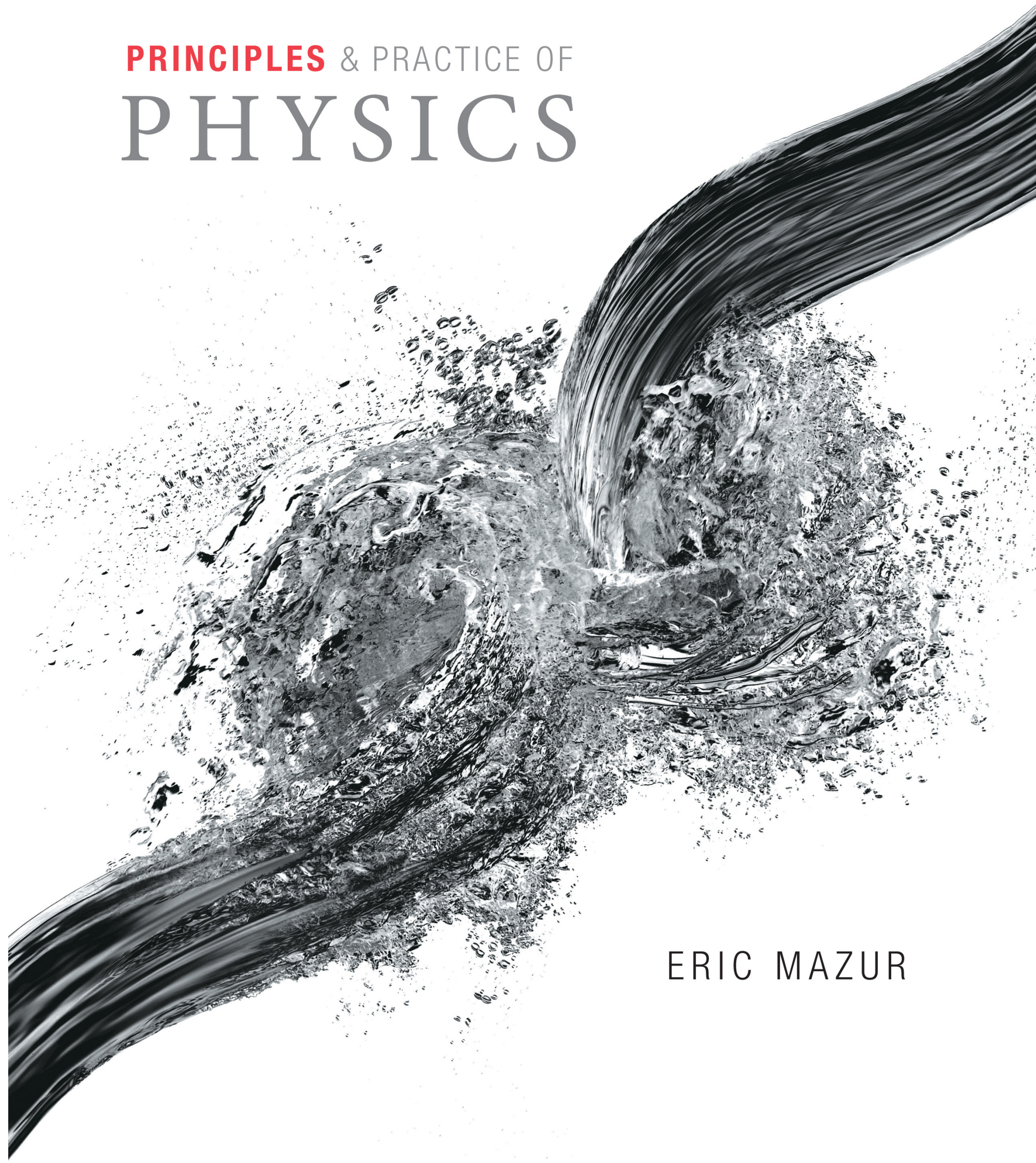
Chapter 4
Momentum



Things

Various & Sundry topics

- labs

    read procedure carefully – what is being asked?

    “human error” is not a thing. do it again ☺

    there *are* important reasons for inaccuracy

- office hours

    you can show up. you are not bothering me

    MWF 1-2, TuTh 3:15-4

Reading quiz for today

question 1

A freight car moves along a frictionless level railroad track at constant speed. The freight car is open on the top. A large load of sand is suddenly dumped into the freight car. What happens to the speed of the freight car?

- The speed of the freight car remains the same.
- The speed of the freight car increases.
- The speed of the freight car decreases.
- It is impossible to determine whether the speed of the freight car increases or decreases from the information given.

Reading quiz for today

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- The speed of the freight car increases.
- The speed of the freight car decreases.
- It is impossible to determine whether the speed of the freight car increases or decreases from the information given.

Momentum is conserved (mv). If inertia increases, speed must decrease to keep mv constant

Reading quiz for today

Question 2

When a person steps forward out of a small boat onto a dock, the boat recoils backward in the water. Why does this occu

- The energy of the person is greater than the energy of the boat.
- The momentum of the person is greater than the momentum of the boat.
- The total mechanical energy of the system is conserved.
- The momentum of the boat is greater than the momentum of the person.
- The energy of the boat is greater than the energy of the person.
- The total momentum of the system is conserved.

Reading quiz for today

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- The total mechanical energy of the system is conserved.
- The momentum of the boat is greater than the momentum of the person.
- The energy of the boat is greater than the energy of the person.
- ☆ The total momentum of the system is conserved.

Momentum is conserved (mv). It starts at zero. When the person steps forward, this new momentum must be countered by an equal and opposite momentum.

Reading quiz for today

Question 3

A golf ball has one-tenth the inertia and six times the speed of a baseball.

Part A

What is the ratio of the magnitudes of their momenta?

Express your answer using three significant digits.

$\frac{p_g}{p_b}$ =

Reading quiz for today

Question 3

A golf ball has one-tenth the inertia and six times the speed of a baseball.

Part A

What is the ratio of the magnitudes of their momenta?

Express your answer using three significant digits.

$$m_g = 0.1m_b \text{ and } v_g = 6v_b$$
$$p_g/p_b = (0.1m_b \text{ x } 6v_b)/(m_b \text{ x } v_b) = 0.600$$

Homework

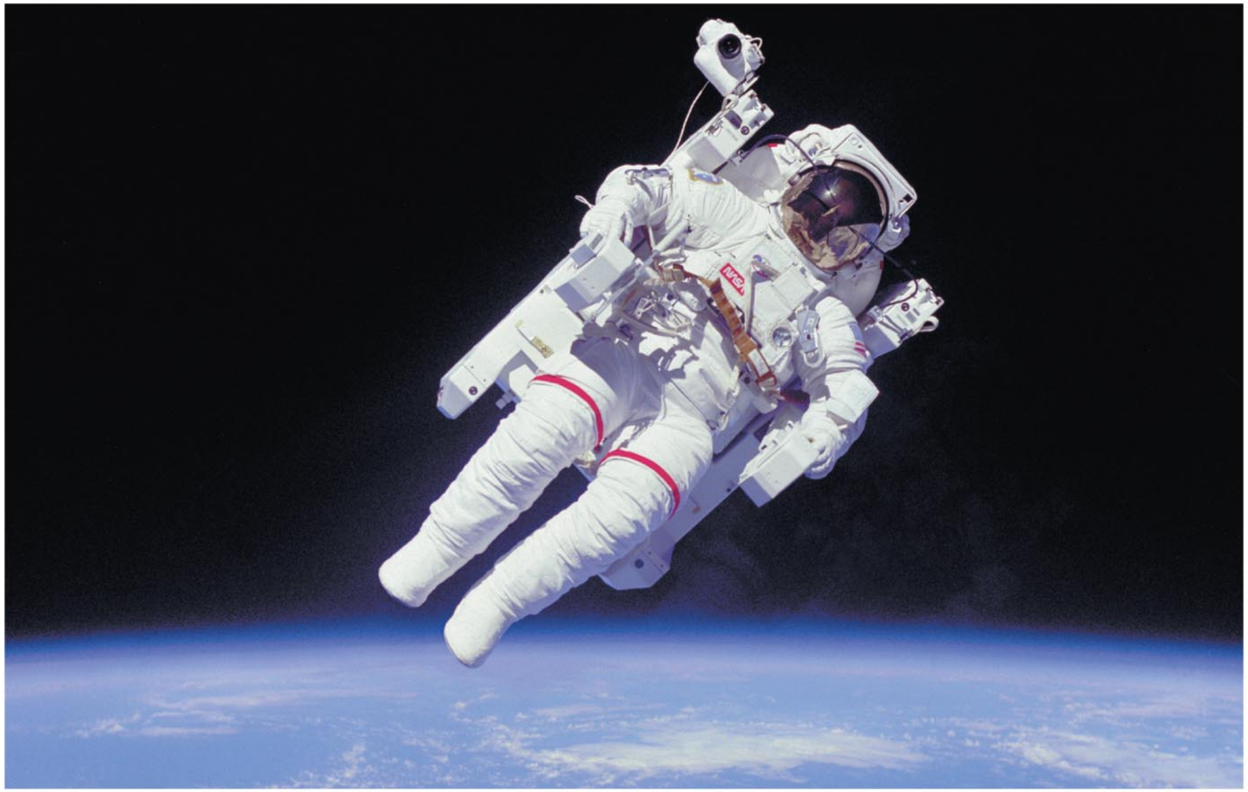
3.54

- Handle the rising and falling portions separately – different accelerations
- First figure out the maximum height, then figure out how fast it reaches the ground falling from that height.
- On the way up v_i=0 (starts from rest) and x_i=0 (choice).
- $x_{top} = ½ at^2$, a=4g. $v_{top} = at$.

- On the way down, the rocket is in free fall, and a=g.
- You know v_{top}, and it falls through a height x_{top}. Equation without time …

- If we make the positive direction upward, x_{top} is negative (downward displacement), but so is the acceleration, so the signs cancel out.

Chapter 4: Momentum



Chapter Goal: To begin a theoretical analysis of motion using the concepts of inertia and momentum.

Usually we do concepts first and worry about math later

If you don’t have an idea how it works already, math won’t help

Section 4.1: Friction

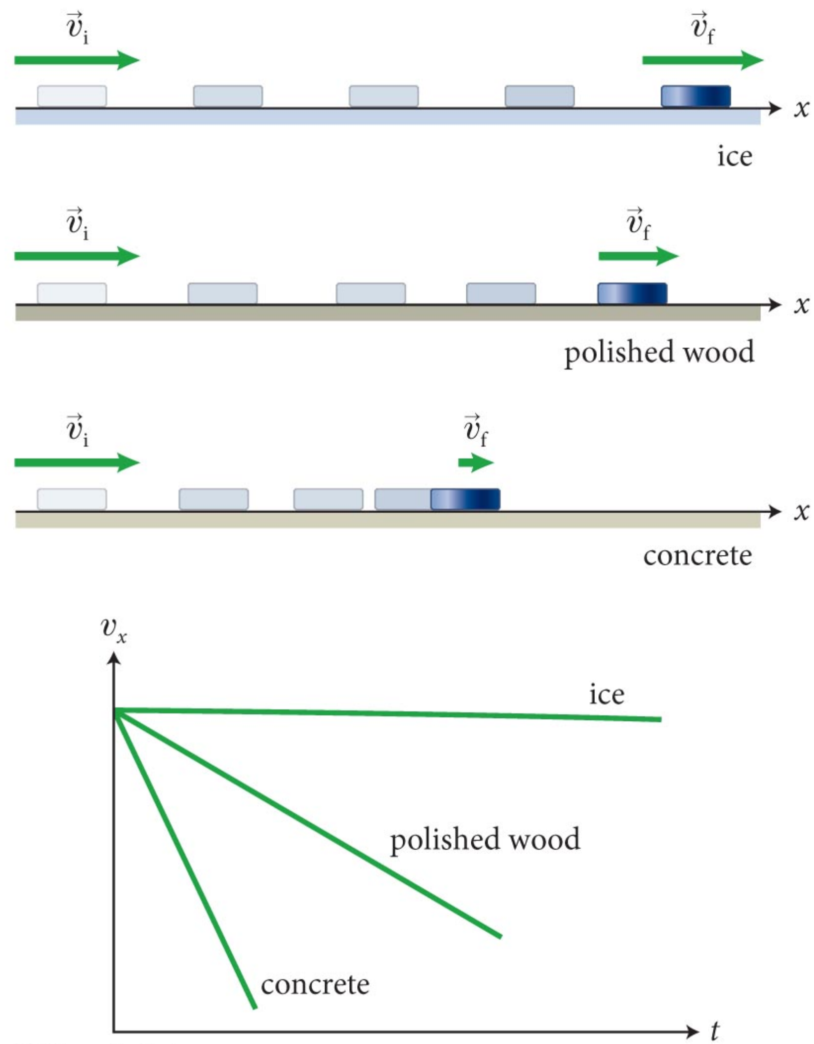
Section Goals

- Identify the effects of **friction** on kinematics
- Generalize the situation of vanishing friction
  fundamental vs circumstantial

- In the absence of friction, objects moving along a horizontal surface keep moving without slowing down.

- *Your intuition includes friction*

- wooden block on three different surfaces.
  - The slowing down is due to **friction**—the resistance to motion that one surface (or object) encounters when moving over another.
  - The block slides easily over ice because there is little friction between the two surfaces.
  - The lower the friction, the longer it takes for the block to come to rest.

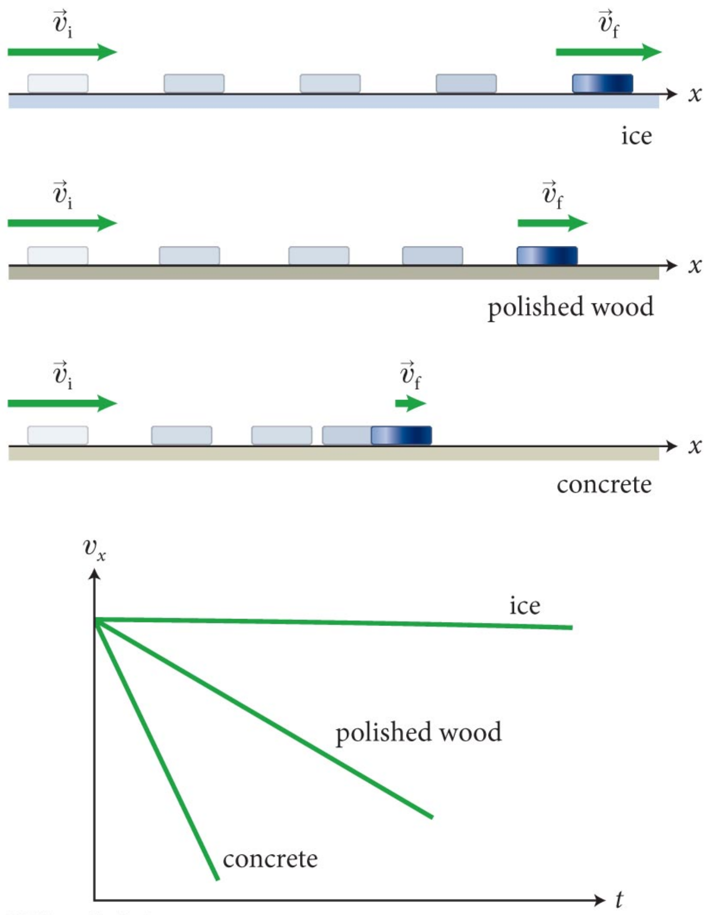


Section 4.1: Friction

- What happens if there is no friction?
  - **In the absence of friction, objects moving along a horizontal track keep moving without slowing.**
  - There are no totally frictionless surfaces.
  - However, using an air track, we can minimize the friction to the point where it can be ignored during an experiment.

Checkpoint

Based on the $v_x(t)$ curves, what specifically is friction doing? What else gives a linear decrease in velocity?



Checkpoint

Velocity decreases at a constant rate?

this means friction causes a *constant acceleration*

it is in the direction opposing motion

Later: this means friction is a *constant force*

How does it work?

interaction of surfaces … irregularities … van der Waals

What does it depend on?

everything. materials, surface finish, air …

Section 4.2: Inertia

Section Goals

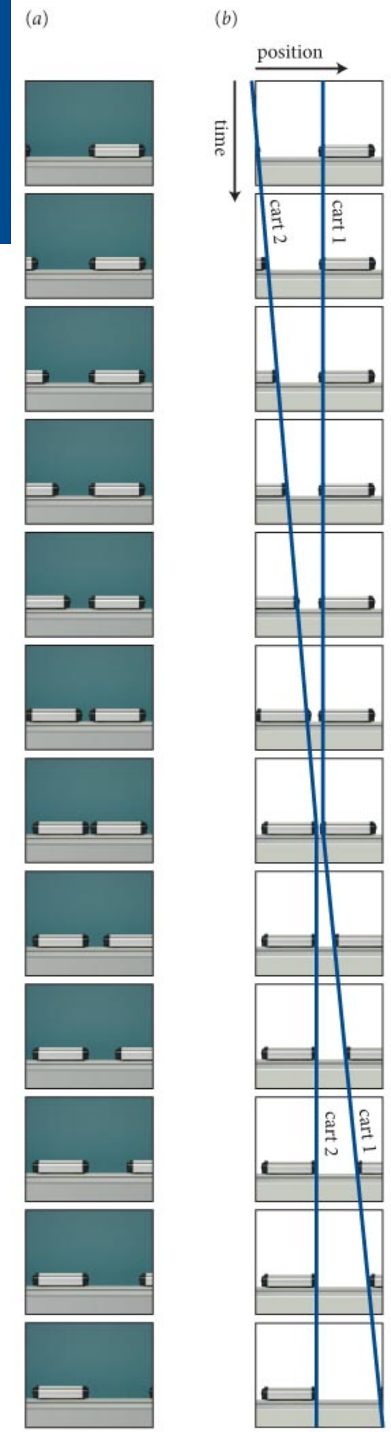
- Determine the changes in velocity of two colliding objects in the absence of friction.
- Display the collision process between two objects using motion diagrams, before-and-after diagrams, and motion graphs.
- Equate **inertia** as a measure of an object's tendency to resist any changes in its velocity.

