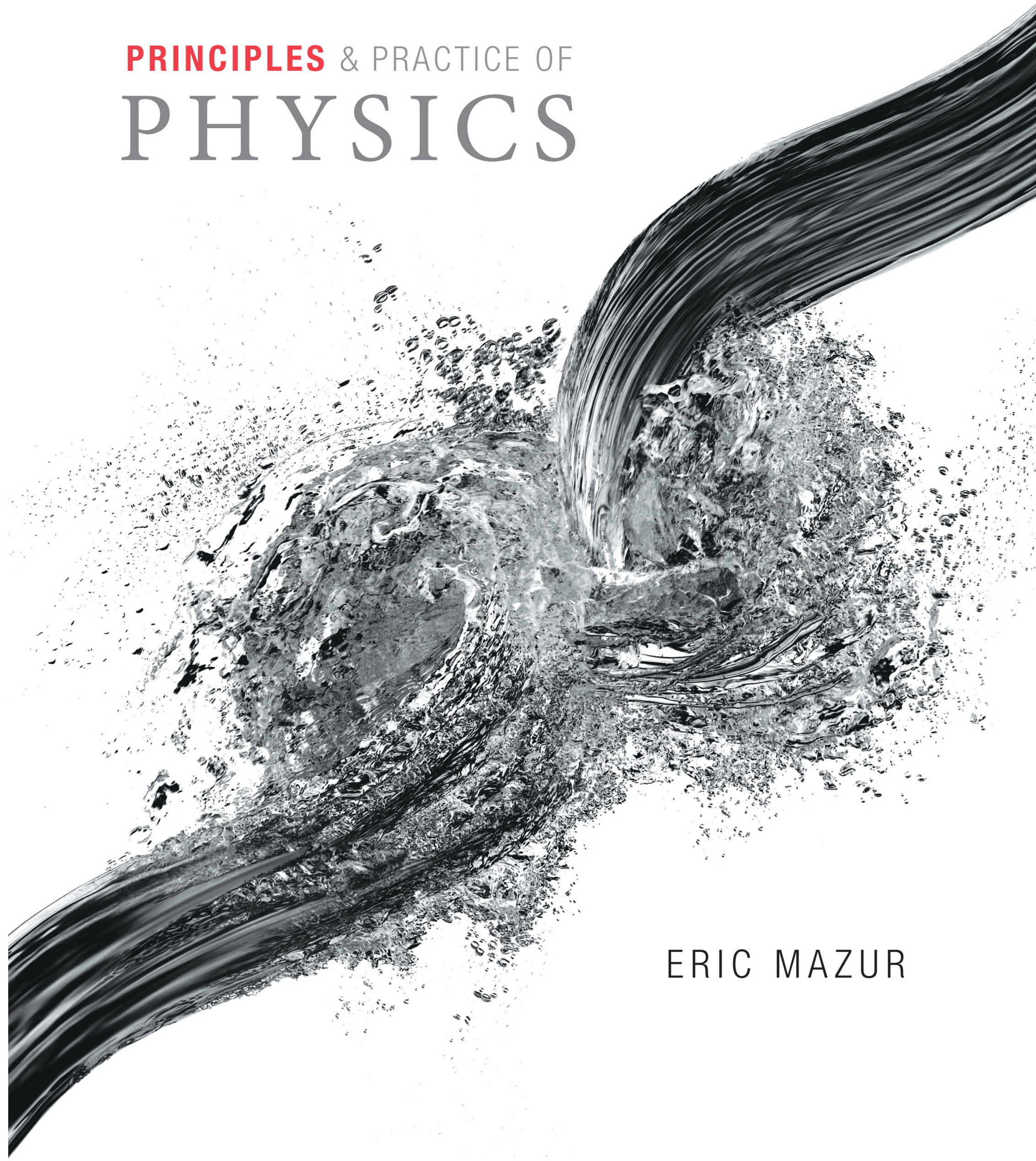
Chapter 9
Work



Chapter 9 Preview

Looking Ahead: Work done by a constant force

- In order for a force to do **work** on an object, the point of application of the force must undergo a **displacement**.
- Work is the ‘useful’ application of a force
- The SI unit of work is the **joule** (J).

Chapter 9: Work

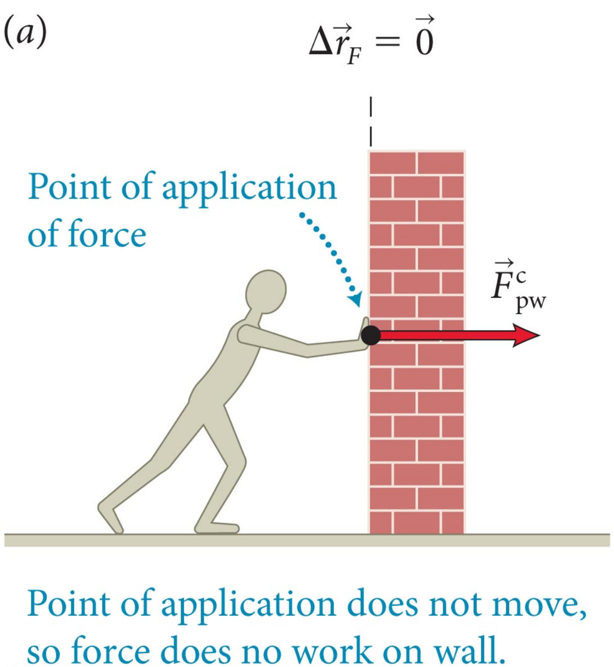
- Forces can change the physical state of an object (internal energy) as well as its state of motion (kinetic energy).
- To describe these changes in energy, physicists use the concept of **work:**
  - **Work is the change in the energy of a system due to external forces.**
- The SI unit of work is the joule (J).

Section 9.1: Force displacement

- **Work** amounts to a **mechanical transfer of energy**, either from a system to the environment or from the environment to a system.
- Do external force *always* cause a change in energy on a system?
  - To answer this question, it is helpful to consider an example

Checkpoint 9.1

9.1 Imagine pushing against a brick wall as shown in Figure 9.1*a*. (*a*) Considering **the wall as the system**, is the force you exert on it internal or external? (*b*) Does this force accelerate the wall? Change its shape? Raise its temperature? (*c*) Does the energy of the wall change as a result of the force you exert on it? (*d*) Does the force you exert on the wall do work on the wall?



Checkpoint 9.1

9.1 Imagine pushing against a brick wall.

(a) Considering the wall as the system, is the force you exert on it internal or external? **external**

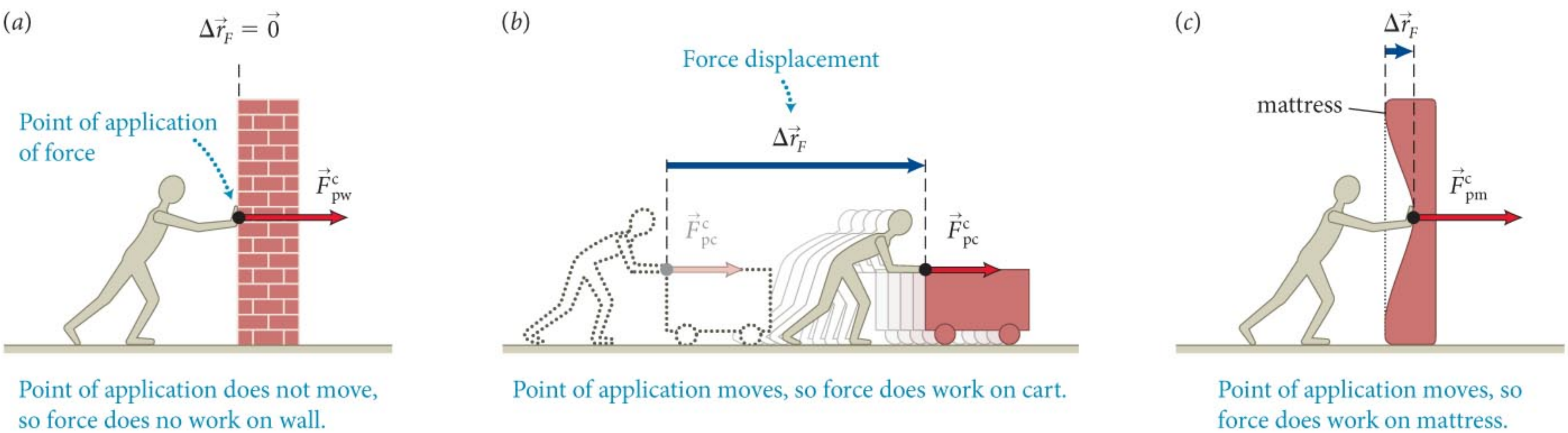
(b) Does this force accelerate the wall? Change its shape? Raise its temperature? **no, no, no**

(c) Does the energy of the wall change as a result of the force you exert on it? **no**

(d) Does the force you exert on the wall do work on the wall?

no

Section 9.1: Force displacement



- Even though the work is zero in (*a*), it is nonzero in (*b*) and (*c*).
- **In order for a force to do work, the point of application of the force must undergo a displacement.**
- The displacement of the point of application of the force is called the **force displacement**.

Exercise 9.1 Displaced forces

For which of the following forces is the force displacement nonzero:

(a) the force exerted by a hand compressing a spring
(b) the force exerted by Earth on a ball thrown upward,
(c) the force exerted by the ground on you at the instant you jump upward,
(d) the force exerted by the floor of an elevator on you as the elevator moves downward at constant speed?

Exercise 9.1 Displaced forces (cont.)

SOLUTION (*a*), (*b*), and (*d*).

(*a*) The point of application of the force is at the hand, which moves to compress the spring.

(*b*) The point of application of the force of gravity exerted by Earth on the ball is at the ball, which moves.

(*c*) The point of application is on the ground, which doesn't move.

(*d*) The point of application is on the floor of the elevator, which moves. ✔

Checkpoint 9.2

9.2 You throw a ball straight up in the air. Which of the following forces do work on the ball while you throw it? Consider the interval from the instant the ball is at rest in your hand to the instant it leaves your hand at speed v.

(a) The force of gravity exerted by Earth on the ball.

(b) The contact force exerted by your hand on the ball.

Checkpoint 9.2

9.2 You throw a ball straight up in the air. Which of the following forces do work on the ball while you throw it? Consider the interval from the instant the ball is at rest in your hand to the instant it leaves your hand at speed v. (*a*) The force of gravity exerted by Earth on the ball. (*b*) The contact force exerted by your hand on the ball.

both do work – for both, the point of application is the ball, and this point moves as you launch the ball

(your hand has to move to launch the ball)

Section 9.1
Question 1

A woman holds a bowling ball in a fixed position. The work she does on the ball

1. depends on the weight of the ball.
2. cannot be calculated without more information.
3. is equal to zero.

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1. depends on the weight of the ball.
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Question 2

A man pushes a very heavy load across a horizontal floor. The work done by gravity on the load

1. depends on the weight of the load.
2. cannot be calculated without more information.
3. is equal to zero.

Section 9.1
Question 2

A man pushes a very heavy load across a horizontal floor. The work done by gravity on the load

1. depends on the weight of the load.
2. cannot be calculated without more information.
3. ✔ is equal to zero.

gravity acts vertically, the displacement is horizontal.
the work is against the frictional force

Section 9.2: Positive and negative work

Section Goal

You will learn to

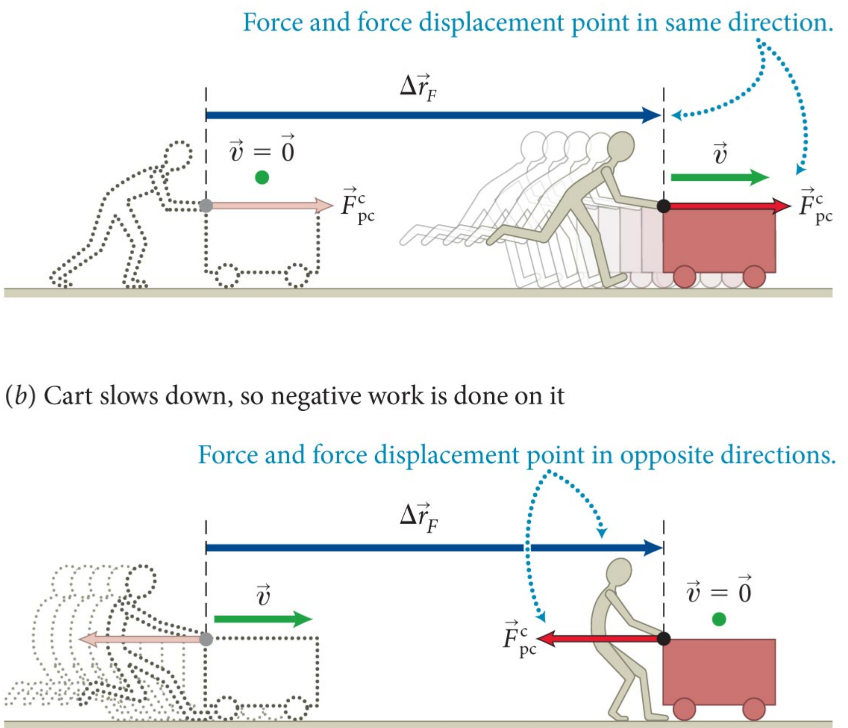
- Determine how the **sign** of the work done depends on the vector relationship between the force and the displacement.

Section 9.2: Positive and negative work

- When the work done by an external force on a system is positive, the change in energy is positive, and when work is negative, the energy change is negative.
- *External force adds or subtracts energy from system*
- Examples of negative and positive work are illustrated in the figure
- **The work done by a force on a system is positive when the force and the force displacement point in the same direction and negative when they point in opposite directions.**

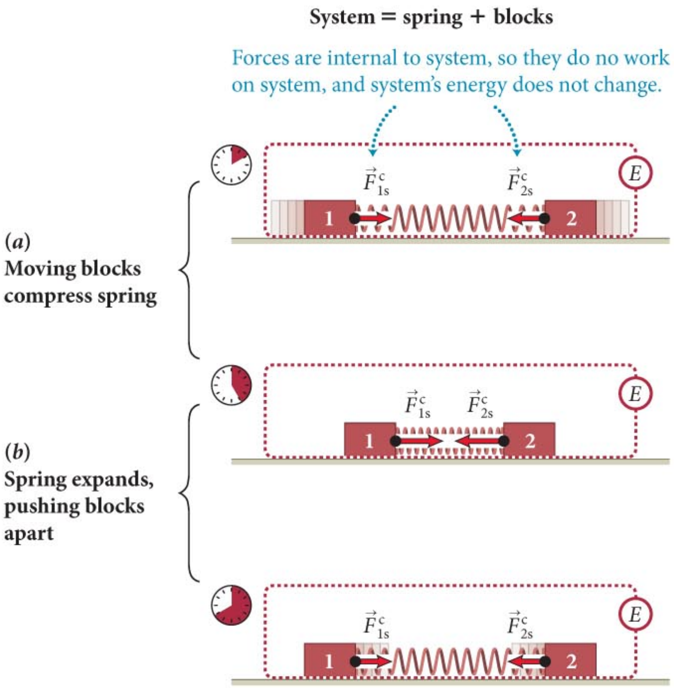
(*a*) Cart speeds up, so positive work is done on it

(*b*) Cart slows down, so negative work is done on it



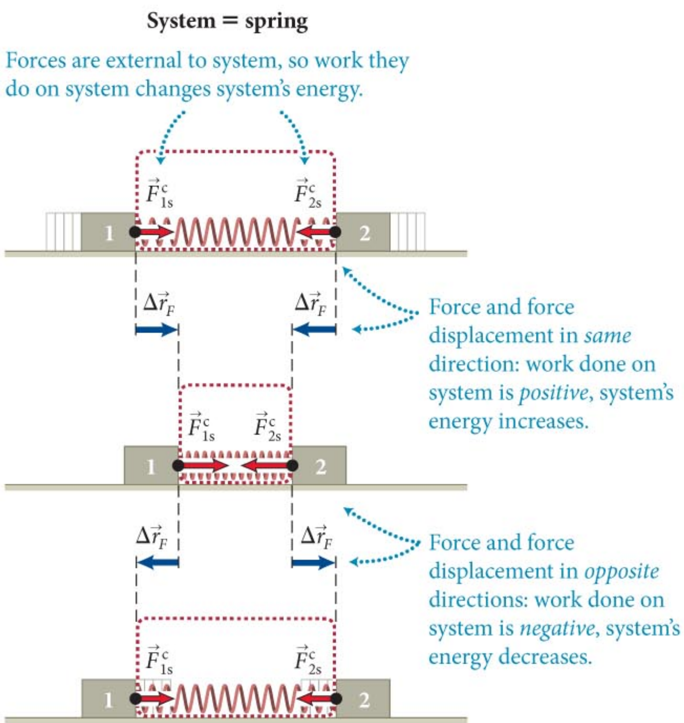
Section 9.2: Positive and negative work

- Let us now consider a situation involving potential energy.
- First, we consider the **spring + blocks to be a closed system.**
- In this case the change in potential energy will manifest as a change in the kinetic energy of the blocks, keeping the total energy constant.
- Because no external forces are exerted on the system, no work is involved.



Section 9.2: Positive and negative work

- Next, consider the **spring by itself as the system.**
- When the compressed spring is released, the decrease in the energy of the spring implies the work done by the block on the spring is negative.
- The force exerted on the spring by the block and the force displacement are in opposite directions, which confirms that the work is negative.



Checkpoint 9.3

9.3 A ball is thrown vertically upward.

(a) As it moves upward, it slows down under the influence of gravity. Considering the changes in energy of the ball, is the work done by Earth on the ball positive or negative?

