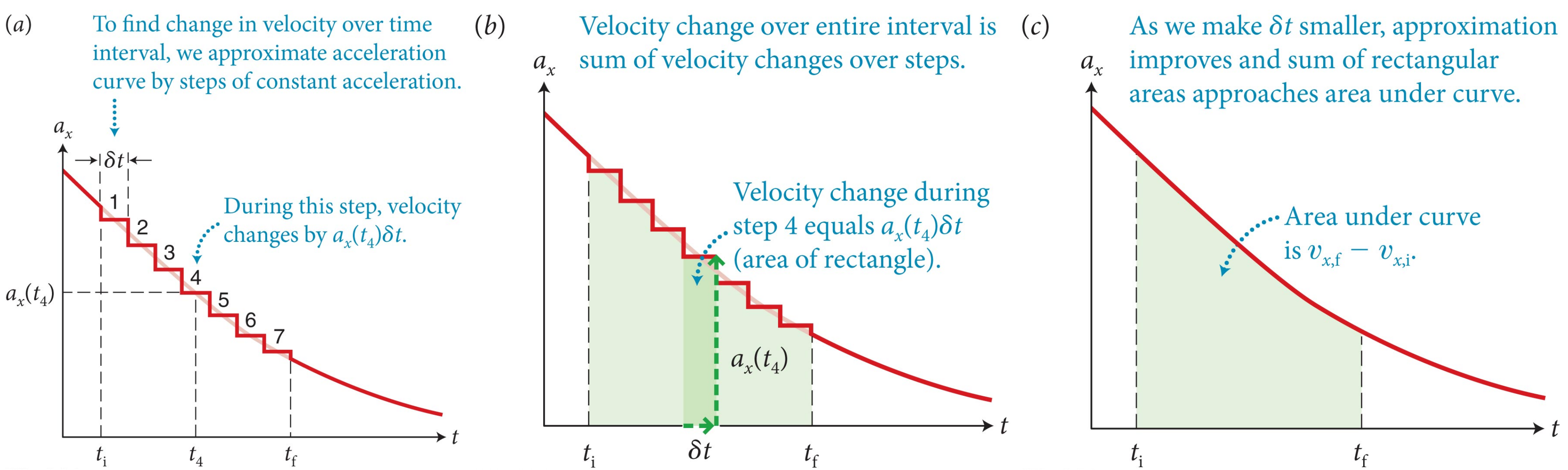
$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) \equiv \frac{d^2x}{dt^2}$$



Section 3.8: Instantaneous acceleration

- To find the change in velocity during the time interval (Δt), we can use the area under the $a_x(t)$ curve in the figure.
- Although, acceleration is not constant, we can divide motion into small intervals of Δt in which it is constant.
- In the limit $\Delta t \longrightarrow 0$, we can find

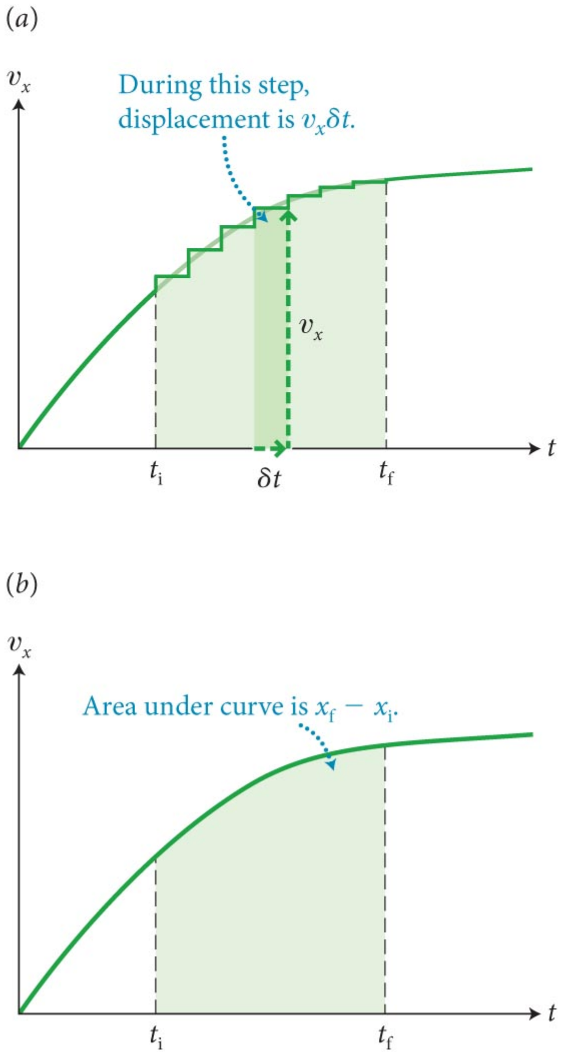
$$\Delta \upsilon_x = \int_{t_i}^{t_f} a_x(t)\,dt$$