Section 4.2: Inertia

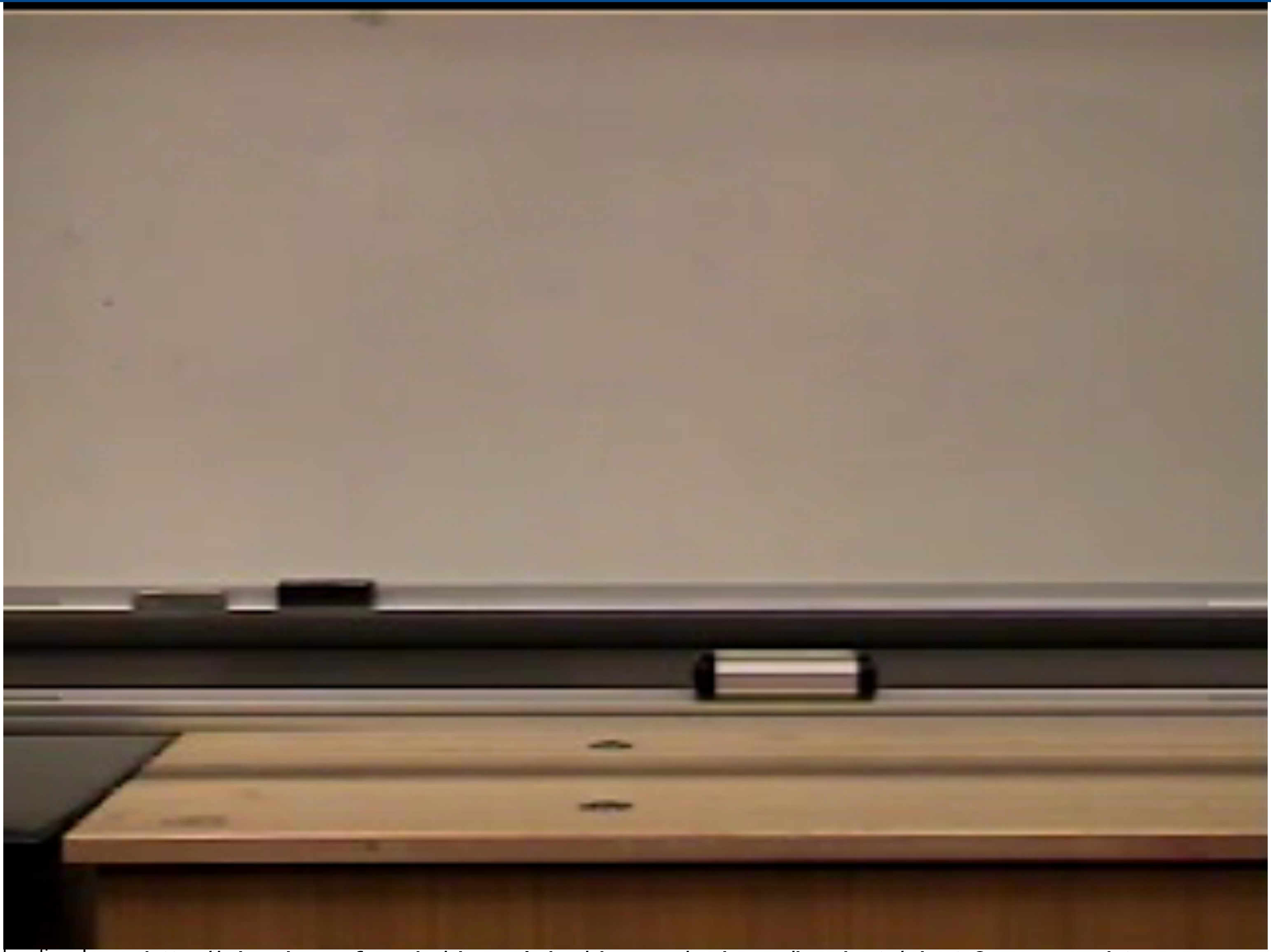
- We can discover one of the most fundamental principles of physics by studying how the velocities of two low-friction carts change when they collide →

- Start with two identical carts

- first cart stops, second starts *as though continuing motion of first*
- like something was transferred …



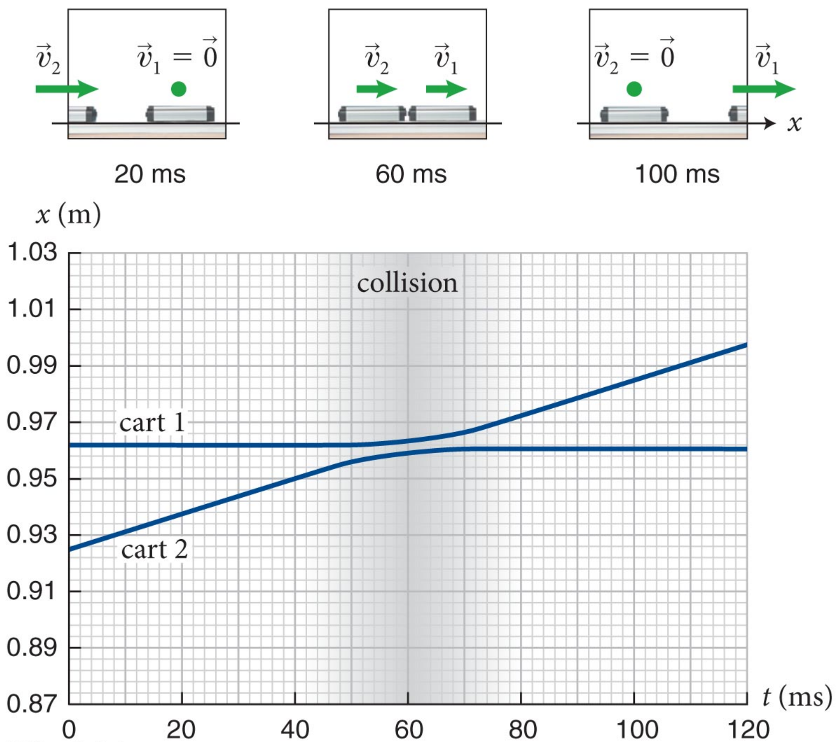
Reality



http://physics.wfu.edu/demolabs/demos/avimov/bychptr/chptr3_energy.htm

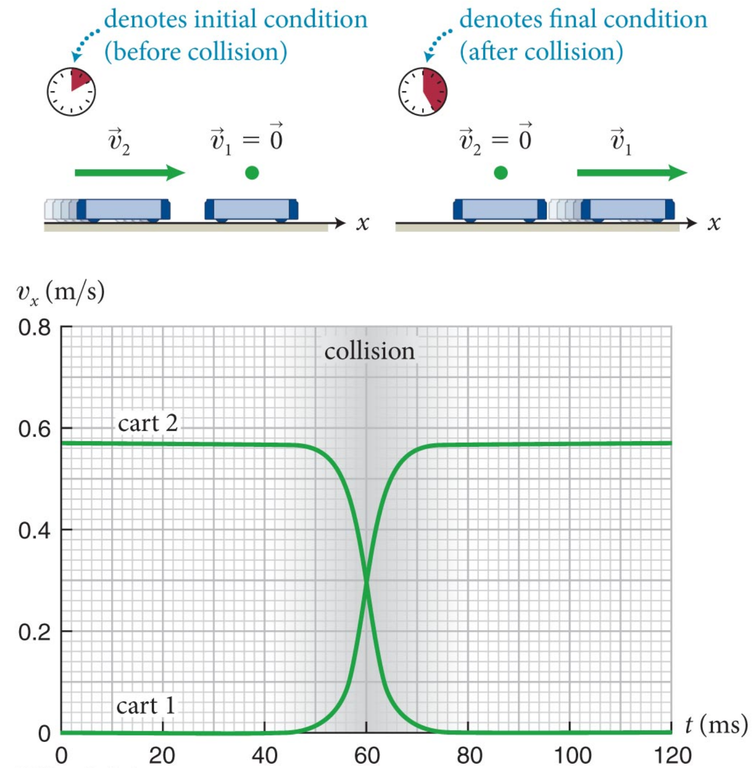
Experiment 1

- Cart 1 initially at rest and Cart 2 moving toward it with velocity v_2.
  - measure x(t) for both
  - collision causes Cart 1 to move to the right, Cart 2 to come to a full stop.
  - as though Cart 1 continues Cart 2's motion after the collision



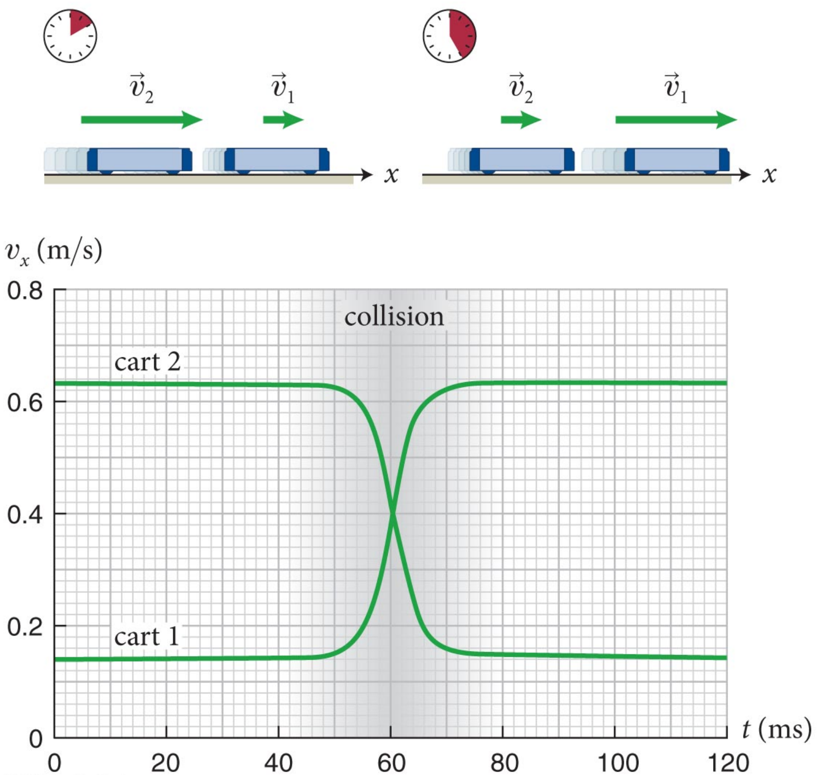
Experiment 1 (cont.)

- Same deal, how about velocity?
  - v(t) for both
  - *velocities are interchanged during the collision.*
  - Further experimentation: *no matter what the initial velocity of Cart 2*, the collision always interchanges the two velocities.
  - again, like something is transferred



Experiment 2

- Now let both carts move to the right. Cart 2 is faster than Cart 1
  - The velocities are once again interchanged during the collision.
  - Repeating this experiment with different initial velocities will yield the same result.
  - something is transferred or ***conserved*** during the collision



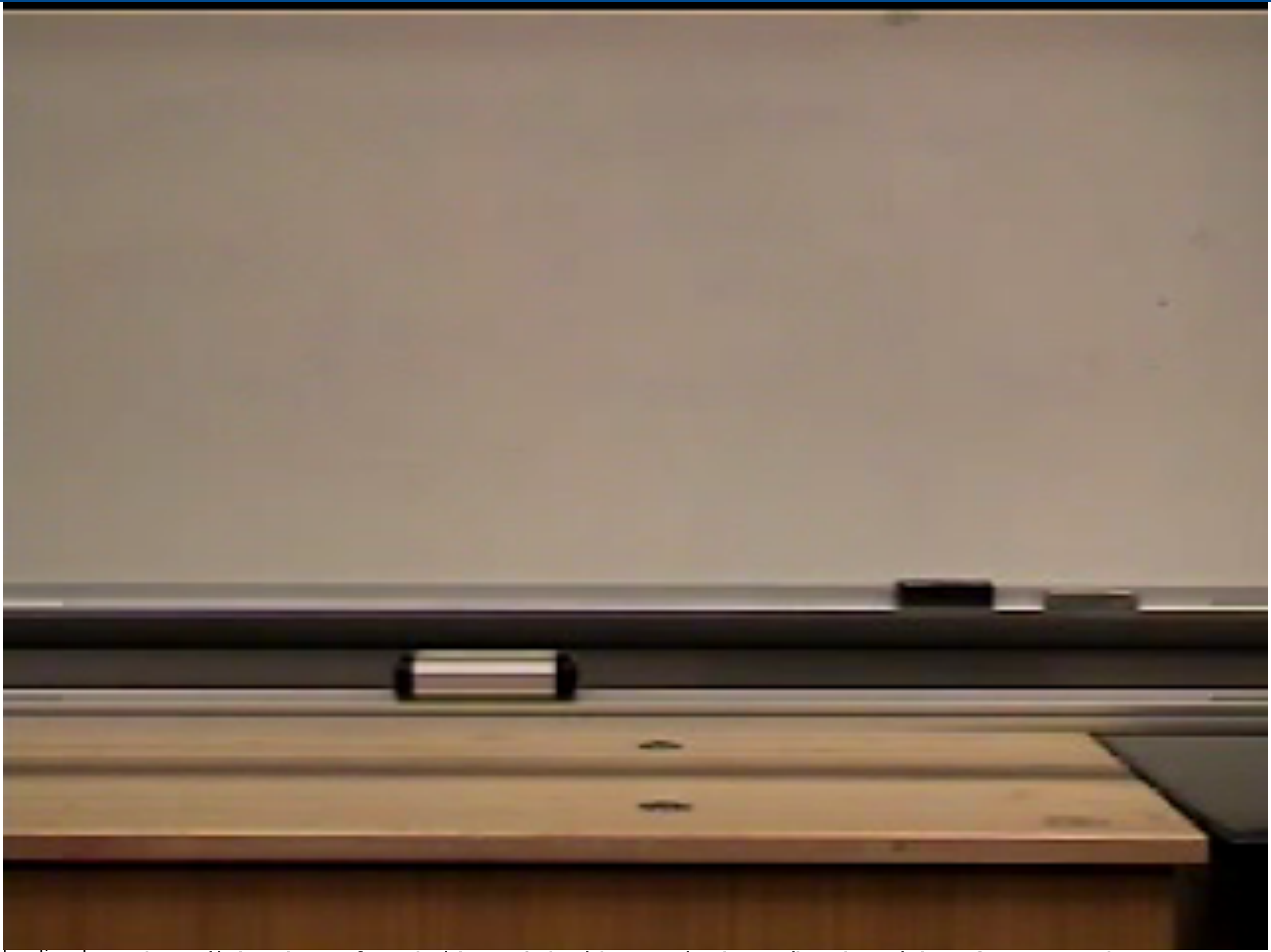
Checkpoint

What would happen if we play the video in reverse?

what if we watched it in a mirror?

Certain *symmetries* about the collision are implied

Is this flipped or played in reverse?



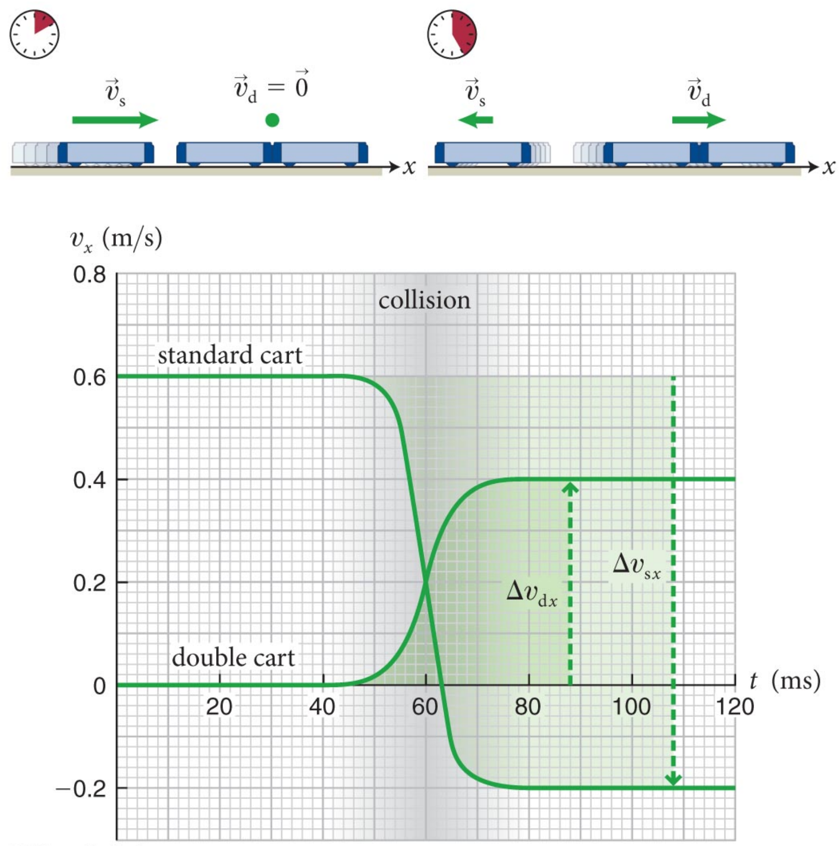
http://physics.wfu.edu/demolabs/demos/avimov/bychptr/chptr3_energy.htm

It was flipped

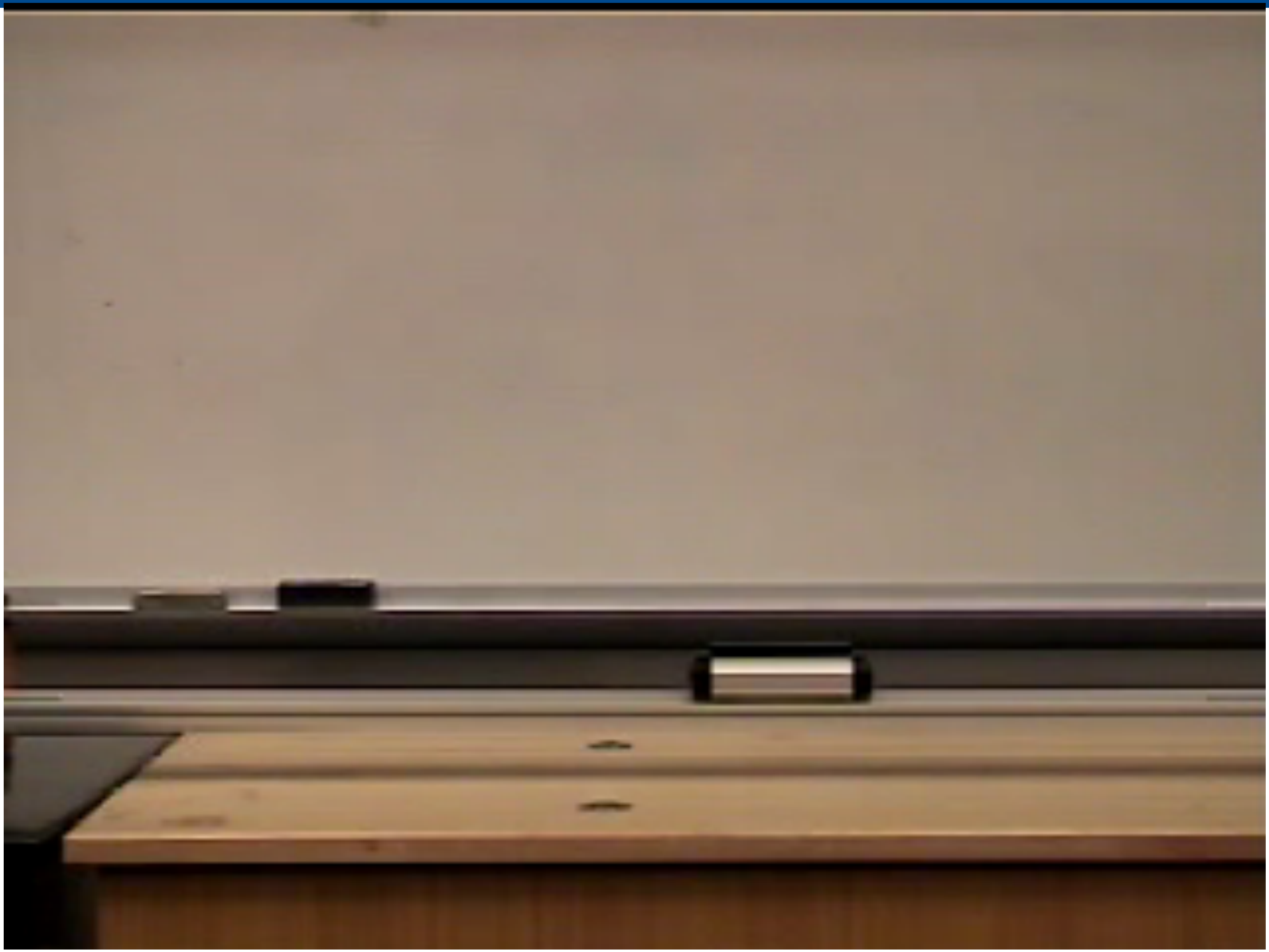
- flipped and played backwards look the same
  - (except for the erasers)
- the system has mirror symmetry
  - left and right are the same
- the system has time reversal symmetry
  - forwards and backwards look the same
  - *reversible* phenomenon

Experiment 3

- Now a standard cart moving towards a double cart at rest.
  - The moving cart reverses direction after the collision.
  - The change in velocity of the double cart is half that of the standard cart.



Reality



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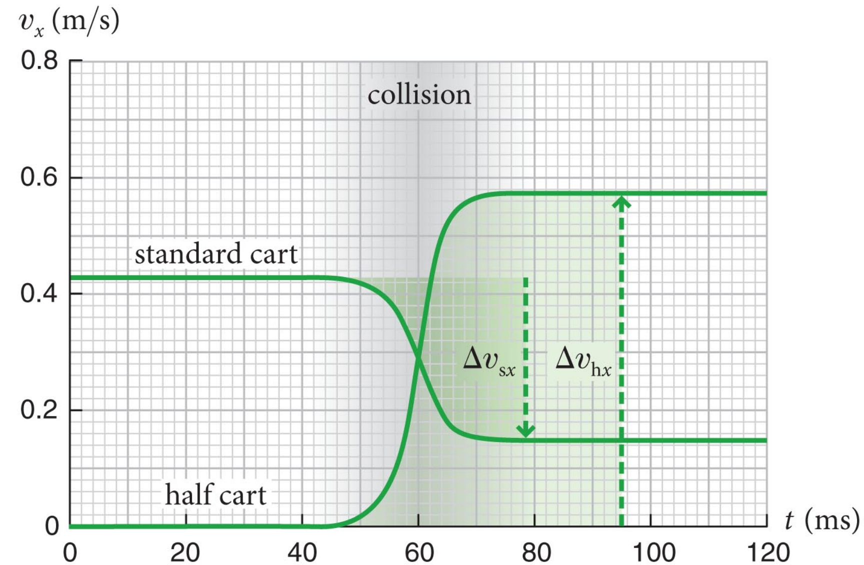
Experiment 4

- Now a standard cart moving toward a *half* cart at rest.
  - The change in velocity of the half cart is twice that of the standard cart.
  - It seems that the amount of material that makes up each cart does affect the motion.