(b) After reaching its highest position, the ball moves downward, gaining speed. Is the work done by the gravitational force exerted on the ball during this motion positive or negative?

Checkpoint 9.3

9.3 A ball is thrown vertically upward.

(a) As it moves upward, it slows down under the influence of gravity. Considering the changes in energy of the ball, is the work done by Earth on the ball positive or negative?

- As it moves upward, KE decreases, E_{int} is constant. The ball's energy ($K + E_{int}$) decreases, so work is negative (force & displacement opposite)

(b) After reaching its highest position, the ball moves downward, gaining speed. Is the work done by the gravitational force exerted on the ball during this motion positive or negative?

- Ball's energy now increases, so work is positive (force & displacement in same direction)

Exercise 9.2 Positive and negative work

Is the work done by the following forces positive, negative, or zero? In each case the **system is the object on which the force is exerted.**

(*a*) the force exerted by a hand compressing a spring,
(*b*) the force exerted by Earth on a ball thrown upward,
(*c*) the force exerted by the ground on you at the instant you jump upward

(*d*) the force exerted by the floor of an elevator on you as the elevator moves downward at constant speed.

Exercise 9.2 Positive and negative work (cont.)

SOLUTION

(*a*) Positive. To compress a spring, I must move my hand in the same direction as I push. ✔

(*b*) Negative. The force exerted by Earth points downward; the point of application moves upward. ✔

(*c*) Zero, because the point of application is on the ground, which doesn't move. ✔

(*d*) Negative. The force exerted by the elevator floor points upward; the point of application moves downward. ✔

Section 9.2
Question 3

You throw a ball up into the air and then catch it. How much work is done by gravity on the ball while it is in the air?

1. A positive amount
2. A negative amount
3. Cannot be determined from the given information
4. Zero

Question 3

You throw a ball up into the air and then catch it. How much work is done by gravity on the ball while it is in the air?

1. A positive amount
2. A negative amount
3. Cannot be determined from the given information
4. ✔ Zero – comes back to where it started

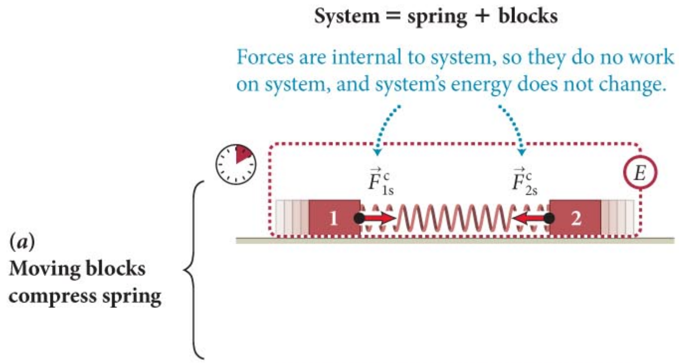
Checkpoint 9.5

9.5 Suppose that instead of the two moving blocks in Figure 9.3*a*, just one block is used to compress the spring while the other end of the spring is held against a wall.

(a) Is the system comprising the block and the spring closed?

(b) When the system is defined as being only the spring, is the work done by the block on the spring positive, negative, or zero? How can you tell?

(c) Is the work done by the wall on the spring positive, negative, or zero?



Checkpoint 9.5

9.5 Suppose that instead of the two moving blocks in Figure 9.3*a*, just one block is used to compress the spring while the other end of the spring is held against a wall.

(a) Is the system comprising the block and the spring closed?

yes – no changes in motion or state in environment

(b) When the system is defined as being only the spring, is the work done by the block on the spring positive, negative, or zero? How can you tell?

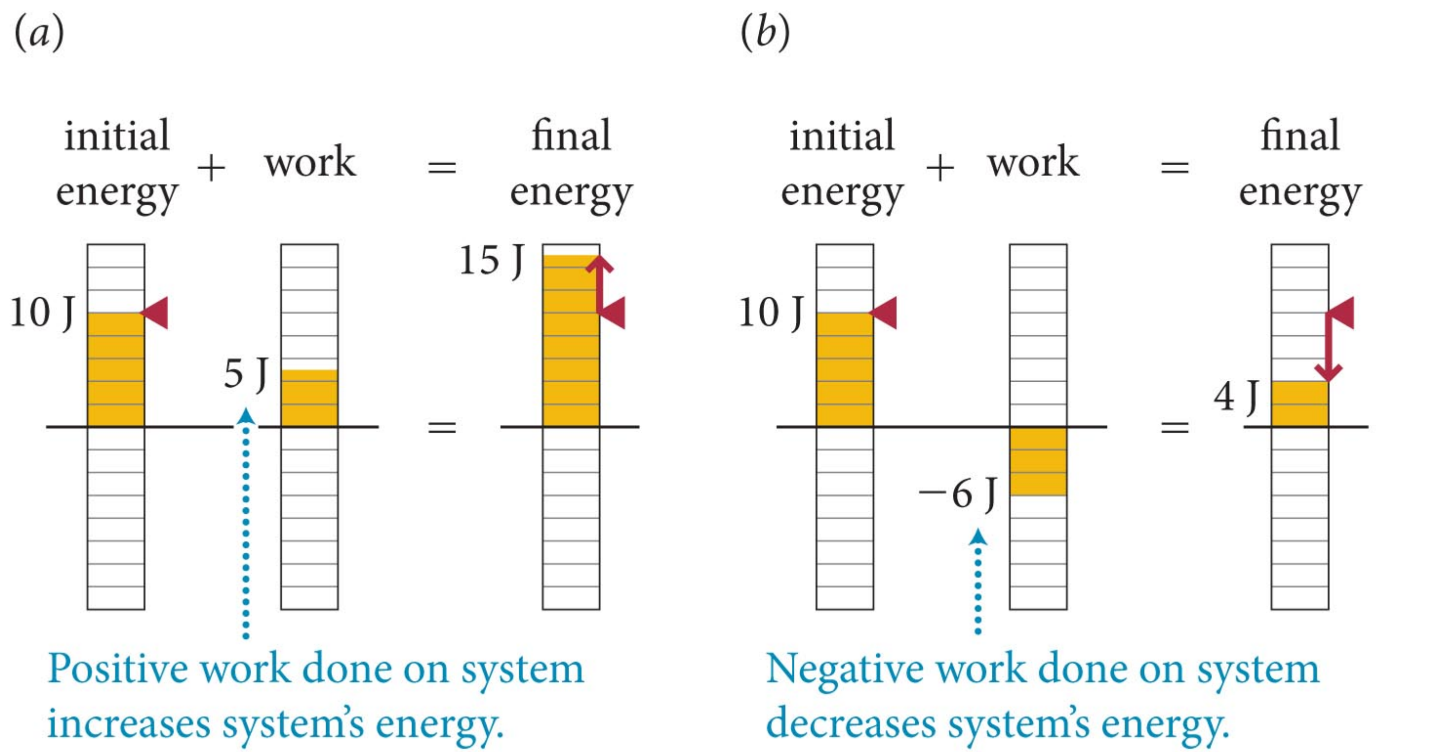
positive – force on spring is in direction point of contact moves

Is the work done by the wall on the spring positive, negative, or zero?

zero – point of contact doesn't move

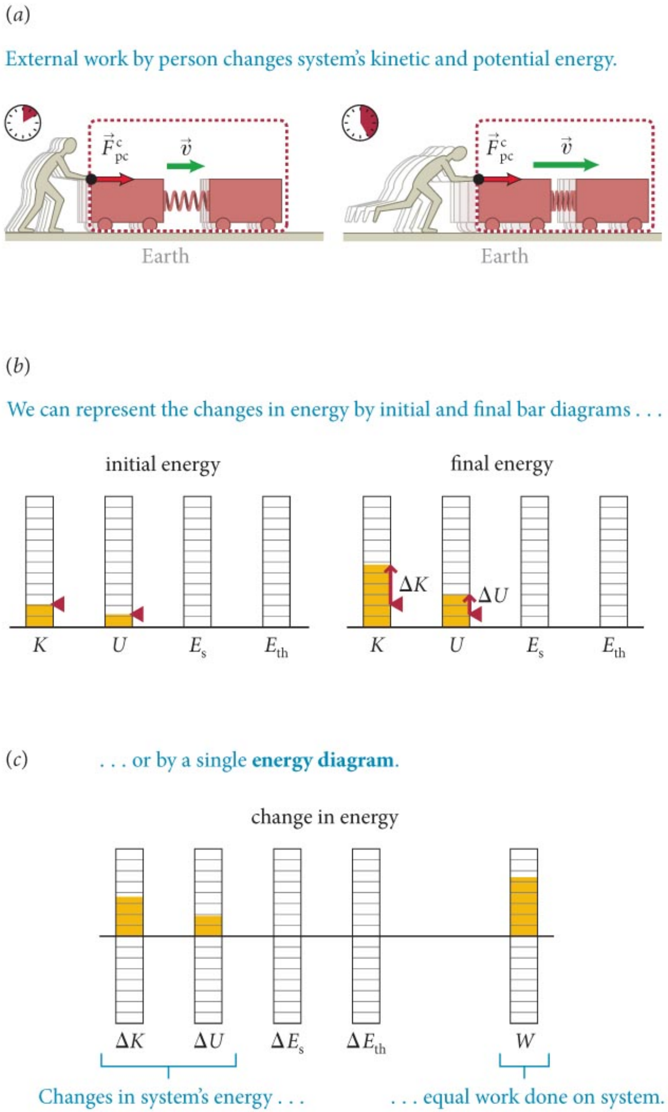
Section 9.3: Energy diagrams

- We can use energy bar charts to visually analyze situations involving work.
- This is why the sign is important



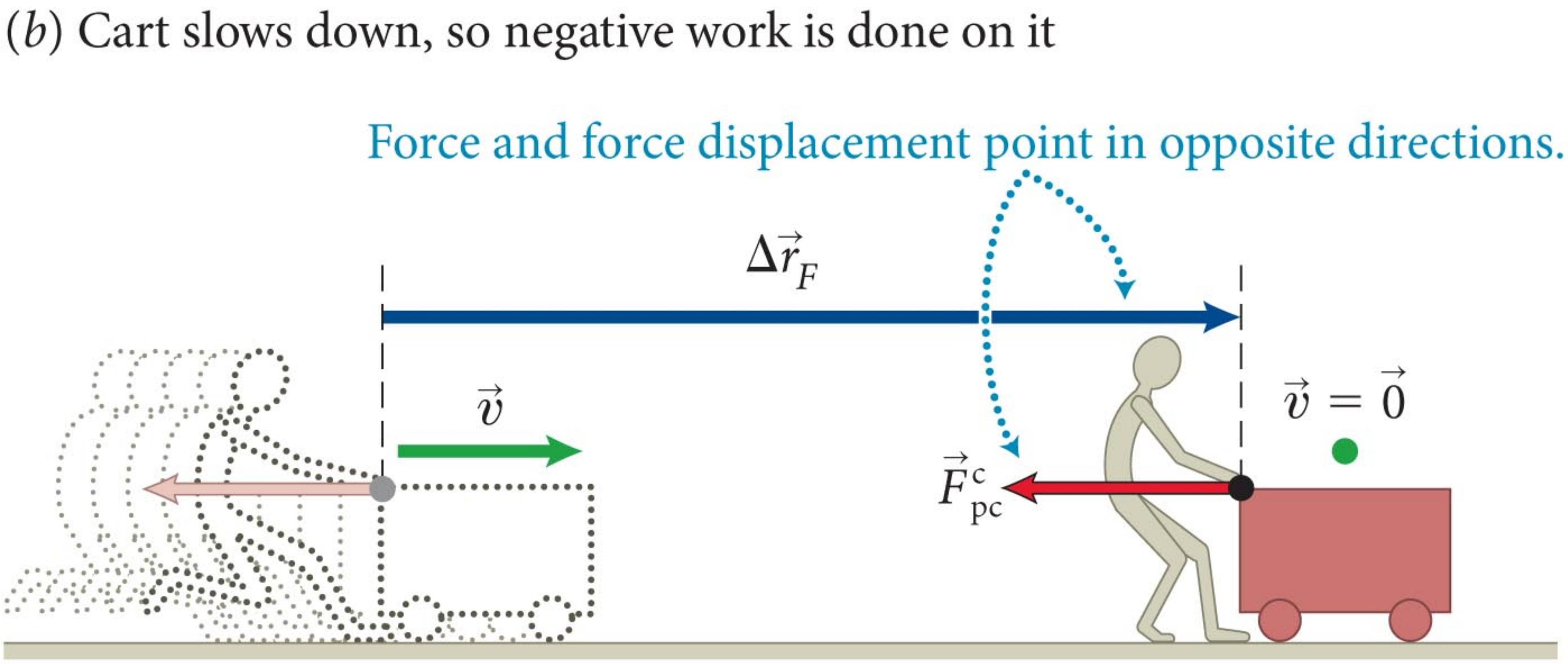
Section 9.3: Energy diagrams

- Because any of the four kinds of energy can change in a given situation, we need more details in our energy bar charts [part (*b*)].
- As shown in part (*c*), we can also draw one set of bars for change in each category of energy, and a fifth bar to represent work done by external forces.
  - These are called **energy diagrams.**



Checkpoint 9.6

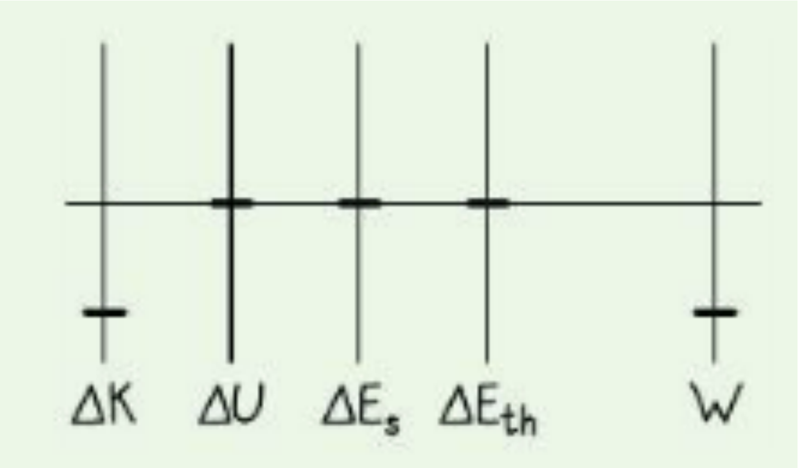
9.6 Draw an energy diagram for the cart in Figure 9.2*b*.



Checkpoint 9.6

9.6

- The cart's KE decreases to zero, no changes in other forms of energy.
- Person's force is to the left, displacement to the right: work is negative.
- Change in KE should be same as W done in magnitude



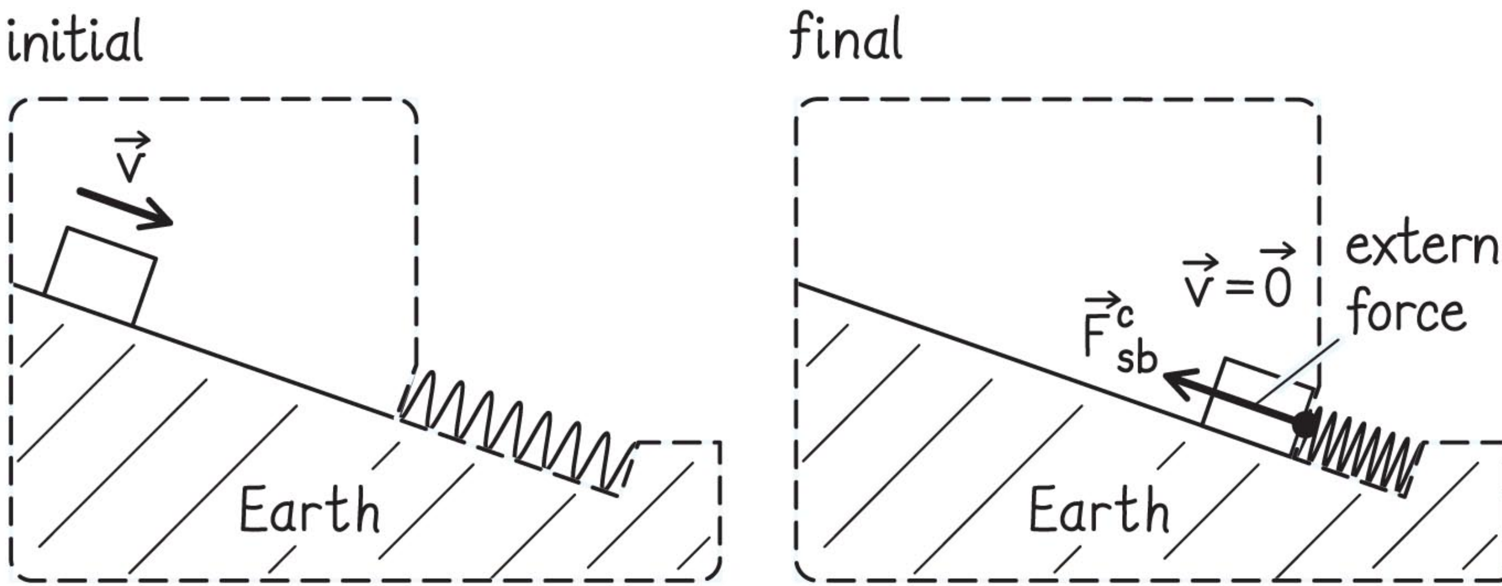
Exercise 9.4 Compressing a spring

A block initially at rest is released on an inclined surface. The block slides down, compressing a spring at the bottom of the incline; there is friction between the surface and the block.

