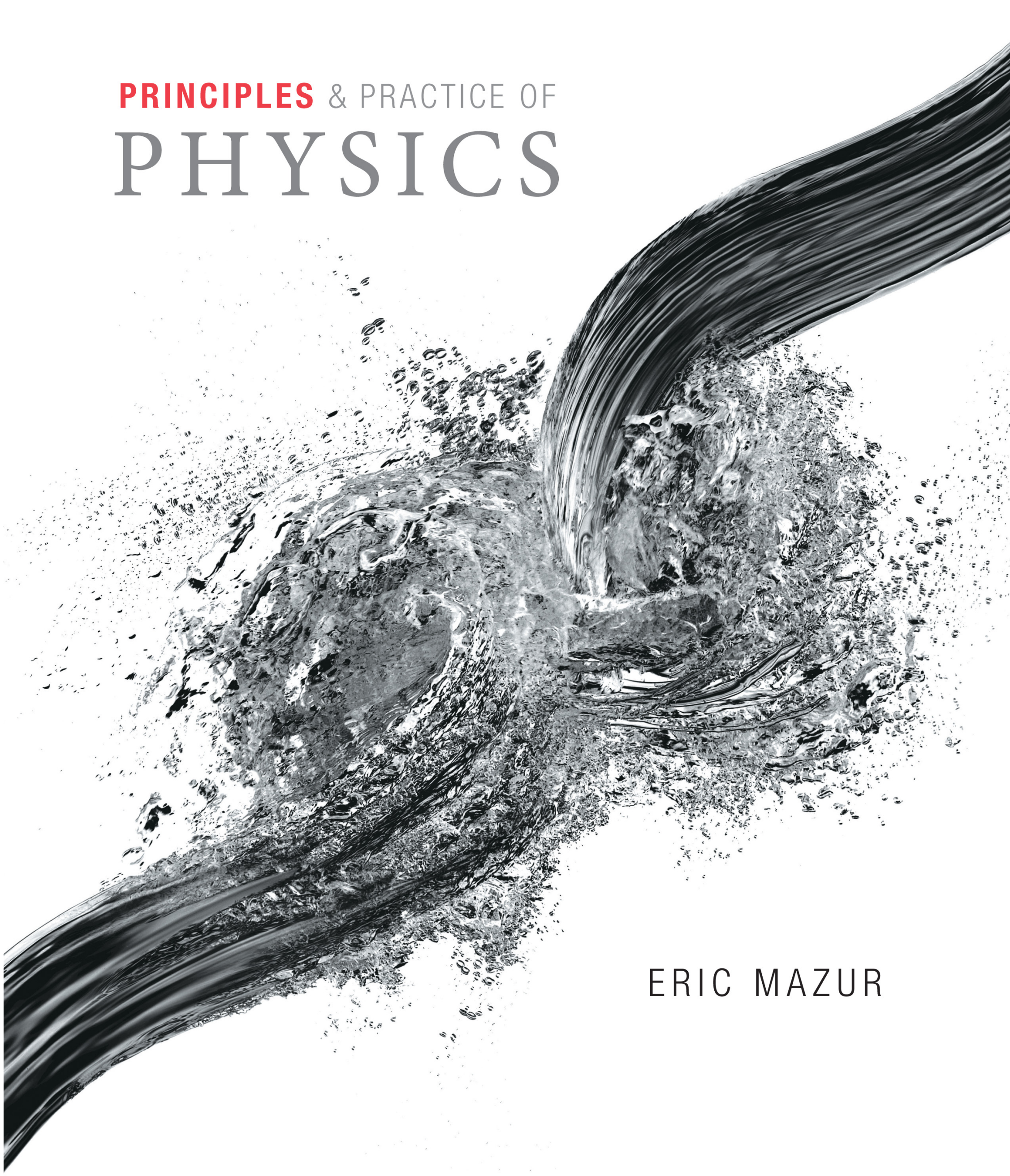
Chapter 10 Motion in a Plane



Exam 2 details

Basically like the last one

- ~20 multiple choice. Chapters 7-10
- Ch. 7.1-10 interactions (energy)
  - 1, 5-7 interaction basics
  - 2, 8-9 potential energy
  - 3-4, 8, 10 dissipative forces
- Ch. 8.1-10 force
  - 1-5, 7 characteristics of force
  - 6, 8, 9 important forces
  - 1, 4-5, 7 effects of forces
  - 10 impulse

Exam 2 details

Basically like the last one

- Ch. 9.1-8 Work
  - 1-2, 5-6 work by constant force
  - 3-4 energy diagrams
  - 7 variable & distributed forces
  - 8 power
- Ch. 10.3-5, 7 Motion in a plane
  - 7 projectile motion
  - 2-3 forces in 2D
  - 4 friction
  - 5 work
  - (need to know 1-2, 6 info on vectors, but not tested directly)

Chapter 10: Motion in a plane

Concepts

Section 10.1: Straight is a relative term

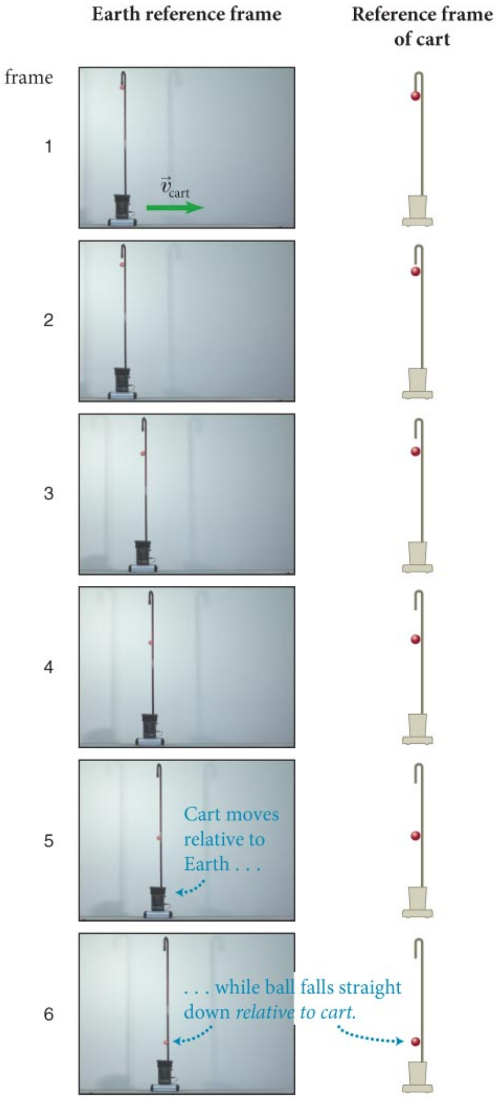
Section Goal

You will learn to

- Recognize that the trajectory followed by a free-falling object depends on the state of motion of the observer.

Section 10.1: Straight is a relative term

- To begin our discussion of motion in a plane, consider the film clip in the figure.
- The ball is dropped from a pole attached to a cart that is moving to the right at a constant speed.

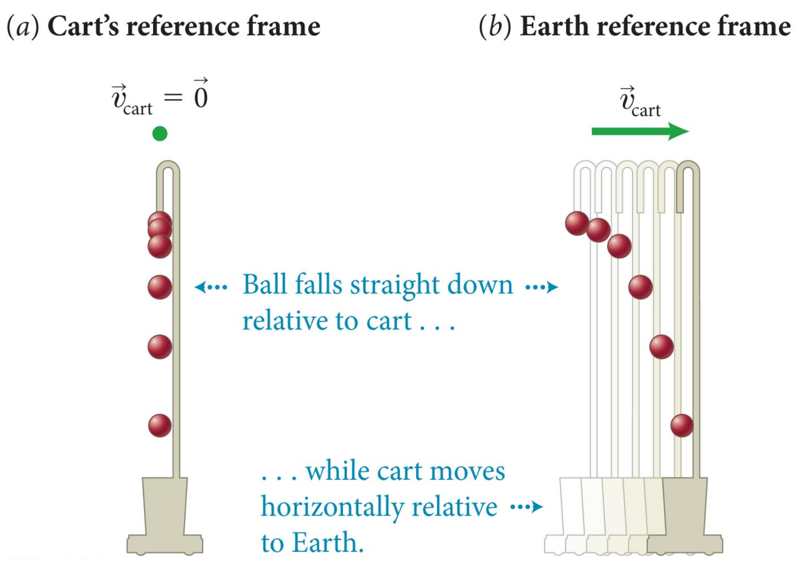


Video …

- https://www.youtube.com/watch?v=JCampZIwL5w

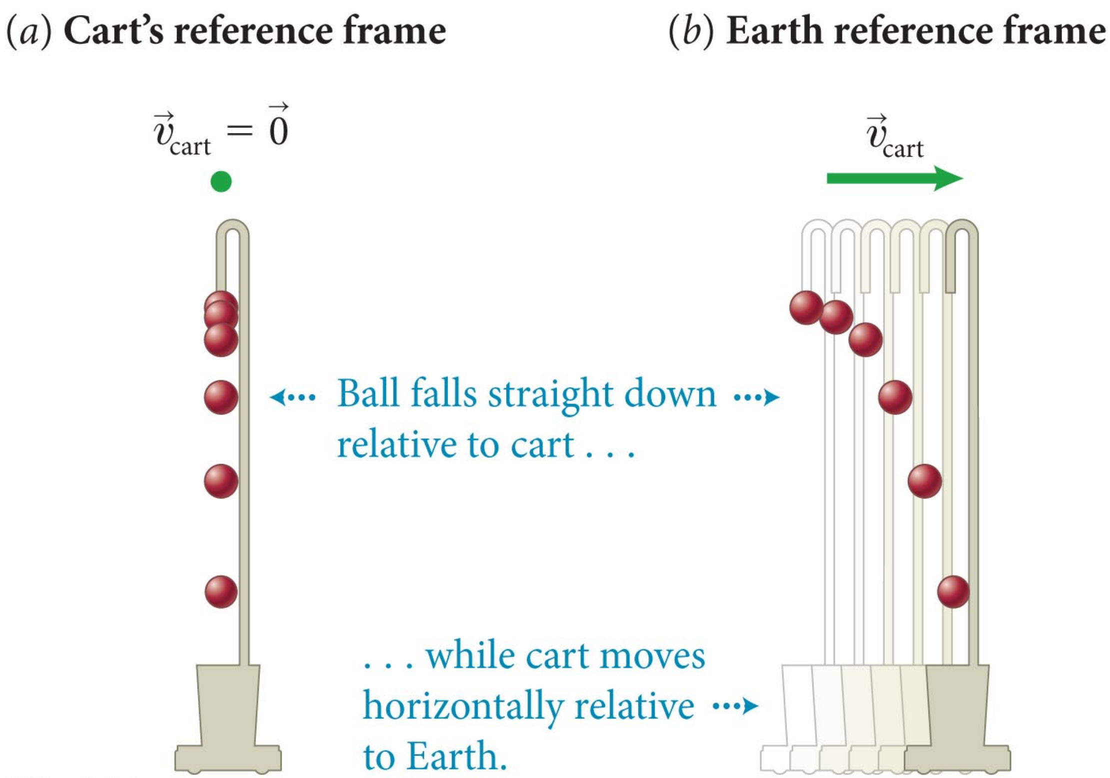
Section 10.1: Straight is a relative term

- The figure shows that
  - (*a*) The ball falls to the ground in a straight line if observed from the cart's reference frame.
  - (*b*) The ball has a horizontal displacement in addition to the straight downward motion when observed from Earth's reference frame.
- The figure shows us that the motion of the ball in Earth's reference frame can be broken down into two parts:
  - Free fall in the vertical direction
  - Constant velocity motion in the horizontal direction



Checkpoint 10.1

10.1 (*a*) In Figure 10.2, what is the ball's velocity the instant before it is released? (*b*) Is the ball's speed in the reference frame of the cart greater than, equal to, or smaller than its speed in the Earth reference frame?



Checkpoint 10.1

10.1

In what reference frame? It depends …

Before release, $v = 0$ relative to cart (attached)

but in Earth's frame, velocity is v_{cart}

After release: ball now has a vertical component of velocity that the cart doesn't. Relative to the earth, its speed is higher

Speed = vector magnitude of velocity

Section 10.1
Question 1

A passenger in a speeding train drops a peanut.

Which is greater?

1. The magnitude of the acceleration of the peanut as measured by the passenger.
2. The magnitude of the acceleration of the peanut as measured by a person standing next to the track.
3. Neither, the accelerations are the same to both observers.

Section 10.1
Question 1

A passenger in a speeding train drops a peanut.

Which is greater?

1. The magnitude of the acceleration of the peanut as measured by the passenger.
2. The magnitude of the acceleration of the peanut as measured by a person standing next to the track.
3. ✔ Neither, the accelerations are the same to both observers – same interaction

Section 10.2: Vectors in a plane

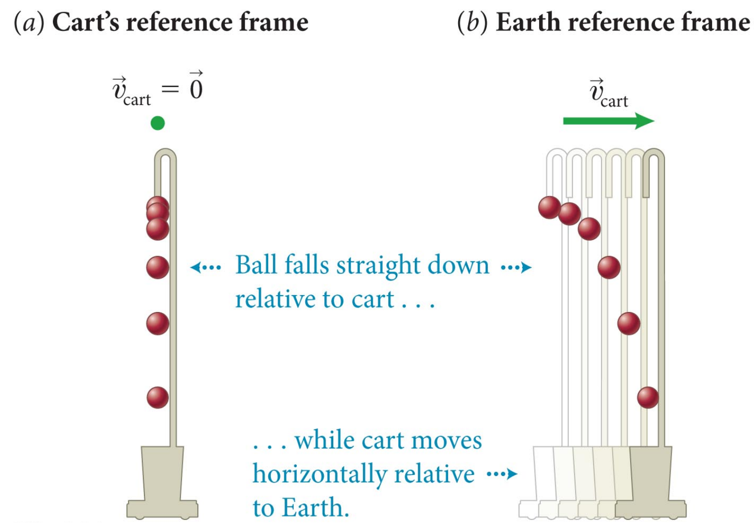
Section Goals

You will learn to

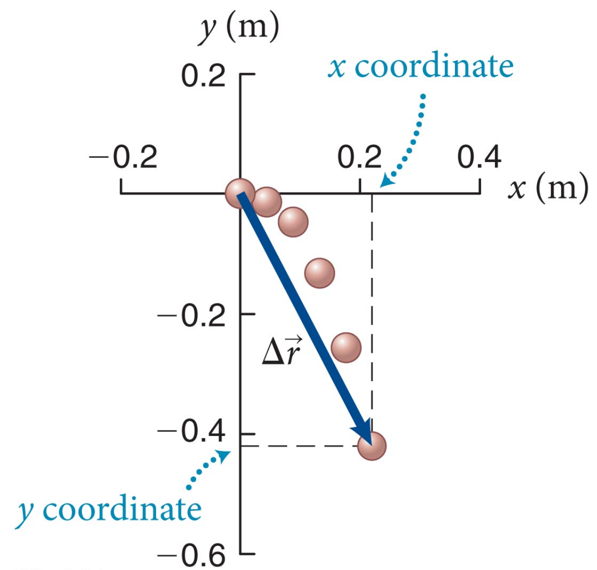
- Determine the sum and the difference of two vectors using a graphical method.
- Develop a procedure to compute the x and y components of a vector using a set of mutually perpendicular axes.
- Understand that the acceleration component parallel to the instantaneous velocity increases the speed of the object, and the perpendicular acceleration component changes the direction of the velocity.

Section 10.2: Vectors in a plane

- To analyze the motion of an object moving in a plane, we need to define two reference axes, as shown on the right.



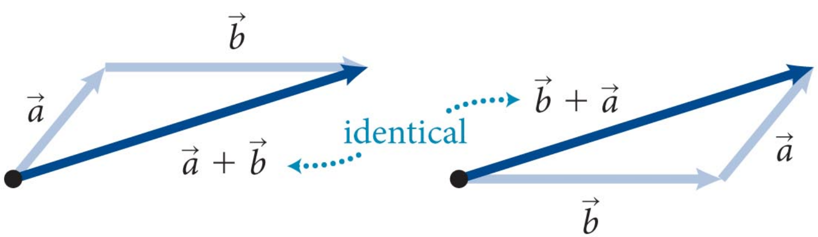
- We see from the figure at right that the ball's displacement in Earth's reference frame is the vector sum of the horizontal displacement $\Delta\vec{x}$ and the vertical displacement $\Delta\vec{y}$.



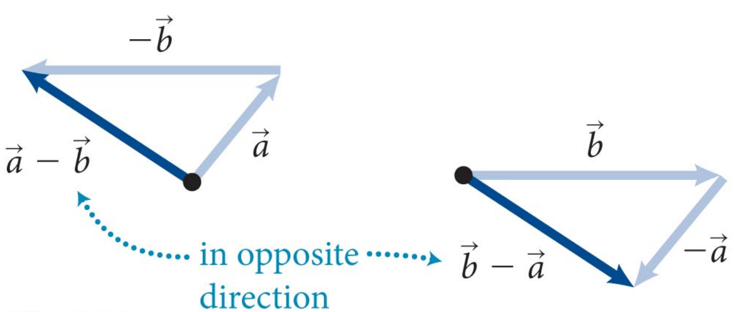
Section 10.2: Vectors in a plane

- The vector sum of two vectors in a plane is obtained by placing the tail of the second vector at the head of the first vector, as illustrated below.
- To subtract a vector $\vec{b}$ from a vector $\vec{a}$, reverse the direction $\vec{b}$ of and then add the reversed $\vec{b}$ to $\vec{a}$.

Vector addition: Vectors may be added in any order: $\vec{a} + \vec{b} = \vec{b} + \vec{a}$



Vector subtraction: Order matters! $\vec{a} - \vec{b} \neq \vec{b} - \vec{a}$



Section 10.2
Question 2

Is vector addition commutative? Is vector subtraction commutative?