Reality

http://physics.wfu.edu/demolabs/demos/avimov/bychptr/chptr3_energy.htm

Section 4.2: Inertia

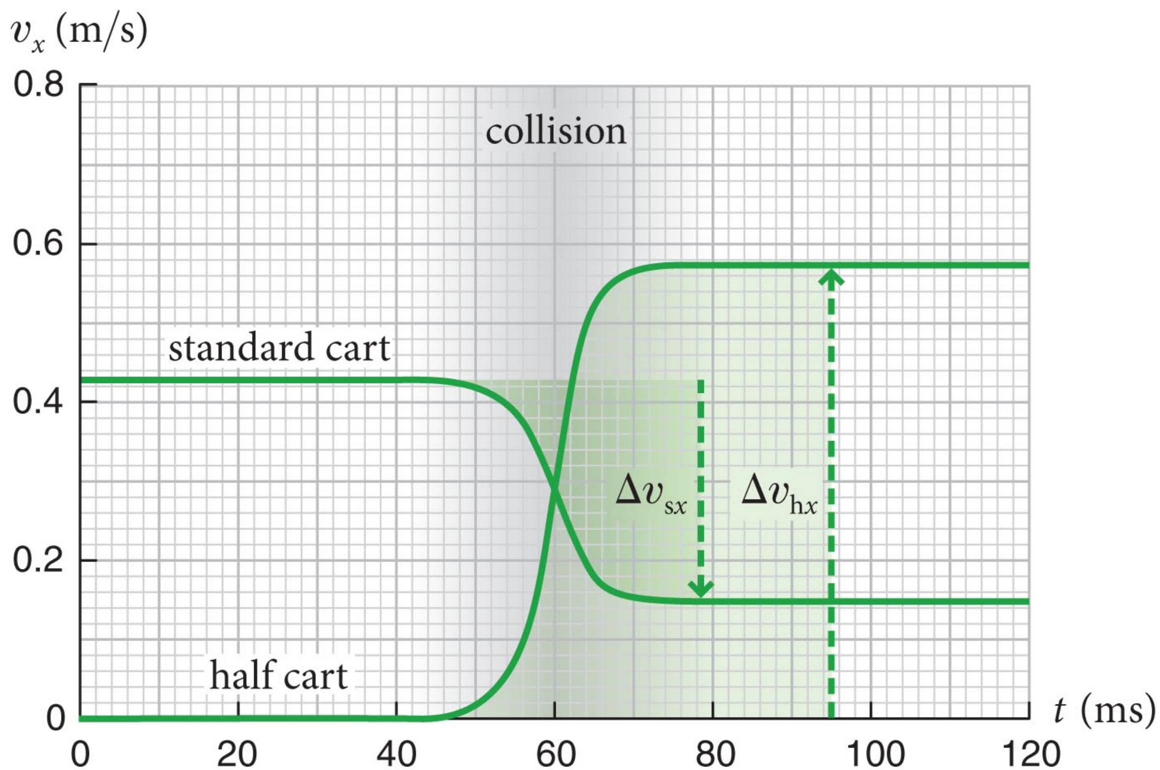
- **Conclusion from our experiments:** For objects made of the same material, the motion of larger objects are harder to change than smaller objects.
- **Inertia** is a measure of an object's tendency to resist any changes in its velocity.
  - The results of our experiments are summarized in Table 4.1.

Table 4.1 Ratio of Velocity Changes in Collisions Between Two Carts

Experiment	Cart 1	Cart 2	$\lvert\Delta v_{1x}\rvert : \lvert\Delta v_{2x}\rvert$
1	standard	standard	1
2	double	standard	0.5
3	half	standard	2

Checkpoint 4.3

4.3 The x component of the final velocity of the standard cart is positive. Can you make it negative or zero by adjusting this cart's initial speed (while still keeping the half cart initially at rest)?

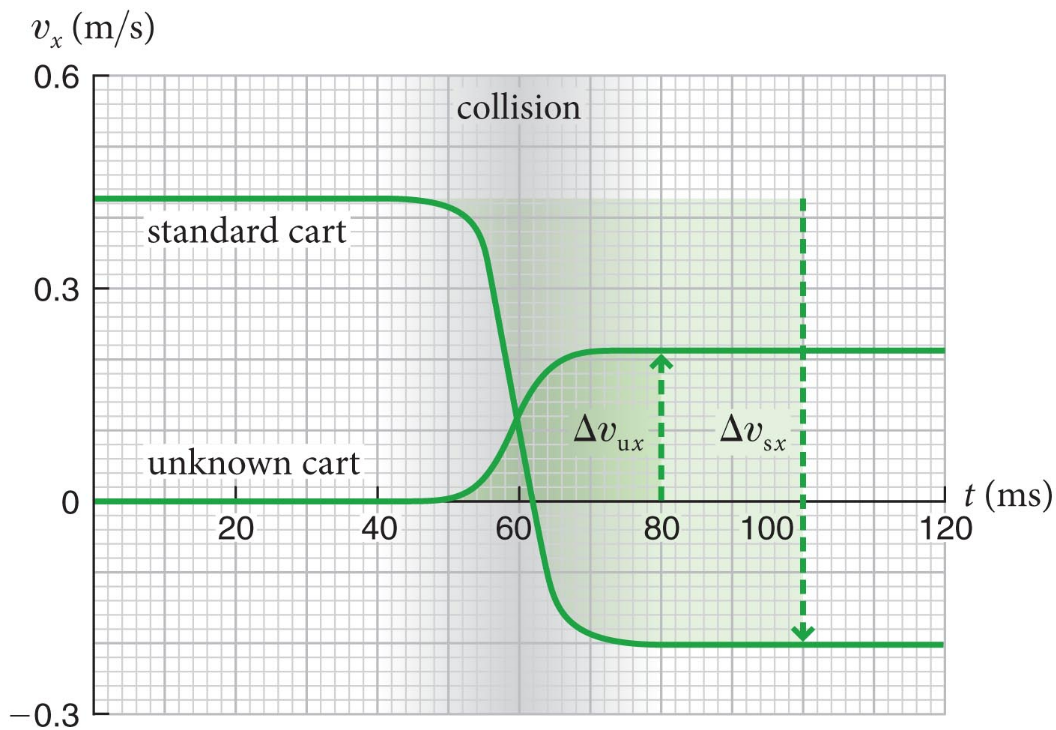


No – the standard cart always has half the velocity change of the half cart.

That means if you decrease the standard cart's velocity, you just shrink the whole graph vertically. The standard cart's velocity remains positive after the collision.

Checkpoint 4.4

- The figure shows a graph of the collision between some unknown cart and a standard cart made of the same material.
- We see that $|\Delta v_{\mathrm{u}\,x}|/|\Delta v_{\mathrm{s}\,x}| \approx 1/3$ for the two carts
- We can conclude that the unknown cart's inertia is three times that of the standard cart.



Section 4.2
Question 1

Carts A and B collide on a horizontal, low-friction track. Cart A has twice the inertia of cart B, and cart B is initially motionless. How does the change in the velocity of A compare with that of B?

1. $\upsilon_A/\upsilon_B = 1$
2. $\upsilon_A/\upsilon_B = -1$
3. $\upsilon_A/\upsilon_B = 1/2$
4. $\upsilon_A/\upsilon_B = -1/2$
5. $\upsilon_A/\upsilon_B = 2$
6. $\upsilon_A/\upsilon_B = -2$

(direction matters!)

Section 4.2
Question 1

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1. $\upsilon_A/\upsilon_B = 1$
2. $\upsilon_A/\upsilon_B = -1$
3. $\upsilon_A/\upsilon_B = 1/2$
4. ✔ $\upsilon_A/\upsilon_B = -1/2$ more inertia = harder to change
5. $\upsilon_A/\upsilon_B = 2$ changes in velocity are in opp dir
6. $\upsilon_A/\upsilon_B = -2$

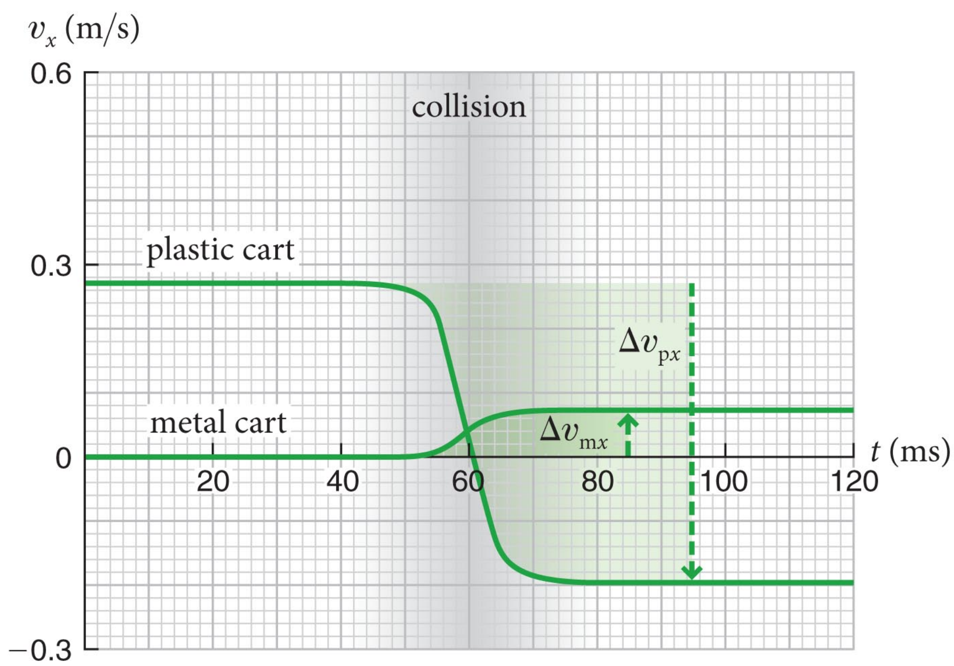
Section 4.3: What determines inertia?

Section Goals

- Extend the analysis of the collision between two objects that are composed of different materials and have different volumes.
- Recognize that the inertia of an object is determined by **the type of material and its volume**.

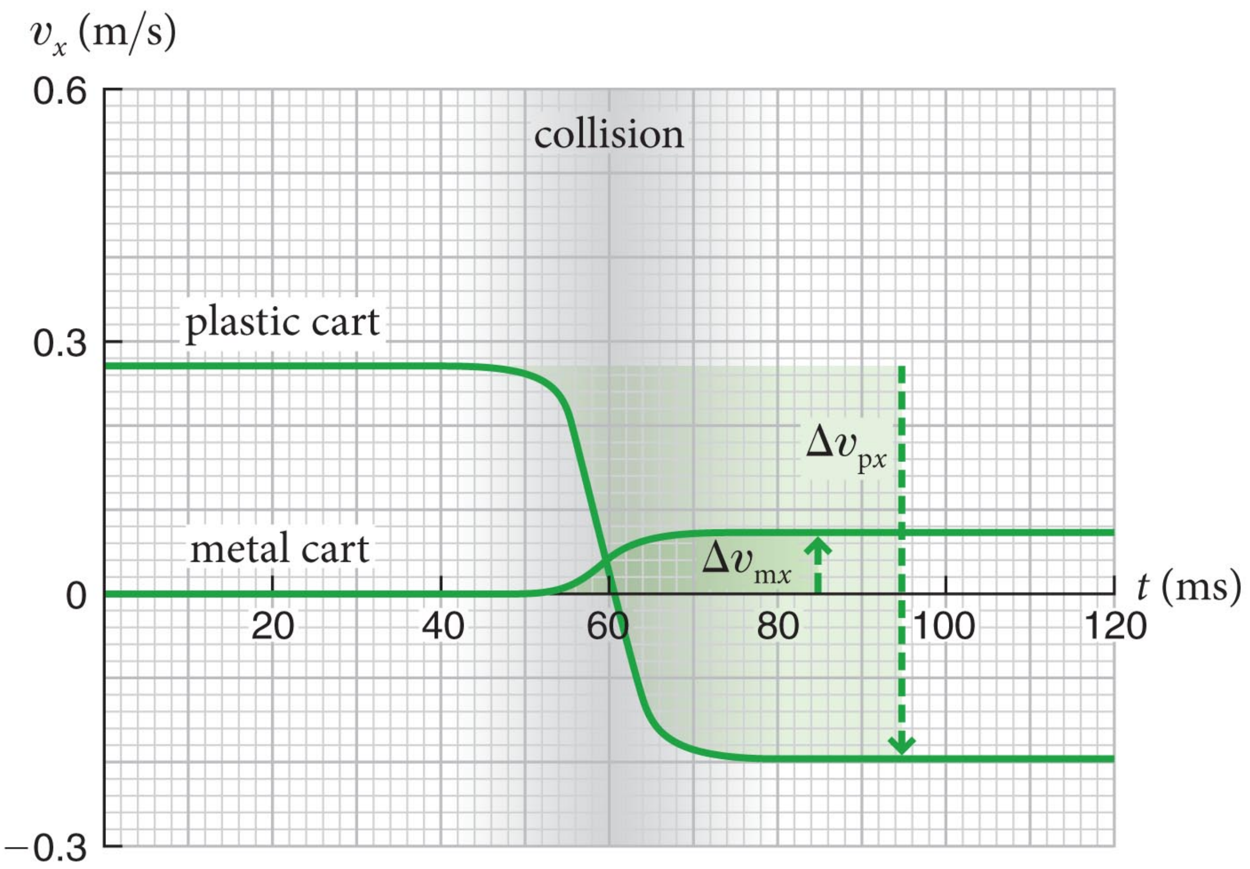
Section 4.3: What determines inertia?

- Is it only the amount of material that determines inertia?
  - Repeat experiment 1 carried out for two carts, one made of plastic and the other metal, but both with an *identical amount* of material
  - The results shown in the figure are very different from the results for two identical carts.



Checkpoint 4.5

- We find $\Delta v_{p\,x}/\Delta v_{m\,x}$ = -6.7
- metal cart hardly budges!
- plastic cart rebounds with large fraction of original speed



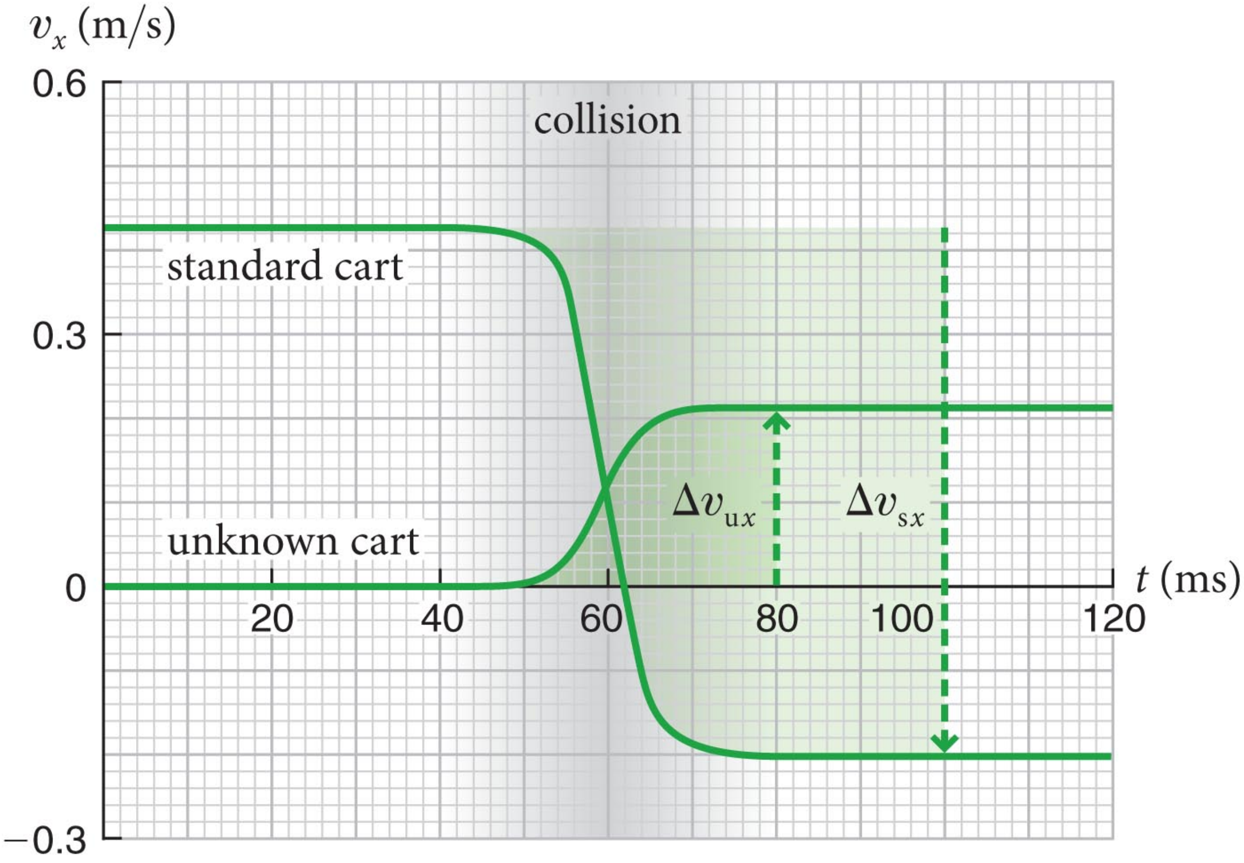
Section 4.3: What determines inertia?

- **Conclusion:** The inertia of an object is determined entirely by the type of material of which the object is made and by the amount of that material contained in the object.

- So why were metal and plastic so different?