Consider the time interval from a little after the release, when the block is moving at some initial speed v, until it comes to rest against the spring. Draw an energy diagram for the **system that comprises the block, surface, and Earth.**

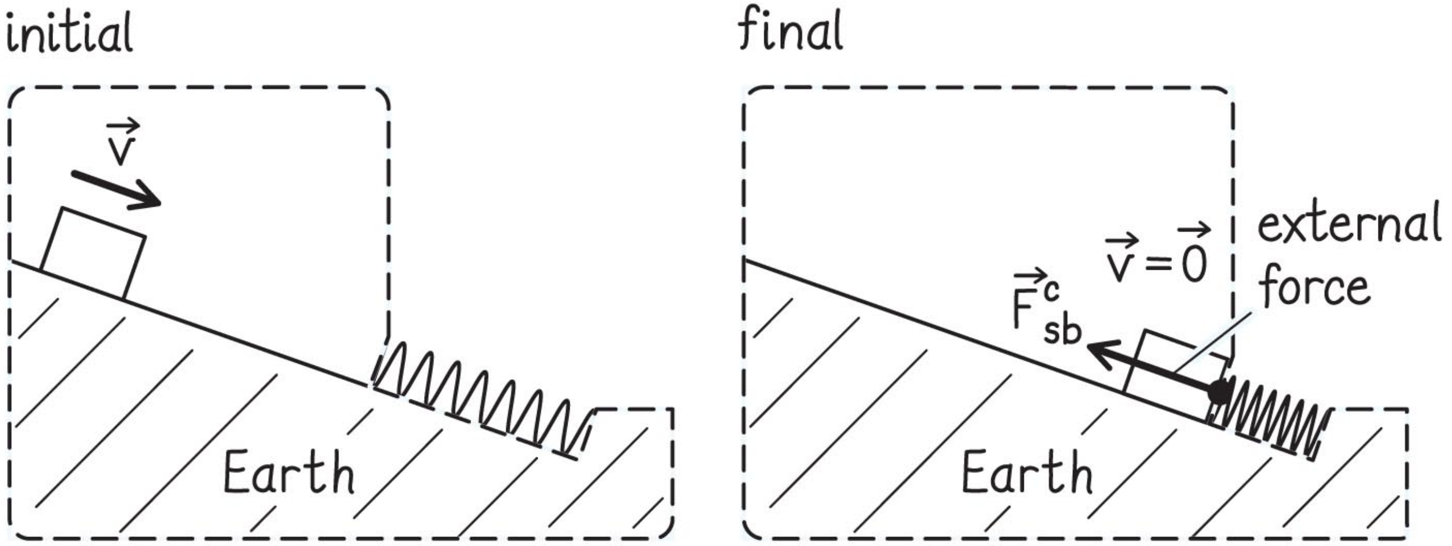
Exercise 9.4 Compressing a spring (cont.)

SOLUTION I begin by listing the objects that make up the system: block, surface, and Earth. Then I sketch the initial and final states of the system (Figure 9.8). The spring exerts external forces on the system.



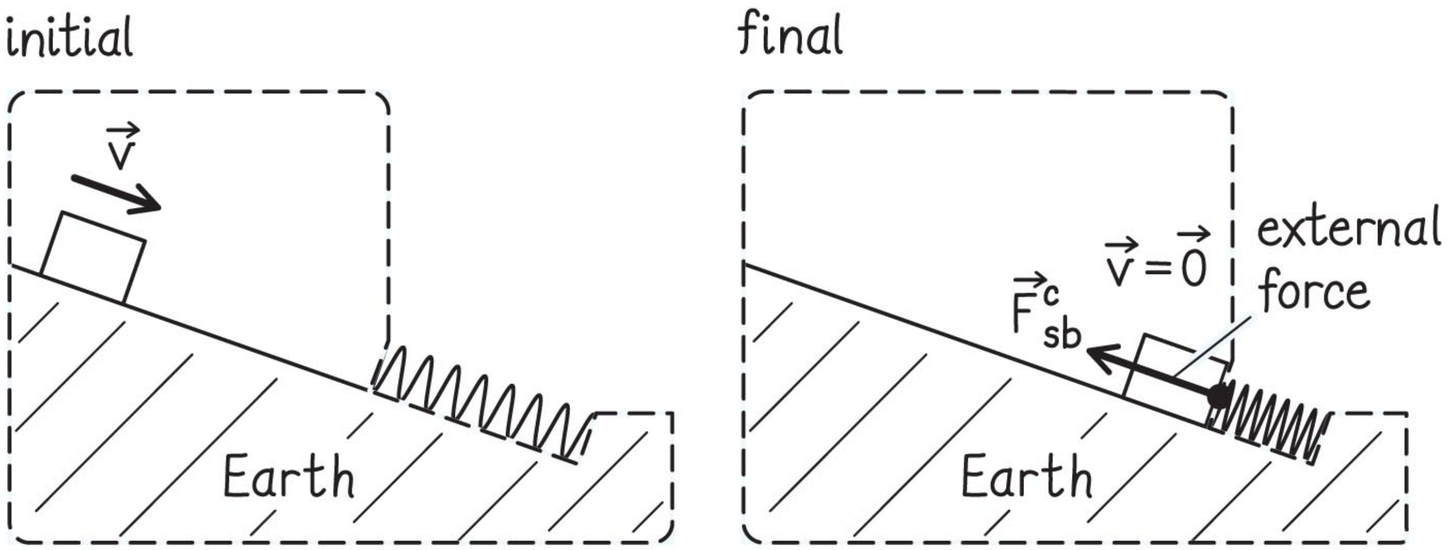
Exercise 9.4 Compressing a spring (cont.)

SOLUTION The bottom end of the spring exerts a force on the surface edge, but this force has a force displacement of zero. The top end of the spring exerts a force $\vec{F}^{c}_{sb}$ on the block. Because this force undergoes a nonzero force displacement, I include it in my diagram and show a dot at its point of application.



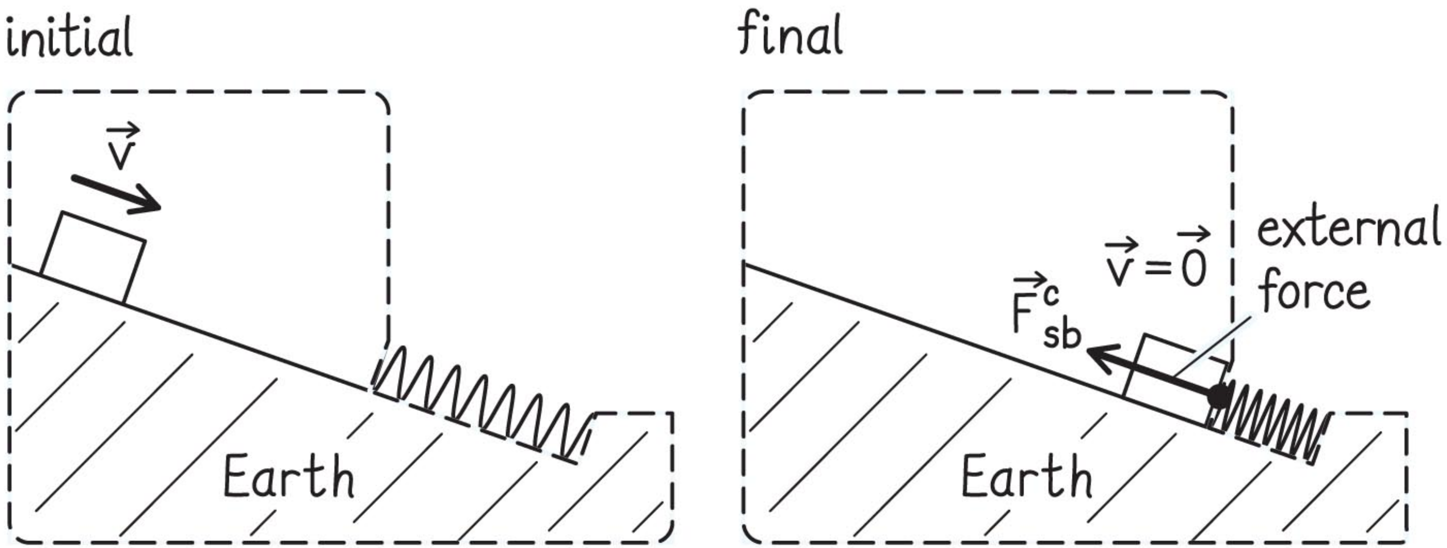
Exercise 9.4 Compressing a spring (cont.)

SOLUTION Next I determine whether there are any energy changes. Kinetic energy: The block's kinetic energy goes to zero, and the kinetic energies of the surface and Earth do not change. Thus the kinetic energy of the system decreases, and ΔK is negative.



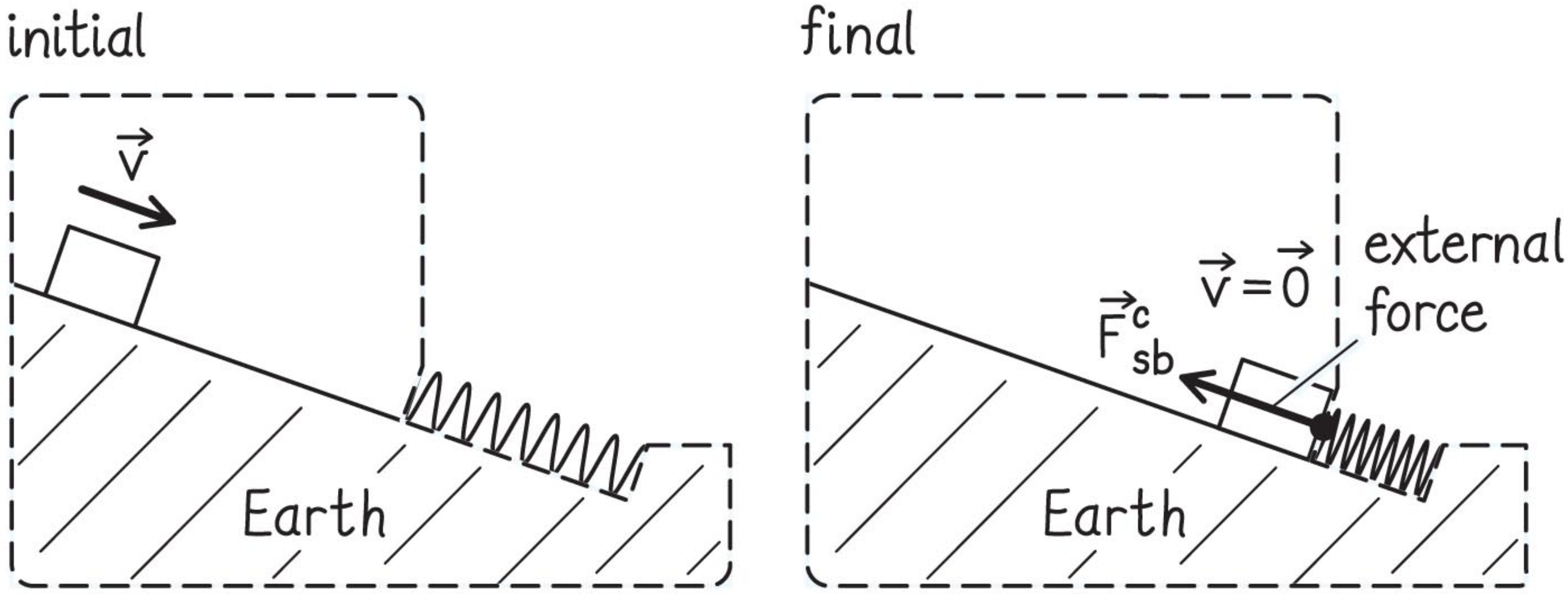
Exercise 9.4 Compressing a spring (cont.)

SOLUTION Potential energy: As the block moves downward, the gravitational potential energy of the block-Earth system decreases, and so ΔU is negative. (Because the spring gets compressed, its elastic potential energy changes, but the spring is not part of the system.)



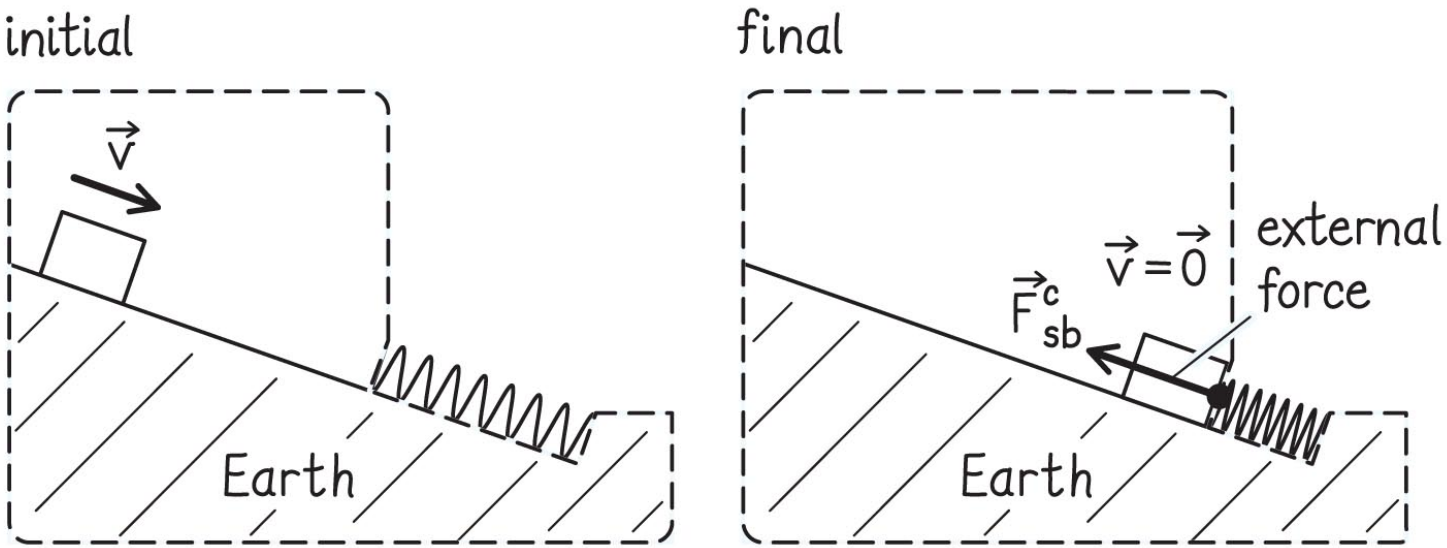
Exercise 9.4 Compressing a spring (cont.)

SOLUTION Source energy: none (no fuel, food, or other source of energy is converted in this problem). Thermal energy: As the block slides, energy is dissipated by the friction between the surface and the block, so ΔE_{th} is positive.



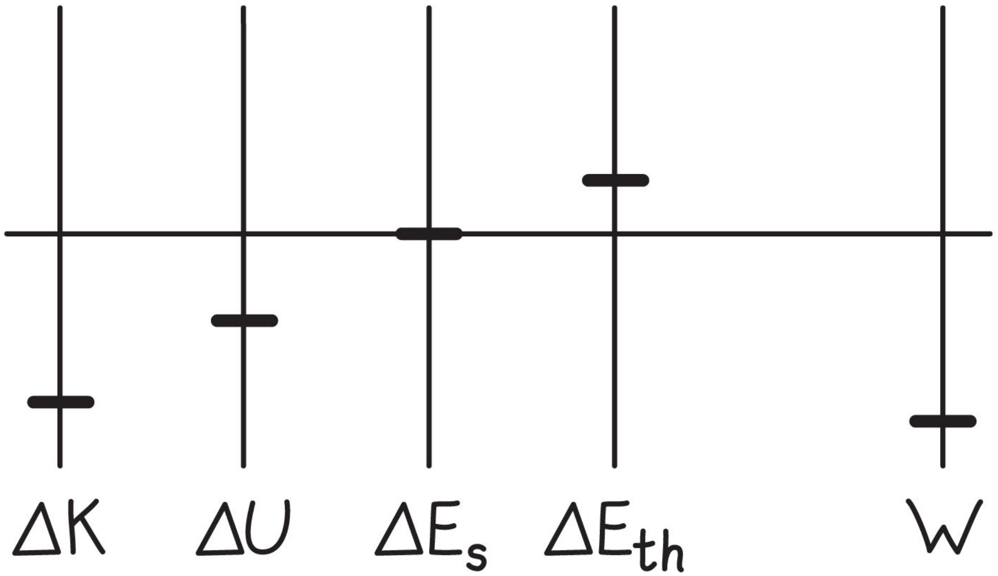
Exercise 9.4 Compressing a spring (cont.)

SOLUTION To determine the work done on the system, I look at the external forces exerted on it. The point of application of the external force $\vec{F}^{c}_{sb}$ exerted by the spring on the block undergoes a force displacement opposite the direction of the force, so that force does negative work on the system.



Exercise 9.4 Compressing a spring (cont.)