1. Yes, yes
2. Yes, no
3. No, yes
4. No, no

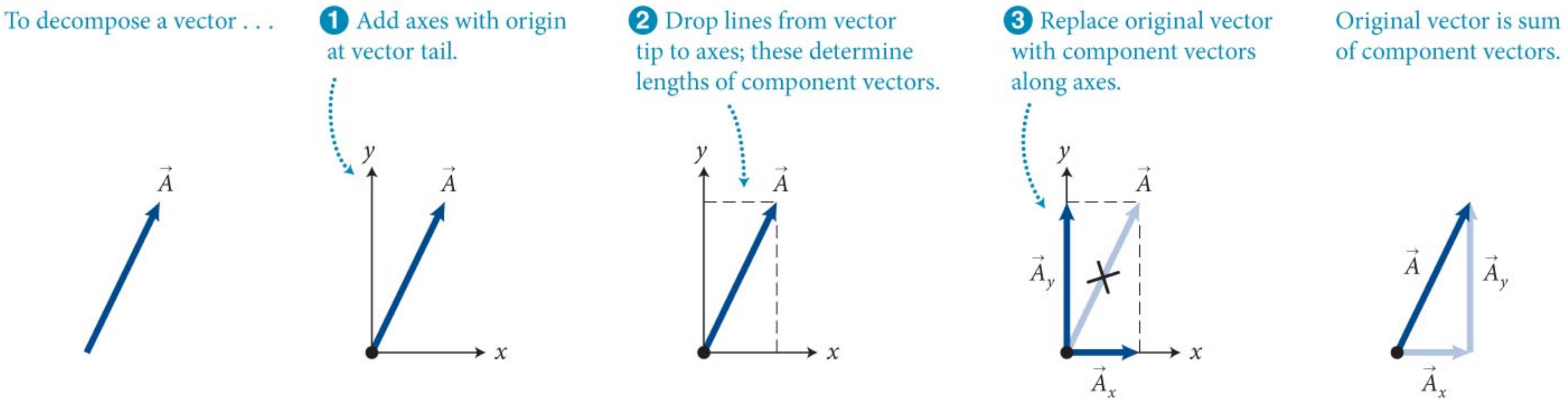
Section 10.2
Question 2

Is vector addition commutative? Is vector subtraction commutative?

1. Yes, yes
2. ✔ Yes, no – order matters for subtraction
3. No, yes
4. No, no

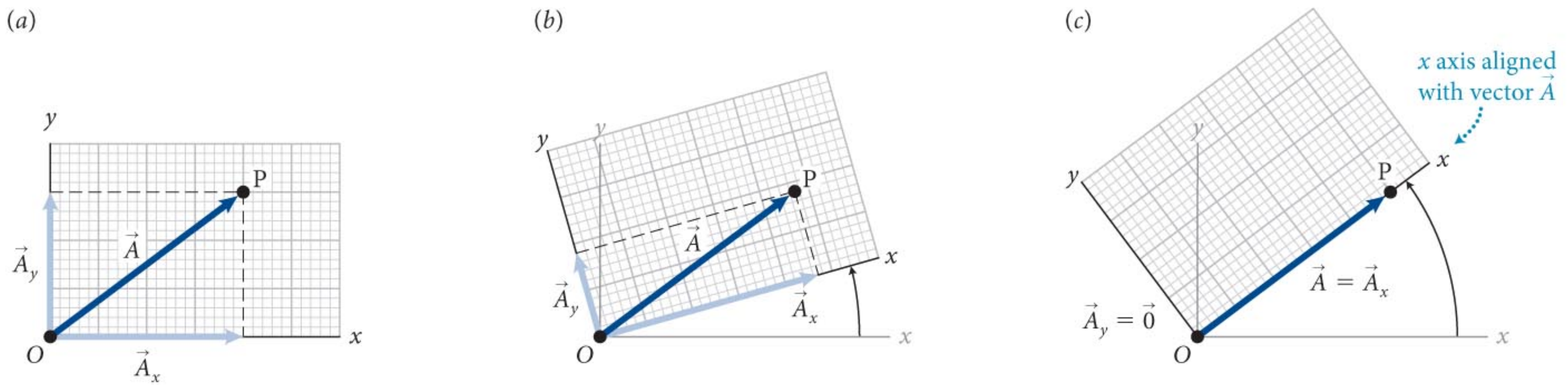
Section 10.2: Vectors in a plane

- Any vector $\vec{A}$ can be decomposed to **component vectors** $\vec{A}_x$ and $\vec{A}_y$ along the axes of some conveniently chosen set of mutually perpendicular axes, called a **rectangular coordinate system**.
- The procedure for decomposing a vector is shown below.



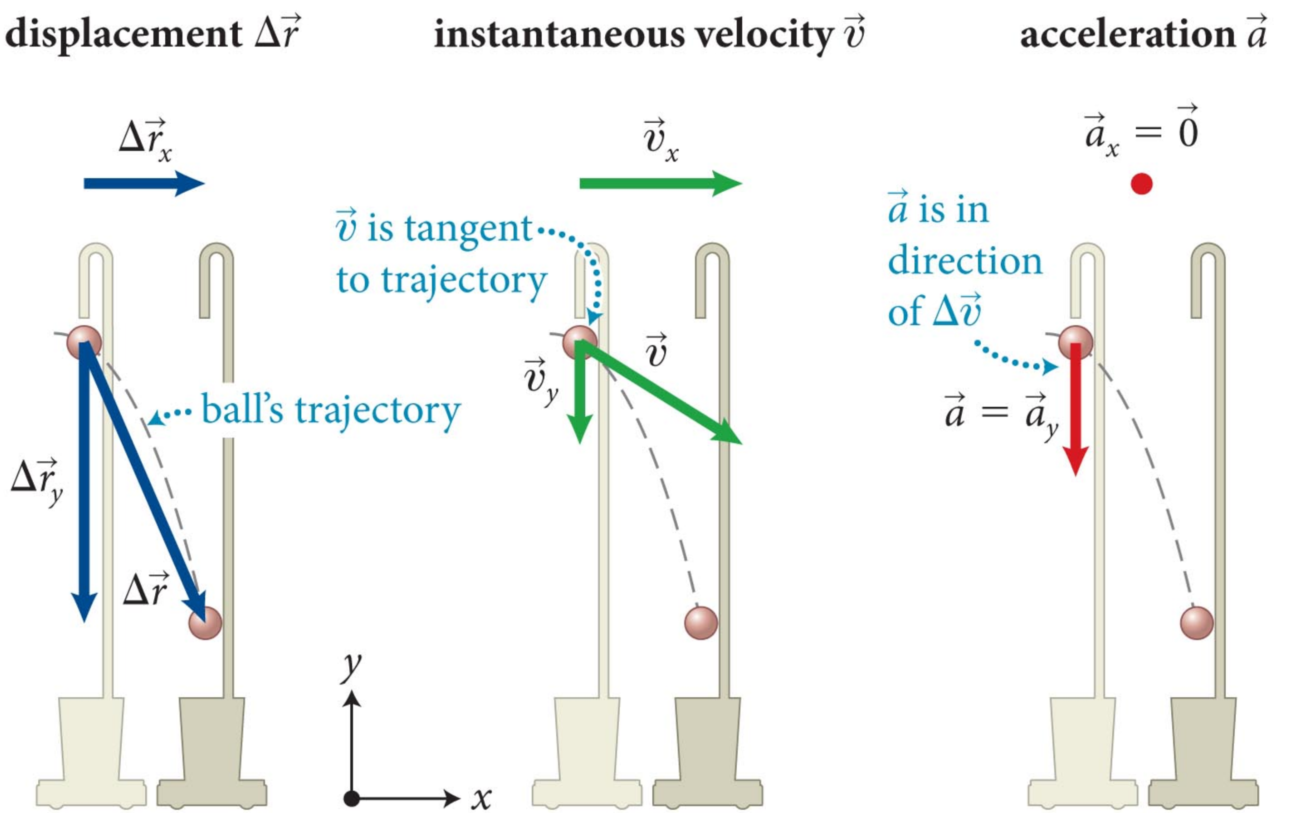
Section 10.2: Vectors in a plane

- The figure below shows the decomposition of a vector $\vec{A}$ in three coordinate systems.
- You need to choose the coordinate system that best suits the problem at hand.



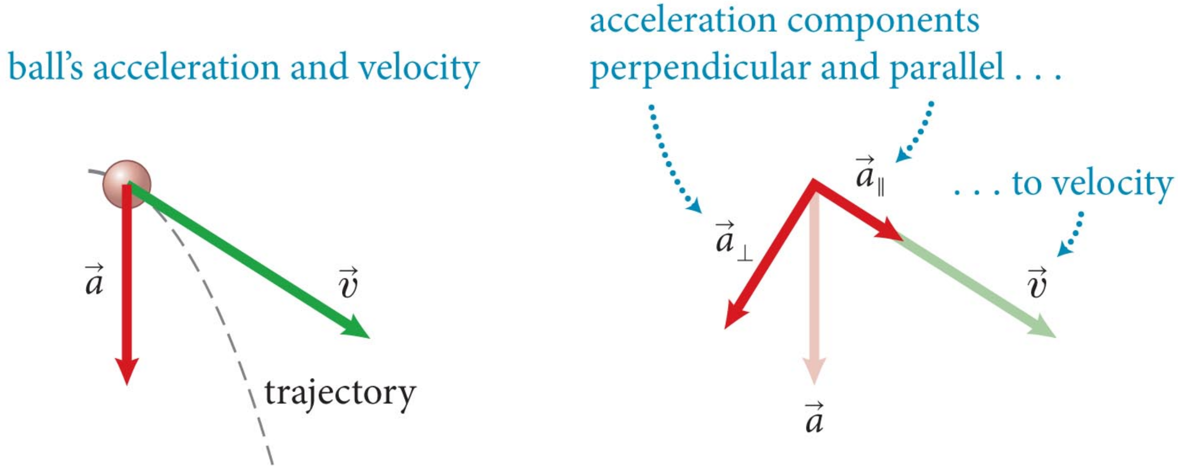
Section 10.2: Vectors in a plane

- The displacement, instantaneous velocity, and acceleration of the ball in the previous section is shown below.
- Notice that the instantaneous velocity and acceleration are not in the same direction. What does this mean?



Section 10.2: Vectors in a plane

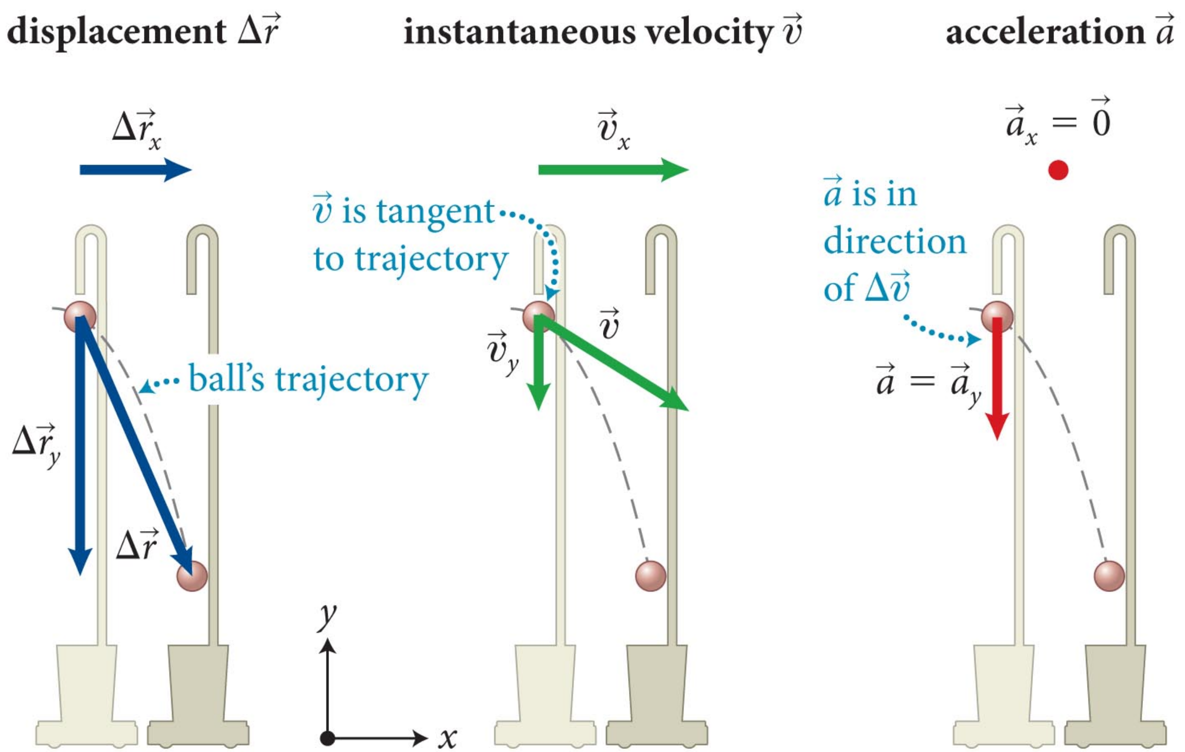
- To answer the question on the previous slide, we will decompose the acceleration vector into two components: one parallel to the instantaneous velocity, and one perpendicular to it.



- **In two-dimensional motion, the component of the acceleration parallel to the instantaneous velocity changes the speed; the component of acceleration perpendicular to the instantaneous velocity changes the direction of the velocity but not its magnitude.**

Checkpoint 10.2

10.2 In Figure 10.10, the ball's instantaneous velocity $\vec{v}$ does not point in the same direction as the displacement $\Delta\vec{r}$ (it points *above* the final position of the ball). Why?



Checkpoint 10.2

10.2

The ball's instantaneous velocity is the displacement over an *infinitesimal time interval*. It is tangential to the trajectory at any point.

The displacement vector is the *entire* net motion of the ball, pointing from initial to final positions.

(tangent vs secant)

Section 10.3: Decomposition of forces

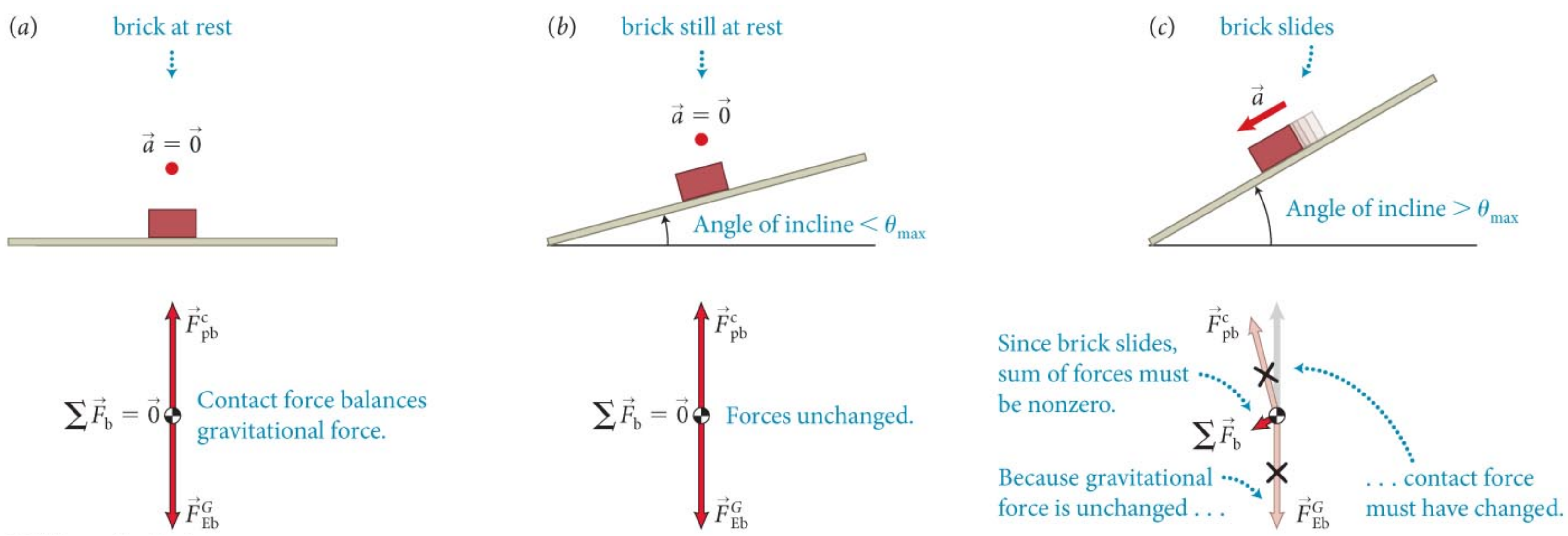
Section Goals

You will learn to

- Apply the vector decomposition technique to analyze the motion of a brick along an inclined surface.
- Realize that choosing a coordinate system such that one of the axes lies along the direction of acceleration of the object allows you to break the problem neatly into two parts.

Section 10.3: Decomposition of forces

- The figure shows a brick lying on a horizontal plank and then the plank is gently tilted.
- When the angle of incline exceeds a θ_{max} the brick accelerates down the incline.
- Then, the vector sum of the forces exerted on the brick must also point down the incline

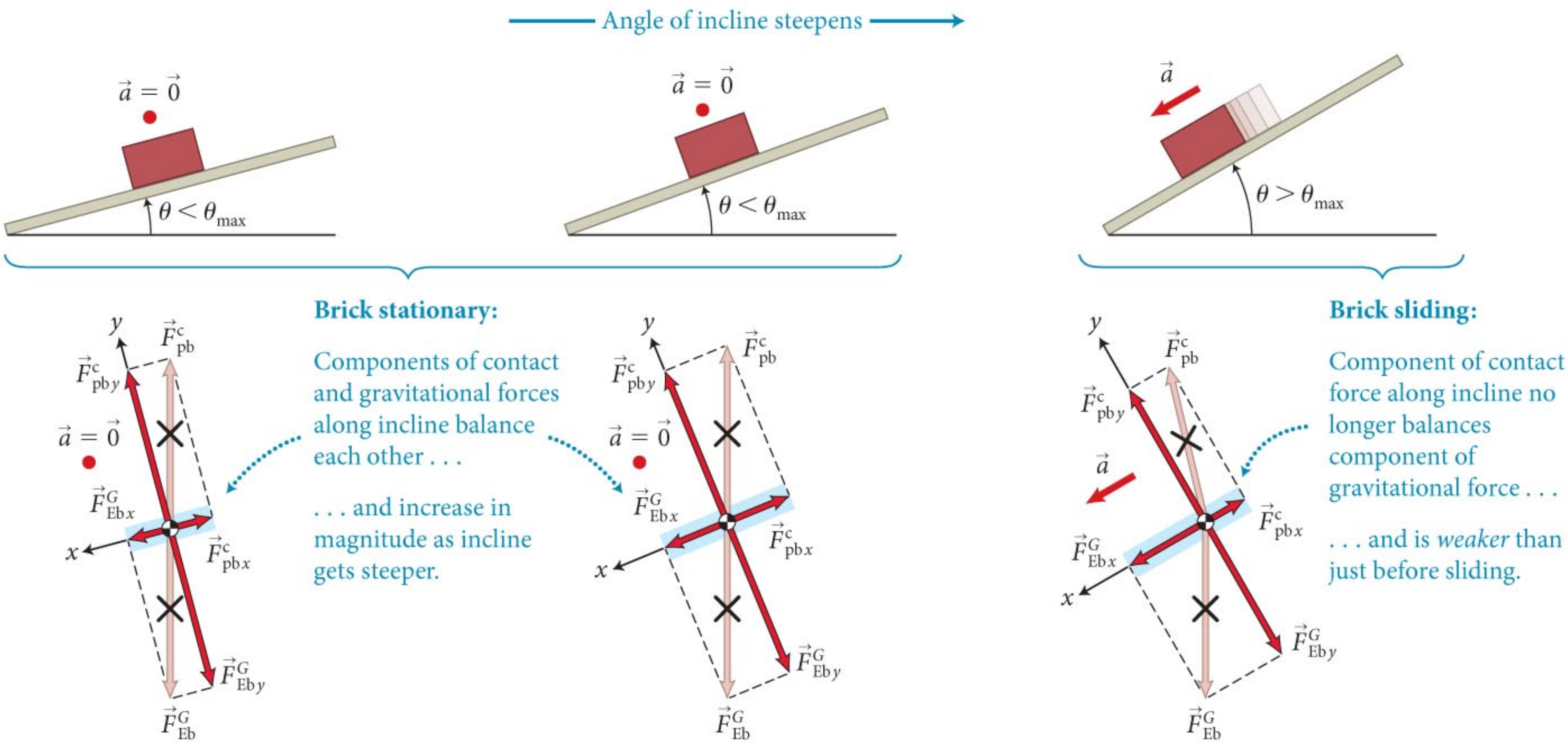


Section 10.3: Decomposition of forces

- Because the brick is constrained to move along the surface of the plank, it make sense to choose the x axis along surface
- The force components parallel to the surface are called **tangential components**.
- The force components perpendicular to the surface are called **normal components**. Normal components must cancel here!
- Contact force contains friction as tangential part