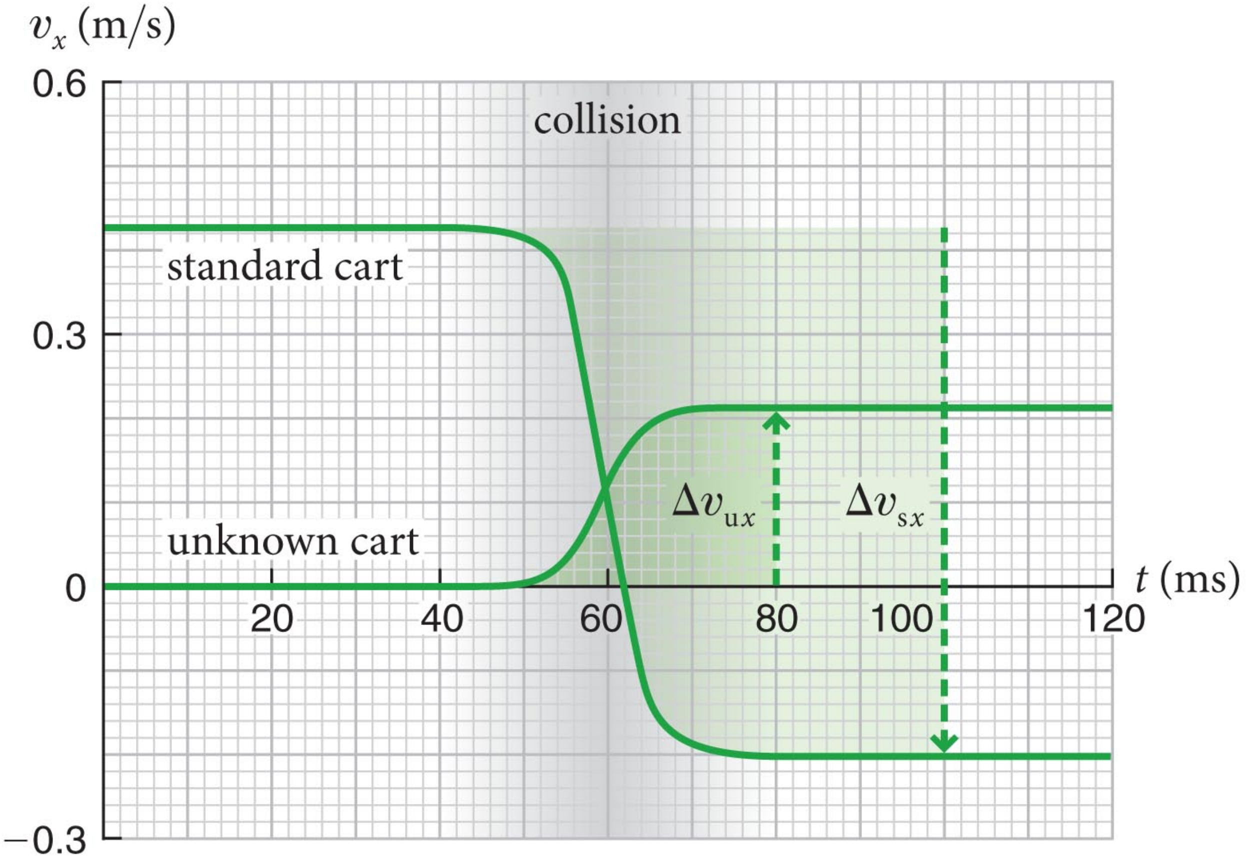
Checkpoint 4.6

4.6 Is the inertia of the cart of unknown inertia in the figure greater or less than that of the standard cart?



Checkpoint 4.6

4.6 The unknown has greater inertia, since its change in velocity is much less

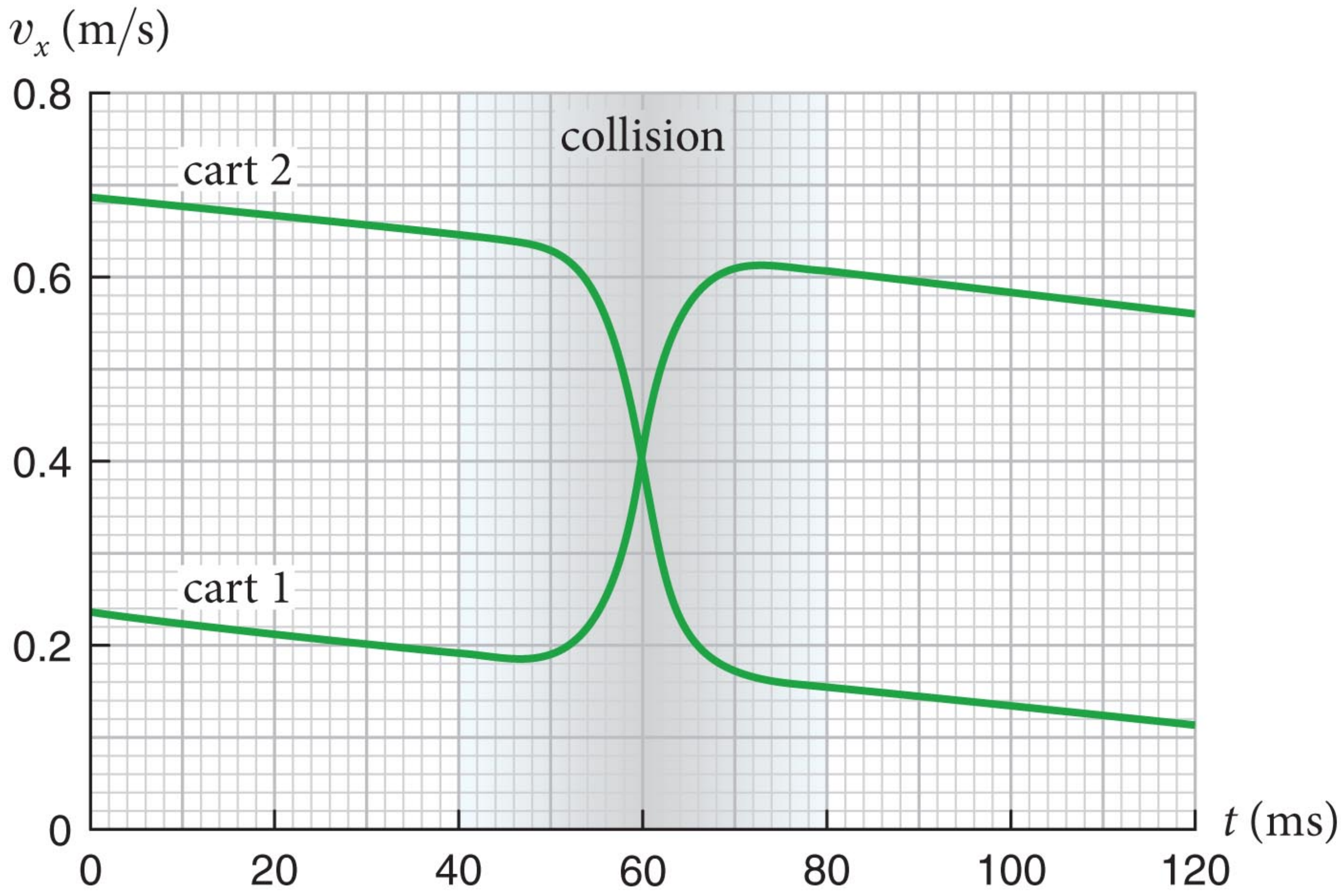


Example 4.1 Friction and collisions

- The figure on the next slide shows the $v_x(t)$ curves for a collision between two identical carts
- Now they are moving on a rough surface, and friction affects their motion.
- Are the changes in the velocity of the carts caused by the collision still equal in magnitude?

- what physics has really changed?

Example 4.1 Friction and collisions (cont.)

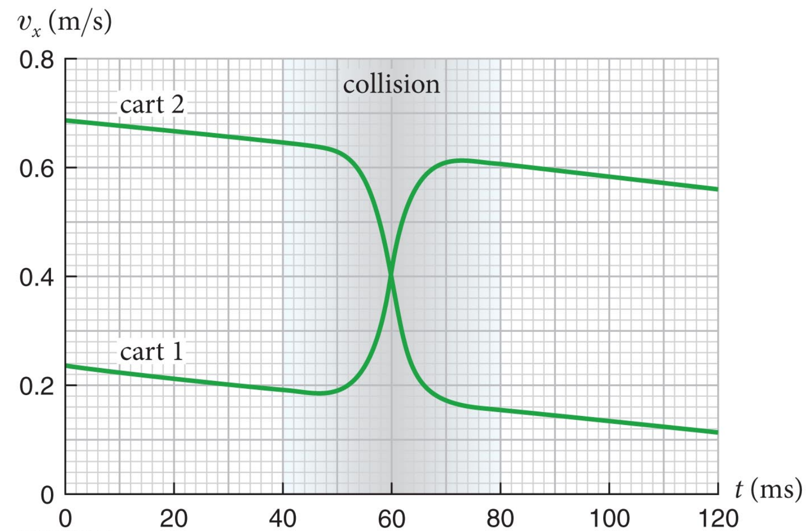


Example 4.1 Friction and collisions (cont.)

❶ GETTING STARTED

We have two bits of physics now:

- collision
  - results in velocity exchange
- friction
  - results in linearly decreasing *v*



reasonable to expect superposition

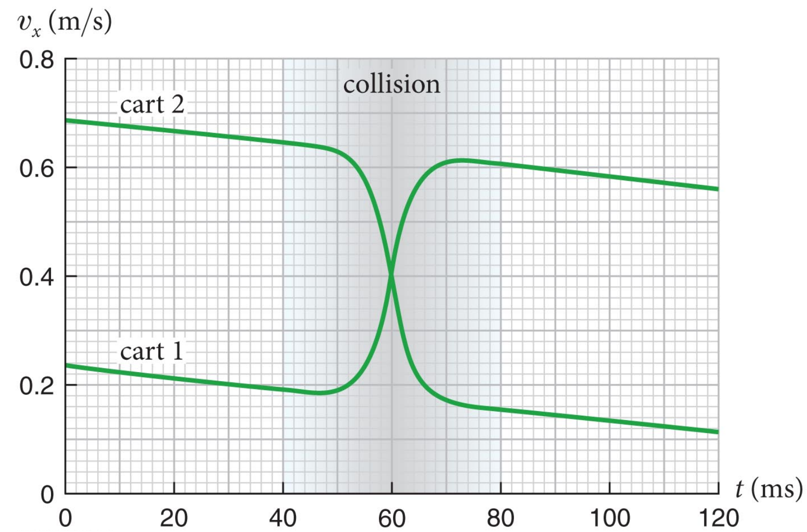
and that's what it looks like …

Example 4.1 Friction and collisions (cont.)

❶ GETTING STARTED

Shape suggests friction acts throughout the motion

If so, how do we figure the change in velocity?



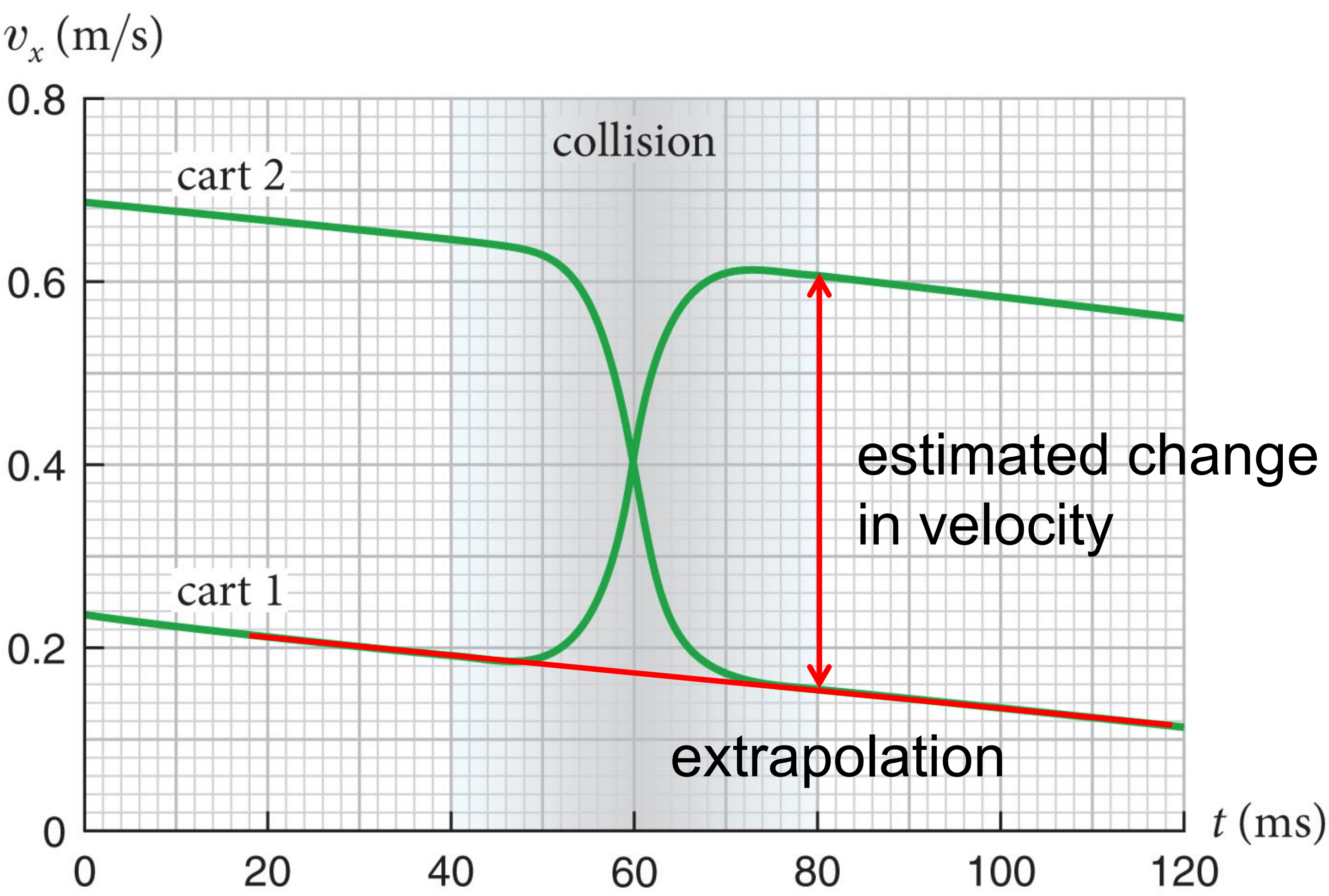
Example 4.1 Friction and collisions (cont.)

❷ DEVISE PLAN

We can extrapolate the $v_x(t)$ curve for cart 1 to calculate what its velocity *would* have been at $t = 80$ ms if the collision had not taken place.

To determine the change in velocity due to the collision, read off the actual value of the velocity of cart 1 at $t = 80$ ms and subtract the two velocities. Repeat for cart 2.

That sounds complicated, but it isn't



Example 4.1 Friction and collisions (cont.)

❸ EXECUTE PLAN

Doing this for cart 1 at t = 80 ms gives +0.15 m/s.

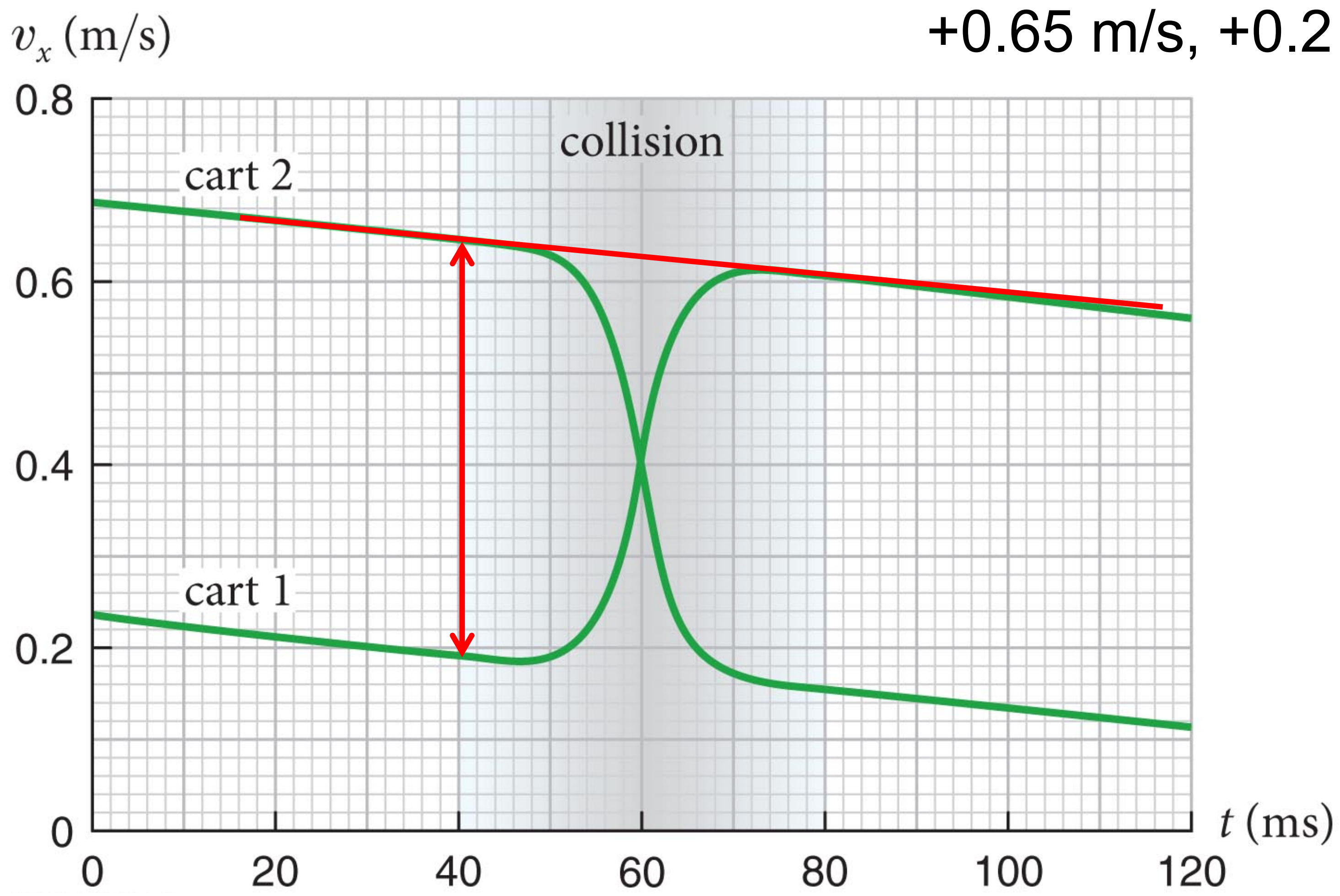
Repeating this procedure for cart 2, about +0.60 m/s.

Do the same just *before* the collision (40 ms) to get velocities before the collision

Section 4.3: What determines inertia?

Now before the collision

+0.65 m/s, +0.2 m/s



Example 4.1 Friction and collisions (cont.)

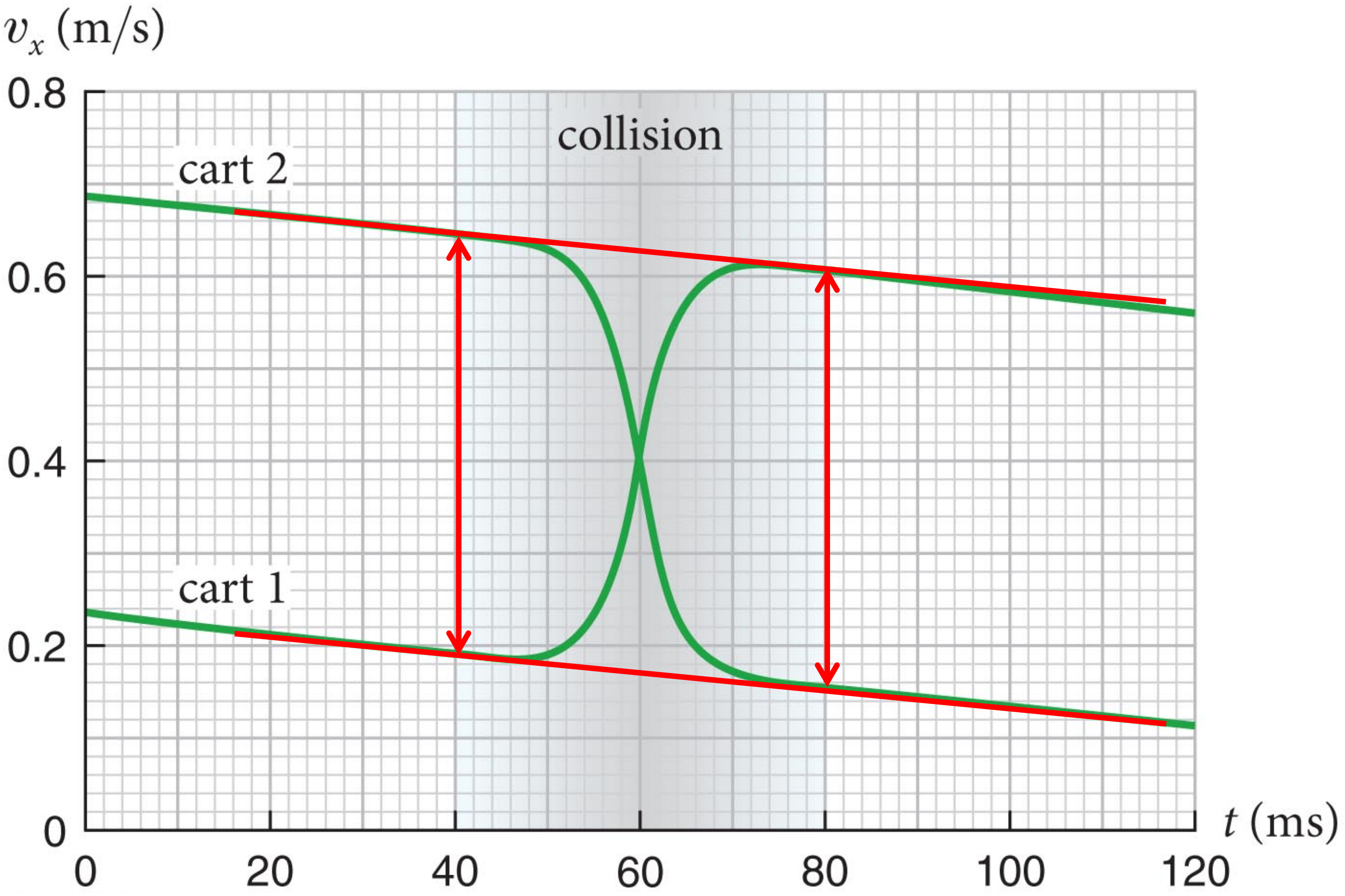
❸ EXECUTE PLAN

Knowing (extrapolated) initial and final velocities, we can find the change in velocity.

We find 0.45 m/s for both carts.✔

- ❖ This tells us that friction is an additive effect, it does not change the underlying fundamental physics.
- ❖ Whatever friction is, it is an *independent* interaction

Arrows are the same size.



Section 4.3
Question

Two objects of identical volume and shape are made of different materials: iron and wood. How do their inertias compare?

1. Inertia of iron = inertia of wood
2. Inertia of iron > inertia of wood
3. Inertia of iron < inertia of wood

Section 4.3
Clicker Question 2

Two objects of identical volume and shape are made of different materials: iron and wood. How do their inertias compare?

1. Inertia of iron = inertia of wood
2. ✔ Inertia of iron > inertia of wood
3. Inertia of iron < inertia of wood

What word are we dancing around by saying “inertia”?

Why are we being careful?

Section 4.4: Systems

Section Goals

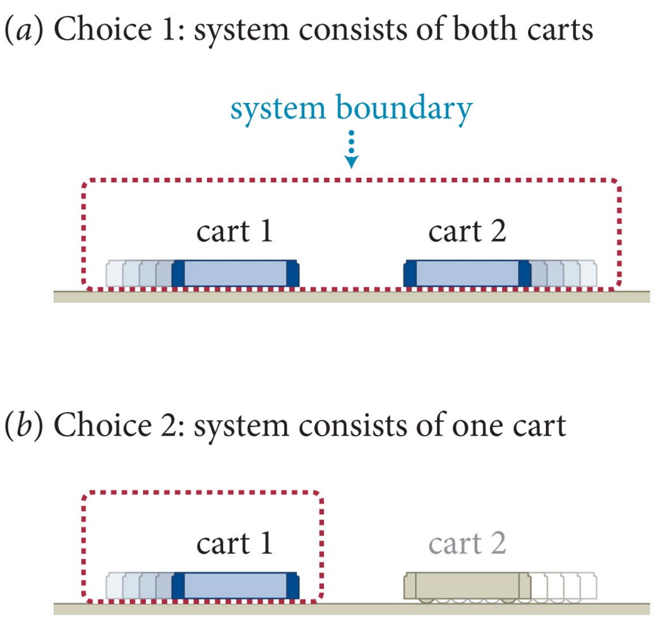
- Classify physical situations as being composed of a **system** of objects of interest and the rest of the universe, **the environment**.
- Choose the appropriate system and environment for physical situations depending on the physics of interest.