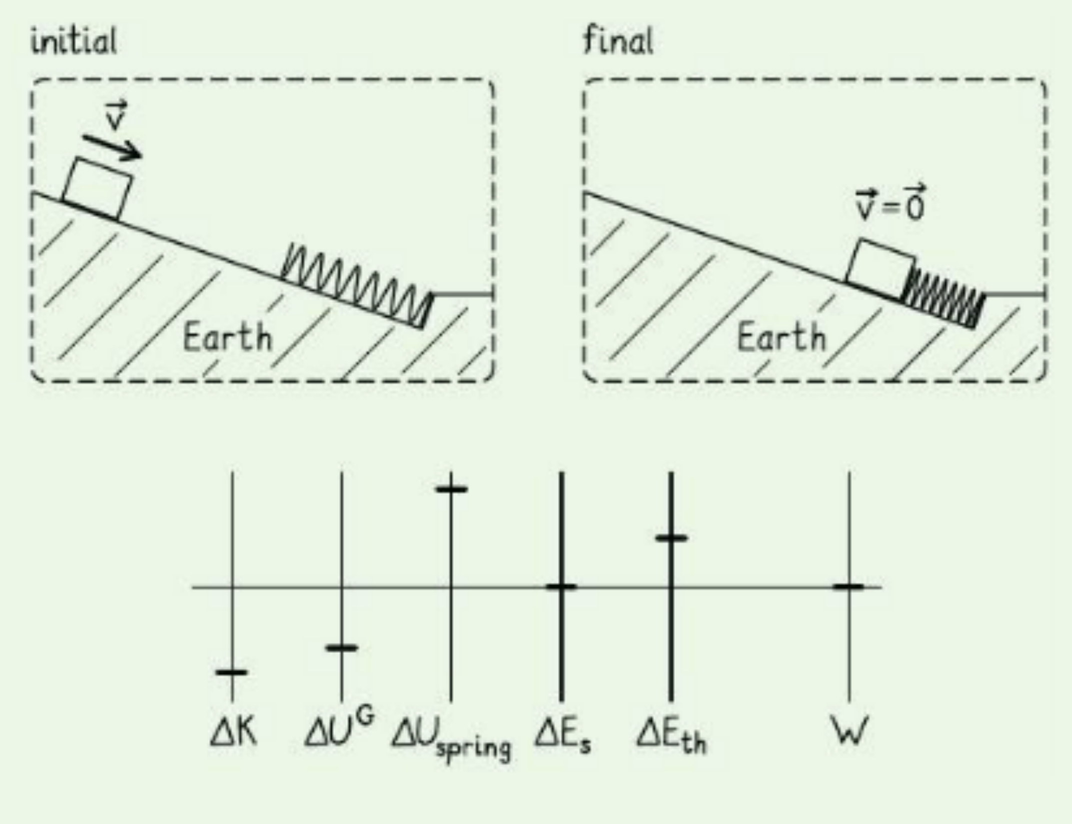
SOLUTION Thus the work done on the system by the external forces is negative, and the *W* bar extends below the baseline (Figure 9.9). I adjust the lengths of the bars so that the length of the *W* bar is equal to the sum of the lengths of the other three bars, yielding the energy diagram shown in Figure 9.9. ✔



Checkpoint 9.7

9.7 Draw an energy diagram for the situation presented in Exercise 9.4, but choose the system that comprises **block, spring, surface, and Earth. (i.e., include the spring now)**

Which changes should be equal? No external force now, no work.



Section 9.4: Choice of system

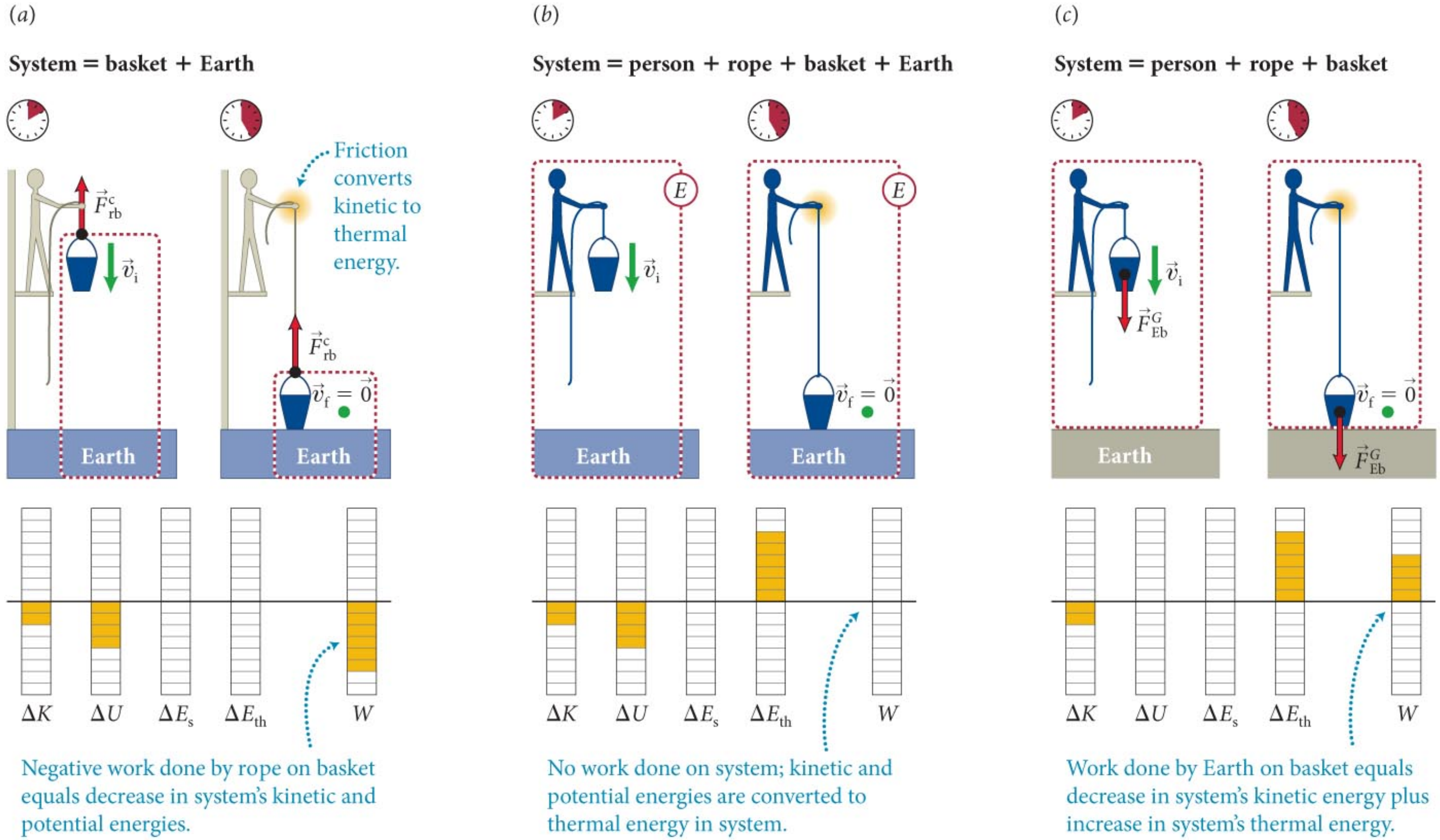
Section Goals

You will learn to

- Choose an **appropriate system** for a physical problem of interest in order to systematically account for the various energy changes.
- Recognize that a system chosen for which **friction** acts across the boundary is **difficult** to analyze. This is because in these situations thermal energy is generated in both the environment and the system, making energy accounting for the system problematic.

Section 9.4: Choice of system

- Different choices of systems lead to different energy diagrams.
- What is "work" in one context is energy conversion in another
- Work is involved when an *external* agent acts with a force

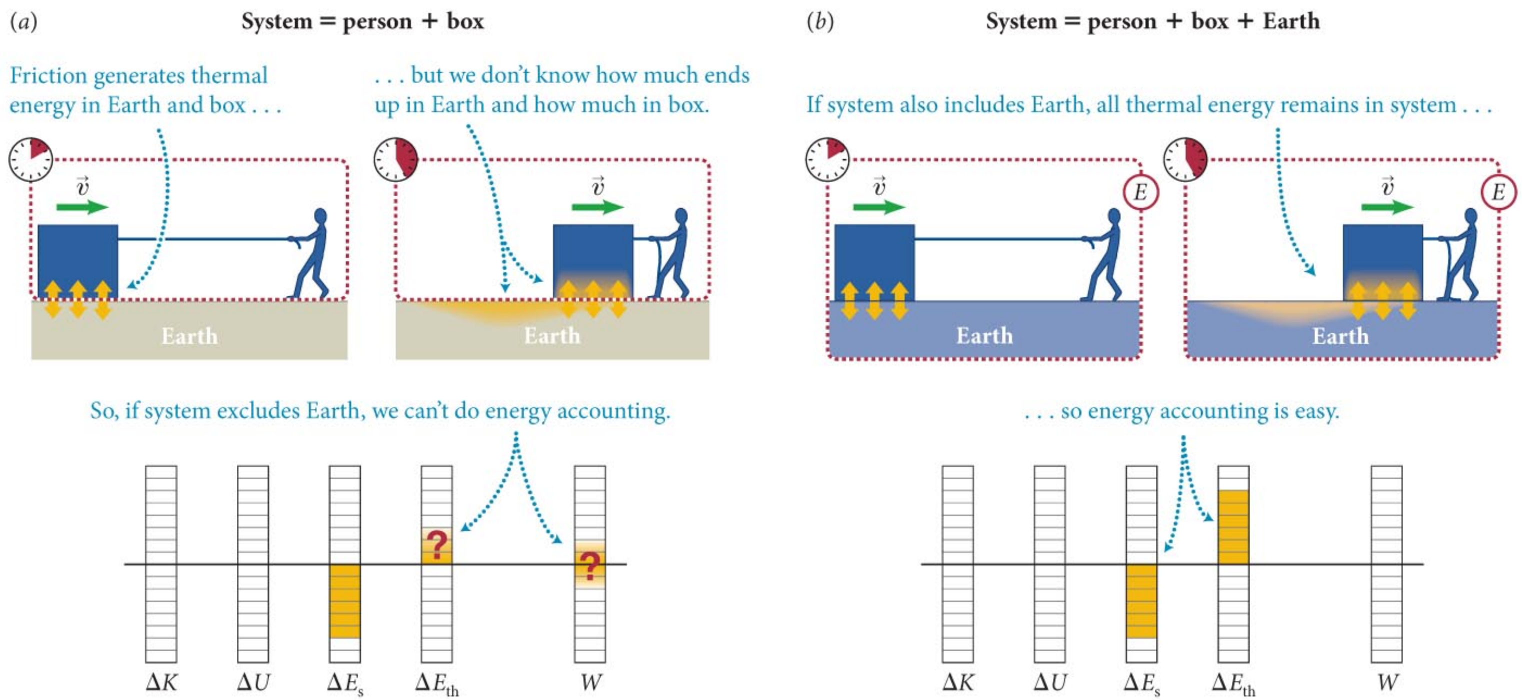


Section 9.4: Choice of system

- Need to be careful not to double count gravitational potential energy!
- It is important to remember the following point:
  - **Gravitational potential energy always refers to the relative position of various parts within a system, never to the relative positions of one component of the system and its environment.**
- In other words, depending on the choice of system, the gravitational interaction with the system can appear in energy diagrams as either a change in gravitational potential energy or work done by Earth, but not both.
- Earth outside system? Probably work (earth = external agent then)

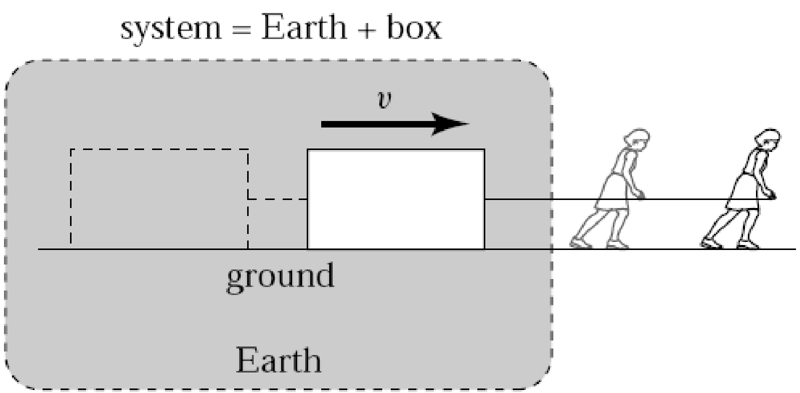
Section 9.4: Choice of system

- As seen in the figure, the thermal energy generated (in this case due to friction) ends up on both surfaces.
- As seen in part (*a*), certain choices of systems lead to complications:
  - **When drawing an energy diagram, do not choose a system for which friction occurs at the boundary of the system.**



Section 9.4
Question 5

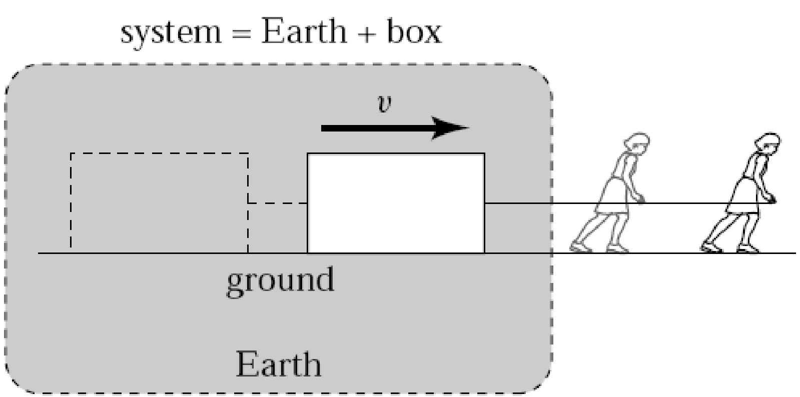
A person pulls a box along the ground at a constant speed. If we consider Earth and the box as our system, what can we say about the net external force on the system?



1. It is zero because the system is isolated.
2. It is nonzero because the system is not isolated.
3. It is zero even though the system is not isolated.
4. It is nonzero even though the system is isolated.
5. None of the above

Section 9.4
Question 5

A person pulls a box along the ground at a **constant speed**. If we consider Earth and the box as our system, what can we say about the net external force on the system?



1. It is zero because the system is isolated.
2. It is nonzero because the system is not isolated.
3. It is zero even though the system is not isolated. ✔
4. It is nonzero even though the system is isolated.
5. None of the above

Chapter 9: Self-Quiz #2

Consider a weightlifter holding a barbell motionless above his head.

(*a*) Is the sum of the forces exerted on the barbell zero?

(*b*) Is the weightlifter exerting a force on the barbell?

(*c*) If the weightlifter exerts a force, does this force do any work on the barbell?

(*d*) Does the energy of the barbell change?

(*e*) Are your answers to parts (*c*) and (*d*) consistent in light of the relationship between work and energy?

Chapter 9: Self-Quiz #2

Answer

(*a*) Yes, because the barbell remains motionless.

(*b*) Yes. He exerts an upward force to counter the downward gravitational force.

(*c*) No, because the point at which the lifter exerts a force on the barbell is not displaced.

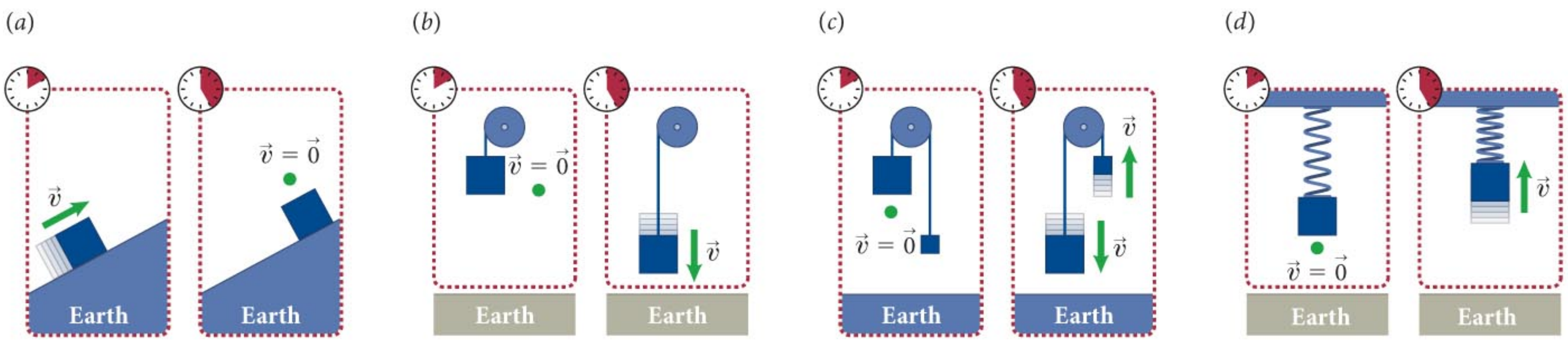
(*d*) No, it just sits there.

(*e*) They are consistent. If no work is done on a system, the energy of the system does not change.

Chapter 9: Self-Quiz #3

Do any of the systems in the figure undergo a change in potential energy?