Section 10.3: Decomposition of forces

- Can resolve all forces into two components:
  - Component along the ramp
  - Component perpendicular to ramp

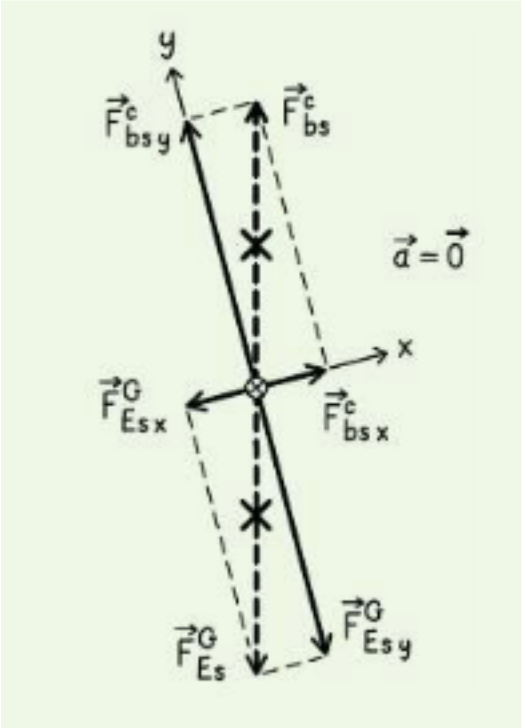


Checkpoint 10.3

10.3 A suitcase being loaded into an airplane moves at constant velocity on an inclined conveyor belt. Draw a free-body diagram for the suitcase as it moves up along with the belt. Show the normal and tangential components of the forces exerted on the suitcase.

Both vert & horz components

must sum to zero!



Section 10.3: Decomposition of forces

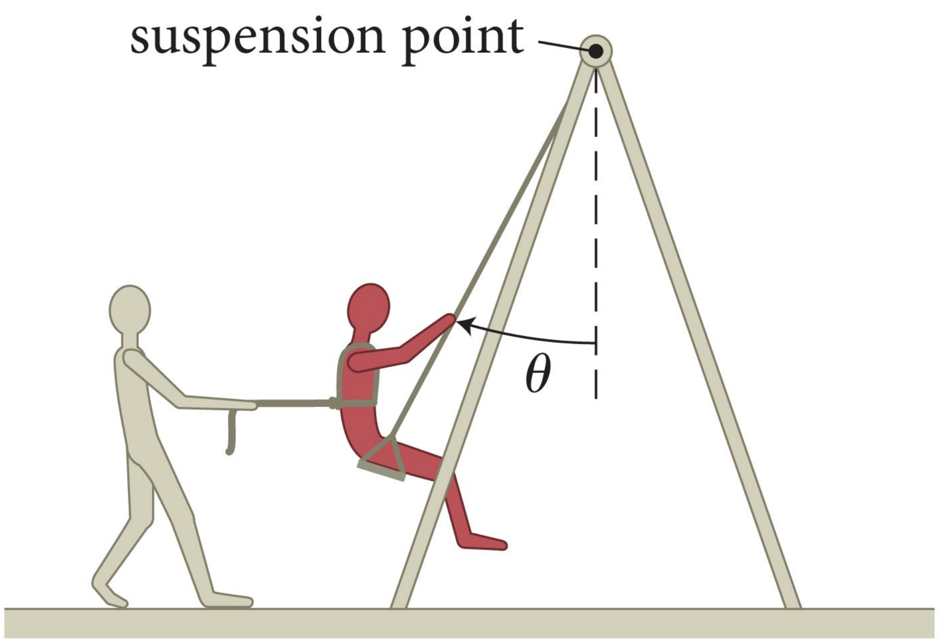
- The brick problem suggests which axes we should choose in a given problem:
  - **If possible, choose a coordinate system such that one of the axes lies along the direction of the acceleration of the object under consideration.**

- **Will it make a difference in the final answer?**
  - **No, but it can make things easier**

Section 10.3: Decomposition of forces

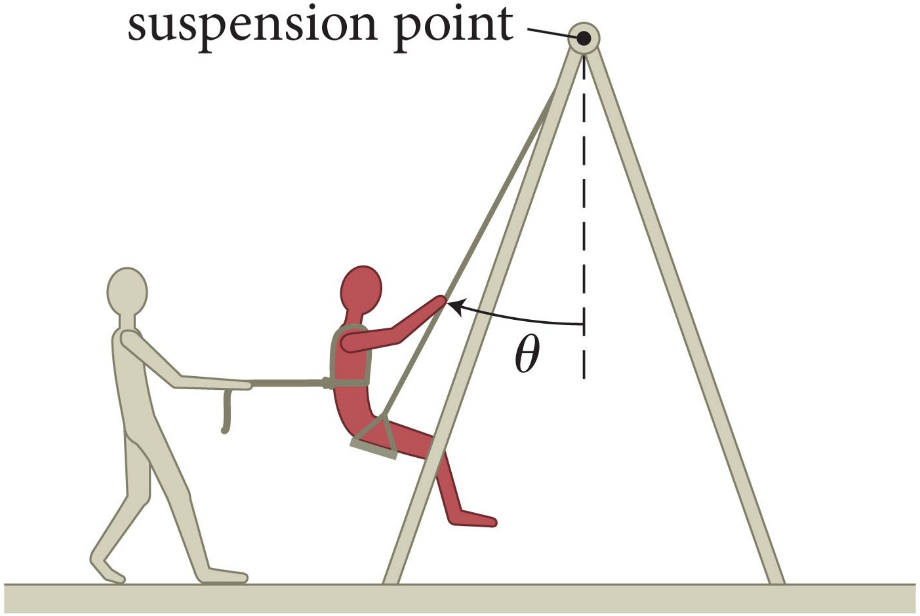
Example 10.2 Pulling a friend on a swing

Using a rope, you pull a friend sitting on a swing (Figure 10.16). (*a*) As you increase the angle θ, does the magnitude of the force $\vec{F}^{c}_{rp}$ required to hold your friend in place increase or decrease?



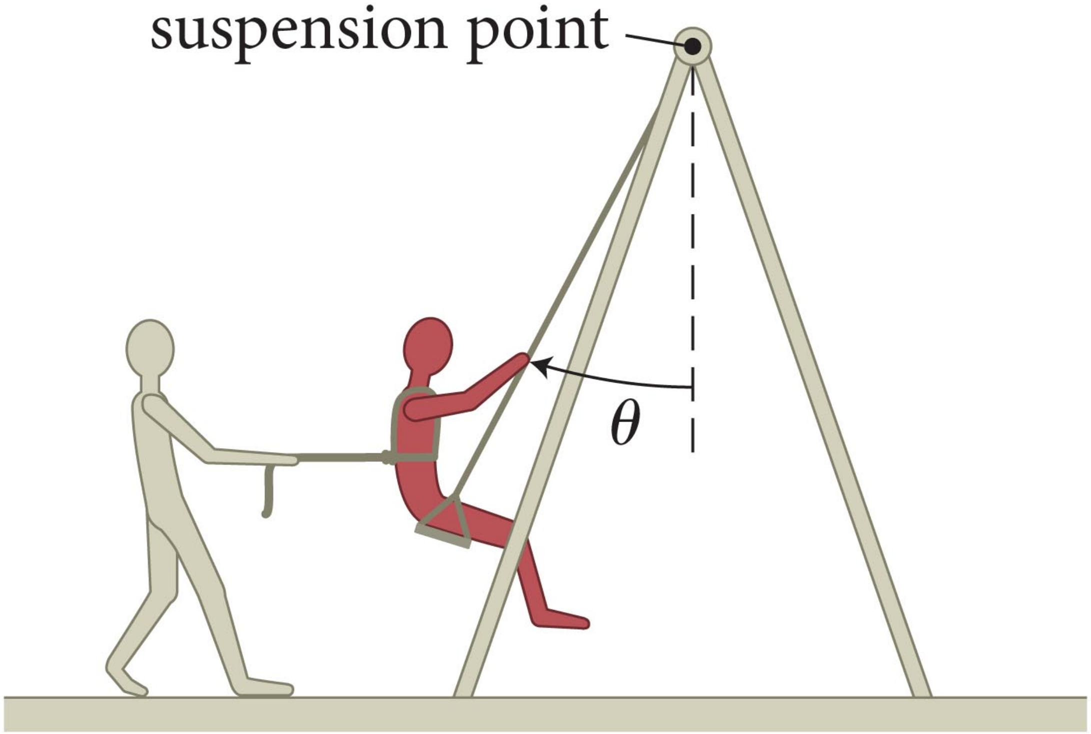
Example 10.2 Pulling a friend on a swing (cont.)

(*b*) Is the magnitude of that force larger than, equal to, or smaller than the magnitude of the gravitational force $\vec{F}^G_{\text{Ep}}$ exerted by Earth on your friend? (Consider the situation for both small and large values of θ.)



Example 10.2 Pulling a friend on a swing (cont.)

(*c*) Is the magnitude of the force $\vec{F}^{c}_{sp}$ exerted by the swing on your friend larger than, equal to, or smaller than F^{G}_{Ep}?



Example 10.2 Pulling a friend on a swing (cont.)

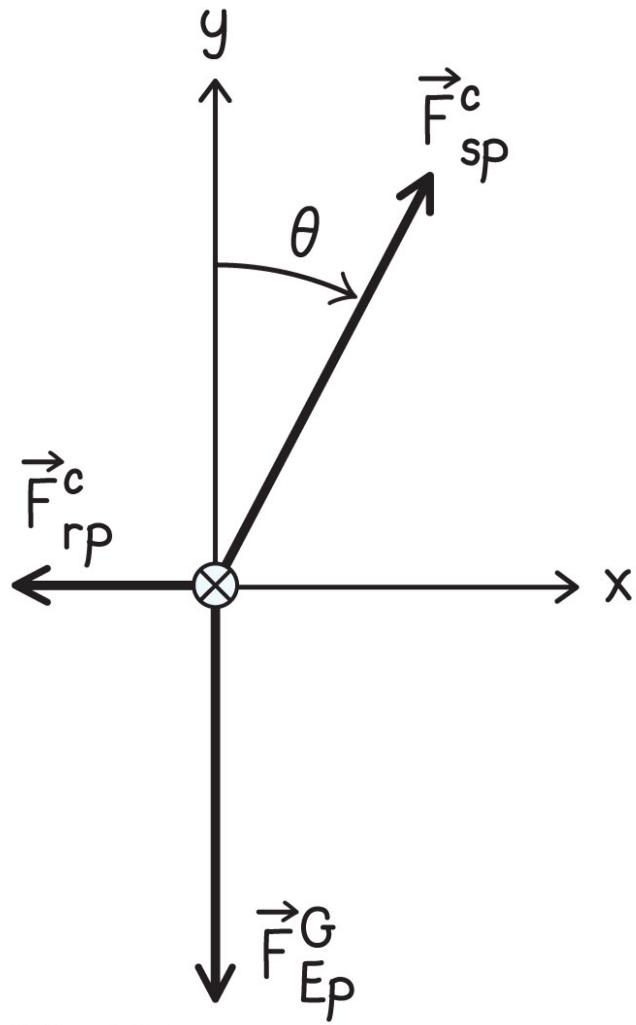
(a) As you increase the angle θ, does the magnitude of the force $\vec{F}^{c}_{rp}$ required to hold your friend in place increase or decrease?

(b) Is the magnitude of that force larger than, equal to, or smaller than the magnitude of the gravitational force $\vec{F}^{G}_{Ep}$ exerted by Earth on your friend?

(c) Is the magnitude of the force $\vec{F}^{c}_{sp}$ exerted by the swing on your friend larger than, equal to, or smaller than F^{G}_{Ep}?

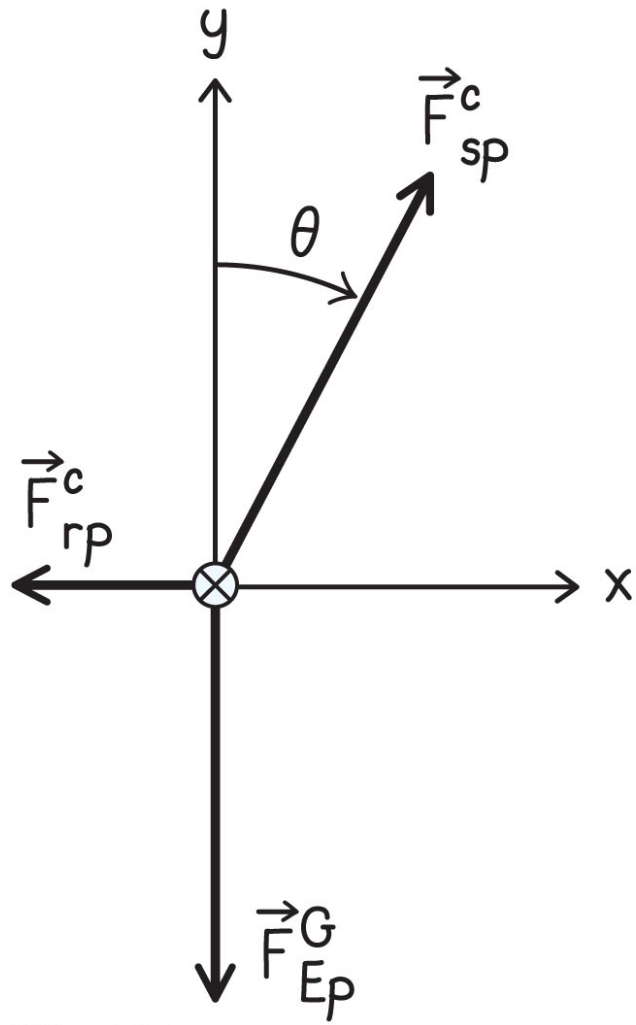
Example 10.2 Pulling a friend on a swing (cont.)

❶ GETTING STARTED I begin by drawing a free-body diagram of your friend (Figure 10.17). Three forces are exerted on him: $\vec{F}^{G}_{Ep}$, the force of gravity directed vertically downward, the horizontal force $\vec{F}^{c}_{rp}$ exerted by the rope, and a force $\vec{F}^{c}_{sp}$ exerted by the swing seat.



Example 10.2 Pulling a friend on a swing (cont.)

❶ GETTING STARTED This latter force $\vec{F}^{c}_{sp}$ is exerted by the suspension point via the chains of the swing and is thus directed along the chains. I therefore choose a horizontal *x* axis and a vertical *y* axis, so that two of the three forces lie along axes. Because your friend's acceleration is zero, the vector sum of the forces must be zero.

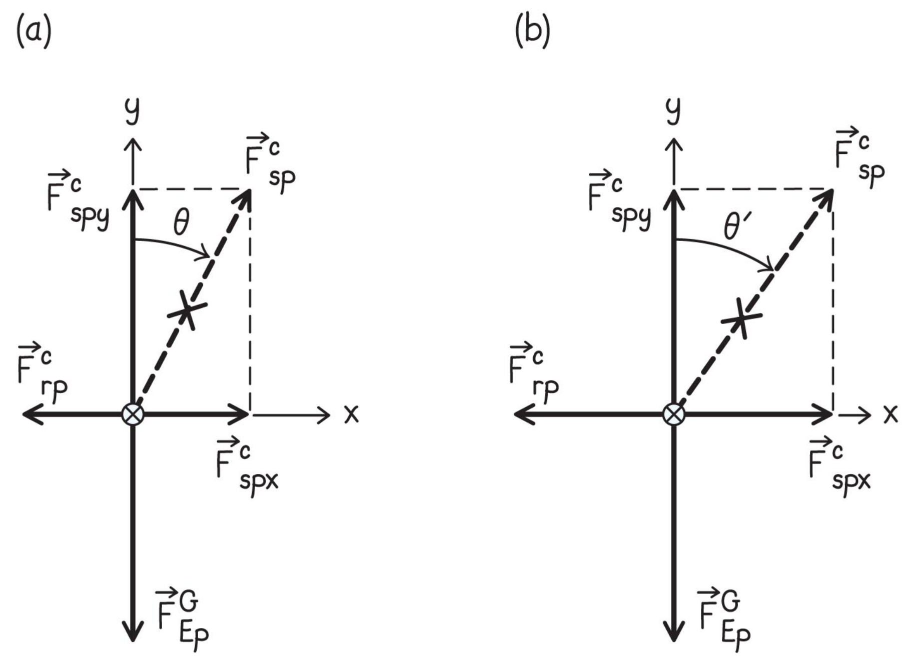


Example 10.2 Pulling a friend on a swing (cont.)

❷ DEVISE PLAN Because your friend is at rest, the vectors along the two axes must add up to zero. The best way to see how the magnitude of the force $\vec{F}^{c}_{rp}$ exerted by the rope must change as θ is increased is to draw free-body diagrams showing different values of θ. To answer parts *b* and *c*, I can compare the various forces in my free-body diagrams.

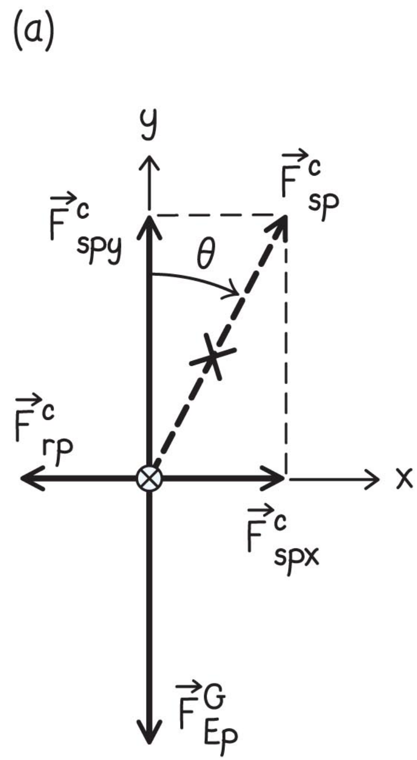
Example 10.2 Pulling a friend on a swing (cont.)