- Decide if the two need to interact at all

    (Hint: you don't want them to)

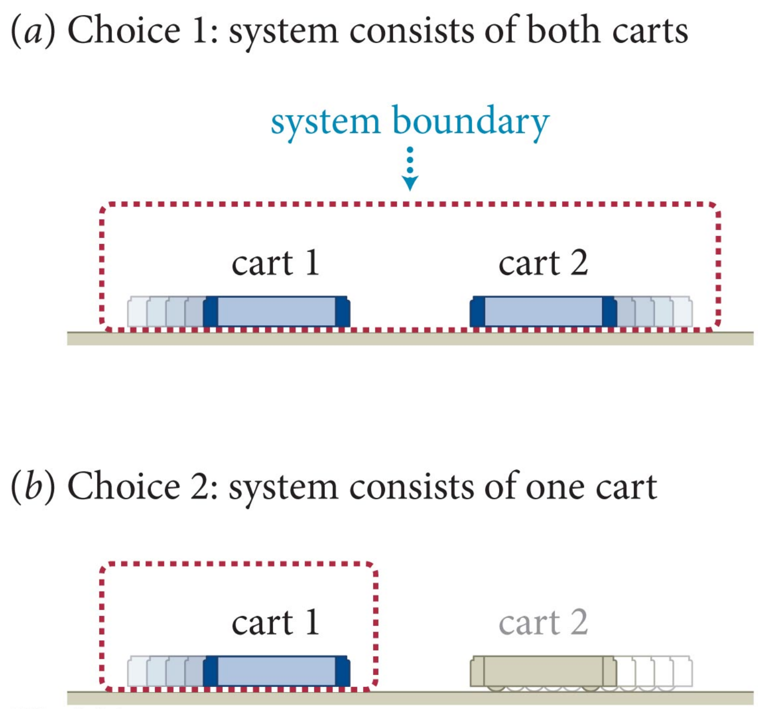
Section 4.4: Systems

- The first step in the analysis: separate the object(s) of interest from the rest of the universe:
  - Any object or group of objects that we can separate, in our minds, from the surrounding environment is a system.



Section 4.4: Systems

- Which choice might be more clever here?
- Why? You want to isolate interactions …
- What would make both choices inconvenient?



Exercise 4.2 Choosing a system

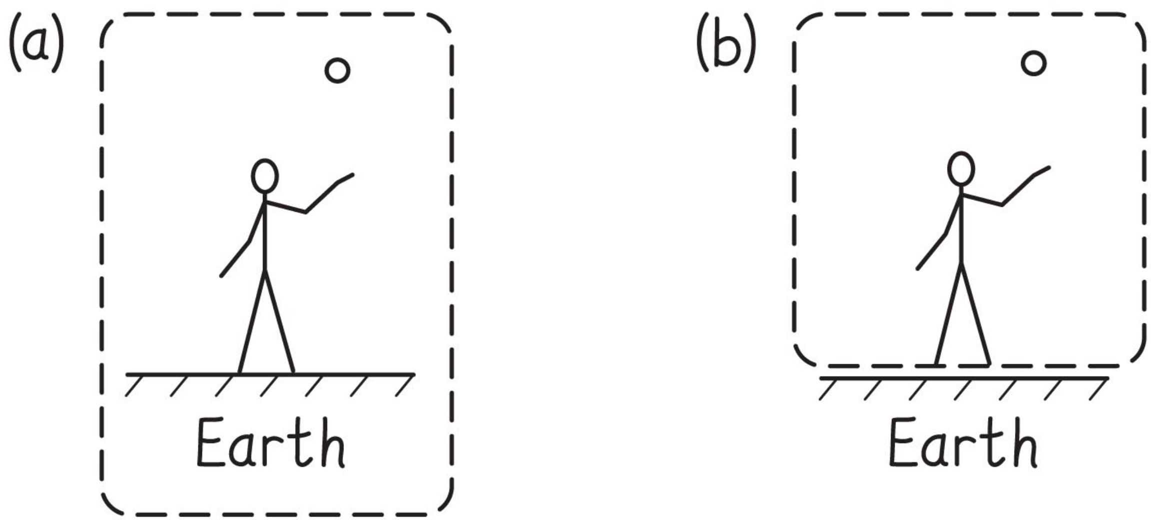
Indicate at least two possible choices of system in each of the following two situations. For each choice, make a sketch showing the system boundary and state which objects are inside the system and which are outside.

(a) After you throw a ball upward, it accelerates downward toward Earth.

(b) A battery is connected to a light bulb that illuminates a room.

Exercise 4.2 Choosing a system (cont.)

SOLUTION (*a*) The description of the situation mentions three objects: the ball, Earth, and you. One option is to include all three of them in the system. As a second choice, include you and the ball in the system. ✔

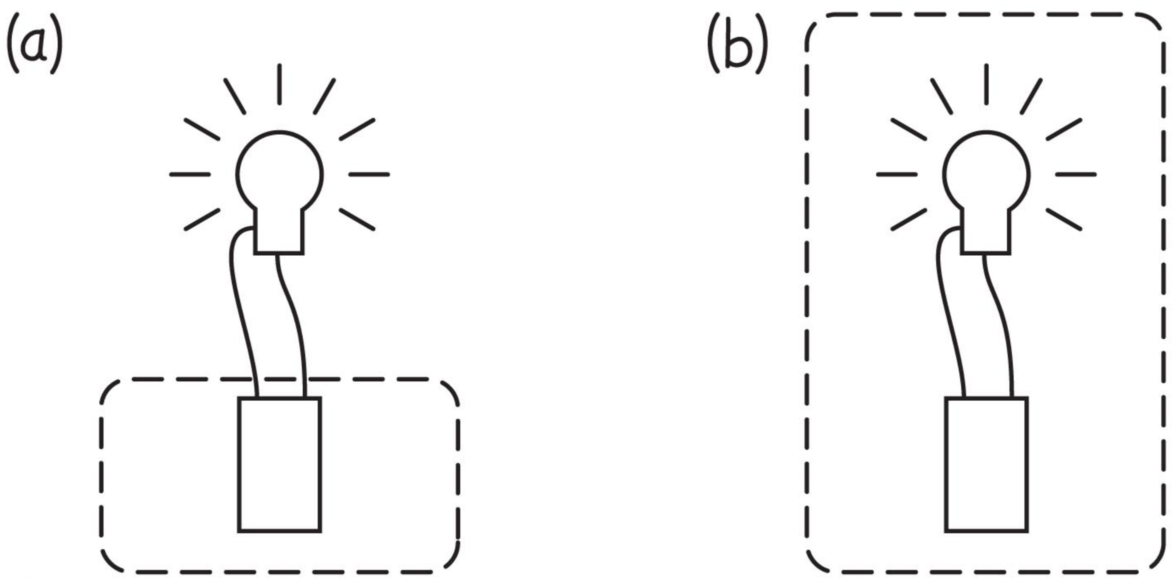


Exercise 4.2 Choosing a system (cont.)

SOLUTION (*b*) 3 objects: battery, light bulb, and room.

I can choose just one of them—the battery—as my system or two of them—the battery and the light bulb.✔

Which is "better" depends on what you're trying to find!



Exercise 4.2 Choosing a system (cont.)

SOLUTION

- Choices of system are arbitrary.
- Nothing in the problem prescribes a choice of system.
- If you tried this problem on your own and made different choices, then your answer is just as “correct”!

(But some “correct” choices are easier than others.)

Section 4.4: Systems

- Once we have chosen a system, we can study how certain quantities associated with the system change over time:
  - **Extensive quantities:** quantities whose value is proportional to the size of the system.
    - examples: volume, energy, length, mass, ***momentum***
  - **Intensive quantities:** quantities that do not depend on the extent of the system.
    - examples: temperature, pressure, density

Section 4.4: Systems

- An **intensive property** is a **bulk property**
  - e.g., when a diamond is cut, pieces retain hardness
- An **extensive property** an additive effect
  - depends on amount of material
  - mass and volume depend on amount of stuff
- Ratio of two extensive properties is intensive!
  - e.g., ratio of extensive mass and volume gives *intensive* **density**

- Who. Cares?

Section 4.4: Systems

- intensive properties are *invariant,* and in some way more fundamental
- extensive properties depend on exact details; circumstantial

- we want to find and first explain *intensive* properties
  - figure out what is fundamental vs circumstantial
  - figure out how to combine parameters to make something intensive
  - e.g., pressure instead of force, density instead of mass

Section 4.4: Systems

- Only four processes can change the value of an extensive quantity: input, output, creation, destruction.
- Then, the change of a certain quantity over a time interval is given by

$$\textit{change} = \textit{input} - \textit{output} + \textit{creation} - \textit{destruction}$$

(*a*)

Extensive quantities can be changed by input, output, creation, and destruction.

Section 4.4: Systems

- More simply:

$$change = (net\ flux\ into\ system) + (net\ appearance\ of\ new\ stuff)$$

(*a*)

Extensive quantities can be changed by input, output, creation, and destruction.

Section 4.4: Systems

- If there is no transfer of an extensive quantity across the boundary of the system (no flux in our out)

$$\textit{change} = \textit{creation} - \textit{destruction}$$

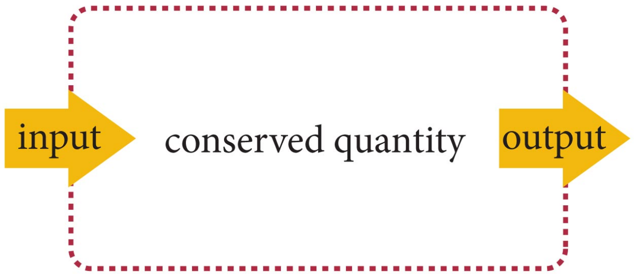
- Any extensive quantity that cannot be created or destroyed is said to be **conserved** (and this is nice):
  - The change in the value of a conserved quantity is

$$\textit{change} = \textit{input} - \textit{output}$$

(*b*)

If there is no input or output, the quantity can be changed only by creation or destruction.

(*c*)



If an extensive quantity cannot be created or destroyed, it is said to be conserved.

Quantitative Tools

Section 4.5: Inertial standard

Section Goals

You will learn to

- Represent the **standard quantity of inertia** by the kilogram in the metric system.
- Compare the inertia of any object with the kilogram standard of inertia.

Section 4.5: Inertial standard

- The inertia of an object is represented by the symbol *m*:
  - *m* is for mass, a concept related to inertia.
- The basic SI unit of inertia is the kilogram (kg):
  - The inertial standard is a platinum-iridium cylinder

- Using the definition of the ratio of inertias of colliding carts and the inertial standard (m_s), we can find inertia of any object (m_u):

$$\frac{m_u}{m_s} \equiv -\frac{\Delta v_{sx}}{\Delta v_{ux}}$$

$$m_u \equiv -\frac{\Delta v_{sx}}{\Delta v_{ux}} m_s$$

  - The minus sign is because the velocity of one of the carts will decrease during collision.

- This implies

$$m_{\mathrm{u}}\Delta v_{\mathrm{u,x}} = -m_s\Delta v_{s,x}$$

- *or* ...

$$m_{\mathrm{u}}\Delta v_{\mathrm{u,x}} + m_s\Delta v_{s,x} = 0$$

- *which implies no net change for the whole system*
- further, noting definition of Δv:

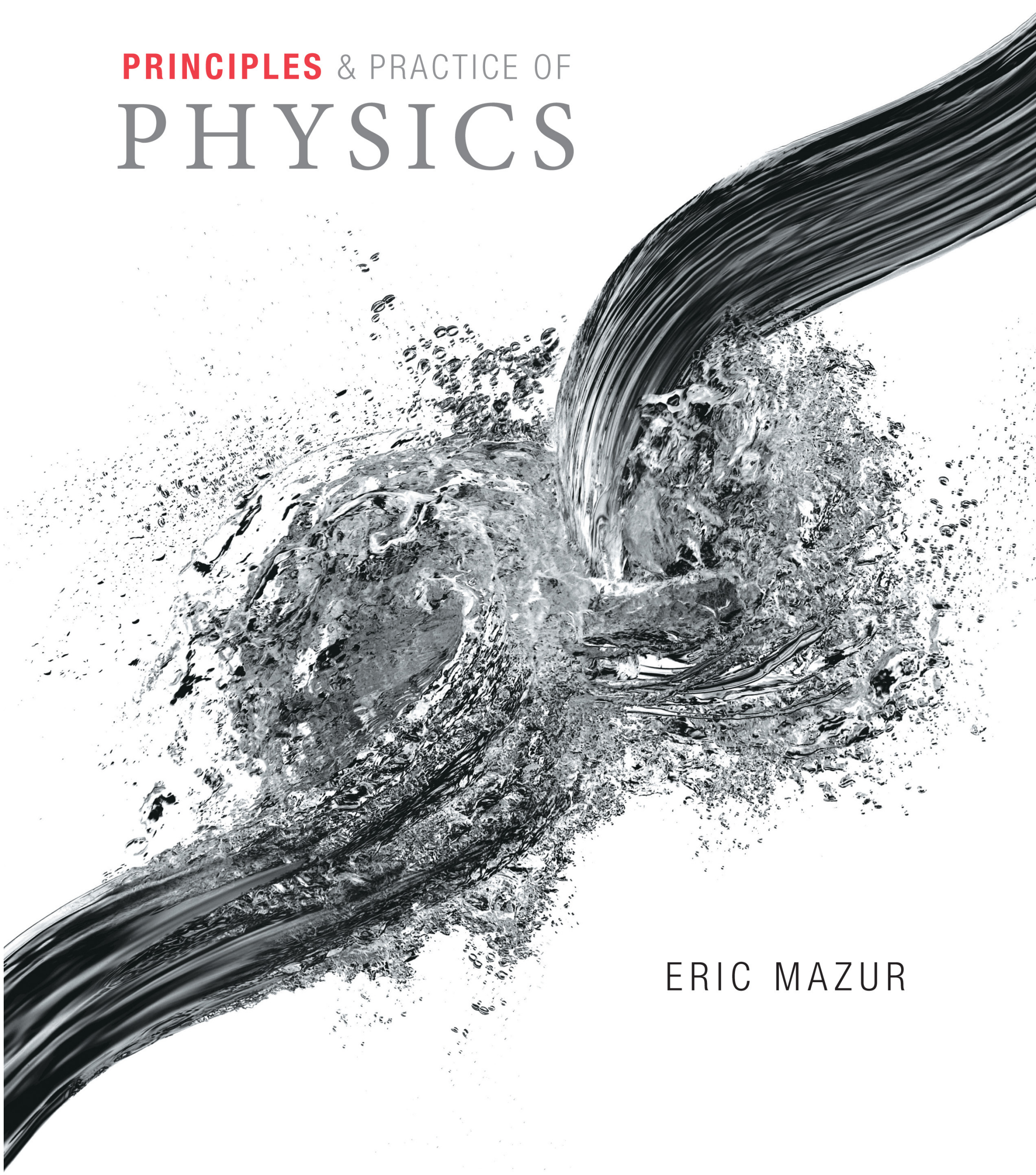
$$m_{\mathrm{u}}v_{\mathrm{u,xi}} + m_{\mathrm{s}}v_{\mathrm{s,xi}} = m_u v_{u,xf} + m_{\mathrm{s}}v_{\mathrm{s,xf}}$$

- total of the product of inertia and velocity is constant
- find *mv* for all objects in closed system. add together.
  - sum never changes – problem solving!

Section 4.5

- the quantity mv is an important one
  - mv = **momentum**
- its sum for the whole system is a constant
- solving problems: write it down before & after!
- think back to identical carts collision …
  - it is the mv that is exchanged by carts
  - in total, the amount of mv stays constant

Momentum



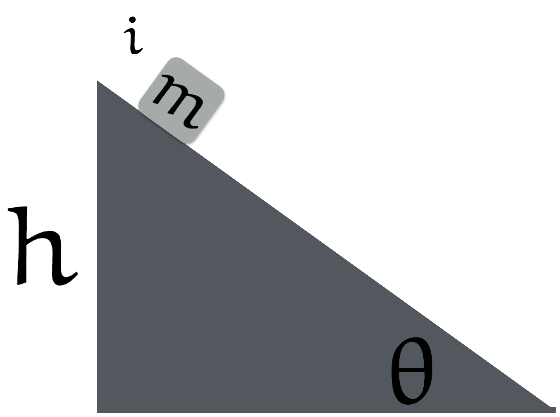
Various

- lab this week – comes a bit early
  - I wrote a preface to explain …
- Homework
  - if you don’t finish part of a question, hit ‘give up’
- today
  - finish up momentum
  - problems
- Thursday
  - most of the energy chapter
  - some exam 1 details

Review problem

Sliding down a ramp

- A block slides down a ramp without friction, starting from rest
- what is its speed at the bottom?



Review problem

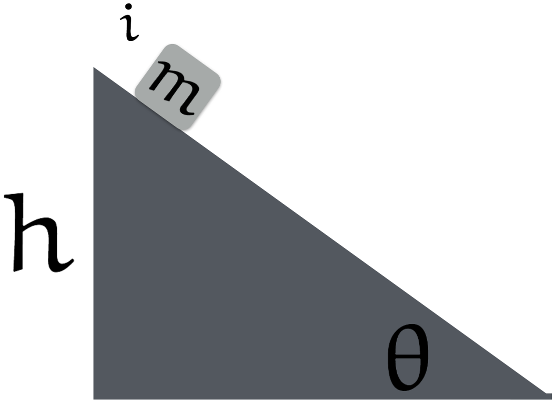
Sliding down a ramp

- We know the initial speed, the displacement along the ramp, and acceleration.
- Let $+x$ be down the ramp

  $\sin\theta = h/\Delta x$ so $\boldsymbol{\Delta x = h/\sin\theta}$

  $\mathbf{a = +g \sin\theta}$

  $\boldsymbol{v_i = 0}$



- We want speed, we don't know time

  $$v_f^2 = v_i^2 + 2a\Delta x$$

Review problem

Sliding down the ramp

- Putting it together:

$\sin\theta = h/\Delta x$ so $\mathbf{\Delta x = h/\sin\theta}$

$\boldsymbol{a = +g \sin\theta}$

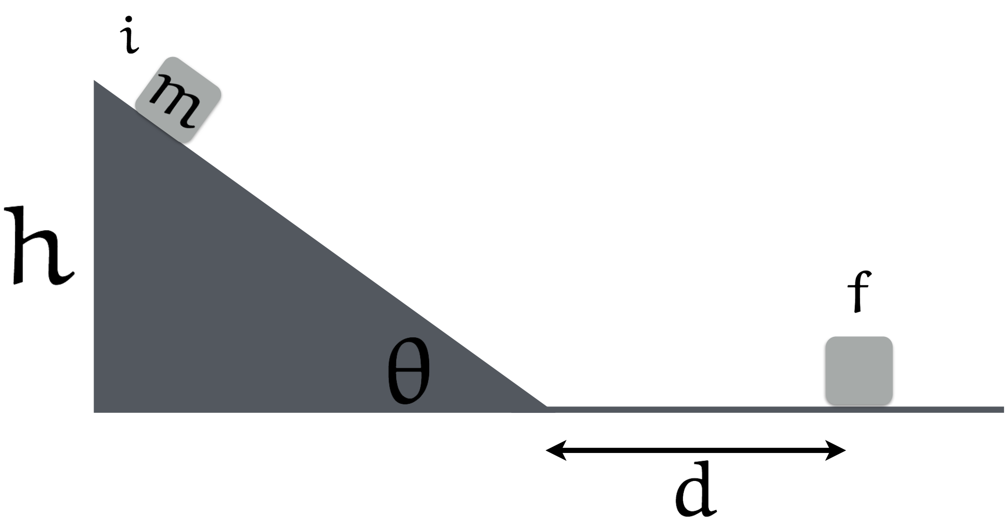
$$v_f^2 = v_i^2 + 2a\Delta x = 2g\sin\theta\,(h/\sin\theta) = 2gh$$

- Curiously, speed does not depend on angle!
  - Only height of ramp matters

Review problem

Sliding down a ramp

- If the flat surface has an acceleration due to friction of μg, how far does the block slide before stopping?



Review problem

Sliding down a ramp

- We know the initial velocity from the earlier part
- We know the acceleration
- We know the final velocity is zero
- Let +*x* be to the right

$$a = -\mu g$$
$$v_i = \sqrt{2gh}$$
$$v_f = 0$$

- Want to find *d*

Review problem

Sliding down a ramp

- Same as last part, only the unknown changed.