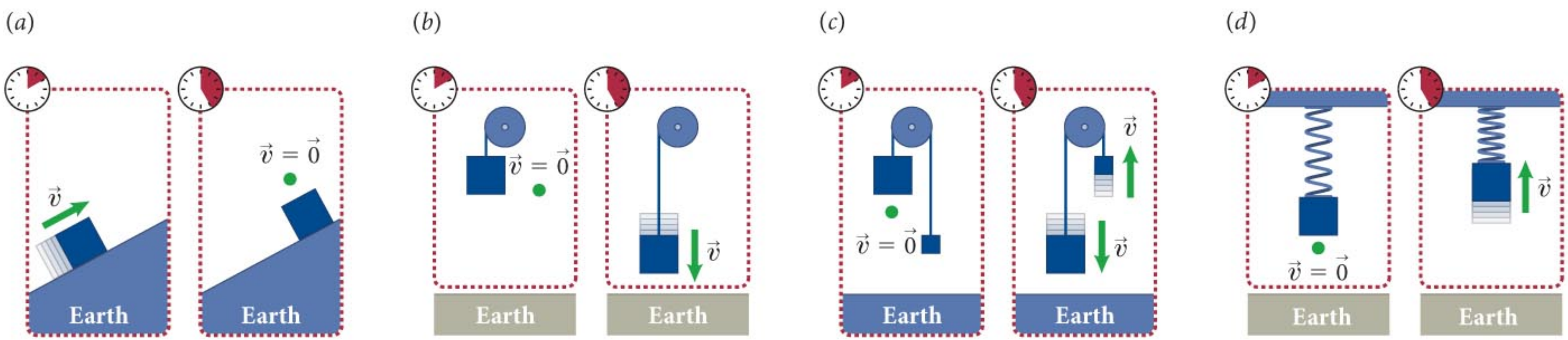
If yes, is the change positive or negative? Ignore any friction.



Chapter 9: Self-Quiz #3

Do any of the systems in the figure undergo a change in potential energy?

If yes, is the change positive or negative? Ignore any friction.



Yes: U^G changes

No: F^G is external

Yes: U^G changes

Yes: U^S changes

Quantitative Tools

Section 9.5: Work done on a single particle

- When work is done by external forces on a system, the energy change in the system is given by the **energy law:**

$$\Delta E = W$$

- To determine the work done by an external force, we will consider the simple case of a **particle:**
  - *Particle* refers to any object with an inertia m and no internal structure ($\Delta E_{\text{int}} = 0$).
- Only the kinetic energy of a particle can change, so

$$\Delta E = \Delta K \text{ (particle)}$$

- The constant force acting on the particle give is it an acceleration given by

$$a_x = \frac{\Sigma F_x}{m} = \frac{F_x}{m}$$

Section 9.5: Work done on a single particle

- Consider the motion of the particle in time interval $\Delta t = t_f - t_i$. From Equations 3.4 and 3.7, we can write

$$\upsilon_{x,f} = \upsilon_{x,i} + a_x \Delta t$$

$$\Delta x = \upsilon_{x,i} \Delta t + \tfrac{1}{2} a_x (\Delta t)^2$$

- The kinetic energy change of the particle is given by

$$\Delta K = K_f - K_i = \tfrac{1}{2} m(\upsilon_f^2 - \upsilon_i^2)$$

- Combining the above equations we get,

$$\Delta K = \tfrac{1}{2} m\left[\left(\upsilon_{x,i} + a_x \Delta t\right)^2 - \upsilon_{x,i}^2\right]$$

$$= \tfrac{1}{2} m\left[2\upsilon_{x,i} a_x \Delta t + a_x^2 (\Delta t)^2\right]$$

$$= m a_x\left[\upsilon_{x,i} \Delta t + \tfrac{1}{2} a_x (\Delta t)^2\right]$$

$$= m a_x \Delta x_F = F_x \Delta x_F$$

where Δx_F is the force displacement.

Section 9.5: Work done on a single particle

- Since $\Delta E = W$, and for a particle $\Delta E = \Delta K$, we get

  $$W = F_x \Delta x_F \text{ (constant force exerted on particle, one dimension)}$$

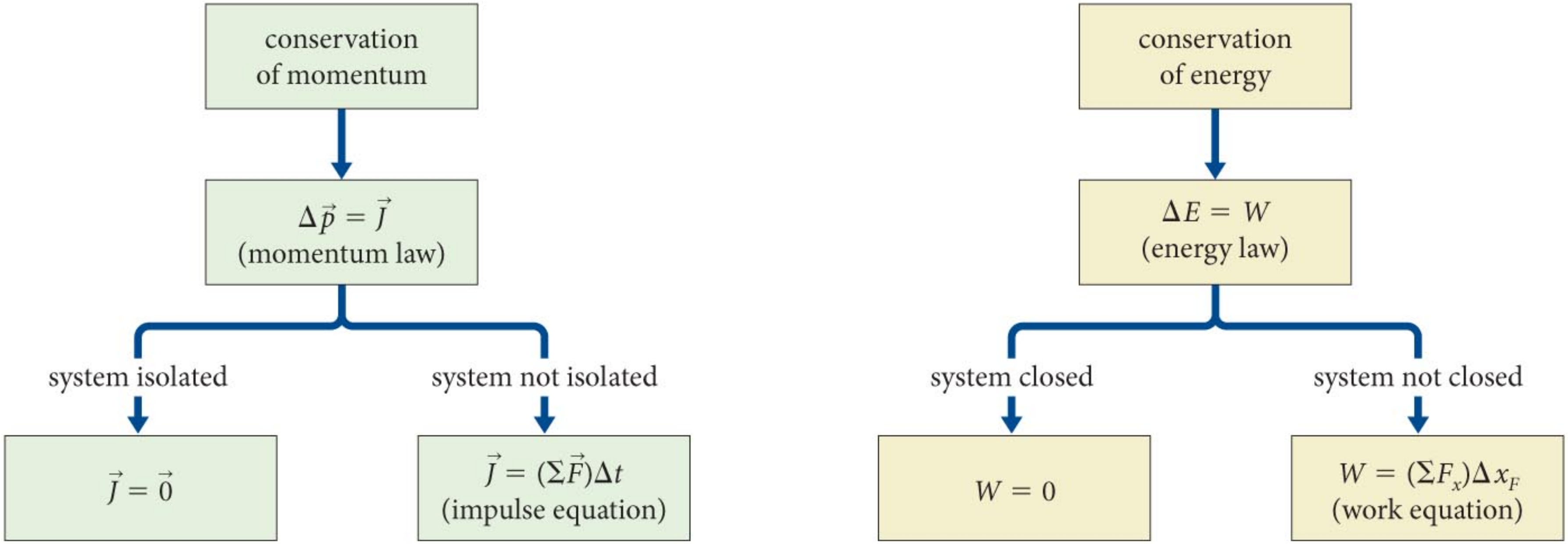
- The equation above in words:
  - **For motion in one dimension, the work done by a constant force exerted on a particle equals the product of the *x* component of the force and the force displacement.**
- If more than one force is exerted on the particle, we get

  $$W = (\Sigma F_x)\Delta x_F \text{ (constant forces exerted on particle, one dimension)}$$

- This is called the **work equation.**

Section 9.5: Work done on a single particle

- Notice the parallel between our treatment of momentum/impulse and energy/work, as illustrated in the figure.



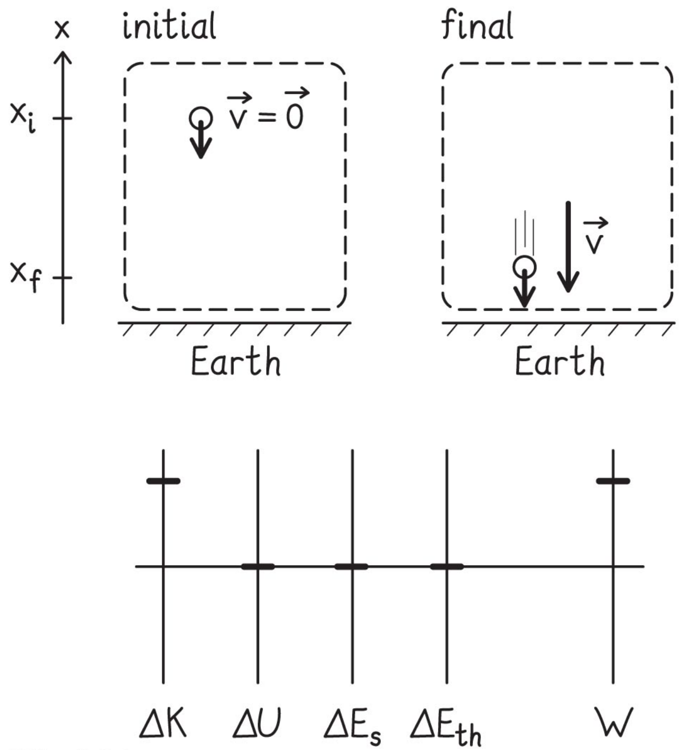
Section 9.5: Work done on a single particle

Example 9.6 Work done by gravity

A ball of inertia m_b is released from rest and falls vertically. What is the ball's final kinetic energy after a displacement $\Delta x = x_f - x_i$?

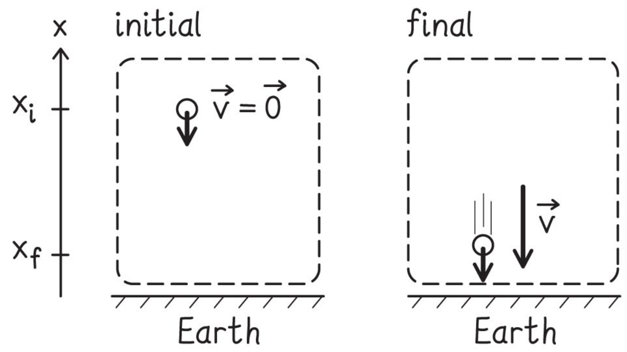
Example 9.6 Work done by gravity (cont.)

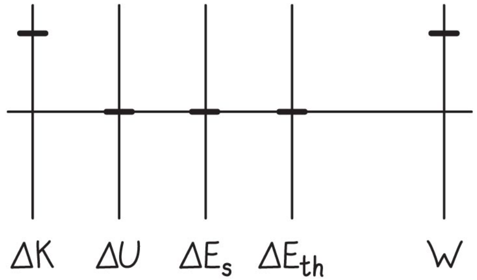
❶ GETTING STARTED I begin by making a sketch of the initial and final conditions and drawing an energy diagram for the ball (Figure 9.17). I choose an x axis pointing upward.



Example 9.6 Work done by gravity (cont.)

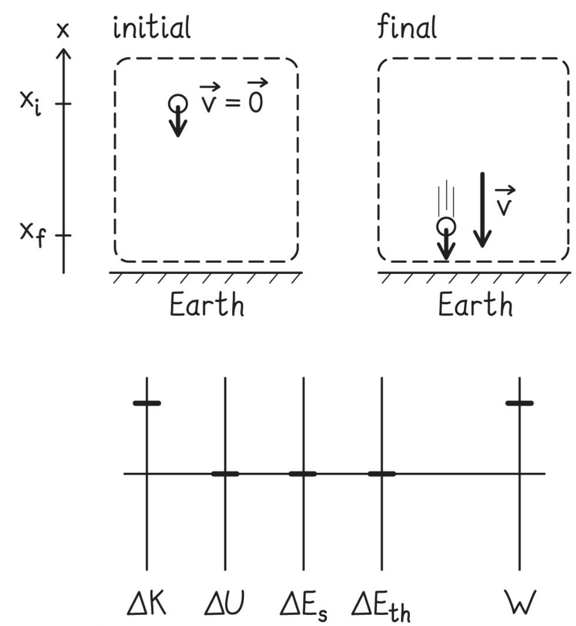
❶ GETTING STARTED Because the ball's internal energy doesn't change as it falls (its shape and temperature do not change), I can treat the ball as a particle. Therefore only its kinetic energy changes.





Example 9.6 Work done by gravity (cont.)

❶ GETTING STARTED I can also assume air resistance is small enough to be ignored, so that the only external force exerted on the ball is a constant gravitational force. This force has a nonzero force displacement and so does work on the ball. I therefore include this force in my diagram.



Example 9.6 Work done by gravity (cont.)

❷ DEVISE PLAN If I treat the ball as a particle, the change in the ball's kinetic energy is equal to the work done on it by the constant force of gravity, the x component of which is:

$$F^G_{\text{Eb}\,x} = -m_\text{b}g.$$

(The minus sign means that the force points in the negative x direction.)

To calculate the work done by this force on the ball, I use Eq. 9.8.

Example 9.6 Work done by gravity (cont.)

❸ EXECUTE PLAN Substituting the x component of the gravitational force exerted on the ball and the force displacement $x_f - x_i$ into Eq. 9.8, I get

$$W = F^G_{\text{Eb}\,x}\Delta_{x_F} = -m_b g(x_f - x_i).$$

Example 9.6 Work done by gravity (cont.)

❸ EXECUTE PLAN Because the work is equal to the change in kinetic energy and the initial kinetic energy is zero, I have $W = \Delta K = K_f - 0 = K_f$, so

$$K_f = -m_b g(x_f - x_i).$$ ✔

Example 9.6 Work done by gravity (cont.)

❹ EVALUATE RESULT Because the ball moves in the negative x direction, $\Delta x = x_f - x_i$ is negative and so the final kinetic energy is positive (as it should be).

Example 9.6 Work done by gravity (cont.)

❹ EVALUATE RESULT An alternative approach is to consider the **closed Earth-ball system**. For that system, the sum of the gravitational potential energy and kinetic energy does not change, and so, from Eq. 7.13,

$$\Delta K + \Delta U^G = \tfrac{1}{2} m_b (\upsilon_f^2 - \upsilon_i^2) + m_b g (x_f - x_i) = 0.$$

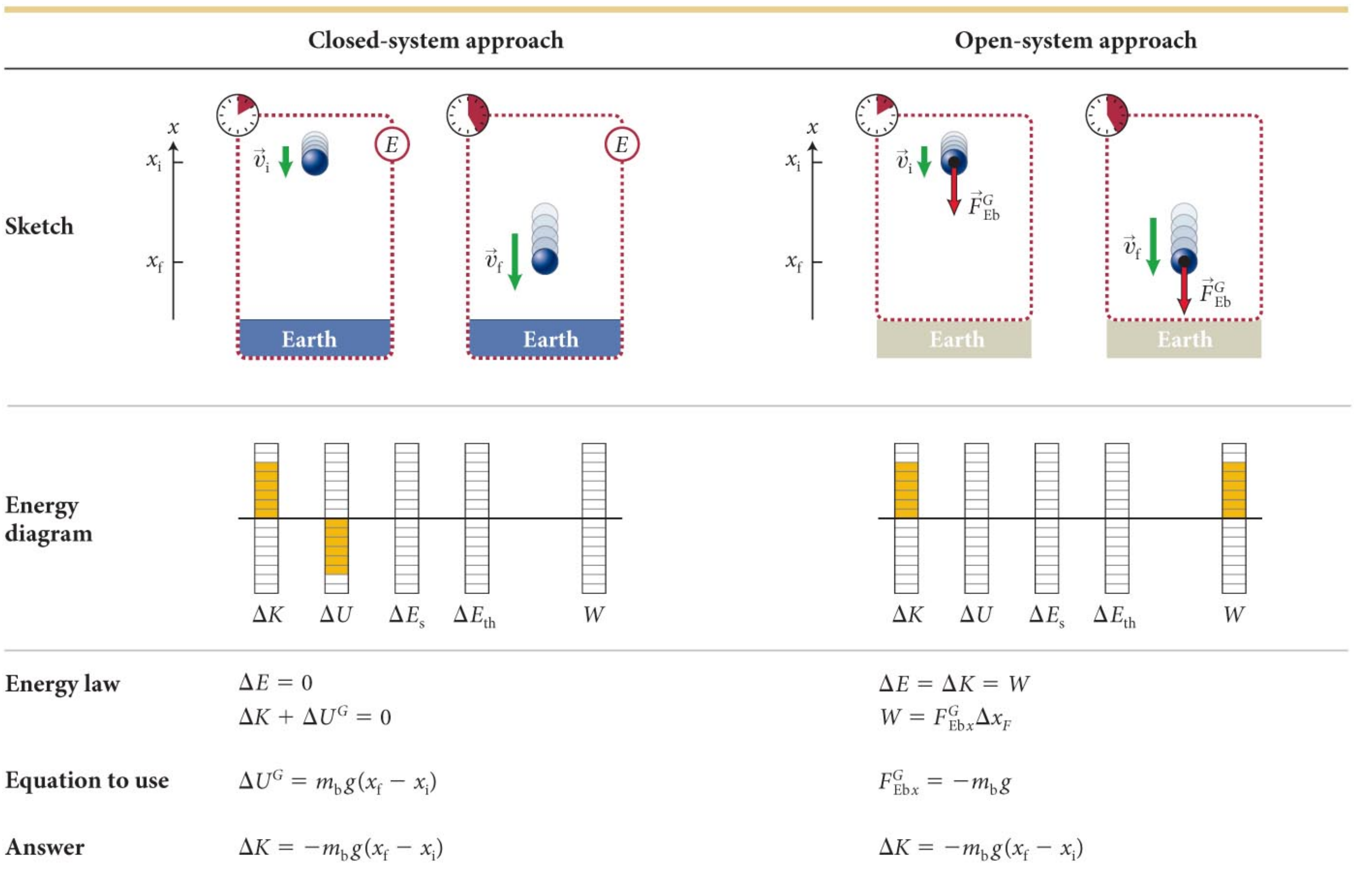
No work done here - the gravitational force is internal

Because the ball starts at rest, $\upsilon_i = 0$, and so I obtain the same result for the final kinetic energy:

$$\tfrac{1}{2} m_b \upsilon_f^2 = -m_b g (x_f - x_i).$$

Section 9.5: Work done on a single particle

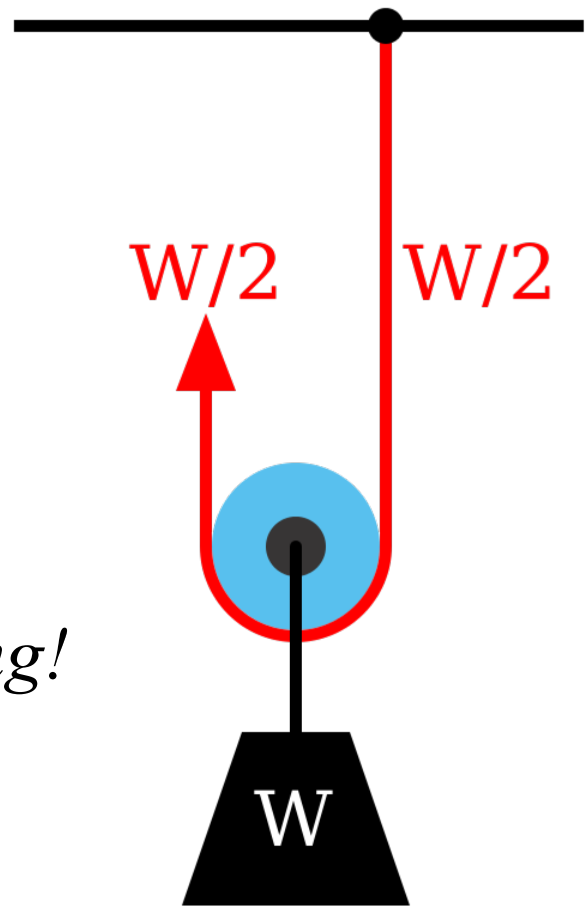
- The two approaches used in the previous example are shown schematically in the figure.