❸ EXECUTE PLAN (*a*) I begin by decomposing $\vec{F}^{c}_{sp}$ into x and y components (Figure 10.18*a*). Because the forces must add to zero along both axes, I conclude from my diagram that $\vec{F}^{c}_{sp\,y}$ must be equal in magnitude to $\vec{F}^{G}_{Ep}$, the downward force of gravity. Likewise, $\vec{F}^{c}_{sp\,x}$ must be equal in magnitude to $\vec{F}^{c}_{rp}$, the horizontal force the rope exerts on your friend.



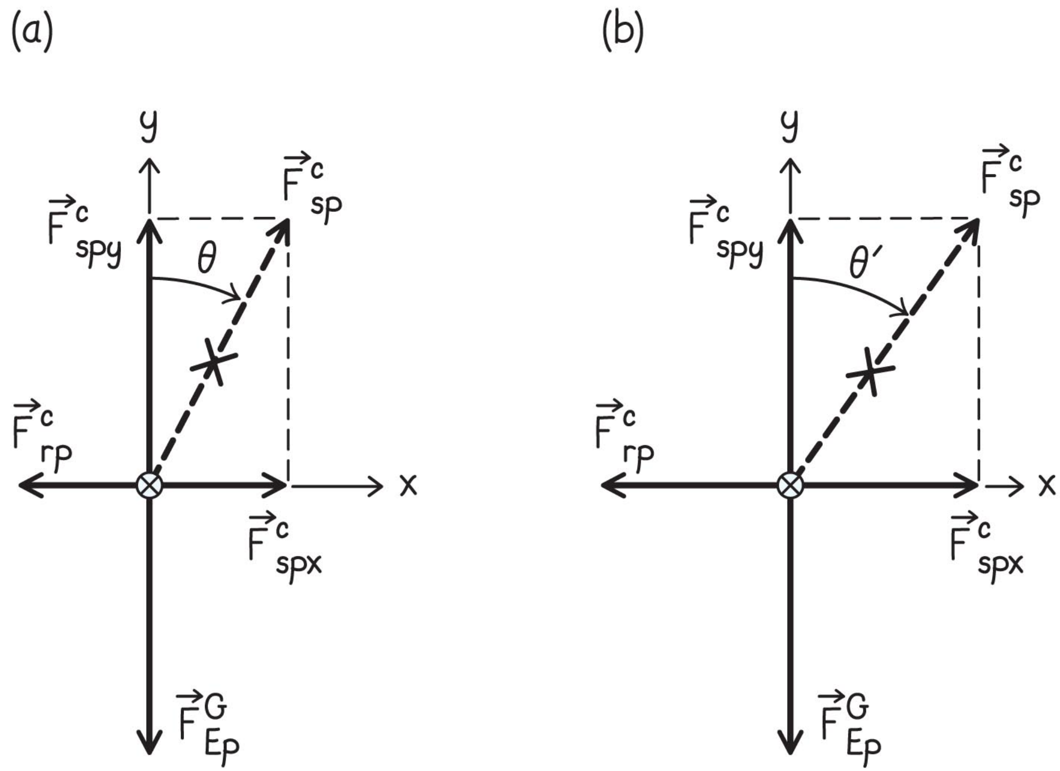
Example 10.2 Pulling a friend on a swing (cont.)

❸ EXECUTE PLAN Next, I draw a second free-body diagram for a larger angle θ (Figure 10.18b). As θ increases, $\vec{F}^{c}_{\text{sp}\,y}$ must remain equal in magnitude to $\vec{F}^{G}_{\text{Ep}}$ (otherwise your friend would accelerate vertically). As Figure 10.18b shows, increasing θ while keeping $\vec{F}^{c}_{\text{sp}\,y}$ constant requires the magnitude of $\vec{F}^{c}_{\text{sp}\,x}$ to increase.



Example 10.2 Pulling a friend on a swing (cont.)

❸ EXECUTE PLAN Because your friend is at rest, the forces in the horizontal direction must add up to zero and so $F^{c}_{rp} = \left|F^{c}_{sp\,x}\right|$. So if the magnitude of $\vec{F}^{c}_{sp\,x}$ increases, the magnitude of $\vec{F}^{c}_{rp}$ must increase, too. ✔



Example 10.2 Pulling a friend on a swing (cont.)

from the figure: $\tan\theta = \dfrac{|F^c_{sp,x}|}{|F^c_{sp,y}|}$

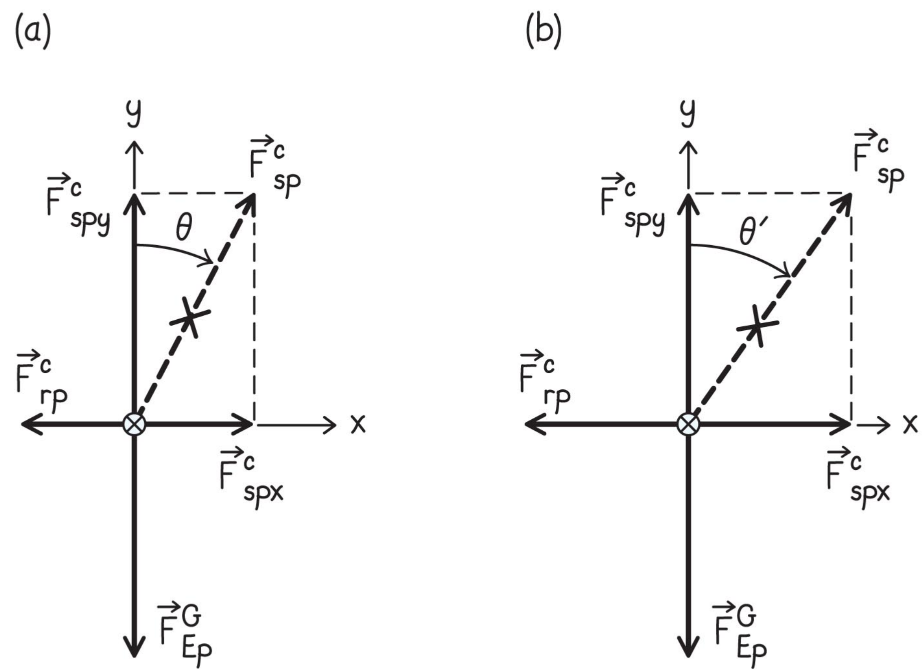
for $\theta < 45°$, $\tan\theta < 1$, so $|F^c_{sp,x}| < |F^c_{sp,y}|$

because $|F^c_{sp,y}| = F^G_{Ep}$ and $|F^c_{sp,x}| = F^c_{rp}$

that means for $\theta < 45°$ $F^c_{rp} < F^G_{Ep}$

when $\theta > 45°$, $\tan\theta > 1$ and thus

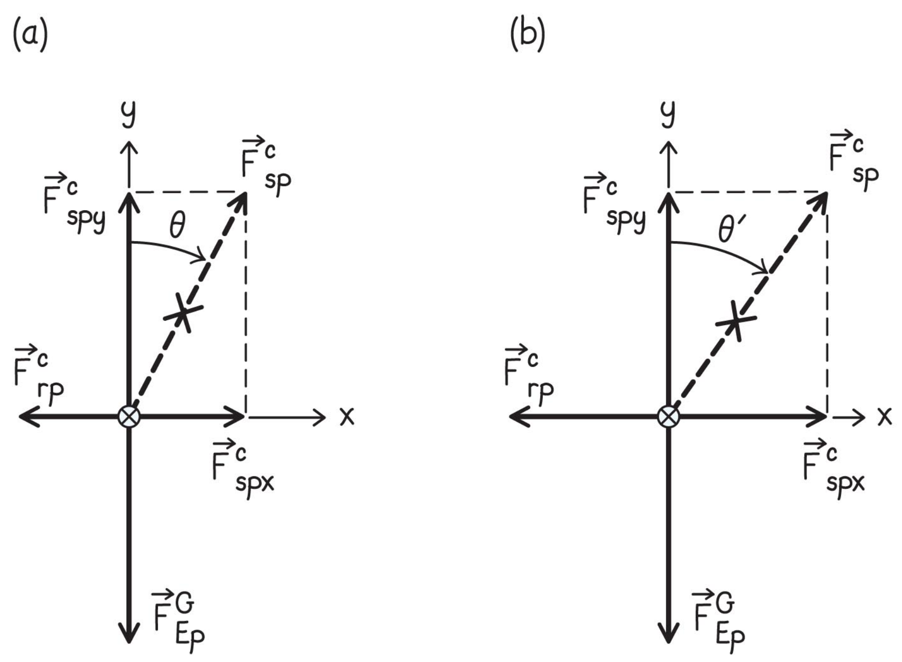
$|F^c_{sp,x}| > |F^c_{sp,y}|$ and $|F^c_{rp}| > |F^G_{Ep}|$



Example 10.2 Pulling a friend on a swing (cont.)

❸ EXECUTE PLAN (*c*) $\left|\vec{F}^{c}_{\text{sp}\,y}\right| = F^{G}_{\text{Ep}}$ and $F^{c}_{\text{sp}} = \sqrt{\left(F^{c}_{\text{sp}\,x}\right)^2 + \left(F^{c}_{\text{sp}\,y}\right)^2}$.

Therefore, F^{c}_{sp} must always be larger than F^{G}_{Ep} when $\theta \neq 0$. ✔



Since the y component of the force of the seat already equals the force of gravity, the *total* force of the seat must be always greater

Example 10.2 Pulling a friend on a swing (cont.)

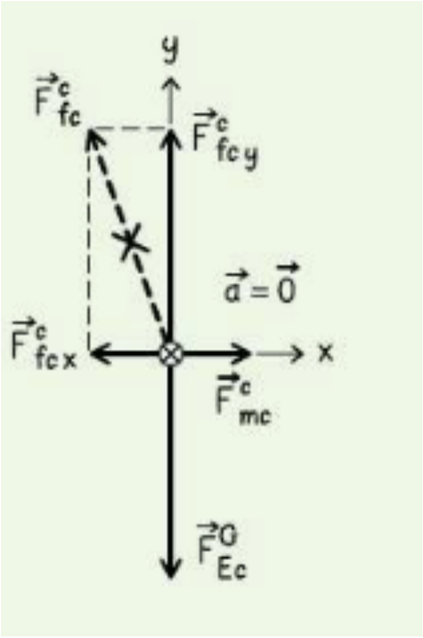
❹ EVALUATE RESULT I know from experience that you have to pull harder to move a swing farther from its equilibrium position, and so my answer to part *a* makes sense. With regard to part *b*, when the swing is at rest at 45°, the forces $\vec{F}^{c}_{rp}$ and $\vec{F}^{G}_{Ep}$ on your friend make the same angle with the force $\vec{F}^{c}_{sp}$, and so $\vec{F}^{c}_{rp}$ and $\vec{F}^{G}_{Ep}$ should be equal in magnitude.

Example 10.2 Pulling a friend on a swing (cont.)

❹ EVALUATE RESULT The force of gravity is independent of the angle, but the force exerted by the rope increases with increasing angle, and so it makes sense that for angles larger than 45°, $\vec{F}^{c}_{rp}$ is larger than $\vec{F}^{G}_{Ep}$. In part *c*, because the vertical component of the force $\vec{F}^{c}_{sp}$ exerted by the seat on your friend always has to be equal to the force of gravity, adding a horizontal component makes $\vec{F}^{c}_{sp}$ larger than $\vec{F}^{G}_{Ep}$, as I found.

Checkpoint 10.4

10.4 You decide to move a heavy file cabinet by sliding it across the floor. You push against the cabinet, but it doesn't budge. Draw a free-body diagram for it.



You apply a horizontal force, but it still doesn't move. That means the contact force must have a horizontal component as well as a vertical component!

Vertical = weight

Horizontal = friction

Section 10.4: Friction

Section Goals

You will learn to

- Recognize that when the contact forces acting between two interacting surfaces are decomposed, the tangential component is the **force of friction**. The normal component is called the **normal force**.
- Classify the friction force present when the surfaces are not moving relative to each other as **static friction**, and when they move relative to each other as **kinetic friction**.

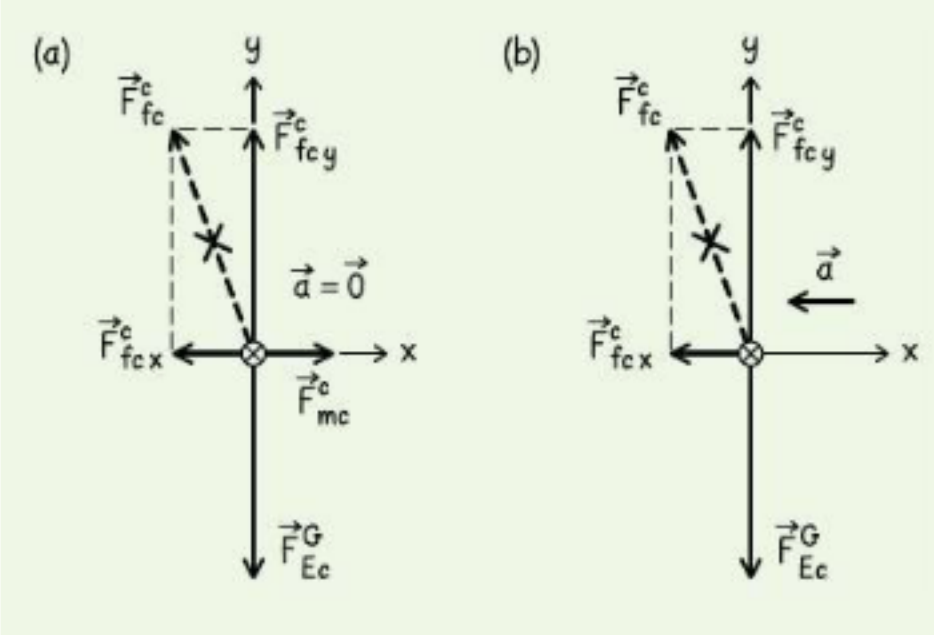
Checkpoint 10.5

- Consider pushing a heavy file cabinet across a floor.
- The tangential component of the contact force exerted by the floor on the cabinet has to do with friction.

10.5 (*a*) Suppose you push the file cabinet just enough to keep it moving at constant speed. Draw a free-body diagram for the cabinet while it slides at constant speed. (*b*) Suddenly you stop pushing. Draw a free-body diagram for the file cabinet at this instant.

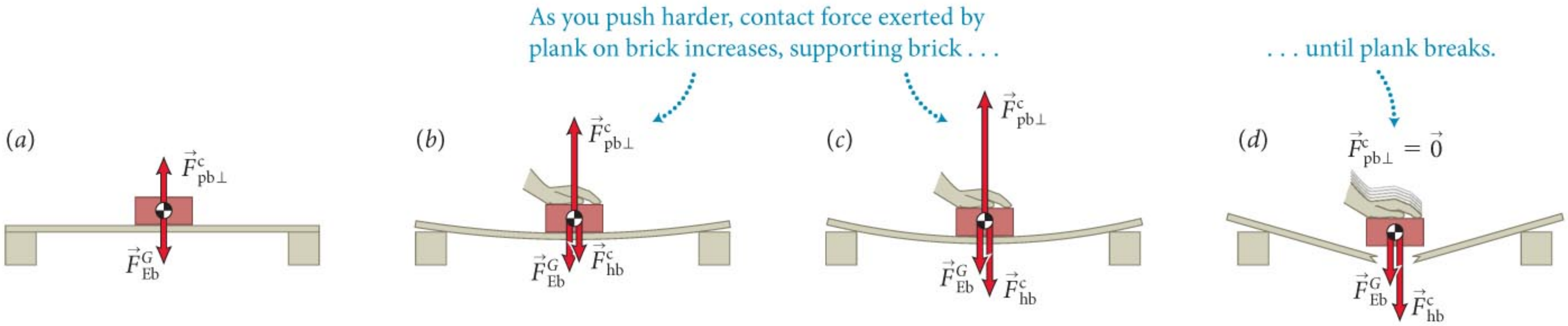
slides at constant speed while pushing

stopped pushing (but still in motion)



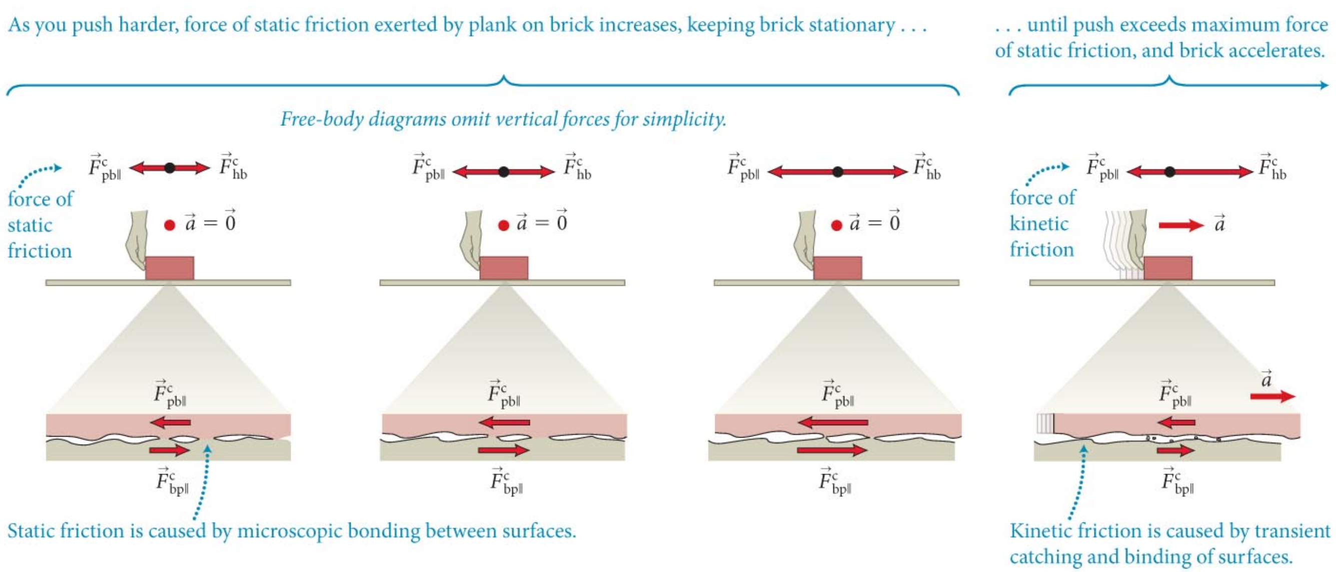
Section 10.4: Friction

- Consider the contact force exerted by the floor on the cabinet:
  - The normal component is called the **normal force** and the tangential component is the **force of friction**.
  - To understand the difference between normal and frictional forces, consider the brick on a horizontal wooden plank.
  - As seen in the figure, the normal force takes on whatever value to balance the net downward force, up to the breaking point.