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$0 = 2gh - 2\mu gd$$

$$h = \mu d$$

$$d = \frac{h}{\mu}$$

Review problem

Sliding down a ramp

- Sensible?
  - ✓ higher the ramp, faster at the bottom, farther it goes
  - ✓ depends on how strong *g* is
  - ✓ if friction goes away, $\mu \to 0$ and $d \to \infty$
    - ✓ (the object doesn't stop if there is no friction)

Section 4.6: Momentum

Section Goals

You will learn to

- Identify **momentum** as the ability of an object to affect the motion of other objects in a collision.
- Calculate momentum for individual objects and systems of interacting objects

Section 4.6: Momentum

- recall:

$$m_{\mathrm{u}}\Delta v_{\mathrm{u}\,x} + m_{\mathrm{s}}\Delta v_{\mathrm{s}\,x} = 0$$

- Using $\Delta v_x = v_{x,\mathrm{f}} - v_{x,\mathrm{i}}$, we get

$$m_{\mathrm{u}}v_{\mathrm{u}\,x,\mathrm{f}} - m_{\mathrm{u}}v_{\mathrm{u}\,x,\mathrm{i}} + m_{\mathrm{s}}v_{\mathrm{s}\,x,\mathrm{f}} - m_{\mathrm{s}}v_{\mathrm{s}\,x,\mathrm{i}} = 0$$

- This equation suggests that the product of inertia and velocity is an important quantity referred to as ***momentum***:

$$\vec{p} \equiv m\vec{\upsilon}$$

where the x component of momentum is

$$p_x \equiv m\upsilon_x$$

Section 4.6: Momentum

- With this definition of momentum, we can rewrite in the form

$$\Delta p_{\mathrm{u}\,x} + \Delta p_{\mathrm{s}\,x} = 0$$

where $\Delta p_x = p_{x,\mathrm{f}} - p_{x,\mathrm{i}}$.

- This equation can be rewritten in vectorial form as

$$\Delta\vec{p}_{\mathrm{u}} + \Delta\vec{p}_{\mathrm{s}} = \vec{0}$$

Section 4.6
Question

Consider these situations:
(i) a ball moving at speed v is brought to rest;
(ii) the same ball is projected from rest so that it moves at speed v;
(iii) the same ball moving at speed v is brought to rest and then projected backward to its original speed.

In which case(s) does the ball undergo the largest change in momentum?

1. *(i)*
2. *(i)* and *(ii)*
3. *(i)*, *(ii)*, and *(iii)*
4. *(ii)*
5. *(ii)* and *(iii)*
6. *(iii)*

Section 4.6
Question

Consider these situations:
(i) a ball moving at speed v is brought to rest;
(ii) the same ball is projected from rest so that it moves at speed v;
(iii) the same ball moving at speed v is brought to rest and then projected backward to its original speed.

In which case(s) does the ball undergo the largest change in momentum?

1. *(i)*
2. *(i)* and *(ii)*
3. *(i)*, *(ii)*, and *(iii)*
4. *(ii)*
5. *(ii)* and *(iii)*
6. *(iii)*

Section 4.7: Isolated systems

Section Goals

You will learn to

- Develop an accounting scheme for the **total momentum** of a system of interacting objects.
- Identify that when a system interacts with the environment, the momentum of the system changes.
- Recognize that for **isolated** systems, that is, ones that have no external influences on the environment, the momentum of the system **does not change**.

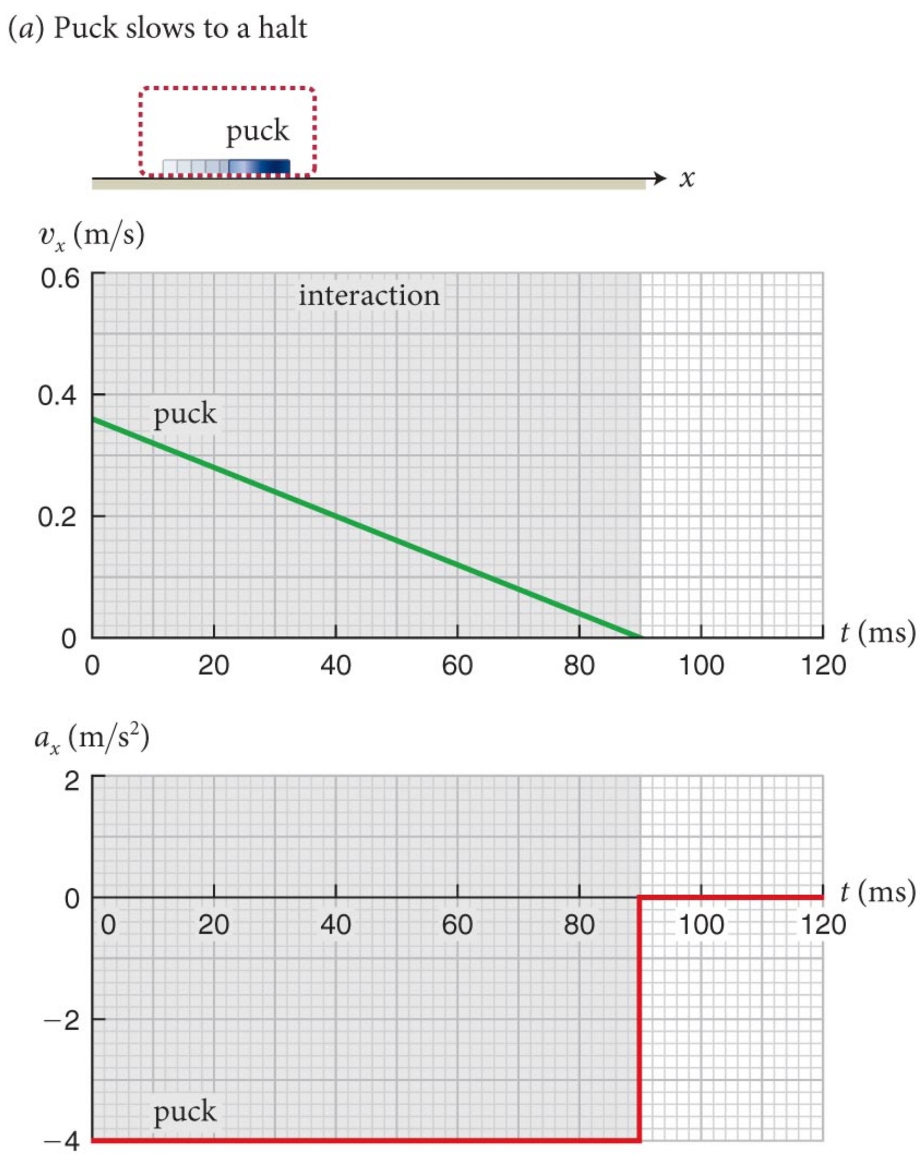
Section 4.7: Isolated systems

- You can add up the momentum of all objects in a system to obtain the *momentum of the system*. Therefore, the momentum of a system of two moving carts is

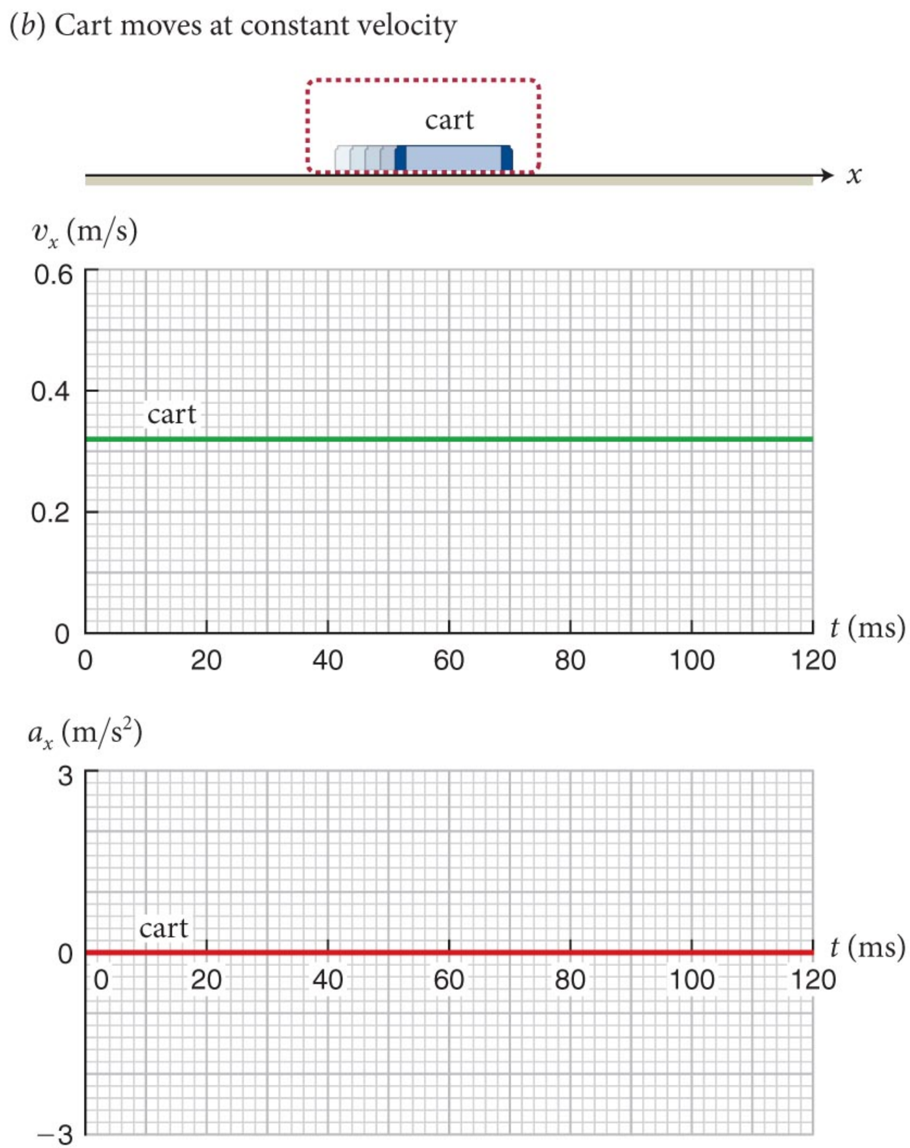
$$\vec{p} \equiv \vec{p}_1 + \vec{p}_2$$

- With this definition we can begin to develop an accounting scheme for the momentum of a system.
- Let us begin by examining the four specific situations seen in the figure on the next four slides.

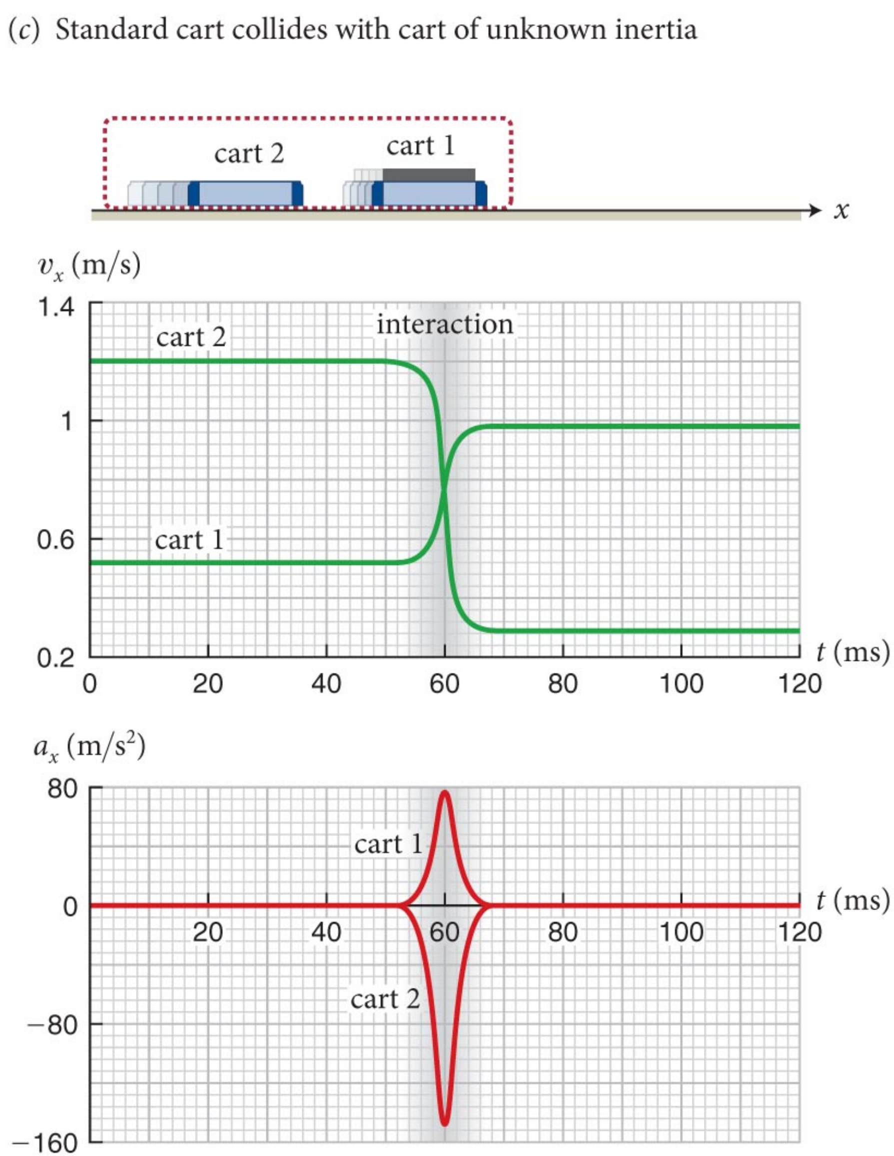
Section 4.7: Isolated systems



Section 4.7: Isolated systems

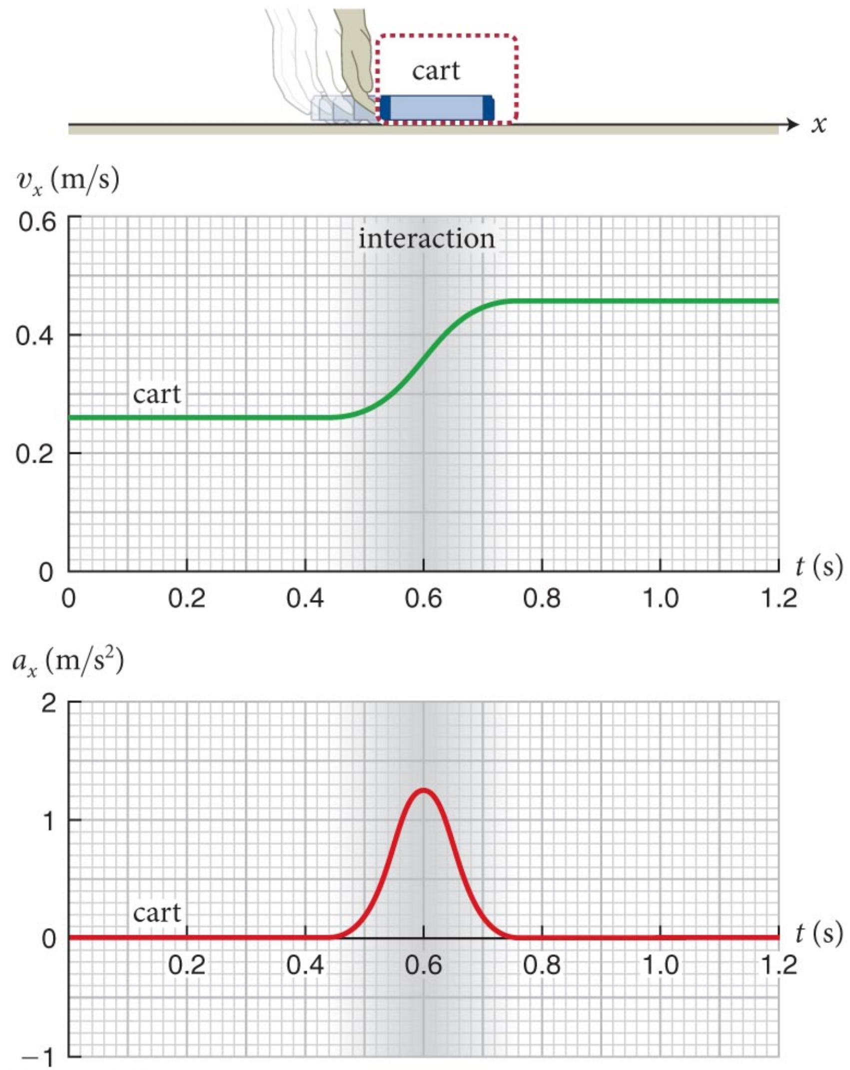


Section 4.7: Isolated systems

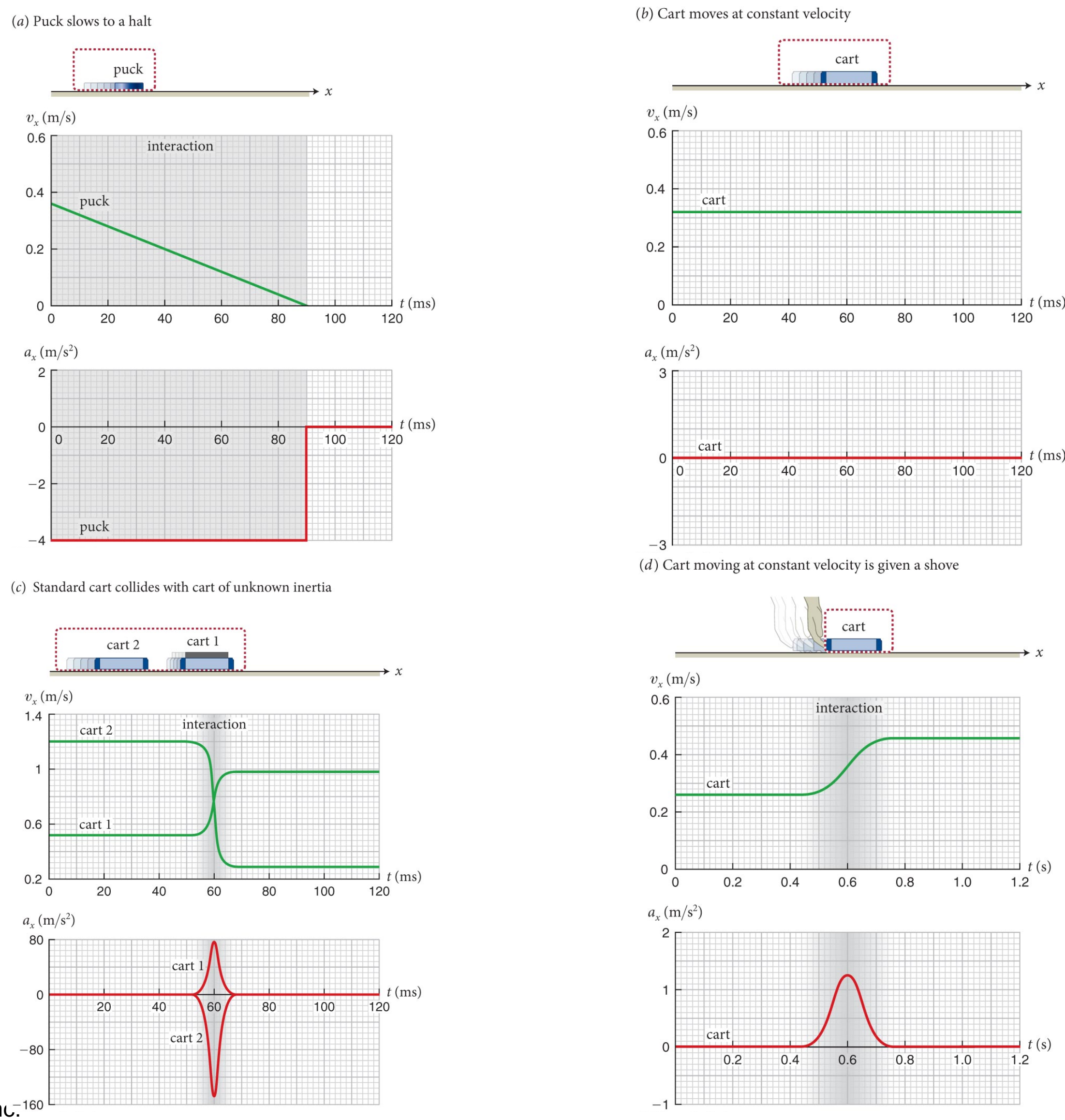


Section 4.7: Isolated systems

(*d*) Cart moving at constant velocity is given a shove



Which are isolated? Interact with environment?



Section 4.7: Isolated systems

Table 4.2 Interactions and momentum changes in Checkpoint 4.11

	Situation	Interacting objects	System	System interacting?	$\Delta\vec{p}$
Slides to a stop	*a*	floor ↔ puck	puck	yes	nonzero
Constant v	*b*	none	cart	no	$\vec{0}$
Collision {	*c*	cart 1 ↔ cart 2	cart 1	yes	nonzero
	d	cart 1 ↔ cart 2	carts 1 & 2	no	$\vec{0}$
Hand pushes	*e*	hand ↔ cart	cart	yes	nonzero

- Note that whenever the system interacts with the environment, the momentum changes.
- A system for which there are no external interactions is said to be **isolated**:
  - For such systems, $\Delta\vec{p} = \vec{0}$.

Section 4.7: Isolated systems

- The figure and table below illustrate two system choices for studying a collision.
- The system containing both cars is isolated and $\Delta\vec{p} = \vec{0}$.
- The system containing just one cart is not isolated and $\Delta\vec{p} \neq \vec{0}$.