Interlude: what is a pulley good for?

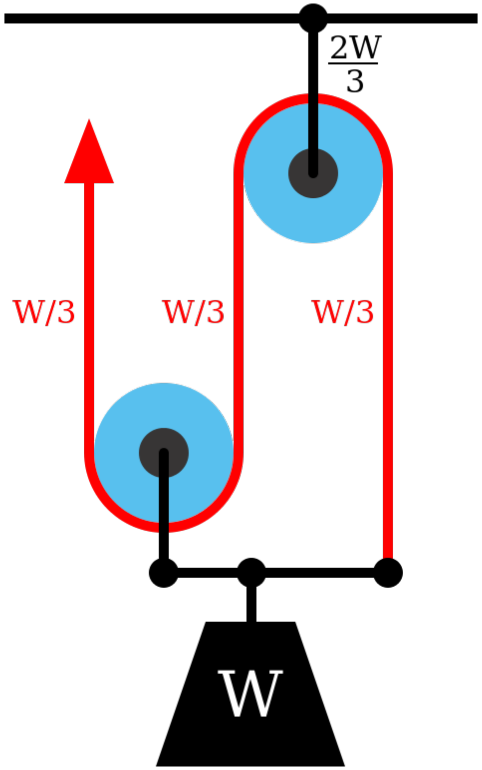
- Why use a pulley?
- Simple pulley redirects force
- Pull down to move object up
- But now you use two ropes
  - each has same tension
- Net downward force: W
- Net upward: W/2 + W/2

- *let the ceiling do half the pulling!*



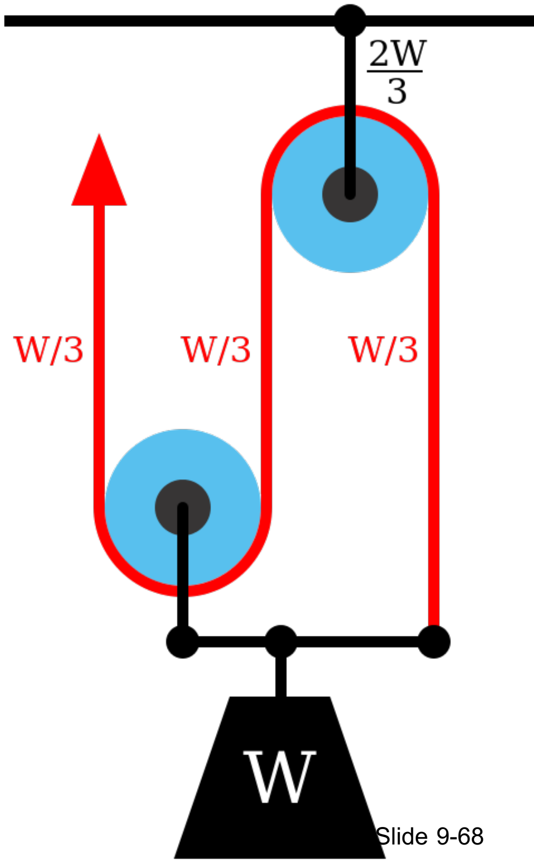
Pulley

- Compound pulley? Why not use more ropes!
- Same rope, same tension, but split it up
- 3 sections, pull with 1/3 weight
- tension same everywhere in the rope if it is light!



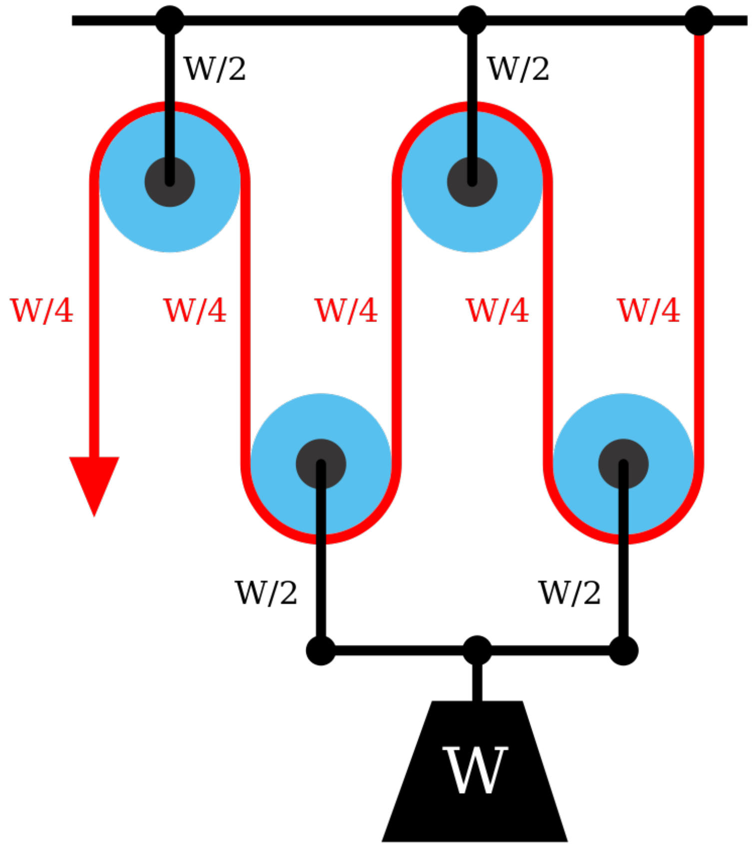
Pulley

- Work?
- if load moves by x, you have to pull L
- Work done by you = work by gravity
- (W/3)L = Wx so L = 3x
- pull with 1/3 the force
- pay with 3x the distance

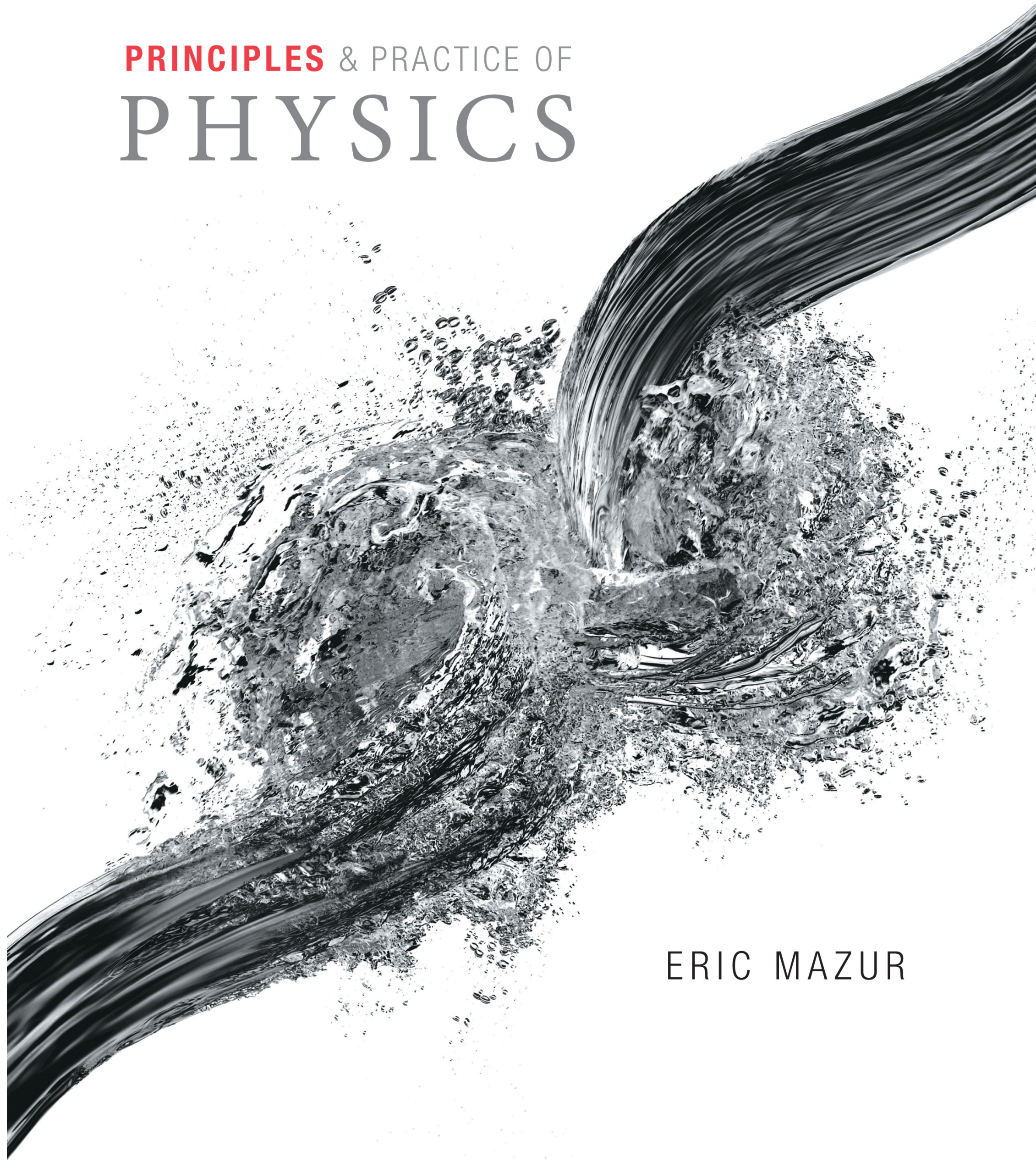


Pulley

- Pull with ¼ the force, but 4 times as far
- Mechanical advantage – trade force for distance



Work



Drag forces

- Resistance in a fluid or gas
- Depends on:
  - Speed v (laminar) or v^2 (turbulent)
  - Shape (factor D, all speeds)
  - Cross sectional area (A)
  - Surface finish (D, esp. high speed "skin friction")
  - Density of fluid (ρ high speed)
  - Viscosity of fluid (η low speed)

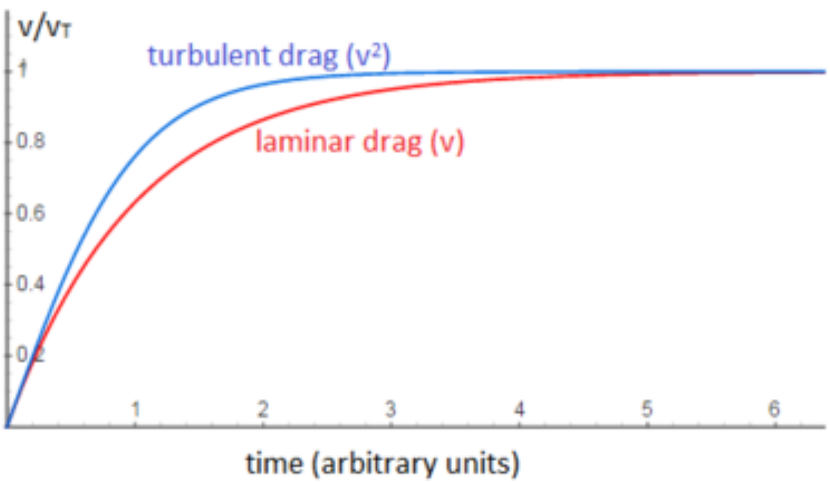
$$F_{\text{drag}} = \begin{cases} -\frac{1}{2}\rho v^2 D A & \text{high speed / turbulent} \\ -6\pi\eta r|v| & \text{low speed / laminar} \end{cases}$$

Low speed drag in 1D: solvable

- acceleration is $a_{\text{drag}} \sim -bv$
- add to this acceleration of +g due to gravity

$$v(t) = \frac{g}{b}\left(1 - e^{-bt}\right) = v_{\text{term}}\left(1 - e^{-bt}\right)$$

- Qualitatively similar for high speeds
- Leads to a *terminal velocity*



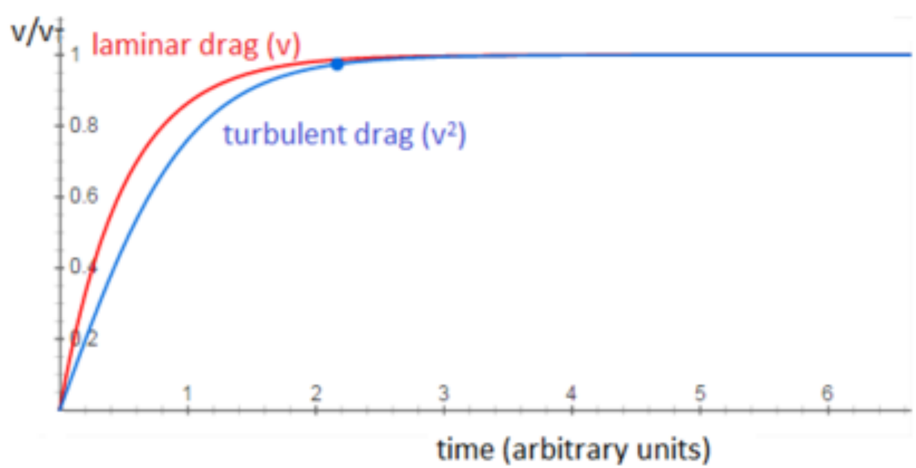
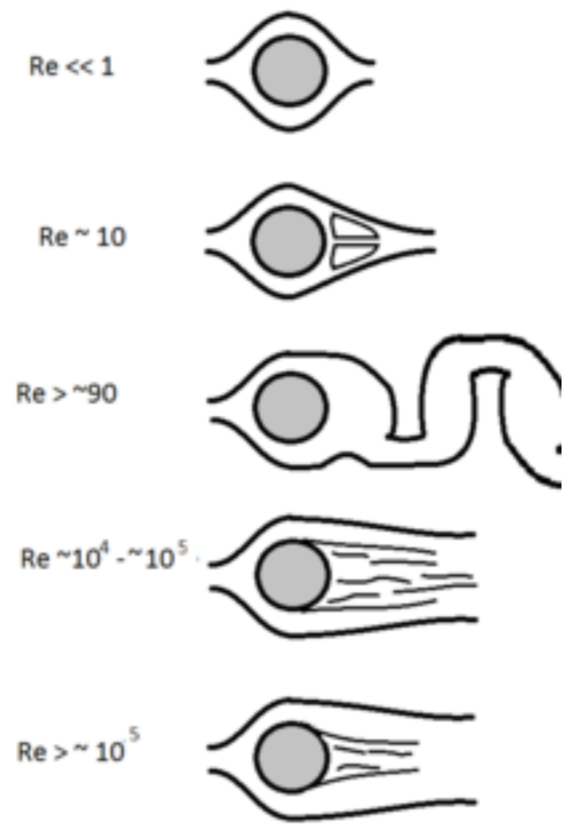


Image: wikipedia, "Drag (physics)

So what is "high speed"

- Depends on balance of inertial and viscous forces
- If the object has characteristic length *L*, characterized by *Reynolds number*

$$\mathrm{Re} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho v L}{\eta}$$



- For Re << 1, nice laminar flow
- For Re ~ 10, turbulent
- Small viscosity
  - more easily turbulent

Image: wikipedia, "Reynolds number"

So?

Say we have a 1mm particle in air at 10 m/s (22 mph)

$$\mathrm{Re} = \begin{cases} 355 & \text{water} \\ 18000 & \text{air} \end{cases}$$

Turbulent! How about a 1um particle?

$$\mathrm{Re} = \begin{cases} 11 & \text{water} \\ 588 & \text{air} \end{cases}$$

Only laminar below ~1m/s. Pollen is already 10-100um

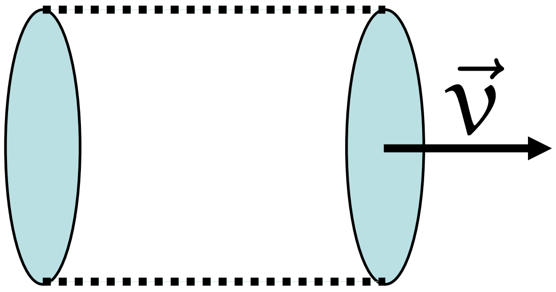
Laminar? Very fine dust, bacteria swimming

most stuff is turbulent, and this is just terrible

Where does high speed drag come from?

Have to move air out of the way.

Mass of air to move?



$$\Delta x = v\Delta t$$

$$m = \rho V = \rho A \Delta x = \rho A v \Delta t$$

Change in momentum?

$$\Delta p = m\Delta v = (\rho A v \Delta t)(v - 0)$$

Force = time rate of change

$$F_{drag} = \frac{\Delta p}{\Delta t} = \rho A v^2$$

(add a ‘fudge factor’ D to handle shape/etc.)