Section 3.8: Instantaneous acceleration

- Once we know the velocity, we can use the same approach to obtain displacement:

$$\Delta x = \int_{t_i}^{t_f} v_x(t)\,dt$$



Checkpoint 3.14

3.14 Take the first and second time derivatives of x_f in Eq. 3.9. What do you notice?

$$x_f = x_i + v_{x,i} t_f + \tfrac{1}{2} a_x t_f^2 \quad \text{(constant acceleration)}$$

$$\frac{dx_f}{dt_f} = v_{x,i} + a_x t_f$$

$$\frac{d^2 x_f}{dt_f^2} = a_x$$

Concepts: Accelerated motion

- If the velocity of an object is changing, the object is **accelerating**. The x component of an object's **average acceleration** is the change in the x component of its velocity divided by the time interval during which this change takes place.
- The x component of the object's **instantaneous acceleration** is the x component of its acceleration at any given instant.
- A **motion diagram** shows the positions of a moving object at equally spaced time intervals.

Chapter 3: Summary

Quantitative Tools: Accelerated motion

- The x component of the **average acceleration** is

$$a_{x,\text{av}} \equiv \frac{\Delta \upsilon_x}{\Delta t} = \frac{\upsilon_{x,\text{f}} - \upsilon_{x,\text{i}}}{t_\text{f} - t_\text{i}}$$

- The x component of the **instantaneous acceleration** is

$$a_x \equiv \frac{d\upsilon_x}{dt} = \frac{d^2x}{dt^2}$$

- The x component of the change in velocity over a time interval is given by

$$\Delta \upsilon_x = \int_{t_\text{i}}^{t_\text{f}} a_x(t)\,dt$$

- The x component of the displacement over a time interval is given by

$$\Delta x = \int_{t_\text{i}}^{t_\text{f}} \upsilon_x(t)\,dt$$

Concepts: Motion with constant acceleration

- If an object has constant acceleration, the $\upsilon_x(t)$ curve is a straight line that has a nonzero slope and the $a_x(t)$ curve is a horizontal line.

Quantitative Tools: Motion with constant acceleration

- If an object moves in the x direction with constant acceleration a_x starting at $t = 0$, with initial velocity $\upsilon_{x,\mathrm{i}}$ at initial position x_i, its x coordinate at any instant t is given by

$$x(t) = x_\mathrm{i} + \upsilon_{x,\mathrm{i}} t + \tfrac{1}{2} a_x t^2$$

- The x component of its instantaneous velocity is given by

$$\upsilon_x(t) = \upsilon_{x,\mathrm{i}} + a_x t$$

- And the x component of its final velocity is given by

$$\upsilon_{x,\mathrm{f}}^2 = \upsilon_{x,\mathrm{i}}^2 + 2a_x \Delta x$$

Concepts: Free fall and projectile motion

- An object subject only to gravity is in **free fall.** All objects in free fall near the surface of Earth have the same acceleration, which is directed downward. We call this acceleration the **acceleration due to gravity** and denote its magnitude by the letter *g*.
- An object that is launched but not self-propelled is in **projectile motion.** Once it is launched, it is in free fall. The it follows is called its **trajectory.**

Quantitative Tools: Free fall and projectile motion

- The magnitude g of the downward acceleration due to gravity is

$$g = \left|\vec{a}_{\text{free fall}}\right| = 9.8 \text{ m/s}^2 \text{ (near Earth's surface)}$$

Chapter 3: Summary

Concepts: Motion along an inclined plane

- An object moving up or down an inclined plane on which friction is negligible has a constant acceleration that is directed parallel to the surface of the plane and points downward along the surface.

Quantitative Tools: Free fall and projectile motion

- When friction is negligible, the x component of acceleration a_x for an object moving on an inclined plane that rises at an angle θ above the horizontal is

$$a_x = +g \sin \theta$$

when the x axis is directed downward along the plane.