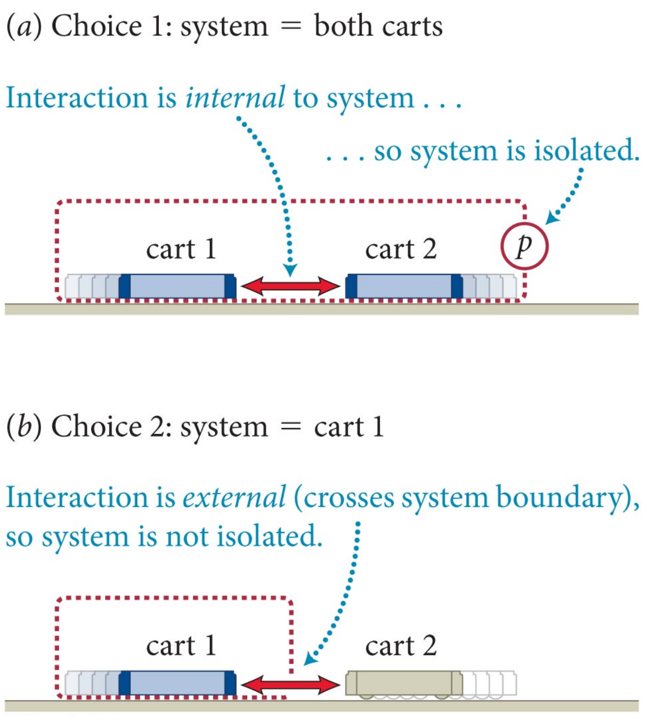
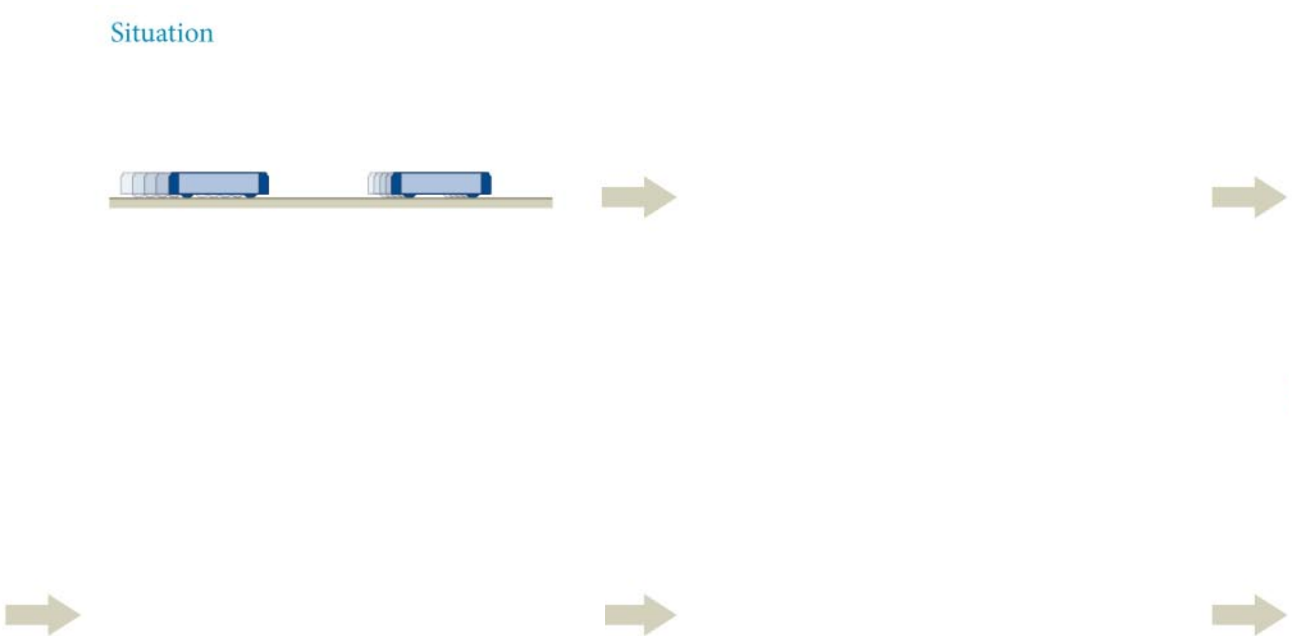


Table 4.3 Two choices of system for carts colliding

	Choice 1	Choice 2
System:	carts 1 & 2	cart 1
Environment:	track	cart 2 & track
Interactions:	internal	external
System isolated?	yes	no
Momentum changing?	no	yes

Section 4.7: Isolated systems

- In the figure, the procedure for choosing an isolated system for a collision between two carts is illustrated.



Procedure: Choosing an isolated system

When you analyze momentum changes in a problem, it is convenient to choose a system for which no momentum is transferred into or out of the system (an isolated system). To do so, follow these steps:

Procedure: Choosing an isolated system (cont.)

1. Separate all objects named in the problem from one another.
2. Identify all possible interactions among these objects and between these objects and their environment (the air, Earth, etc.).

Procedure: Choosing an isolated system (cont.)

3. Consider each interaction individually and determine whether it causes the interacting objects to accelerate. Eliminate any interaction that does not affect (or has only a negligible effect on) the objects' accelerations during the time interval of interest.

Procedure: Choosing an isolated system (cont.)

4. Choose a system that includes the object or objects that are the subject of the problem (for example, a cart whose momentum you are interested in) in such a way that none of the remaining interactions cross the system boundary. Draw a dashed line around the objects in your choice of system to represent the system boundary. None of the remaining interactions should cross this line.

Procedure: Choosing an isolated system (cont.)

5. Make a system diagram showing the initial and final states of the system and its environment.

Problem

An archer stands on a frozen pond. Her inertia is 60kg.

She fires a 0.5kg arrow at 50 m/s

What is her velocity afterwards?

Problem

Can neglect friction on ice

→ isolated system of archer and arrow

That means:

interactions are internal

momentum does not change

total momentum is constant

Problem

- Momentum is conserved
- Initially: zero
- After?

  still zero! If arrow moves in +x, she moves in –x
- Set up conservation. Arrow has v_a, girl v_g

$$p_i = p_f = 0$$

$$p_f = m_g v_{g,f} + m_a v_{a,f} = 0$$

$$v_{g,f} = -\left(\frac{m_a}{m_g} v_{a,f}\right)$$

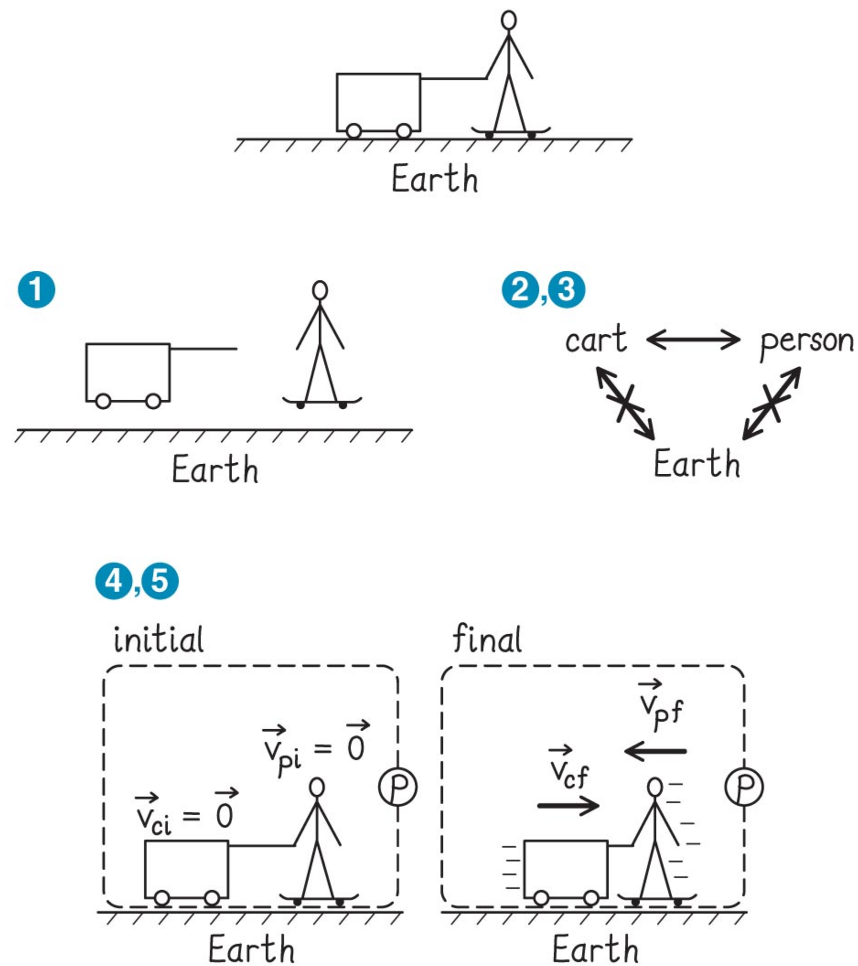
$$v_{g,f} \approx -0.42\,\mathrm{m/s}$$

Exercise 4.7 Who's pulling whom?

A person standing on a skateboard on horizontal ground pulls on a rope fastened to a cart. Both the person and the cart are initially at rest. Identify an isolated system and make a system diagram.

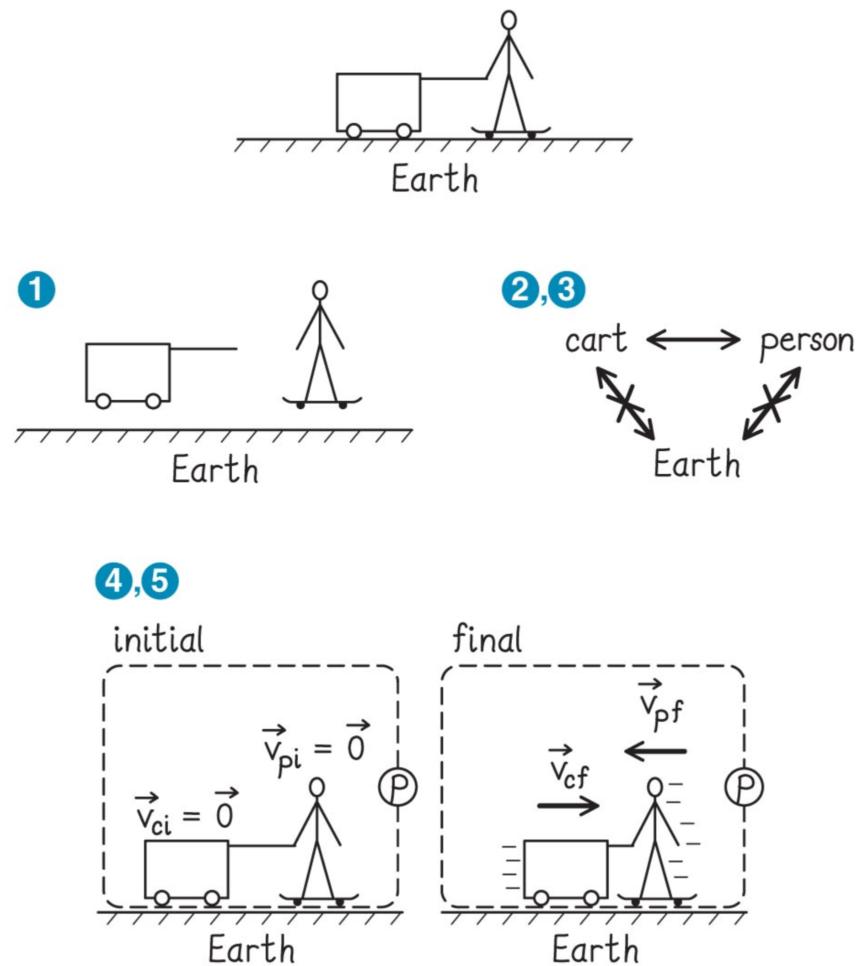
Exercise 4.7 Who’s pulling whom? (cont.)

SOLUTION Begin by separating the objects in the problem: the person, the cart, and Earth. (I could always go into more detail—include the air, the rope, and the skateboard—but it pays to keep things as simple as possible. For that reason, I consider the skateboard to be part of the person and the rope to be part of the cart.)



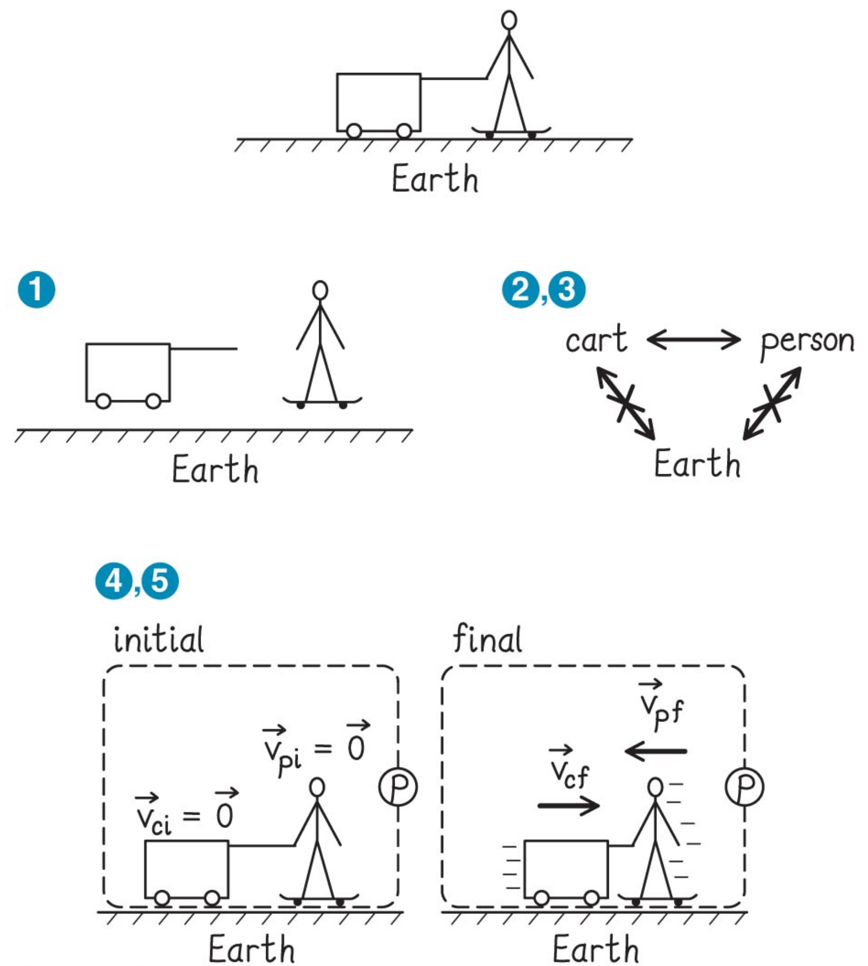
Exercise 4.7 Who's pulling whom? (cont.)

SOLUTION The cart interacts with Earth and the person; the person interacts with the cart and Earth. Ignoring friction in the wheels of the cart, I know that the interaction between the cart and Earth has no effect on any motion, and so I can eliminate it from the analysis. The same holds for the interaction between the person (the skateboard) and Earth.



Exercise 4.7 Who's pulling whom? (cont.)

SOLUTION I then draw a boundary around the person and the cart, making the interaction between the two internal. Because there are no external interactions, this system is isolated. Finally I draw a system diagram showing the initial and final conditions of the system with the person and cart initially at rest and then moving. ✔



Section 4.8: Conservation of momentum

Section Goals

You will learn to

- Recognize that for an isolated system there is **no creation or destruction** of momentum inside the system, only transfer between the objects of the system.
- Calculate the kinematics of isolated systems using the conservation of momentum law.

Section 4.8: Conservation of momentum

- We saw that

$$\Delta\vec{p} = \vec{0} \quad \text{(isolated system)}$$

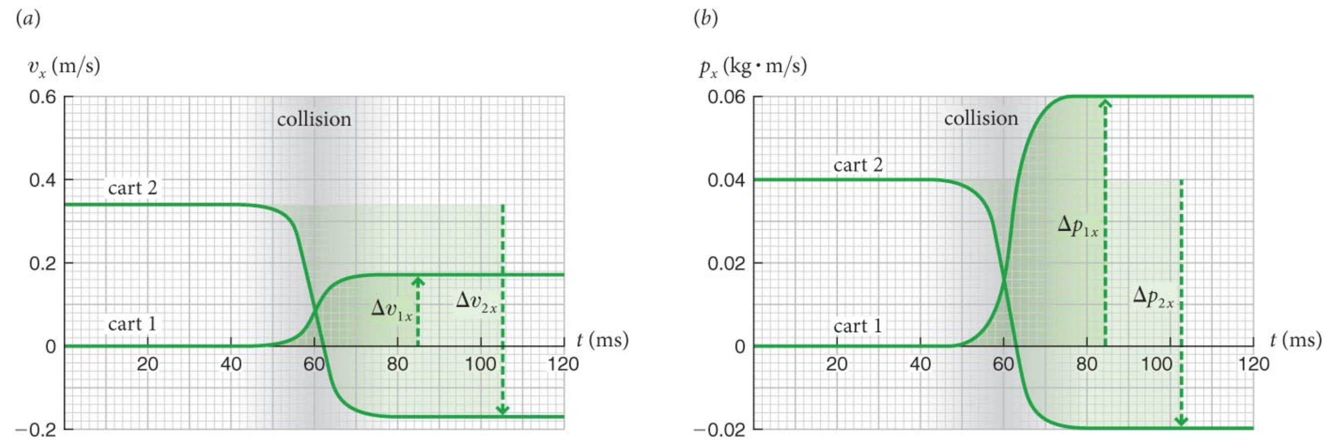
- So, for any two objects colliding should we expect to get

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = \vec{0}\,?$$

- The only way to verify this is to do an experiment.

Section 4.8: Conservation of momentum

- The figure shows the results of a collision experiment.



- Cart 1: $m_1 = 0.36$ kg, $v_{1x,\text{i}} = 0$ m/s, $v_{1x,\text{f}} = +0.17$ m/s:

$$\Delta p_{1x} = m_1(v_{1x,\text{f}} - v_{1x,\text{i}}) = 0.0061 \text{ kg} \cdot \text{m/s}$$

- Cart 2: $m_2 = 0.12$ kg, $v_{2x,\text{i}} = +0.34$ m/s, $v_{1x,\text{f}} = –0.17$ m/s:

$$\Delta p_{1x} = m_2(v_{2x,\text{f}} - v_{2x,\text{i}}) = –0.0061 \text{ kg} \cdot \text{m/s}$$

- Consequently the momentum of the systems does not change:

$$\Delta\vec{p} \equiv \Delta\vec{p}_1 + \Delta\vec{p}_2 = \vec{0}$$

Section 4.8: Conservation of momentum

- Repeating the experiment with any other pair of objects yields the same result.
- We must therefore conclude that **momentum can be transferred from one object to another, but it cannot be created or destroyed.**
- This statement is referred to as the **conservation of momentum**, and for isolated systems this means

$$\Delta\vec{p} \equiv \Delta\vec{p}_1 + \Delta\vec{p}_2 = \vec{0}$$

or

$$\vec{p}_\mathrm{f} = \vec{p}_\mathrm{i} \text{ (isolated system)}$$

- For systems that are not isolated, we have the **momentum law**:

$$\Delta \mathrm{p} = \mathrm{J}$$

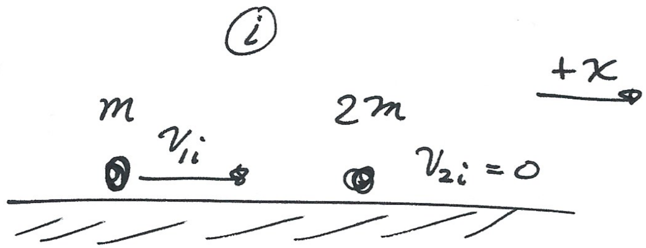
- where $\vec{J}$ represents the momentum transfer from the environment to the system; this is called the **impulse.**

Collision!

- Two identical balls collide. One is at rest with inertia $2m$, the other with inertia m is moving toward it with speed v.

- If the first ball rebounds with half its initial velocity, what is the velocity of the second ball after the collision?