Section 10.4: Friction

- Now consider gently pushing the brick to the right, as shown.
  - The horizontal frictional force is caused by microscopic bonds between the surfaces in contact.
  - As you push the brick, the net effect of these microscopic forces is to hold the brick in place.
  - As you increase your push force, this tangential component of the contact force grows.



Section 10.4: Friction

- The friction exerted by the surfaces that are not moving relative to each other is called **static friction**.
- When the horizontal push force exceeds the maximum force of static friction, the brick will accelerate.
- The friction force exerted by the surfaces when they move relative to each other is call **kinetic friction**, which is caused by transient microscopic bonds between the two surfaces.

Section 10.4
Question 3

You push horizontally on a crate at rest on the floor, gently at first and then with increasing force until you cannot push harder. The crate does not move. What happens to the force of static friction between crate and floor during this process?

1. It is always zero.
2. It remains constant in magnitude.
3. It increases in magnitude.
4. It decreases in magnitude.
5. More information is needed to decide.

Section 10.4
Question 3

You push horizontally on a crate at rest on the floor, gently at first and then with increasing force until you cannot push harder. The crate does not move. What happens to the force of static friction between crate and floor during this process?

1. It is always zero.
2. It remains constant in magnitude.
3. ✔ It increases in magnitude.
4. It decreases in magnitude.
5. More information is needed to decide.

Section 10.4: Friction

- The main differences between the normal force and the force of static friction are
  - The maximum value of the force of static friction is generally much smaller than the maximum value of the normal force.
  - Once the maximum value of the normal force is reached, the normal force disappears, but once the maximum value of the force of static friction is reached, there still is a smaller but nonzero force of kinetic friction.

Section 10.10
Question 9

In a panic situation, many drivers make the mistake of locking their brakes and skidding to a stop rather than applying the brakes gently. A skidding car often takes longer to stop. Why?

1. This is a misconception; it doesn't matter how the brakes are applied.
2. The coefficient of static friction is larger than the coefficient of kinetic friction.
3. The coefficient of kinetic friction is larger than the coefficient of static friction.

Question 9

In a panic situation, many drivers make the mistake of locking their brakes and skidding to a stop rather than applying the brakes gently. A skidding car often takes longer to stop. Why?

1. This is a misconception; it doesn't matter how the brakes are applied.
2. ✔ The coefficient of static friction is larger than the coefficient of kinetic friction. Skidding means you use kinetic friction, which provides less force!
3. The coefficient of kinetic friction is larger than the coefficient of static friction.

Section 10.5: Work and Friction

Section Goal

You will learn to

- Recognize that kinetic force causes energy dissipation and static friction causes no energy dissipation.

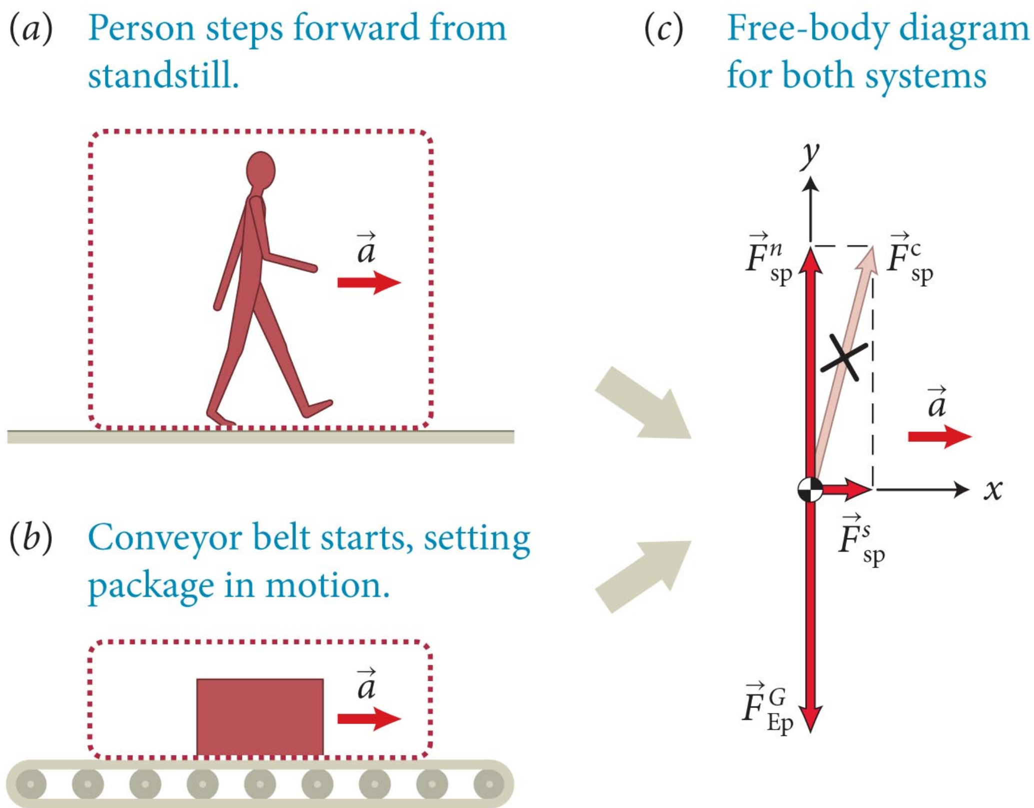
Section 10.5: Work and Friction

- Like normal force, the force of static friction is an **elastic force** that causes no irreversible change.
- The force of elastic friction causes no energy dissipation (no force displacement)

- Kinetic friction does cause irreversible change, including causing microscopic damage to the surfaces.
- The force of kinetic friction is not an elastic force and so causes energy dissipation.

Section 10.5: Work and Friction

- Consider the two cases shown in the figure.
- Both of these cases are examples where the object is accelerated by static friction.



Chapter 10: Self-Quiz #1

In the diagram below, the velocity of an object is given along with the vector representing a force exerted on the object. For each case, determine whether the object's speed increases, decreases, or remains constant. Also determine whether the object's direction changes in the clockwise direction, changes in the counterclockwise direction, or doesn't change.

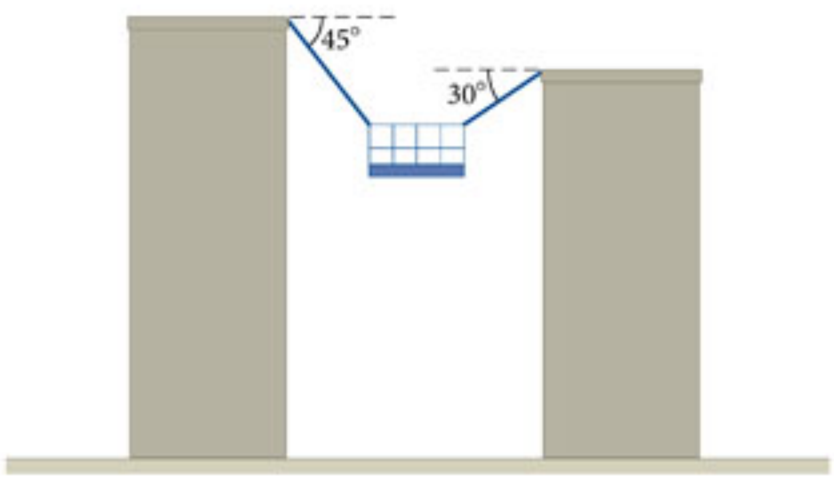
Chapter 10: Self-Quiz #1

In the diagram below, the velocity of an object is given along with the vector representing a force exerted on the object. For each case, determine whether the object's speed increases, decreases, or remains constant. Also determine whether the object's direction changes in the clockwise direction, changes in the counterclockwise direction, or doesn't change.

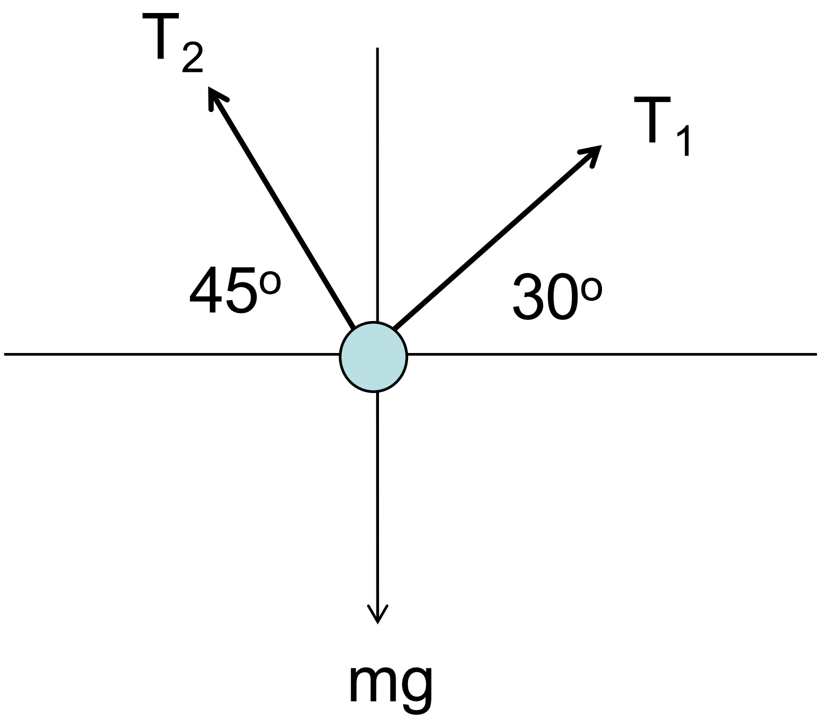
(a)	(b)	(c)	(d)
increases turns CW	decreases turns CCW	constant turns CW	decreases no turning

10.30

- Tension in the right cable is 790N
- What is the tension in the left cable?
- What is the inertia of the platform?

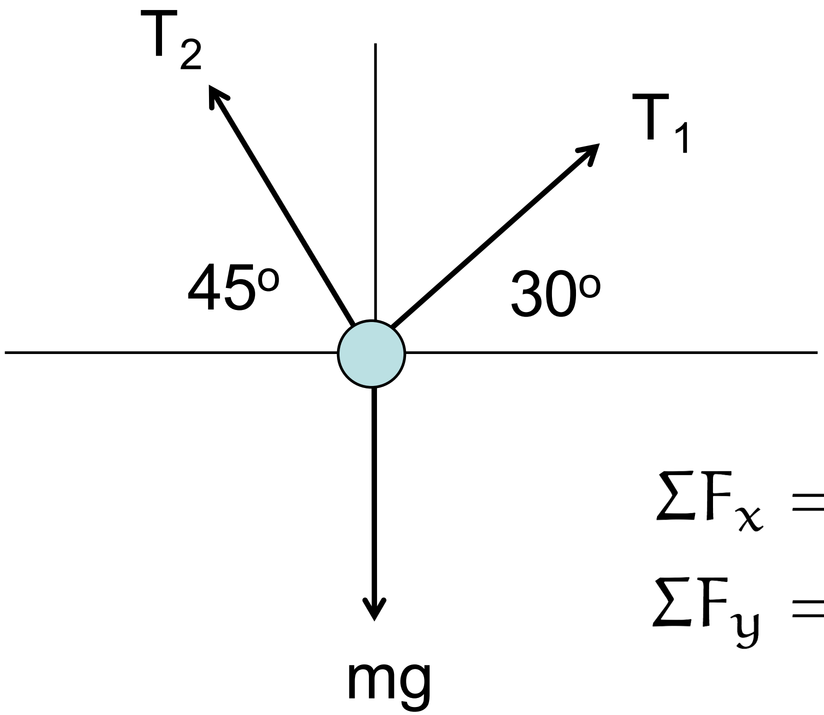


Free body diagram



Static situation

- Forces sum to zero as vectors
- Sum of components along each axis also zero!

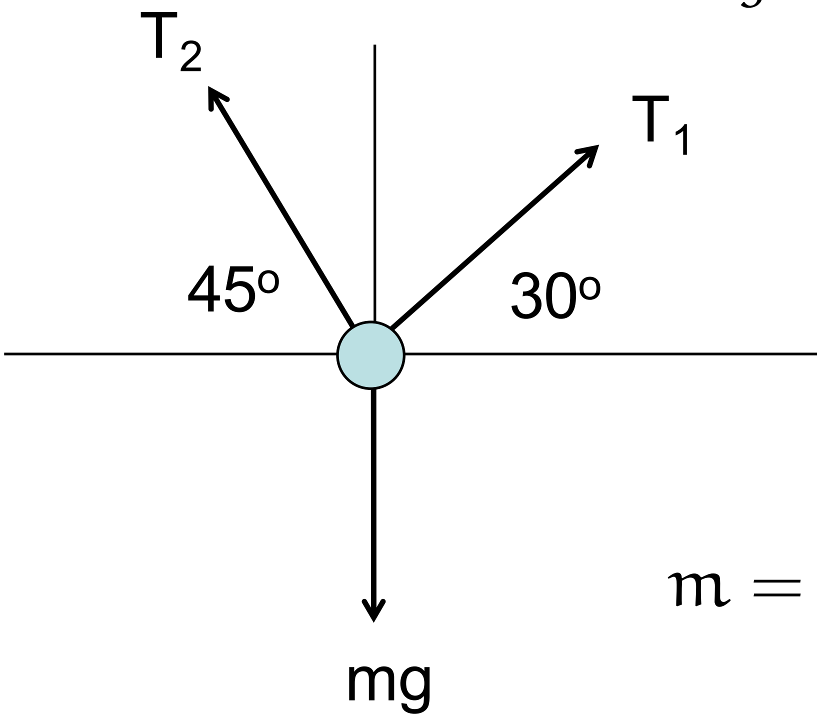


$$\Sigma F_x = T_1 \cos 30^\circ - T_2 \cos 45^\circ = 0$$

$$\Sigma F_y = T_1 \sin 30^\circ + T_2 \sin 45^\circ - mg = 0$$

Static situation

- Know $T_1 = 790\text{N}$



$$\Sigma F_x = T_1 \cos 30^\circ - T_2 \cos 45^\circ = 0$$

$$\Sigma F_y = T_1 \sin 30^\circ + T_2 \sin 45^\circ - mg = 0$$

$$T_2 = T_1 \frac{\cos 30}{\cos 45} \approx 970\,\text{N}$$

$$m = \frac{1}{g}\left(T_1 \sin 30 + T_2 \sin 45\right) \approx 110\,\text{kg}$$

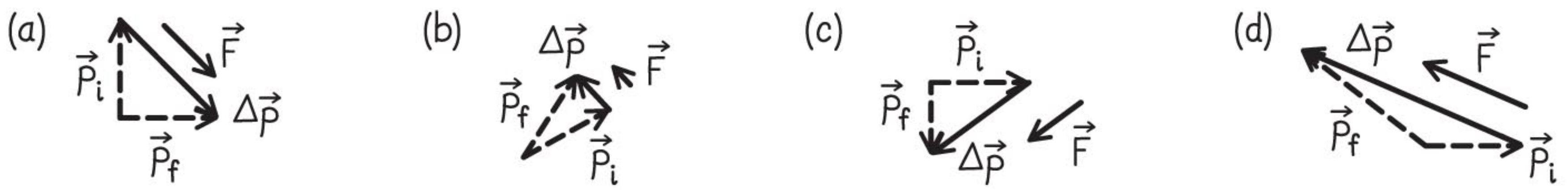
Chapter 10: Self-Quiz #3

Each diagram in the figure below indicates the momentum of an object before and after a force is exerted on it. For each case determine the direction of the force.

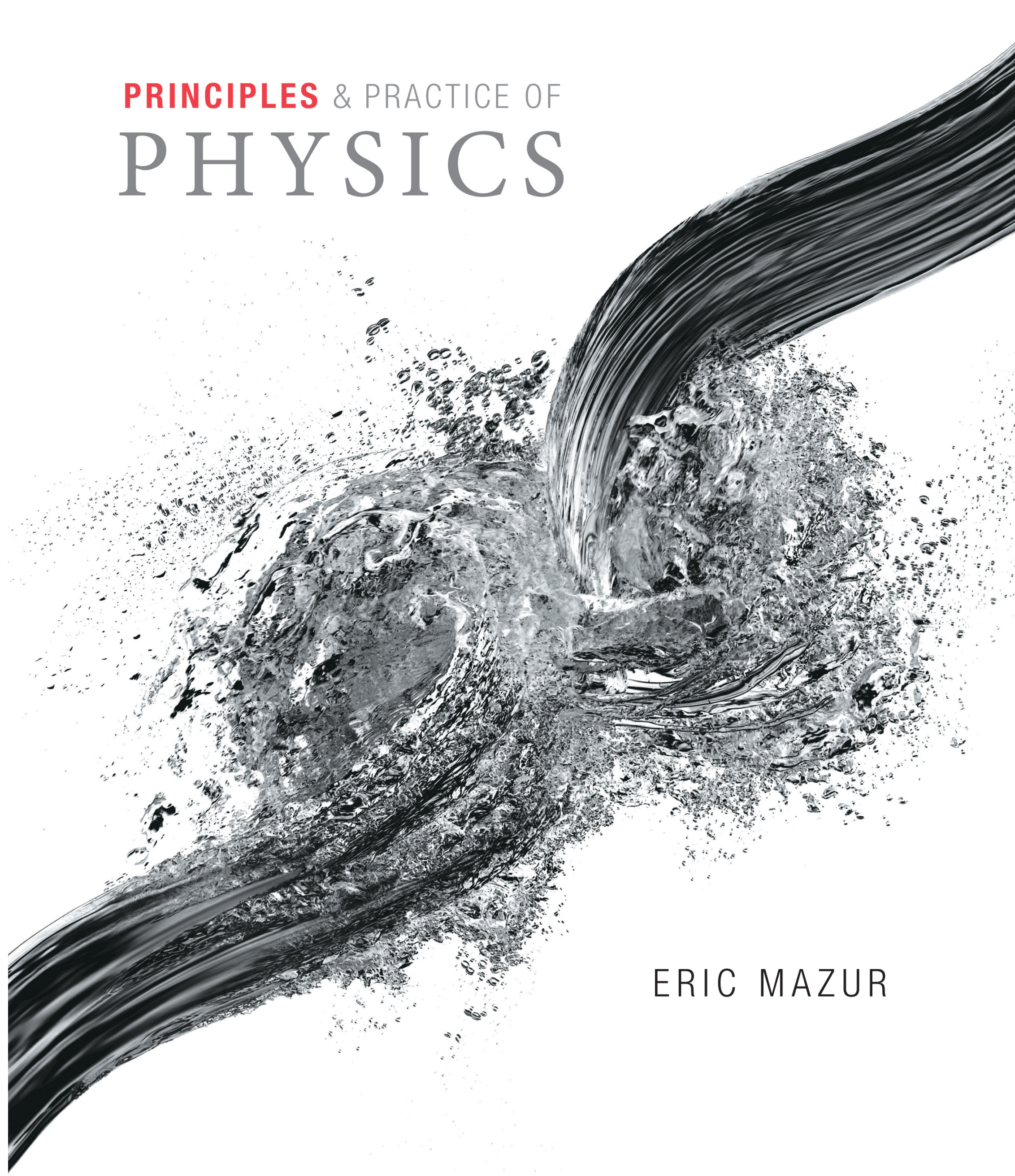
Chapter 10: Self-Quiz #3

Answer

In each case the force is parallel to $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$ (Figure 10.26).