How to calculate?

- Drag forces for turbulent flow don't often admit analytic solutions
- Make computers do the work. Let's say we start at rest at the origin at time t=0, with only gravity

$a[0] = -g$

$v[0] = 0$

$x[0] = 0$

- How about some infinitesimally small time later?

A time *dt* later

$$a = -g$$

$$v[dt] = v[0] + g\,dt = g\,dt$$

$$x[dt] = v[dt]\,dt + \frac{1}{2}g(dt)^2$$

How about one step later?

$$a = -g$$

$$v[2dt] = v[dt] + g\,dt$$

$$x[2dt] = x[dt] + v[2dt]\,dt + \frac{1}{2}g(dt)^2$$

Notice a pattern

$$a = -g$$

$$v_{\text{now}} = v_{\text{prev}} + g\,dt$$

$$x_{\text{now}} = x_{\text{prev}} + v_{\text{now}}\,dt + \frac{1}{2}g(dt)^2$$

- We can use a known starting point and just increment one step *dt* at a time!
- Need stopping condition (e.g., x = -h to hit ground)
- Advantage? Can trivially add any acceleration

Example (python)

```
import math

#1D motion with drag for a dropped object

g=9.81          #grav accel defined as constant
v0=0.0          #starting velocity
h=100           #starting position
dt = 0.001      #time step; smaller=more accurate but slower
b = 0.1         #drag coefficient

def trajectory(v,h):
    y=h
    t=0.0
    a=-g

    while y>0:      #iterate until v change is tiny, terminal velocity reached
        a = -g + b*v**2     #drag is opposite gravity (in dir of v)
        v += a*dt
        y += v*dt+0.5*a*dt*dt
        t+=dt
    return (t,v)

time,speed=trajectory(v0,h)     #run with given starting speed and height

print "Time of flight (2)"
print "%.2f s" % (time)

print "Terminal velocity"
print "%.2f m/s" % (speed)
```

```
MacBook-Pro-82:python pleclair$ python ./1D-v2.py
Time of flight (2)
10.79 s
Terminal velocity
-9.90 m/s
MacBook-Pro-82:python pleclair$
```

So what?

- You can do this on your phone
- You are mostly science & engineering majors, this will totally come up again
- Don’t be afraid of code. Much better than labs, you can’t break anything.

- Some examples:
- http://faculty.mint.ua.edu/~pleclair/ph125/python/

Section 9.5
Question 6

When you do positive work on a particle, its kinetic energy

1. increases.
2. decreases.
3. remains the same.
4. We need more information about the way the work was done.

Section 9.5
Question 6

When you do positive work on a particle, its kinetic energy

✔ 1. increases.
2. decreases.
3. remains the same.
4. We need more information about the way the work was done.

Section 9.6: Work done on a many-particle system

Section Goals

You will learn to

- Extend the work-force-displacement relationship for single objects to **systems of interacting objects**.
- Recognize that **only** external forces contribute to the work done for many particle systems. Since the **internal** forces are members of an interaction pair the **work done** by that pair of forces always sums to **zero**.

Example 9.7 Landing on his feet

A 60-kg person jumps off a chair and lands on the floor at a speed of 1.2 m/s. Once his feet touch the floor surface, he slows down with constant acceleration by bending his knees. During the slowing down, his center of mass travels 0.25 m.

Determine the magnitude of the force exerted by the floor surface on the person and the work done by this force on him.

What?

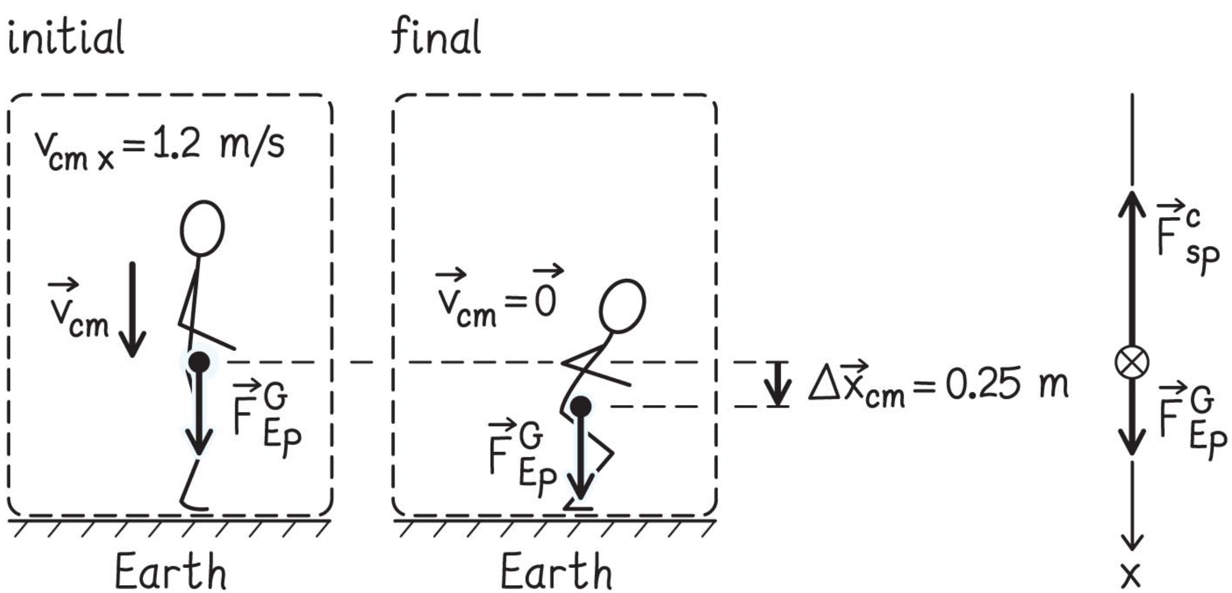
I have no idea what to do.

- What is the physics?
  - person's KE changes
  - external forces cause this change
  - the change in KE must be due to the work done by these forces

- Figure the change in KE and the work done by gravity slowing down. From work you get force, knowing the displacement.

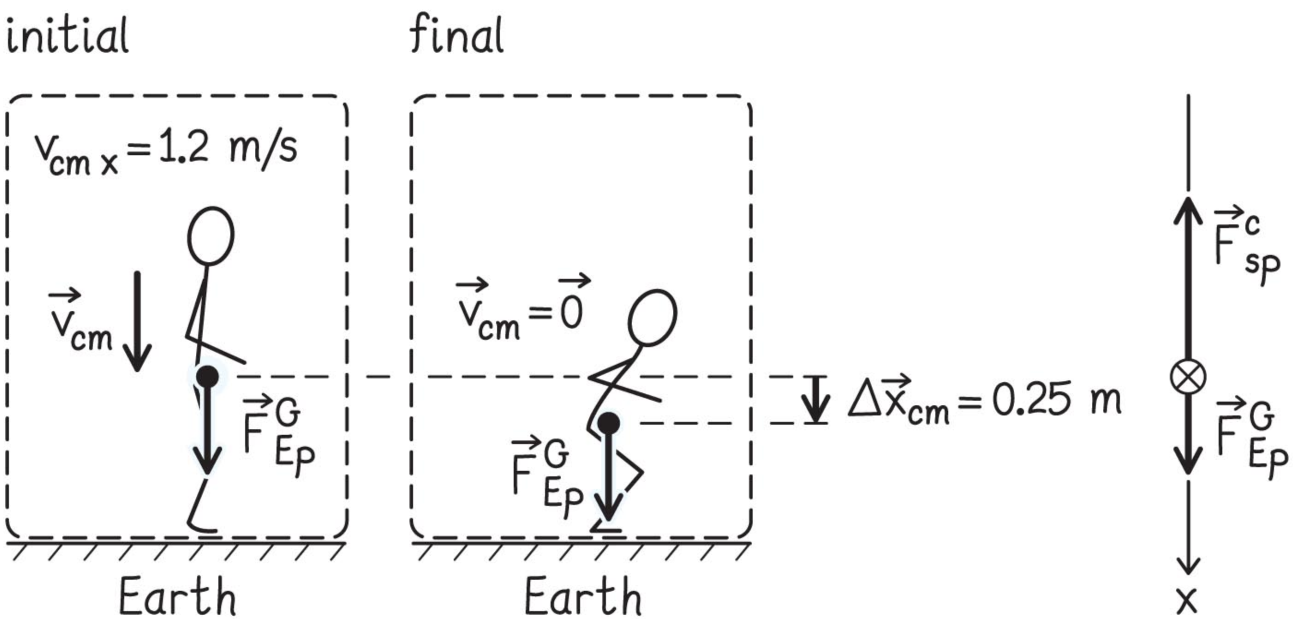
Example 9.7 Landing on his feet (cont.)

❶ GETTING STARTED I begin by making a sketch of the initial and final conditions, choosing my person as the system and assuming the motion to be entirely vertical (Figure 9.21).



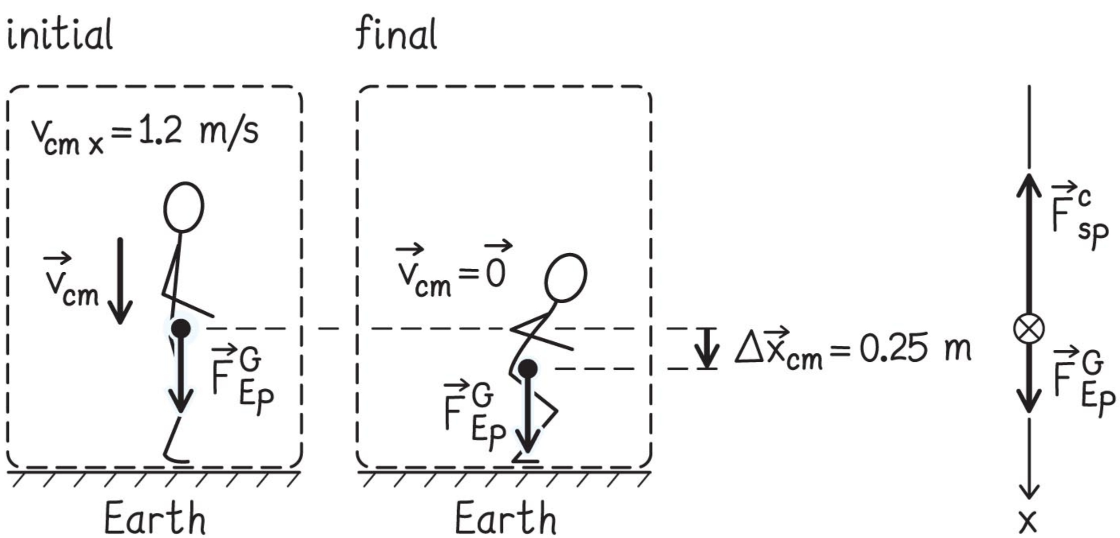
Example 9.7 Landing on his feet (cont.)

❶ GETTING STARTED I point the x axis downward in the direction of motion, which means that the x components of both the displacement and the velocity of the center of mass are positive: $\Delta x_{\text{cm}} = +0.25$ m and $v_{\text{cm}\,x} = +1.2$ m/s.



Example 9.7 Landing on his feet (cont.)

❶ GETTING STARTED Two external forces are exerted on the person: a downward force of gravity $\vec{F}^{G}_{Ep}$ exerted by Earth and an upward contact force $\vec{F}^{c}_{sp}$ exerted by the floor surface. Only the point of application of the force of gravity undergoes a displacement, and so I need to include only that force in my diagram.



Example 9.7 Landing on his feet (cont.)

❷ DEVISE PLAN Knowing the initial center-of-mass velocity, I can use Eq. 9.13 to calculate the change in the person's translational kinetic energy ΔK_{cm}.

This change in kinetic energy must equal the work done by the net external force.

From that and the displacement $\Delta x_{cm} = +0.25$ m we can find the vector sum of the forces exerted on the person.

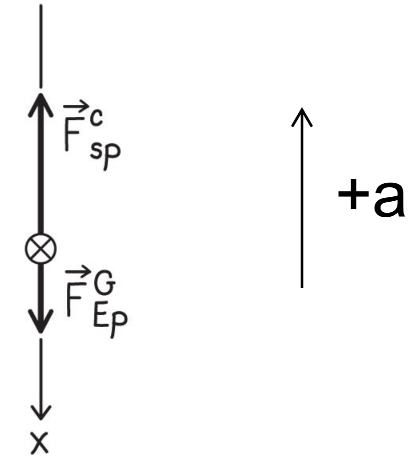
Example 9.7 Landing on his feet (cont.)

❷ DEVISE PLAN If I then subtract the force of gravity, I obtain the force exerted by the floor surface on the person. To determine the work done by this force on the person, I need to multiply it by the force displacement.

Section 9.6: Work done on a many-particle system

Example 9.7 Landing on his feet (cont.)

❷ DEVISE PLAN Because the person slows down as he travels downward, his acceleration is upward and so the vector sum of the forces is upward too. To remind myself of this, I draw a free-body diagram in which the arrow for the upward contact force is longer than the arrow for the downward force of gravity.



Section 9.6: Work done on a many-particle system

Example 9.7 Landing on his feet (cont.)

❸ EXECUTE PLAN Because the person ends at rest, his final translational kinetic energy is zero

$$\Delta K_{\text{cm}} = 0 - \tfrac{1}{2} m \upsilon_{\text{cm,i}}^2 = \tfrac{1}{2}(60\,kg)(1.2\ \text{m/s})^2 = -43\ \text{J}.$$

Example 9.7 Landing on his feet (cont.)

❸ EXECUTE PLAN Substituting this value and the displacement of the center of mass into Eq. 9.14 yields

$$\Sigma F_{\text{ext}\,x} = \frac{\Delta K_{\text{cm}}}{\Delta x_{\text{cm}}} = \frac{-43\ \text{J}}{0.25\ \text{m}} = -170\ \text{N}.$$

This is the *net* force. We need the free body diagram to disentangle the forces.

Example 9.7 Landing on his feet (cont.)

❸ EXECUTE PLAN To obtain the force exerted by the floor from this vector sum, look back to the free body diagram: just two forces.

$$\Sigma F_{\text{ext}\,x} = F^{G}_{\text{Ep}\,x} + F^{c}_{\text{sp}\,x}$$

and so $F^{c}_{\text{sp}\,x} = \Sigma F_{\text{ext}\,x} - F^{G}_{\text{Ep}\,x}$. The x component of the force of gravity is $F^{G}_{\text{Ep}\,x} = mg = (60\text{ kg})(9.8\ (\text{m/s}^2)) = +590\text{ N}$ and so $F^{c}_{\text{sp}\,x} = -170\text{ N} - 590\text{ N} = -760\text{ N}$. ✔

Example 9.7 Landing on his feet (cont.)

❸ EXECUTE PLAN To determine the work done by this force on the person, I must multiply the *x* component of the force by the force displacement.

The point of application is the floor, which doesn't move. This means that the force displacement is zero, so the work done on the person by the surface is zero too: $W = 0$. ✔

Example 9.7 Landing on his feet (cont.)

❹ EVALUATE RESULT The contact force $F^{c}_{\text{sp}\,x}$ is negative because it is directed upward, as I expect. Its magnitude is larger than that of the force of gravity, as it should be in order to slow the person down.

Section 9.7: Variable and distributed forces

Section Goals

You will learn to

- Derive the relationship for the **work** done by a **variable force**.
- Interpret the work done by a variable force **graphically**.
- Understand that distributed forces, like friction, have **no single point of application** on an object.