Collision!



choice of axes

$$P_i = m v_{1i} \qquad P_f = -m v_{1f} + 2m v_{2f}$$

$$P_i = P_f$$

$$m v_{1i} = -m v_{1f} + 2m v_{2f}$$

$$v_{1i} = -v_{1f} + 2v_{2f}$$

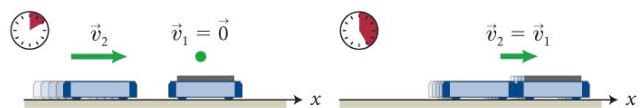
$$v_{2f} = (v_{1i} + v_{1f}) \cdot \frac{1}{2} \qquad v_{1i} = v, \; v_{1f} = \frac{1}{2}v$$

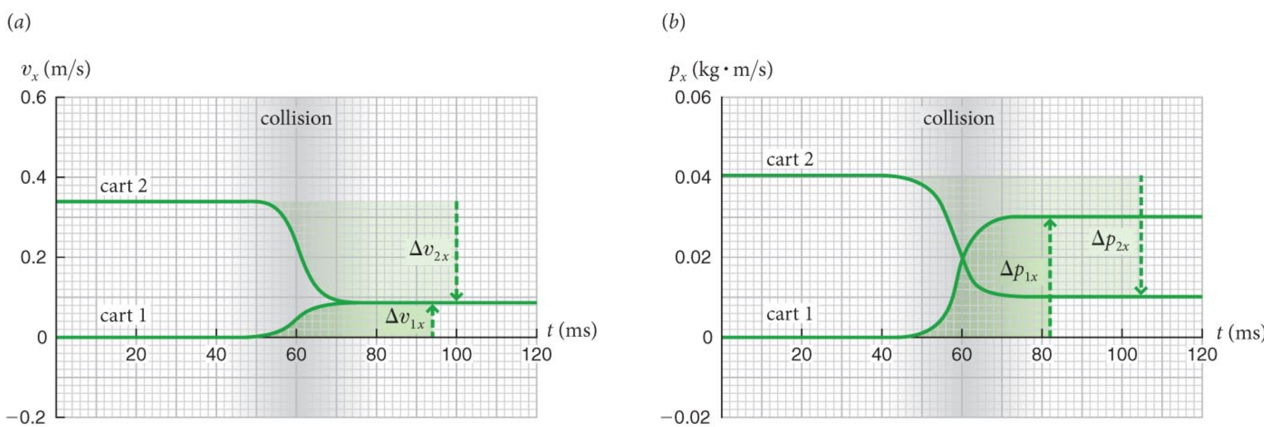
$$v_{2f} = \frac{3}{4}v \;\checkmark$$

Section 4.8: Conservation of momentum

- Let us look at a somewhat different experiment:
  - Let carts 1 and 2 collide with the same initial velocities as before, but this time they **stick together** after the collision.
  - The two carts will have the same final velocity.
  - Very different sort of collision …
  - The data confirms the momentum of the system is unchanged:

$$\vec{p}_\mathrm{i} = \vec{p}_{1,\mathrm{i}} + \vec{p}_{2,\mathrm{i}} + \vec{p}_{3,\mathrm{i}} = \vec{p}_{1,\mathrm{f}} + \vec{p}_{2,\mathrm{f}} + \vec{p}_{3,\mathrm{f}} = \vec{p}_\mathrm{f} \text{ (isolated system)}$$





Section 4.8
Question

Suppose the entire population of the world gathers in one spot and, at the sounding of a prearranged signal, everyone jumps up. While all the people are in the air, does Earth gain momentum in the opposite direction?

1. No: The inertial mass of Earth is so large that the planet's change in motion is imperceptible.
2. Yes: Because of its much larger inertial mass, however, the change in momentum of Earth is much less than that of all the jumping people.
3. Yes: Earth recoils, like a rifle firing a bullet, with a change in momentum equal to and opposite that of the people.
4. It depends.

Section 4.8
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2. Yes: Because of its much larger inertial mass, however, the change in momentum of Earth is much less than that of all the jumping people.
3. ✔ Yes: Earth recoils, like a rifle firing a bullet, with a change in momentum equal to and opposite that of the people.
4. It depends.

How much

Mass of average human ~70kg

Number of humans ~7×10^9

Total mass of humans ~5×10^{11}

Total mass of earth ~6×10^{24}

The ratio of velocity changes will be about 10^{-13}

Section 4.8
Question

A car accelerates from rest. In doing so the car gains a certain amount of momentum and Earth gains

1. more momentum.
2. the same amount of momentum.
3. less momentum.
4. The answer depends on the interaction between the two.

Question

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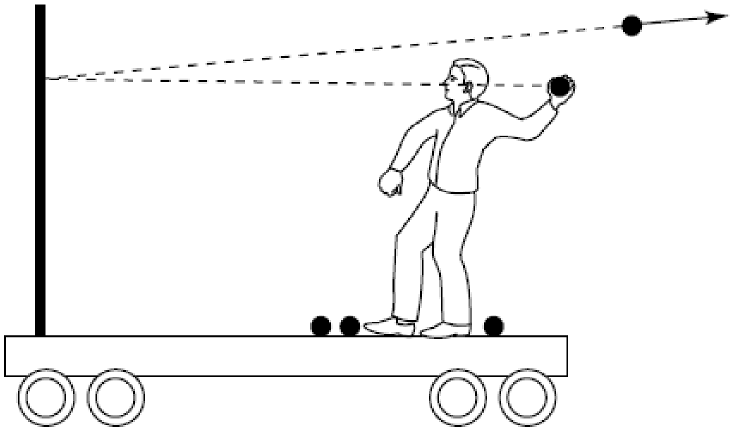
Section 4.8
Question

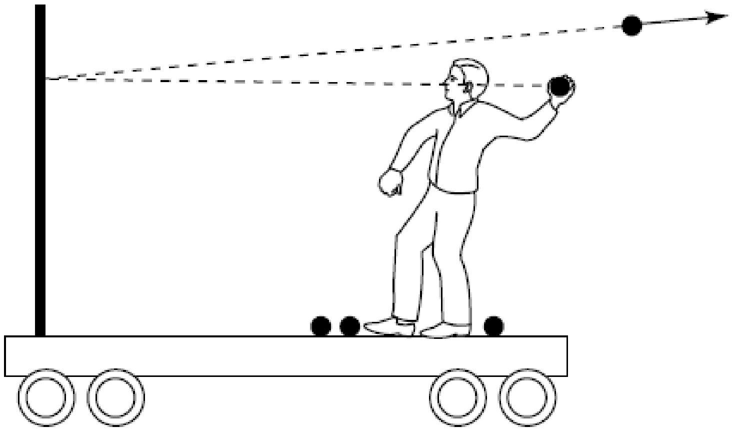
Suppose you are on a cart that is initially at rest on a track with very little friction. You throw balls at a partition that is rigidly mounted on the cart. If the balls bounce straight back as shown in the figure, is the cart put in motion?



1. Yes, it moves to the right.
2. Yes, it moves to the left.
3. No, it remains in place.

Section 4.8
Clicker Question 12

Suppose you are on a cart that is initially at rest on a track with very little friction. You throw balls at a partition that is rigidly mounted on the cart. If the balls bounce straight back as shown in the figure, is the cart put in motion?



1. Yes, it moves to the right.
2. Yes, it moves to the left.
3. No, it remains in place.

Section 4.8
Question

Is it possible for a stationary object that is struck by a moving object to have a larger final momentum than the initial momentum of the incoming object?

1. Yes.
2. No, because such an occurrence would violate the law of conservation of momentum.

Question

Is it possible for a stationary object that is struck by a moving object to have a larger final momentum than the initial momentum of the incoming object?

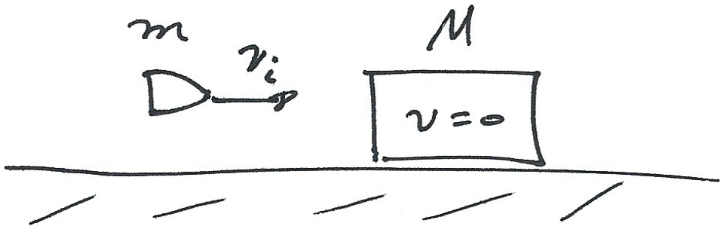
1. Yes.
2. No, because such an occurrence would violate the law of conservation of momentum.

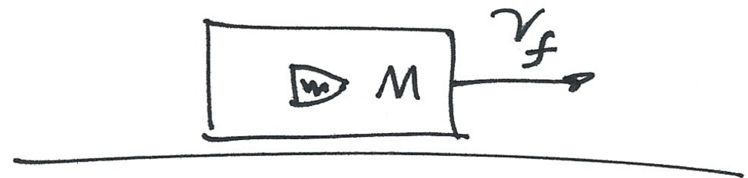
Problem

A bullet of mass m is fired at velocity v_i into a wooden block of mass M initially at rest. The bullet embeds itself in the block.

What is the velocity of the bullet & block after the collision?

Problem





$$P_i = mv_i + M\cdot 0 = mv_i$$

$$P_f = mv_f + Mv_f = (m+M)v_f$$

$$mv_i = (m+M)v_f$$

$$v_f = \left(\frac{m}{m+M}\right)v_i \checkmark$$

Check:

units

$m=M$, $v_f = v_i/2$

$M=0$, $v_f = v_i$

Example 4.8 Bounce

A 0.20-kg rubber ball is dropped from a height of 1.0 m onto a hard floor and bounces straight up.

Assuming the speed with which the ball rebounds from the floor is the same as the speed it has just before hitting the floor, determine the impulse delivered by the floor to the rubber ball.

Example 4.8 Bounce (cont.)

❶ GETTING STARTED I define the ball to be my system in this problem. The impulse delivered to the ball is then given by the change in its momentum. I need to develop a way to determine this change in momentum.

Example 4.8 Bounce (cont.)

❷ DEVISE PLAN To solve this problem I need to first determine the velocity of the ball just before it hits the floor.

I therefore break the problem into two parts: the downward fall of the ball and its collision with the floor.

I can use 1D motion to determine the time interval it takes the ball to fall from its initial height (assuming the ball is initially at rest).

Example 4.8 Bounce (cont.)

❷ DEVISE PLAN As it falls, the ball undergoes constant acceleration, so I can use 1D motion to calculate its velocity just before it hits the floor.

Because its speed is not changing as it rebounds, I also know its velocity after it bounces up.

Knowing the velocities, I can calculate the ball’s momentum before and after the bounce. The change in momentum then gives the impulse.

Example 4.8 Bounce (cont.)

❸ EXECUTE PLAN From Eq. 3.15 I see that it takes an object $t = \sqrt{2h/g} = \sqrt{2\,(1.0\,\mathrm{m})\,/9.81\,\mathrm{m/s^2}} \approx 0.45\,\mathrm{s}$ to fall from a height of 1.0 m.

Choosing the positive *x* axis pointing upward, we can find the velocity

$$v_{x,f} = 0 + (-9.8\,\mathrm{m/s^2})(0.45\,\mathrm{s}) \approx -4.4\,\mathrm{m/s}$$

(note error in textbook. Could also use $v_f^2 = v_i^2 + 2gh$)

Example 4.8 Bounce (cont.)

❸ EXECUTE PLAN Now that I know the ball's velocity just before it hits the ground I can obtain the x component of the momentum of the ball just before it hits the ground by multiplying the ball's velocity by its inertia:

$$p_{x,\mathrm{i}} = (0.20\ \mathrm{kg})(-4.4\ \mathrm{m/s}) = -0.88\ \mathrm{kg \cdot m/s}.$$

(I use the subscript i to indicate that this is the initial momentum of the ball before the collision.)

Example 4.8 Bounce (cont.)

❸ EXECUTE PLAN If the ball rebounds with the same speed, then the *x* component of the momentum after the collision with the floor has the same magnitude but opposite sign: $p_{x,\mathrm{f}} = +0.88\ \mathrm{kg \cdot m/s}$. The change in the ball's momentum is thus

$$\Delta p_x = p_{x,\mathrm{f}} - p_{x,\mathrm{i}} = +0.88\ \mathrm{kg \cdot m/s} - (-0.88\ \mathrm{kg \cdot m/s})$$
$$= +1.76\ \mathrm{kg \cdot m/s}.$$

Example 4.8 Bounce (cont.)

❸ EXECUTE PLAN The interaction with the ground changes the momentum of the ball, making it rebound. The ball does not constitute an isolated system, and the change in its momentum is due to an impulse delivered by Earth to the ball. To determine the impulse, I substitute the change in momentum of the ball into Eq. 4.18:

$$\vec{J} = \Delta\vec{p} = \Delta p_x\,\hat{\imath} = (+1.76\,\mathrm{kg \cdot m/s})\,\hat{\imath} \quad ✔$$

Example 4.8 Bounce (cont.)

❹ EVALUATE RESULT

The x component of the velocity of the ball just before it hits the floor is negative because the ball moves downward, in the negative x direction.

After the collision it moves in the opposite direction, and consequently the x components of the changes in velocity and momentum are both positive.

Example 4.8 Bounce (cont.)

❹ EVALUATE RESULT

Because the x component of the change in the ball's momentum is positive, the impulse is directed upward (in the positive x direction). This makes sense because this impulse changes the direction of travel of the ball from downward to upward.✔

Section 4.8: Conservation of momentum

Worked Problem 4.5 Forensic physics

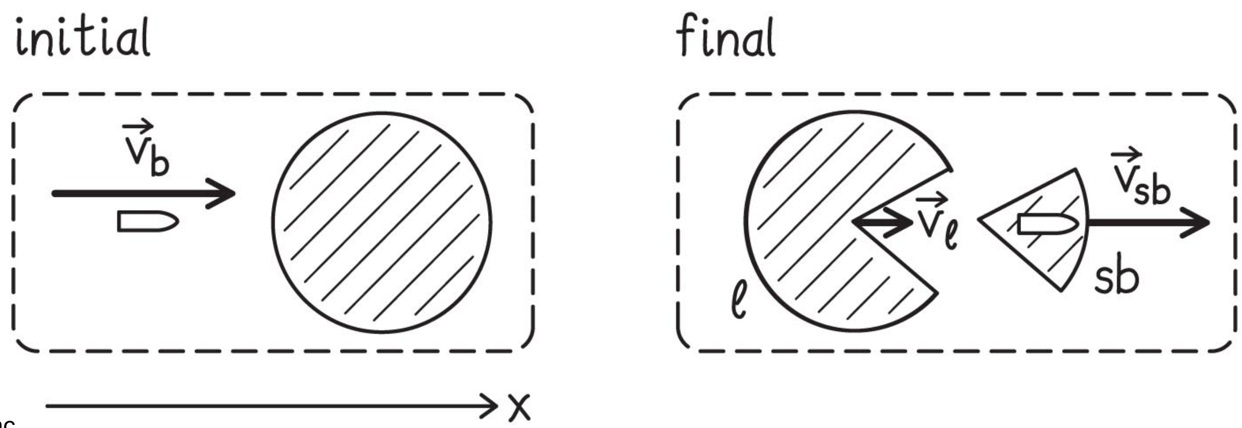
Your friend from law enforcement claims that, in certain cases, a piece of an object hit by gunfire can move toward the shooter rather than away from the shooter. You decide to investigate whether this is possible by firing a target rifle at some melons. Suppose that in one of your tests an 8.0-g bullet is fired at 400 m/s toward a 1.20-kg melon several meters away, splitting the melon into two pieces of unequal size. The bullet lodges in the smaller piece and propels it forward (that is, in the direction the bullet originally traveled) at 9.2 m/s. If the combined inertia of this piece and the lodged bullet is 0.45 kg, determine the final velocity of the larger piece.

Worked Problem 4.5 Forensic physics (cont.)

❶ GETTING STARTED As usual, we begin by identifying an isolated system. What about one made up of the bullet and melon? There may be some interaction with the ground or the support upon which the melon sits, but if we choose our time interval to start immediately before the bullet strikes the melon and end immediately after the smaller piece breaks loose, we can ignore this interaction. During this very short time interval, we can treat the system as isolated.

Worked Problem 4.5 Forensic physics (cont.)

❶ GETTING STARTED Next we sketch a system diagram showing the initial and final conditions of the system (Figure WG4.4). We arbitrarily choose the direction in which the smaller piece moves (to the right in the figure) as our positive *x* direction. We don't know which way the larger piece moves, and so we draw it as going forward, too. Our calculation will let us know whether this guess is right or wrong. The momentum of the system remains constant during this collision because the system is isolated.



Worked Problem 4.5 Forensic physics (cont.)

❷ DEVISE PLAN We can translate the information given into an equation that requires no change in the momentum of the system. There are two objects in the system in the final condition, but they are different from the ones in the initial condition. We designate the inertia of the bullet as m_b and the x component of its velocity as $v_{b\,x}$, and we note that the intact melon had an initial speed $v_{m,i} = 0$ and inertia m_m.

Worked Problem 4.5 Forensic physics (cont.)

❷ DEVISE PLAN We have

$$\Delta\vec{p} = 0 \Rightarrow \vec{p}_\mathrm{i} = \vec{p}_\mathrm{f}$$

$$m_\mathrm{b}\upsilon_{\mathrm{b}x,\mathrm{i}} + m_\mathrm{m}\upsilon_{\mathrm{m}x,\mathrm{i}} = (m_\mathrm{s} + m_\mathrm{b})\upsilon_{\mathrm{sb}x,\mathrm{f}} + m_\ell\upsilon_{\ell x,\mathrm{f}}$$

where the subscript s denotes the smaller piece, the subscript ℓ denotes the larger piece, and the subscript sb denotes the combination of the bullet and the smaller piece. We are looking for $v_{\ell x,\mathrm{f}}$, and because all our other variables have known values, the planning is finished.

Worked Problem 4.5 Forensic physics (cont.)

❸ EXECUTE PLAN We isolate the desired unknown and then insert numerical values. Recall that all of the velocity components in our diagram are in the positive direction.

$$m_\ell \upsilon_{\ell x,\mathrm{f}} = m_\mathrm{b} \upsilon_{\mathrm{b}x,\mathrm{i}} + m_\mathrm{m} \upsilon_{\mathrm{m}x,\mathrm{i}} - (m_\mathrm{s} + m_\mathrm{b}) \upsilon_{\mathrm{sb}x,\mathrm{f}}$$

$$\upsilon_{\ell x,\mathrm{f}} = \frac{m_\mathrm{b}(+|\vec{\upsilon}_{\mathrm{b,i}}|) - (m_\mathrm{s} + m_\mathrm{b})(+|\vec{\upsilon}_{\mathrm{sb,f}}|)}{m_\ell}$$

$$= \frac{(0.0080\ \mathrm{kg})(400\ \mathrm{m/s}) - (0.45\ \mathrm{kg})(9.2\ \mathrm{m/s})}{1.20\ \mathrm{kg} - (0.45\ \mathrm{kg} - 0.0080\ \mathrm{kg})}$$

$$= -1.240\ \mathrm{m/s} = -1.2\ \mathrm{m/s}.$$ ✔

Worked Problem 4.5 Forensic physics (cont.)

❹ EVALUATE RESULT The negative sign means that the larger piece moves in the negative x direction, which means that our initial guess for the direction of this motion was wrong. Note, however, that the algebra nicely informed us of the incorrect assumption without pain or confusion. The larger piece really does move back toward the rifle. The magnitude of this velocity is fairly small, as we would expect, but it certainly would be noticeable.

Concepts: Inertia

- *Friction* is the resistance to motion that one surface encounters when moving over another surface. In the absence of friction, objects moving along a horizontal track keep moving without slowing down.
- **Inertia** is a measure of an object's tendency to resist a change in its velocity. Inertia is determined entirely by the type of material of which the object is made and by the amount of that material contained in the object. Inertia is related to *mass*, and for this reason we use the symbol m to represent it. The SI unit of inertia is the **kilogram** (kg).

Quantitative Tools: Inertia

- If an object of unknown inertia m_u collides with an inertial standard of inertia m_s, the ratio of the inertias is related to the changes in the velocities by

$$\frac{m_u}{m_s} \equiv -\frac{\Delta v_{sx}}{\Delta v_{ux}}$$

Concepts: Systems and momentum

- A **system** is any object or group of objects that can be separated, in our minds, from the surrounding environment.
- The **environment** is everything that is not part of the system. You can choose the system however you want, but once you decide to include a certain object in the system, that object must remain a part of the system throughout your analysis.

Concepts: Systems and momentum

- A system for which there are no external interactions is called an **isolated system.**
- An **extensive quantity** is one whose value is proportional to the size or "extent" of the system.
- An **intensive quantity** is one that does not depend on the extent of the system.
- A **system diagram** shows the initial and final conditions of a system.

Quantitative Tools: Systems and momentum

- The **momentum** $\vec{p}$ of an object is the product of its inertia and velocity:

$$\vec{p} \equiv m\vec{\upsilon}$$

- The momentum of a system of objects is the sum of the momenta of its constituents:

$$\vec{p} \equiv \vec{p}_1 + \vec{p}_2 + \cdots$$

Concepts: Conservation of momentum

- Any extensive quantity that cannot be created or destroyed is said to be **conserved,** and the amount of any *conserved* quantity in an isolated system is *constant*.
- Momentum is a conserved quantity, and therefore the momentum of an isolated system is constant.
- The momentum can be transferred from one object to another in the system, but the momentum of the system cannot change.

Quantitative Tools: Conservation of momentum

- The momentum of an isolated system is constant:

$$\Delta\vec{p} = \vec{0}$$

- Another way to say this is that for an isolated system, the initial momentum is equal to the final momentum:

$$\vec{p}_{\mathrm{i}} = \vec{p}_{\mathrm{f}}$$

- The **impulse** $\vec{J}$ delivered to a system is equal to the change in momentum of the system:

$$\vec{J} = \Delta\vec{p}$$

- For an isolated system, $\vec{J} = \vec{0}$.