Motion in a plane



Quantitative Tools

Section 10.6: Vector algebra

Section Goals

You will learn to

- Represent vectors using polar coordinates and rectangular coordinates.
- Express rectangular coordinates in terms of polar coordinates and vice versa using trigonometry.
- Determine the sum of vectors using their components.

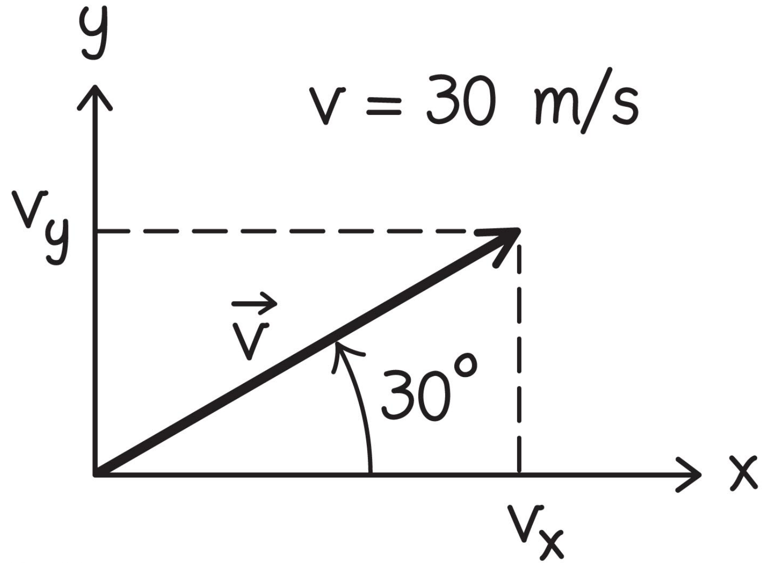
Section 10.6: Vector algebra

Example 10.4 Speeding ball

A ball is thrown at an angle of 30° to the horizontal at a speed of 30 m/s. Write the ball's velocity in terms of rectangular unit vectors.

Example 10.4 Speeding ball (cont.)

❶ GETTING STARTED I begin by making a sketch showing the velocity vector $\vec{v}$ and its decomposition in a rectangular coordinate system (Figure 10.30). I position the x axis along the horizontal in the direction of motion and the y axis along the vertical. I label the x and y components of the ball's velocity v_x and v_y, respectively.



Example 10.4 Speeding ball (cont.)

❷ DEVISE PLAN Equation 10.5 tells me that the velocity vector can be written as $\vec{v} = v_x \hat{i} + v_y \hat{j}$. To determine the x and y components, I apply trigonometry to the triangle made up by v, v_x, and v_y.

Example 10.4 Speeding ball (cont.)

❸ EXECUTE PLAN From my sketch I see that $\cos\theta = \upsilon_x/\upsilon$ and $\sin\theta = \upsilon_y/\upsilon$, where $\upsilon = 30$ m/s and $\theta = 30^\circ$. Therefore $\upsilon_x = \upsilon\cos\theta$ and $\upsilon_y = \upsilon\sin\theta$. Substituting the values given for υ and θ, I calculate the rectangular components:

$$\upsilon_x = (30\text{ m/s})(\cos 30^\circ) = (30\text{ m/s})(0.87) = +26\text{ m/s}$$

$$\upsilon_y = (30\text{ m/s})(\sin 30^\circ) = (30\text{ m/s})(0.50) = +15\text{ m/s}.$$

The velocity in terms of unit vectors is thus

$$\vec{\upsilon} = \upsilon_x\hat{\imath} + \upsilon_y\hat{\jmath} = (+26\text{ m/s})\hat{\imath} + (+15\text{ m/s})\hat{\jmath}.$$

Example 10.4 Speeding ball (cont.)

❹ EVALUATE RESULT The x and y components are both positive, as I expect because I chose the direction of the axis in the direction that the ball is moving, and the magnitude of υ_x is larger than that of υ_y, as it should be for a launch angle that is smaller than 45°. I can quickly check my math by using Eq. 10.6 to calculate the magnitude of the velocity:

$$\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2} = \sqrt{(26\ \text{m/s})^2 + (15\ \text{m/s})^2} = 30\ \text{m/s},$$

which is the speed at which the ball is launched.

Section 10.7: Projectile motion in two dimensions

Section Goals

You will learn to

- Infer that knowing how to decompose vectors allows us to separate motion in a plane to two one-dimensional problems.
- Apply the equations for constant acceleration motion we developed in Chapter 3 to projectile motion.

Section 10.7: Projectile motion in two dimensions

- The position vector of an object moving in two dimensions is

$$\vec{r} = x\hat{\imath} + y\hat{\jmath}$$

- The object's instantaneous velocity is

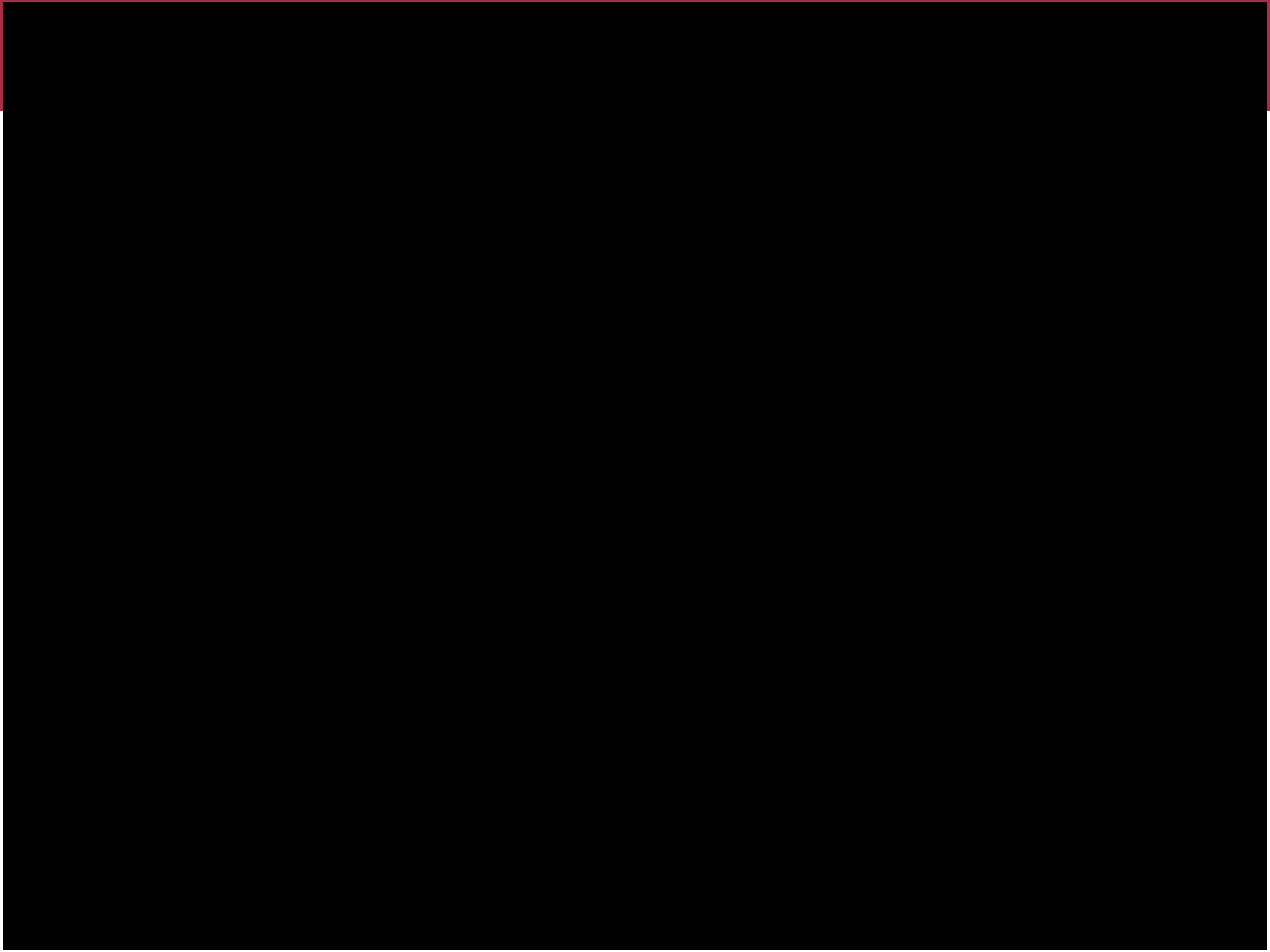
$$\vec{\upsilon} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\hat{\imath} + \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}\hat{\jmath} = \upsilon_x \hat{\imath} + \upsilon_y \hat{\jmath}$$

$$\upsilon_x = \frac{dx}{dt} \quad \text{and} \quad \upsilon_y = \frac{dy}{dt}$$

- Similarly, instantaneous acceleration components are

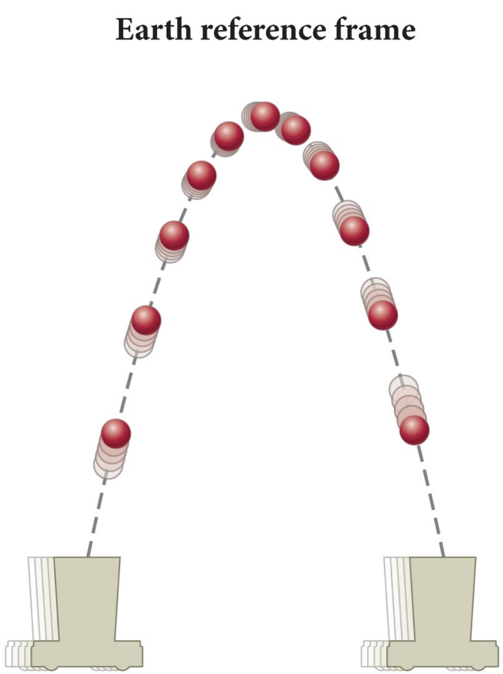
$$a_x = \frac{d\upsilon_x}{dt} = \frac{d^2x}{dt^2} \quad \text{and} \quad a_y = \frac{d\upsilon_y}{dt} = \frac{d^2y}{dt^2}$$

- Decomposing vectors into components allows us to separate motion in a plane into two one-dimensional problems.



Section 10.7: Projectile motion in two dimensions

- Consider the motion of a ball launched straight up from a cart moving at a constant velocity (see the figure on the previous slide).
- The resulting two-dimensional projectile motion in Earth's reference frame is the curved trajectory shown in the figure.
- The balls has a constant downward acceleration of $a_y = -g$.



Section 10.7: Projectile motion in two dimensions

- In Earth's reference frame, the ball's initial velocity is

$$\vec{\upsilon}_{\rm i} = \upsilon_{x,\rm i}\hat{i} + \upsilon_{y,\rm i}\hat{j}$$

- The ball's launch angle relative to the x axis is

$$\tan\theta = \frac{\upsilon_{y,\rm i}}{\upsilon_{x,\rm i}}$$

- Using $a_x = 0$ and $a_y = -g$ in Equations 3.4 and 3.8, we get

$$\upsilon_{x,\rm f} = \upsilon_{x,\rm i} \quad \text{(constant velocity)}$$

$$\upsilon_{y,\rm f} = \upsilon_{y,\rm i} - g\Delta t \quad \text{(constant acceleration)}$$

$$x_{\rm f} = x_{\rm i} + \upsilon_{x,\rm i}\,\Delta t \quad \text{(constant velocity)}$$

$$y_{\rm f} = y_{\rm i} + \upsilon_{y,\rm i}\Delta t - \tfrac{1}{2}g(\Delta t)^2 \quad \text{(constant acceleration)}$$

Example 10.5 Position of highest point

The ball of Figure 10.32 is launched from the origin of an *xy* coordinate system. Write expressions giving, at the top of its trajectory, the ball's rectangular coordinates in terms of its initial speed v_i and the acceleration due to gravity g.

Section 10.7: Projectile motion in two dimensions

Example 10.5 Position of highest point (cont.)

❶ GETTING STARTED Because the ball is launched from the origin, $x_i = 0$ and $y_i = 0$. As the ball moves upward, the vertical component of its velocity, v_y, is positive. After crossing its highest position, the ball moves downward, and so now v_y is negative. As the ball passes through its highest position, therefore, v_y reverses sign, so at that position $v_y = 0$.

Section 10.7: Projectile motion in two dimensions

Example 10.5 Position of highest point (cont.)

❷ DEVISE PLAN Taking the highest position as my final position and then substituting $v_{y,\mathrm{f}} = 0$ into Eq. 10.18, I can determine the time interval Δt_{top} it takes the ball to travel to this position. Once I know Δt_{top}, I can use Eqs. 10.19 and 10.20 to obtain the ball's coordinates at the top.

Example 10.5 Position of highest point (cont.)

❸ EXECUTE PLAN Substituting $v_{y,\mathrm{f}} = 0$ into Eq. 10.18, I get

$$0 = v_{y,\mathrm{i}} - g\Delta t_{\mathrm{top}}.$$

Solving for Δt_{top} then yields

$$\Delta t_{\mathrm{top}} = \frac{v_{y,\mathrm{i}}}{g}. \qquad (1)$$

Example 10.5 Position of highest point (cont.)

❸ EXECUTE PLAN Using $x_i = 0$, $y_i = 0$ in Eqs. 10.19 and 10.20, I then calculate the location of the highest position:

$$x_{top} = 0 + \upsilon_{x,i}\left(\frac{\upsilon_{y,i}}{g}\right) = \frac{\upsilon_{x,i}\upsilon_{y,i}}{g}$$

and

$$y_{top} = 0 + \upsilon_{y,i}\left(\frac{\upsilon_{y,i}}{g}\right) - \tfrac{1}{2}g\left(\frac{\upsilon_{y,i}}{g}\right)^2$$

$$= \frac{\upsilon_{y,i}^2}{g} - \tfrac{1}{2}\frac{\upsilon_{y,i}^2}{g} = \tfrac{1}{2}\frac{\upsilon_{y,i}^2}{g}. \checkmark \quad \text{or} \quad x_{top} = \frac{v_i^2 \sin^2\theta}{2g}$$

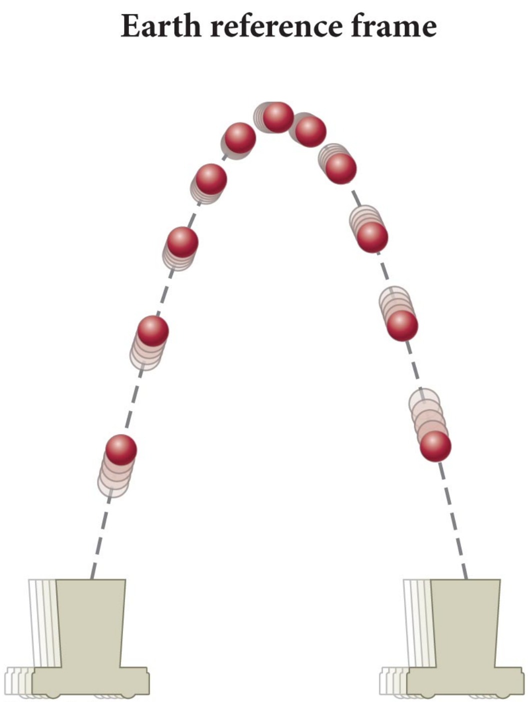
Example 10.5 Position of highest point (cont.)

❹ EVALUATE RESULT Because the ball moves at constant velocity in the horizontal direction, the x coordinate of the highest position is the horizontal velocity component $v_{x,\mathrm{i}}$ multiplied by the time interval it takes to reach the top (given by Eq. 1). The y coordinate of the highest position does not depend on $v_{x,\mathrm{i}}$ as I expect because the vertical and horizontal motions are independent of each other.

Section 10.7: Projectile motion in two dimensions

Example 10.6 Range of projectile

How far from the launch position is the position at which the ball of Figure 10.32 is once again back in the cart? (This distance is called the *horizontal range* of the projectile.)



Section 10.7: Projectile motion in two dimensions

Example 10.6 Range of projectile (cont.)

❶ GETTING STARTED As we saw in Example 3.6, the time interval taken by a projectile to return to its launch position from the top of the trajectory is equal to the time interval Δt_{top} it takes to travel from the launch position to the top. I can therefore use the Δt_{top} expression I found in Example 10.5 to solve this problem.

Example 10.6 Range of projectile (cont.)

❷ DEVISE PLAN If the time interval it takes the ball to travel from its launch position to the top is Δt_{top}, then the time interval the ball is in the air is $2\Delta t_{top}$. Because the ball travels at constant velocity in the horizontal direction, I can use Eq. 10.19 to obtain the ball's range.

Example 10.6 Range of projectile (cont.)

❸ EXECUTE PLAN From Example 10.5, I have $\Delta t_{\text{top}} = \upsilon_{y,\text{i}}/g$, and so the time interval spent in the air is $\Delta t_{\text{flight}} = 2\upsilon_{y,\text{i}}/g$. Substituting this value into Eq. 10.19 yields

$$x_{\text{f}} = 0 + \upsilon_{x,\text{i}}\left(\frac{2\upsilon_{y,\text{i}}}{g}\right) = \frac{2\upsilon_{x,\text{i}}\upsilon_{y,\text{i}}}{g}. \checkmark$$

or $$x_{\text{f}} = \frac{2v_{\text{i}}\cos\theta v_{\text{i}}\sin\theta}{g} = \frac{v_{\text{i}}^2\sin 2\theta}{g}$$

Section 10.7: Projectile motion in two dimensions

Example 10.6 Range of projectile (cont.)

❹ EVALUATE RESULT Because the trajectory is an inverted parabola, the top of the parabola lies midway between the two locations where the parabola intercepts the horizontal axis. So the location at which the parabola returns to the horizontal axis lies a horizontal distance twice as far from the origin as the horizontal distance at the top. In Example 10.5 I found that $x_{\text{top}} = \upsilon_{x,\text{i}}\upsilon_{y,\text{i}}/g$. The answer I get for the horizontal range is indeed twice this value.