Section 9.7: Variable and distributed forces

- The acceleration of the center of mass of a system consisting of many interacting particles is given by

$$\vec{a}_{\text{cm}} = \frac{\Sigma \vec{F}_{\text{ext}}}{m}$$

- Following the same derivation as in the single-particle system, we can write

$$\Delta K_{\text{cm}} = m a_{\text{cm}\,x} \Delta x_{\text{cm}} = (\Sigma F_{\text{ext}\,x}) \Delta x_{\text{cm}} \text{ (constant forces, one dimension)}$$

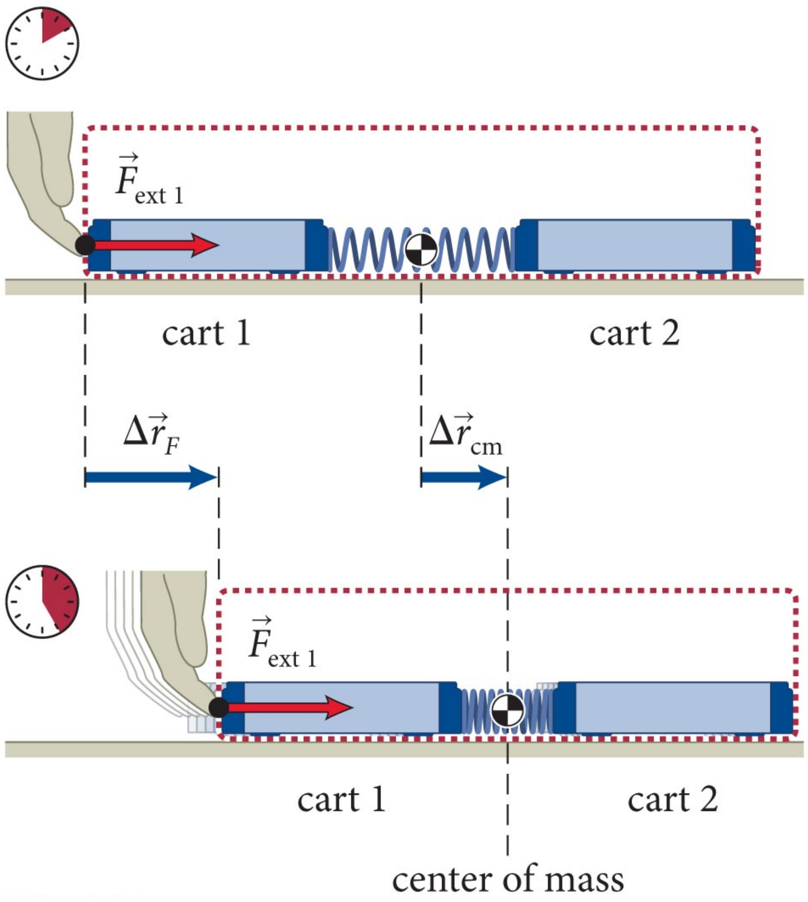
- So?
  - treat object as a point particle, concentrated at center of mass
  - work done in moving the center of mass gives change in center of mass kinetic energy
  - if you can calculate a ball, you can calculate a wrench

Section 9.7: Variable and distributed forces

- For a system of many particles $K = K_{cm} + K_{conv}$ – there is some KE due to internal motion of constituents
- Therefore, $\Delta K_{cm} \neq \Delta E$, and since $\Delta E = W$, we can see that

$$\Delta K_{cm} \neq W \text{ (many-particle system)}$$

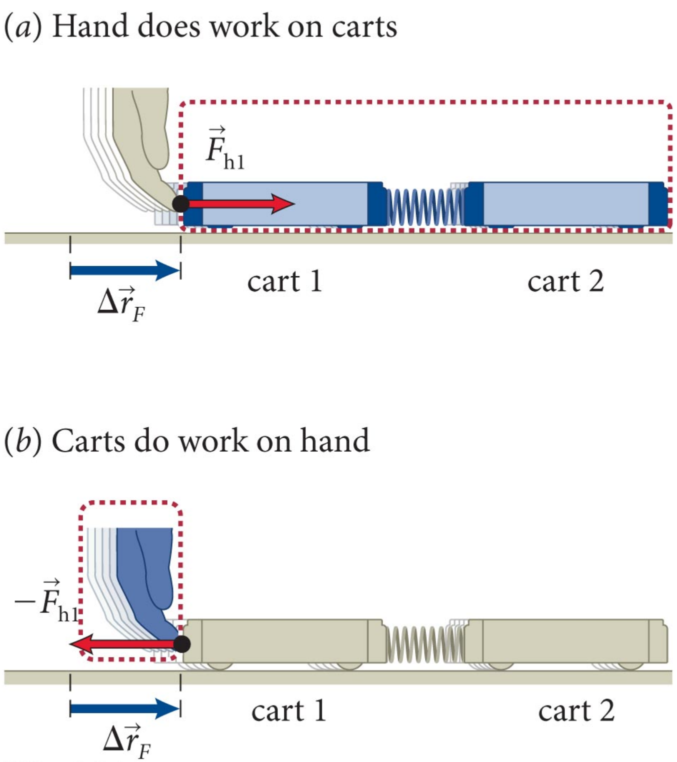
- This is explicitly illustrated in the example shown in the figure.
- The external force on cart 1 increases the kinetic energy **and** the internal energy of the system.



Section 9.7: Variable and distributed forces

- To determine the work done by external forces on a many particle system, we can use the fact that $W_{env} = -W_{sys}$.
- This makes sense – work is what crosses the boundary, and a loss to the environment is the same as a gain to the system (& vice versa)
- We can see from the figure that the work done by the two-cart system on the hand is $= -F_{h1x}\Delta x_F$.
- Then the work done by the external force on the two-cart system is

$$W = F_{ext\,1x}\Delta x_F \text{ (constant nondissipative force, one dimension)}$$



Section 9.7: Variable and distributed forces

- Generalizing this work equation to many-particle systems subject to several constant forces, we get

$$W = W_1 + W_2 + \cdots = F_{\text{ext}\,1x}\Delta x_{F1} + F_{\text{ext}\,2x}\Delta x_{F2} + \cdots$$

or

$$W = \sum_n (F_{\text{ext}\,nx}\Delta x_{Fn}) \text{ (constant nondissipative forces, one dimension)}$$

If we consider a varying force $F(x)$, we take infinitesimal displacements, and this becomes an integral

$$W = \int_{x_\text{i}}^{x_\text{f}} F_x(x)\,dx \text{ (nondissipative force, one dimension)}$$

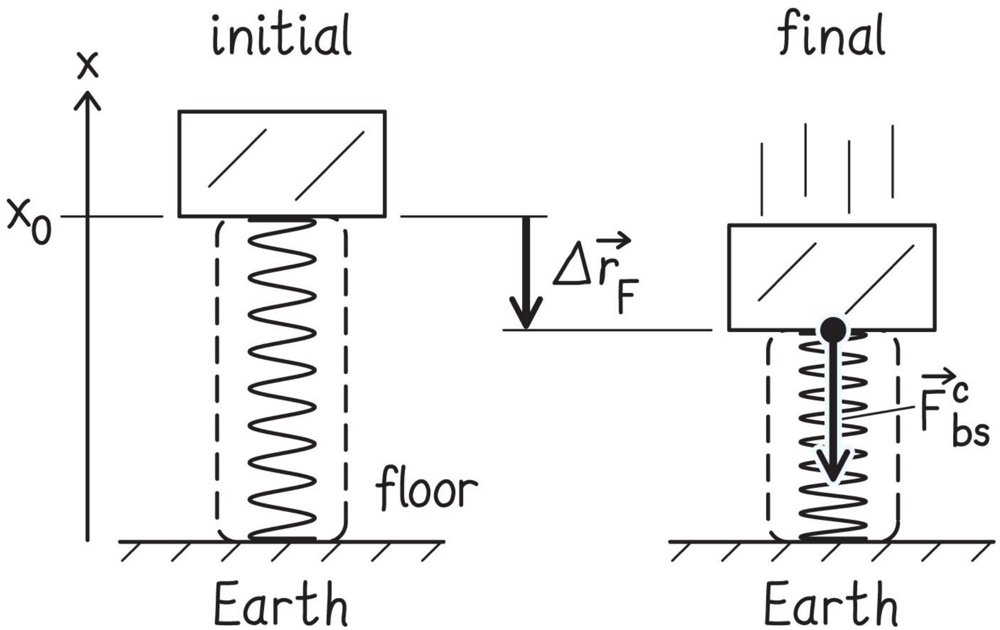
work is the area under the force-displacement curve!

Example 9.8 Spring work

A brick of inertia m compresses a spring of spring constant k so that the free end of the spring is displaced from its relaxed position. What is the work done by the brick on the spring during the compression?

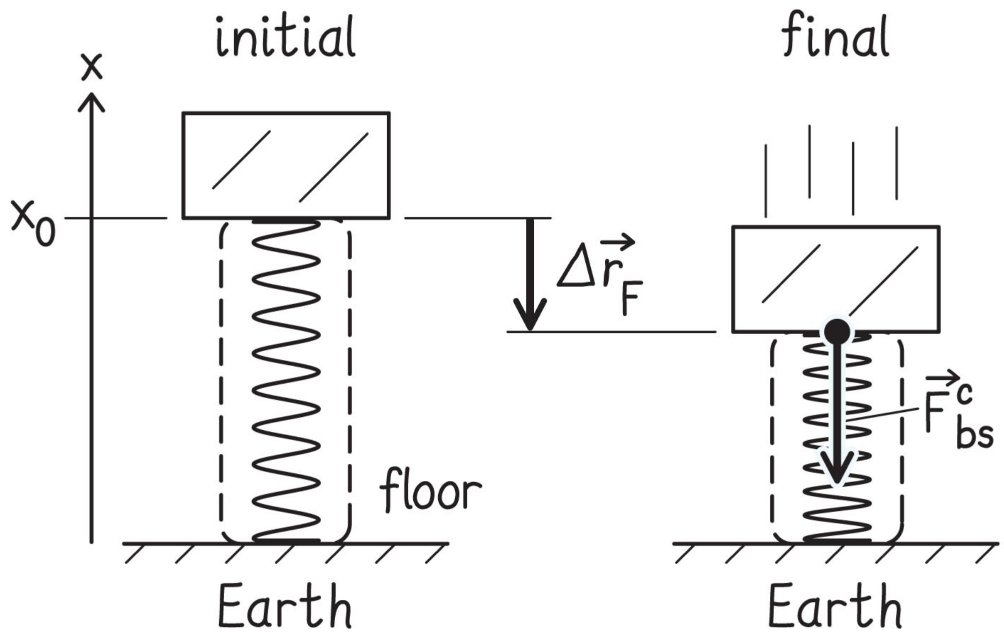
Example 9.8 Spring work (cont.)

❶ GETTING STARTED I begin by making a sketch of the situation as the free end of the spring is compressed from its relaxed position x_0 to a position x (Figure 9.23). Because I need to calculate the work done by the brick on the spring, I choose the spring as my system.



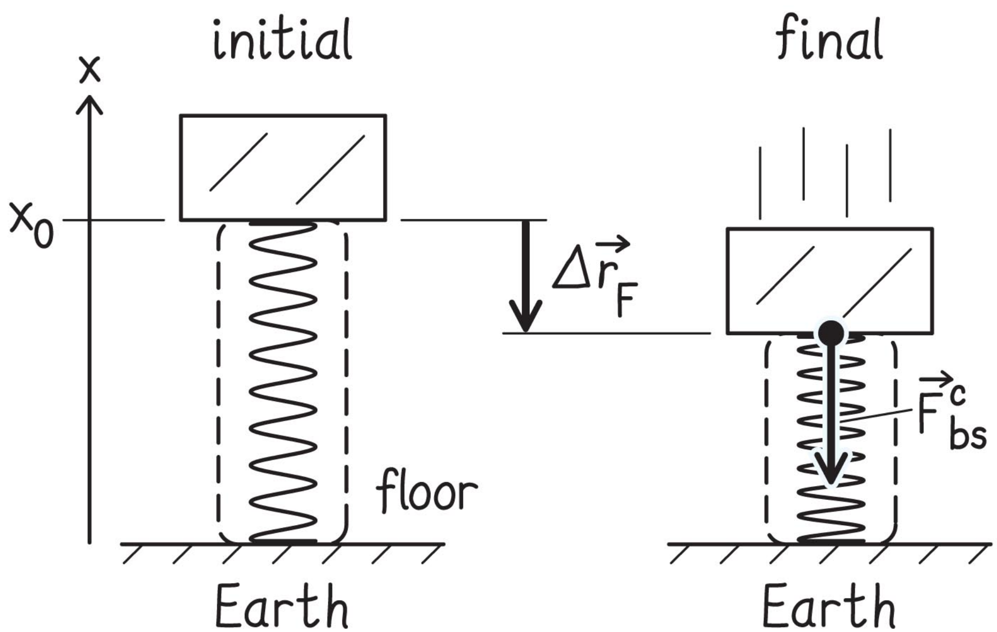
Example 9.8 Spring work (cont.)

❶ GETTING STARTED Three forces are exerted on the spring: contact forces exerted by the brick and by the floor, and the force of gravity. As usual when dealing with compressed springs, I ignore the force of gravity exerted on the spring (see Section 8.6).



Example 9.8 Spring work (cont.)

❶ GETTING STARTED Only the force $\vec{F}^{c}_{bs}$ exerted by the brick on the spring undergoes a nonzero force displacement, so I need to show only that force in my diagram. Because the brick and spring do not exert any forces on each other when the spring is in the relaxed position, I do not draw this force in the initial state.



Example 9.8 Spring work (cont.)

❷ DEVISE PLAN I need to calculate the work done by the brick on the spring. I could use Eq. 9.22 if I knew the force the brick exerts on the spring.

That force is not given, but the **force exerted by the brick on the spring and the force exerted by the spring on the brick form an interaction pair:** $\vec{F}^{c}_{bs} = -\vec{F}^{c}_{sb}$. I can use Eq. 8.20 to determine $\vec{F}^{c}_{sb}$ and then use it in Eq. 9.22.

Example 9.8 Spring work (cont.)

❸ EXECUTE PLAN Equation 8.20 tells me that the x component of the force exerted by the spring on the brick varies depending on how far the spring is compressed:

$$F_{\text{sb}\,x} = -k(x - x_0),$$

where x_0 is the coordinate of the relaxed position of the free end of the spring.

Example 9.8 Spring work (cont.)

❸ EXECUTE PLAN The x component of the force exerted by the brick on the spring is thus

$$F_{\text{bs}\,x} = +k(x - x_0). \qquad (1)$$

Because $x_0 > x$, $F_{\text{bs}\,x}$ is negative, which means that $\vec{F}^{\text{c}}_{\text{bs}}$ points in the same direction as the force displacement. Thus the work done by the brick on the spring is positive.

Example 9.8 Spring work (cont.)

❸ EXECUTE PLAN Now I substitute Eq. 1 into Eq. 9.22 and work out the integral to determine the work done by the brick on the spring:

$$W_{\text{bs}} = \int_{x_0}^{x} F_{\text{bs}\,x}(x)\,dx = \int_{x_0}^{x} k(x - x_0)\,dx$$

$$= \left[\tfrac{1}{2}kx^2 - kx_0x\right]_{x_0}^{x} = \tfrac{1}{2}k(x - x_0)^2. \checkmark \qquad (2)$$

limits of integration? start & end points of motion

Example 9.8 Spring work (cont.)

❹ Evaluate result Because the spring constant k is always positive (see Section 8.9 on Hooke's law), the work done by the brick on the spring is also positive. This is what I expect because the work done in compressing the spring is stored as potential energy in the spring.

Section 9.7
Question 7

When you plot the force exerted on a particle as a function of the particle's position, what feature of the graph represents the work done on the particle?

1. The maximum numerical value of the force
2. The area under the curve
3. The value of the displacement
4. You need more information about the way the work was done.

Question 7

When you plot the force exerted on a particle as a function of the particle's position, what feature of the graph represents the work done on the particle?

1. The maximum numerical value of the force
2. ✔ The area under the curve
3. The value of the displacement
4. You need more information about the way the work was done.

Section 9.8
Question 8

A sports car accelerates from zero to 30 mph in 1.5 s. How long does it take for it to accelerate to 60 mph, assuming that the power delivered by the engine is independent of velocity and neglecting friction?

1. 2.0 s
2. 3.0 s
3. 4.5 s
4. 6.0 s
5. 9.0 s
6. 12.0 s

Clicker Question 8

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1. 2.0 s
2. 3.0 s
3. 4.5 s
4. ✔ 6.0 s twice the speed, four times as long.
5. 9.0 s
6. 12.0 s

Concepts: Work done by a constant force

- In order for a force to do work on an object, the point of application of the force must undergo a displacement.
- The SI unit of work is the **joule** (J).
- The work done by a force is positive when the force and the force displacement are in the same direction and negative when they are in opposite directions.

Chapter 9: Summary

Quantitative Tools: Work done by a constant force

- When one or more constant forces cause a particle or a rigid object to undergo a displacement Δx in one dimension, the work done by the force or forces on the particle or object is given by the **work equation**:

$$W = \left(\sum F_x\right)\Delta x_F$$

- In one dimension, the work done by a set of constant nondissipative forces on a system of particles or on a deformable object is

$$W = \sum_n (F_{\text{ext}\, nx} \Delta x_{Fn})$$

Quantitative Tools: Work done by a constant force

- If an external force does work W on a system, the **energy law** says that the energy of the system changes by an amount

$$\Delta E = W$$

- For a closed system, $W = 0$ and so $\Delta E = 0$.
- For a particle or rigid object, $\Delta E_{\text{int}} = 0$ and so

$$\Delta E = \Delta K$$

- For a system of particles or a deformable object,

$$\Delta K_{\text{cm}} = \left(\sum F_{\text{ext}\,x}\right)\Delta x_{\text{cm}}$$

Concepts: Energy diagrams

- An **energy diagram** shows how the various types of energy in a system change because of work done on the system.
- In choosing a system for an energy diagram, avoid systems for which friction occurs at the boundary because then you cannot tell how much of the thermal energy generated by friction goes into the system.

Chapter 9: Summary

Concepts: Variable and distributed forces

- The force exerted by a spring is variable (its magnitude and/or direction changes) but nondissipative (no energy is converted to thermal energy).
- The frictional force is dissipative and so causes a change in thermal energy. This force is also a distributed force because there is no single point of application.

Chapter 9: Summary

Quantitative Tools: Variable and distributed forces

- The work done by a variable nondissipative force on a particle or object is

$$W = \int_{x_i}^{x_f} F_x(x)\,dx$$

- If the free end of a spring is displaced from its relaxed position x_0 to position x, the change in its potential energy is

$$\Delta U_{spring} = \tfrac{1}{2}k(x - x_0)^2$$

- If a block travels a distance d_{path} over a surface for which the magnitude of the force of friction is a constant F^f_{sb}, the energy dissipated by friction (the thermal energy) is

$$\Delta E_{th} = F^f_{sb} d_{path}$$

Concepts: Power

- **Power** is the *rate* at which energy is either converted from one form to another or transferred from one object to another.

- The SI unit of power is the **watt** (W), where 1 W = 1 J/s.

Quantitative Tools: Power

- The **instantaneous power** is

$$P = \frac{dE}{dt}$$

- If a constant external force $F_{\text{ext}\,x}$ is exerted on an object and the x component of the velocity at the point where the force is applied is υ_x, the power this force delivers to the object is

$$P = F_{\text{ext}\,x}\upsilon_x$$