Section 10.7
Question 6

A baseball player hits a fly ball that has an initial velocity for which the horizontal component is 30 m/s and the vertical component is 40 m/s. What is the speed of the ball at the highest point of its flight?

1. $\sqrt{[(30\text{ m/s})^2 + 40\text{ m/s})^2]^{1/2}}$
2. Zero
3. 30 m/s
4. 40 m/s

Section 10.7
Question 6

A baseball player hits a fly ball that has an initial velocity for which the horizontal component is 30 m/s and the vertical component is 40 m/s. What is the speed of the ball at the highest point of its flight?

1. $\sqrt{[(30\ \text{m/s})^2 + 40\ \text{m/s})^2]^{1/2}}$
2. Zero
3. ✔ 30 m/s
4. 40 m/s

Section 10.7
Question 7

What is the shape of the path of an object launched at an angle to the vertical, assuming that only the force of gravity is exerted on the object?

1. A straight line
2. A harmonic curve
3. A parabola
4. None of the above

Section 10.7
Question 7

What is the shape of the path of an object launched at an angle to the vertical, assuming that only the force of gravity is exerted on the object?

1. A straight line
2. A harmonic curve
3. ✔ A parabola
4. None of the above

Section 10.7

- We know:

$$x_f = x_i + v_{x,i}\,\Delta t \quad \text{(constant velocity)}$$

$$y_f = y_i + v_{y,i}\Delta t - \tfrac{1}{2}g(\Delta t)^2 \quad \text{(constant acceleration)}$$

- These are parametric equations $x(t)$ and $y(t)$
- We should be able to eliminate t!
- Given $v_{x,i} = v_i \cos\theta$ and $v_{y,i} = v_i \sin\theta$ …

$$y(x) = (\tan\theta)\,x - \frac{gx^2}{2v_i^2\cos^2\theta}$$

- Parabolic path! Presumes launch from (0,0).

Section 10.8: Collisions and momentum in two dimensions

Section Goal

You will learn to

- Apply the conservation of momentum equations to collisions in two dimensions.

Section 10.8: Collisions and momentum in two dimensions

- As we saw in Chapter 5, momentum conservation states that the momentum of an isolated system of colliding objects does not change, or $\Delta\vec{p} = 0$.
- Momentum is a vector, so in two dimensions momentum change must be expressed in terms of the components.
- Conservation of momentum in two dimensions is given by

$$\Delta p_x = \Delta p_{1x} + \Delta p_{2x} = m_1(v_{1x,\mathrm{f}} - v_{1x,\mathrm{i}}) + m_2(v_{2x,\mathrm{f}} - v_{2x,\mathrm{i}}) = 0$$
$$\Delta p_y = \Delta p_{1y} + \Delta p_{2y} = m_1(v_{1y,\mathrm{f}} - v_{1y,\mathrm{i}}) + m_2(v_{2y,\mathrm{f}} - v_{2y,\mathrm{i}}) = 0$$

as with motion equations and forces:

perpendicular axes are treated *completely separately*

Section 10.9: Work as the product of two vectors

Section Goal

You will learn to

- Understand that the work done by gravity is independent of path.
- Define the scalar product of two vectors.
- Associate work done by nondissipative forces with the scalar product of the force vector and the displacement vector.

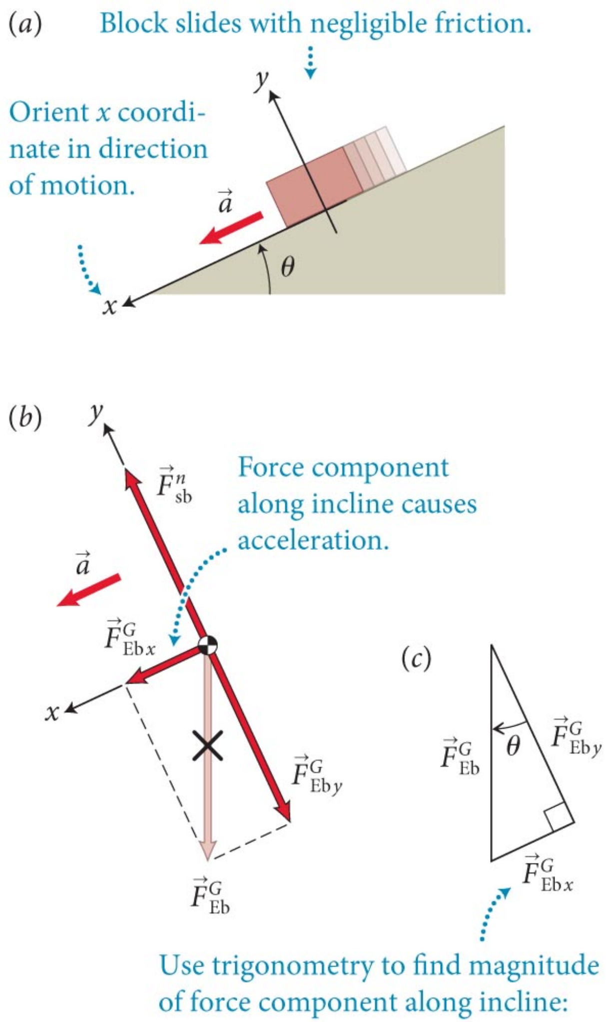
Section 10.9: Work as the product of two vectors

- Consider a block sliding down an incline as shown (ignore friction).
- The x component of the force of gravity is what causes the block to accelerate

$$F^G_{\text{Eb}x} = F^G_{\text{Eb}} \sin\theta = +mg\sin\theta$$

- The magnitude of the resulting acceleration is

$$a_x = \frac{\sum F_x}{m} = \frac{\sum^G_{\text{Eb}\,x}}{m} = \frac{+mg\sin\theta}{m} = +g\sin\theta$$

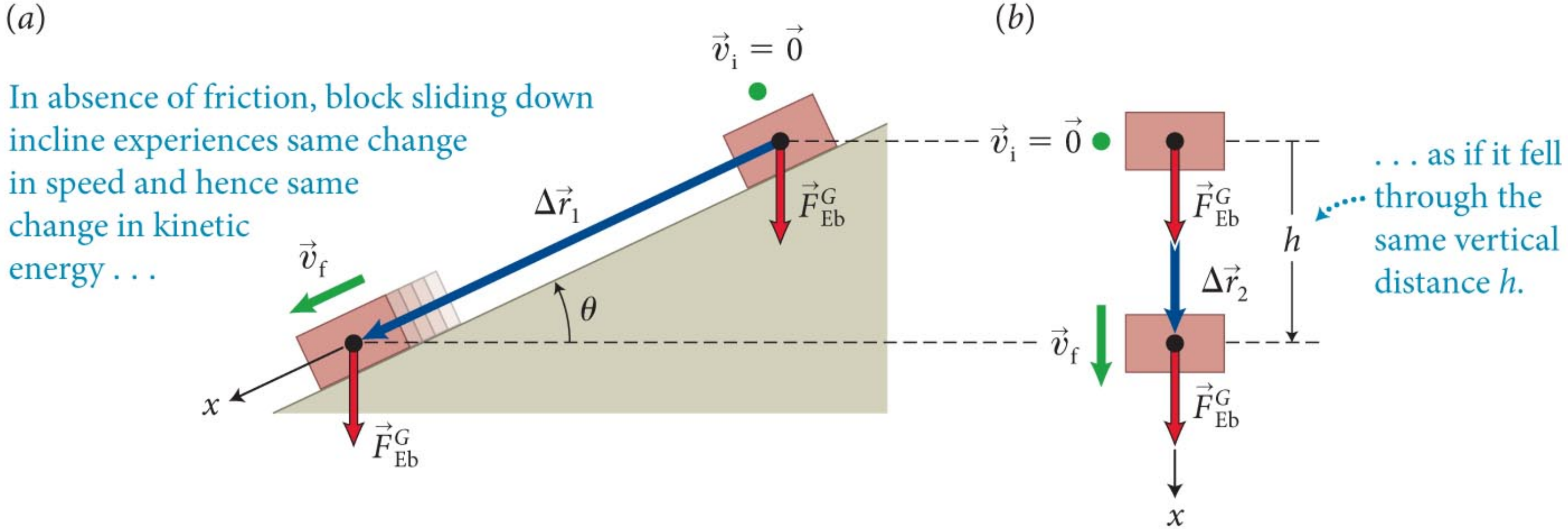


$$F^G_{\text{Eb}x} = F^G_{\text{Eb}} \sin\theta = +mg\sin\theta$$

Section 10.9: Work as the product of two vectors

- As illustrated in the figure, the block begins at rest ($v_{x,i} = 0$) at position $x_i = 0$ and drops a height h.
- The displacement **along the incline** is $\Delta x = +h/\sin\theta$.
- The time the block takes to cover this distance is obtained from Equation 3.7: $\Delta x = v_{x,i}\Delta t + \frac{1}{2}a_x(\Delta t)^2 = 0 + \frac{1}{2}g\sin\theta(\Delta t)^2$

from which we obtain $(\Delta t)^2 = \dfrac{2h}{g\sin^2\theta}$



Section 10.9: Work as the product of two vectors

- Using Equation 3.4 and Equation 10.25, the blocks final kinetic energy is then

$$\tfrac{1}{2}m\upsilon_{x,\mathrm{f}}^2 = \tfrac{1}{2}m(\upsilon_{x,\mathrm{i}} + a_x\Delta t^2) = \tfrac{1}{2}m(0 + g\sin\theta\ \Delta t)^2$$

$$= \tfrac{1}{2}mg^2\sin^2\theta(\Delta t)^2$$

- Substituting in for time:

$$\tfrac{1}{2}m\upsilon_{x,\mathrm{f}}^2 = \tfrac{1}{2}mg^2\sin^2\theta\left(\frac{2h}{g\sin^2\theta}\right) = mgh$$

- Because the blocks initial kinetic energy is zero, we get

$$W = \Delta K = K_\mathrm{f} - K_\mathrm{i} = \tfrac{1}{2}m\upsilon_{x,\mathrm{f}}^2 - 0 = mgh$$

- We can get the same result by using the work equation:

$$W = F_{\mathrm{Eb}\,x}^{G}\Delta x = (+mg\sin\theta)\left(+\frac{h}{\sin\theta}\right) = mgh$$

  - only portion of force parallel to displacement matters!

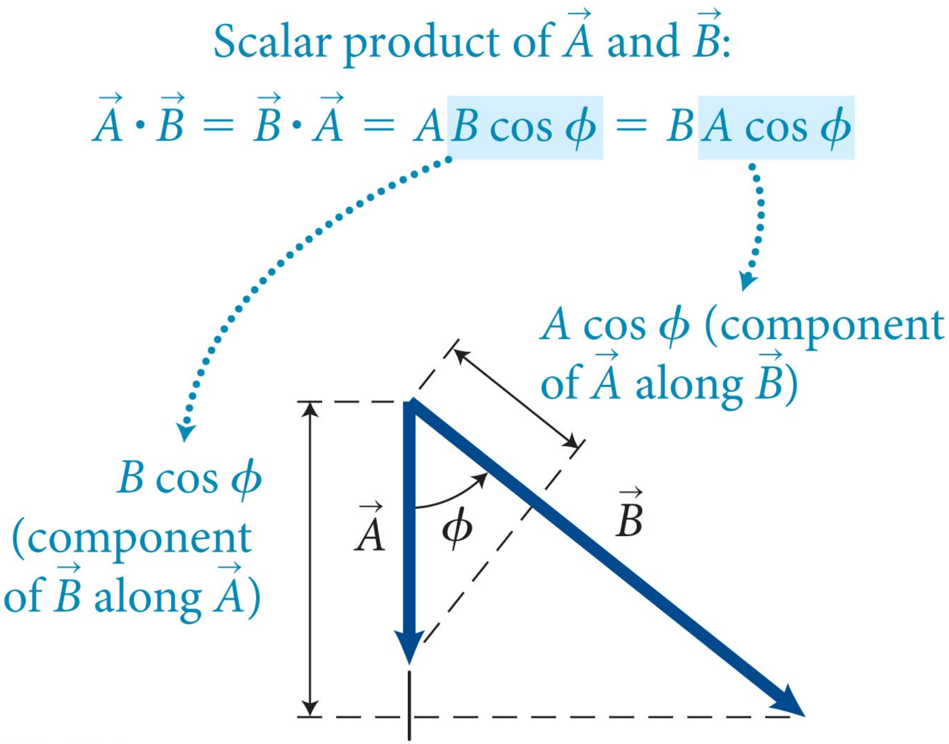
Section 10.9: Work as the product of two vectors

- If the angle between $\vec{A}$ and $\vec{B}$ is ϕ, the **scalar product** of the two vectors is

$$\vec{A} \cdot \vec{B} \equiv AB\cos\phi$$

- The scalar product is commutative:

$$\vec{A} \cdot \vec{B} \equiv \vec{B} \cdot \vec{A} = AB\cos\phi = BA\cos\phi$$



Physically?
Projecting out component of one vector parallel to another!

(A)*(parallel part of B)
(B)*(parallel part of A)

Section 10.9
Question 8

Under what circumstances could the scalar product of two vectors be zero?

1. When the vectors are parallel to each other
2. When the vectors are perpendicular to each other
3. When the vectors make an acute angle with each other
4. None of the above

Section 10.9
Question 8

Under what circumstances could the scalar product of two vectors be zero?

1. When the vectors are parallel to each other
2. ✔ When the vectors are perpendicular to each other
3. When the vectors make an acute angle with each other
4. None of the above

Section 10.9: Work as the product of two vectors

- We can write **work as a scalar product**:

$$W = \vec{F} \cdot \Delta\vec{r}_F \quad \text{(constant nondissipative force)}$$

- The normal force does no work, because $\phi = 90^{\circ}$ between the normal force and the force displacement.
- The work done by gravity on the block is

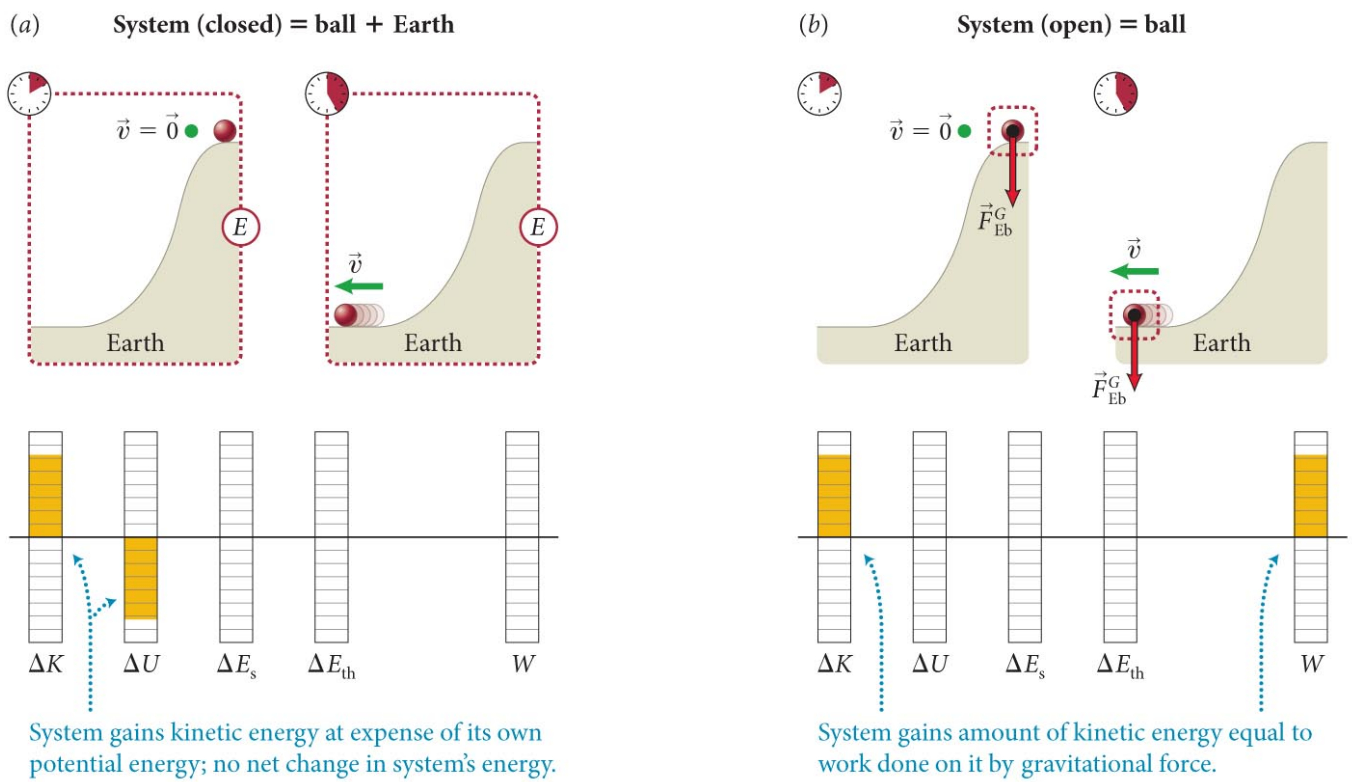
$$W = \vec{F}^G_{\text{Eb}} \cdot \Delta\vec{r}_F = (mg)\left(\frac{h}{\sin\theta}\right)\cos\phi$$

where θ is the angle of the incline.

- However, since $\phi = \pi/2 - \theta$, cos ϕ = sin ϕ, so $W = mgh$.
- If force and force displacement are opposite ($\phi = 180^{\circ}$), then work is negative.

Section 10.9: Work as the product of two vectors

Consider a ball sliding down an incline with negligible friction:



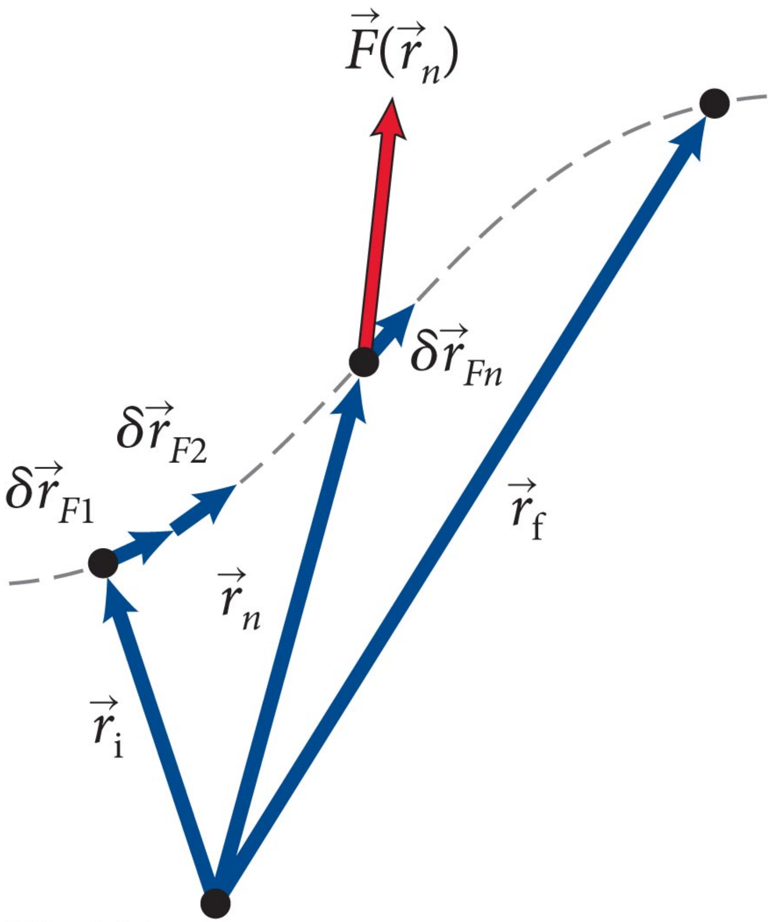
- If the closed system = ball + Earth, $\Delta K_b + \Delta U^G = 0$
- If the closed system = ball, then $\Delta K_b = W$, where $W = mgh$ is work done by gravity.

Section 10.9: Work as the product of two vectors

- Let us generalize the previous result to forces that are not constant, $\vec{F}(\vec{r})$.
- Start by subdividing the force displacement $\Delta\vec{r}_F$ into many small fragments $\delta\vec{r}_{Fn}$. Work done by a variable force $\vec{F}(\vec{r})$ over a small displacement is

$$W_n \approx \vec{F}(\vec{r}_n)\cdot\delta\vec{r}_{Fn}$$

- The work done over the entire force displacement is:



$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}(\vec{r})\cdot d\vec{r} \quad \text{(variable nondissipative force)}$$

Section 10.10: Coefficients of Friction

Section Goal

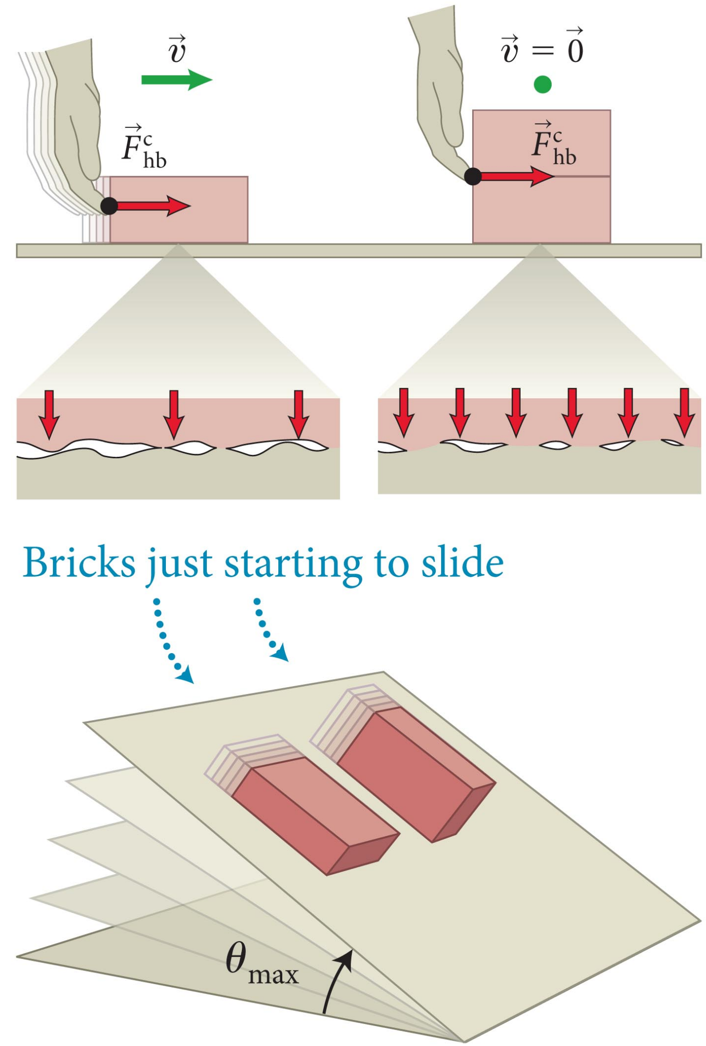
You will learn to

- Understand that the work done by gravity is independent of path.
- Define the scalar product of two vectors.
- Associate work done by nondissipative forces with the scalar product of the force vector and the displacement vector.

Section 10.10: Coefficients of Friction

- Let us now develop a quantitative description of friction:
- The figure illustrates the following observations.
  - **The maximum force of static friction exerted by a surface on an object is proportional to the force with which the object presses the surface.**
  - **The force of static friction does not depend on the contact area.*****

***** so long as the surfaces are otherwise the same**



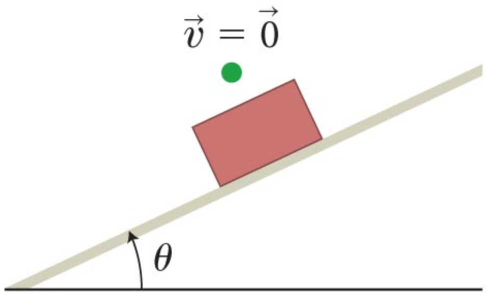
Section 10.10: Coefficients of Friction

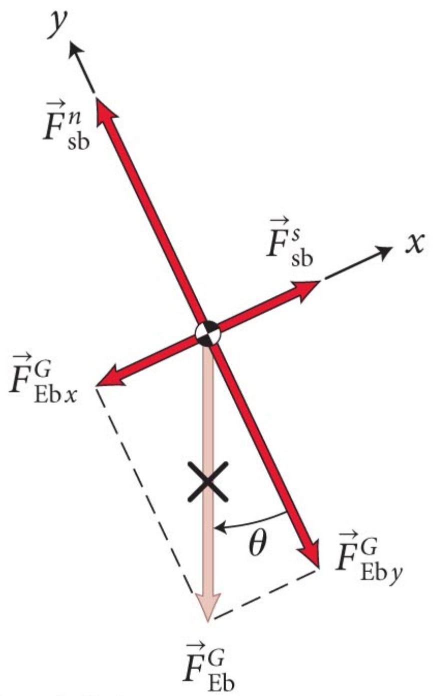
- As we have just determined, the maximum magnitude of the static friction force must be proportional to $\vec{F}^{n}_{\text{sb}}$, as seen in the figure.
- Therefore, for any two surfaces 1 and 2 we have

$$(F^{s}_{12})_{\max} = \mu_s F^{n}_{12}$$

- The unitless proportionality constant μ_s is called the **coefficient of static friction**.
- This upper limit means that the magnitude of static friction must obey the following condition:

$$F^{s}_{12} \leq \mu_s F^{n}_{12}$$





Section 10.10: Coefficients of Friction

Table 10.1 Coefficients of friction

Material 1	Material 2	IVI$_s$	IVI$_k$
aluminum	aluminum	1.1–1.4	1.4
aluminum	steel	0.6	0.5
glass	glass	0.9–1.0	0.4
glass	nickel	0.8	0.6
ice	ice	0.1	0.03
rubber	aluminum	0.5	—
rubber	concrete	1.0–4.0	0.8
rubber	rubber	0.84	—
steel	steel	0.8	0.4
steel	brass	0.5	0.4
steel	copper	0.5	0.4
steel	lead	0.95	0.95
wood	wood	0.25–0.5	0.2

The values given are for clean, dry, smooth surfaces.

Section 10.10: Coefficients of Friction

Procedure: Working with frictional forces

1. Draw a free-body diagram for the object of interest. Choose your x axis parallel to the surface and the y axis perpendicular to it, then decompose your forces along these axes. Indicate the acceleration of the object.

Section 10.10: Coefficients of Friction

Procedure: Working with frictional forces

2. The equation of motion in the y direction allows you to determine the sum of the y components of the forces in that direction:

$$\sum F_y = ma_y.$$

Constraint: Unless the object is accelerating in the normal direction, $a_y = 0$.

Substitute the y components of the forces from your free-body diagram. The resulting equation allows you to determine the normal force.

Procedure: Working with frictional forces

3. The equation of motion in the x direction is

$$\sum F_x = ma_x.$$

Constraint: If the object is not accelerating along the surface, $a_x = 0$.

Substitute the x components of the forces from your free-body diagram. The resulting equation allows you to determine the frictional force.

Section 10.10: Coefficients of Friction

Procedure: Working with frictional forces

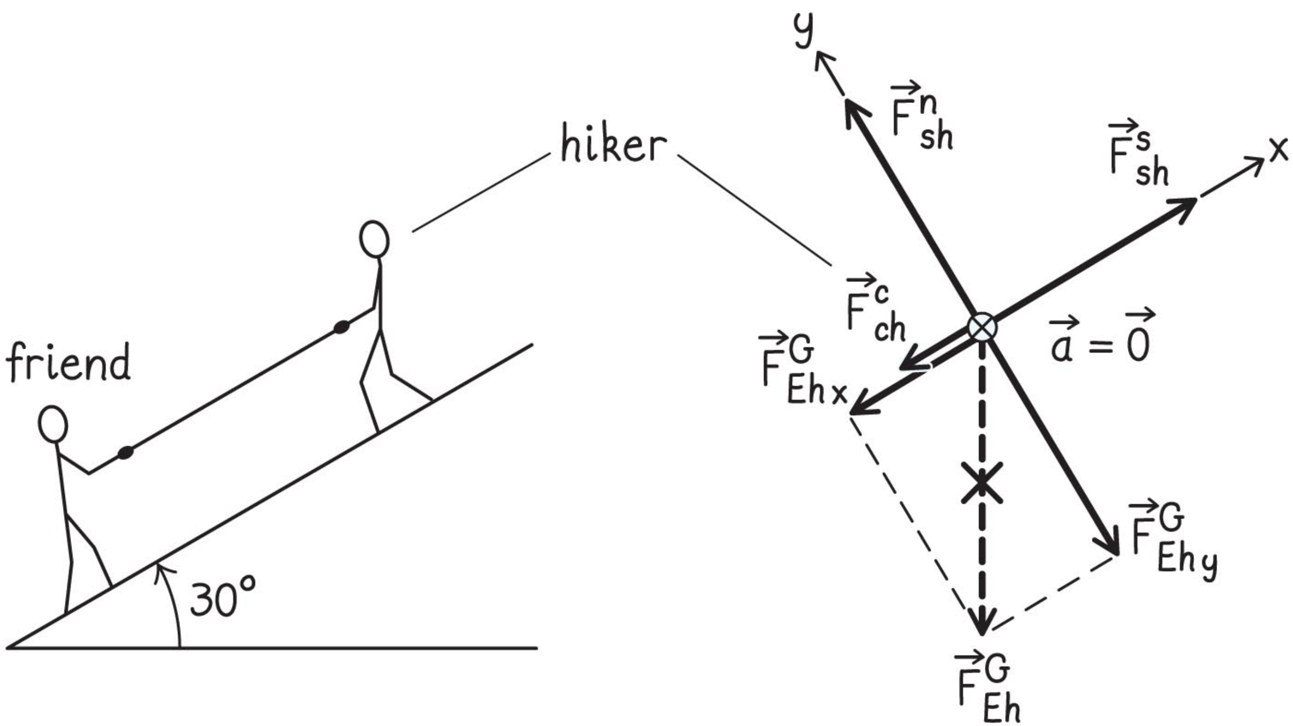
4. If the object is not slipping, the normal force and the force of static friction should obey Inequality 10.54.

Example 10.8 Pulling a friend up a hill

A hiker is helping a friend up a hill that makes an angle of 30^o with level ground. The hiker, who is farther up the hill, is pulling on a cable attached to his friend. The cable is parallel to the hill so that it also makes an angle of 30^o with the horizontal. If the coefficient of static friction between the soles of the hiker's boots and the surface of the hill is 0.80 and his inertia is 65 kg, what is the maximum magnitude of the force he can exert on the cable without slipping?

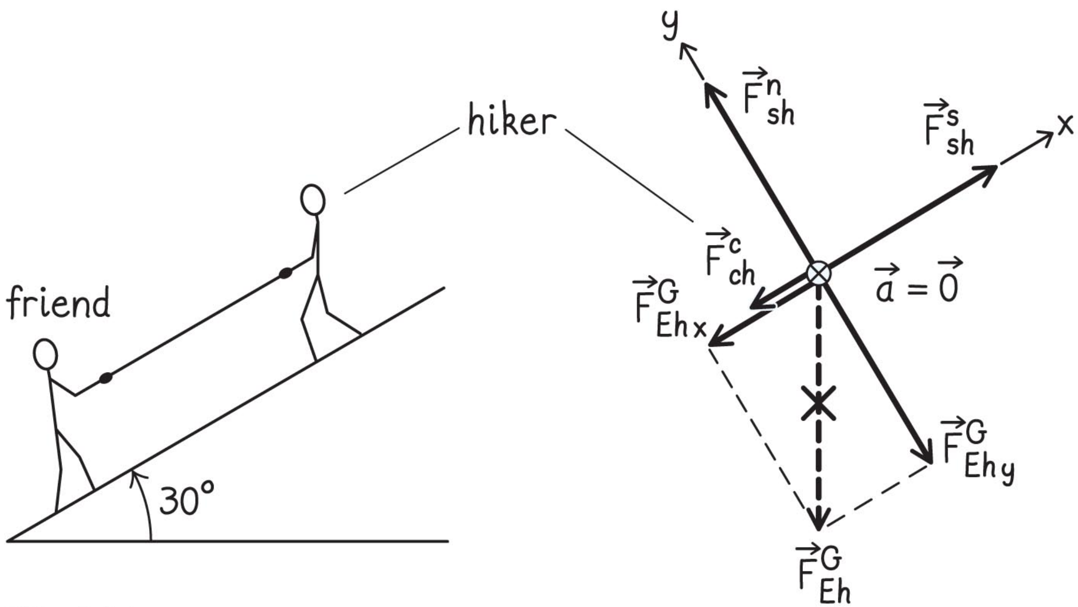
Example 10.8 Pulling a friend up a hill (cont.)

❶ GETTING STARTED I begin by making a sketch and drawing the free-body diagram for the hiker (Figure 10.43). I chose an *x* axis that points up the hill and the *y* axis perpendicular to it.



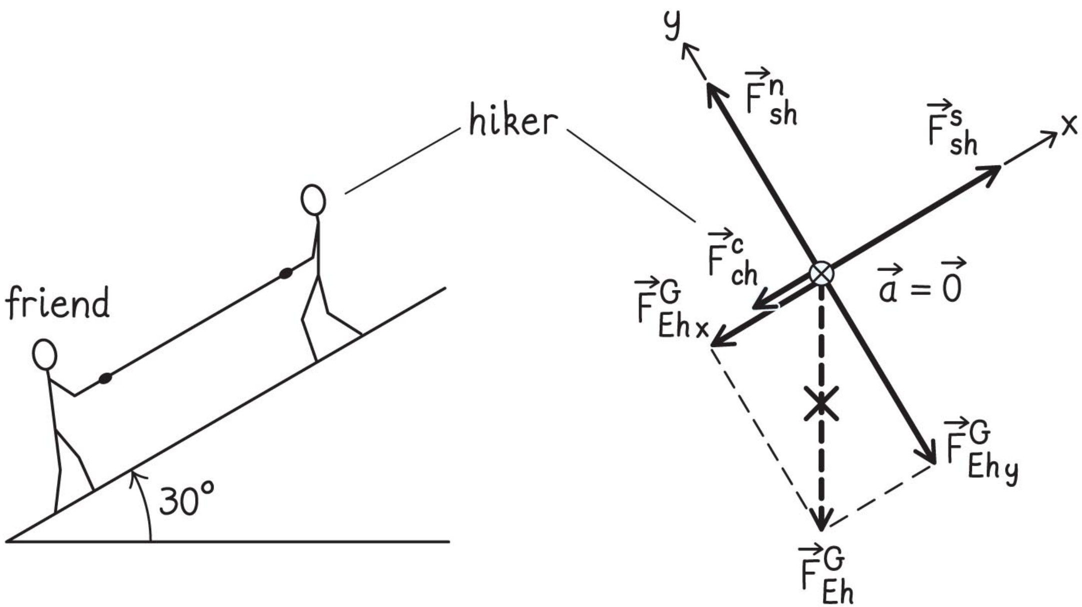
Example 10.8 Pulling a friend up a hill (cont.)

❶ GETTING STARTED The forces exerted on the hiker are a force $\vec{F}^{c}_{ch}$ exerted by the cable and directed down the hill (this force forms an interaction pair with the force the hiker exerts on the cable), the force of gravity $\vec{F}^{G}_{Eh}$, and the contact force exerted by the surface of the hill on the hiker.



Example 10.8 Pulling a friend up a hill (cont.)

❶ GETTING STARTED This last force has two components: the normal force $\vec{F}^{n}_{\text{sh}}$ and the force of static friction $\vec{F}^{s}_{\text{sh}}$. The force of static friction is directed up the hill. If the hiker is not to slip, his acceleration must be zero, and so the forces in the normal (y) and tangential (x) directions must add up to zero.



Example 10.8 Pulling a friend up a hill (cont.)

❷ DEVISE PLAN Following the suggested procedure, I'll write the equation of motion along the two axes, setting the acceleration in each direction to zero.

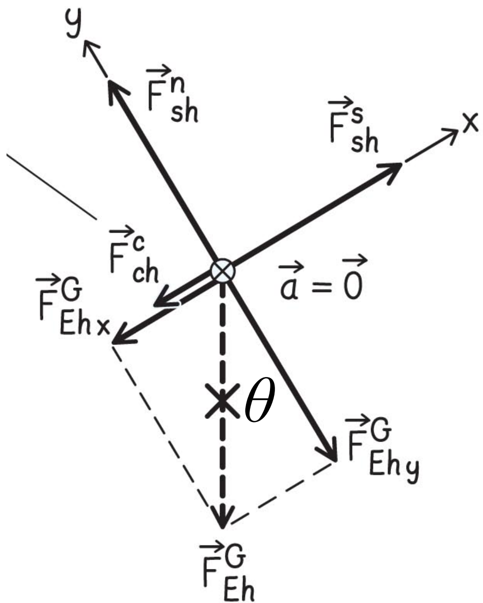
Then I use Inequality 10.54 to determine the maximum force that the hiker can exert without slipping.

Example 10.8 Pulling a friend up a hill (cont.)

❸ EXECUTE PLAN Because the magnitude of the force of gravity exerted on the hiker is mg, I have in the y direction

$$\sum F_y = F^G_{\text{Eh}\,y} + F^n_{\text{sh}} = -mg\cos\theta + F^n_{\text{sh}} = ma_y = 0$$

$$F^n_{\text{sh}} = mg\cos\theta$$



where the minus sign in $-mg\cos\theta$ indicates that $F^G_{\text{Eh}\,y}$ is in the negative y direction.

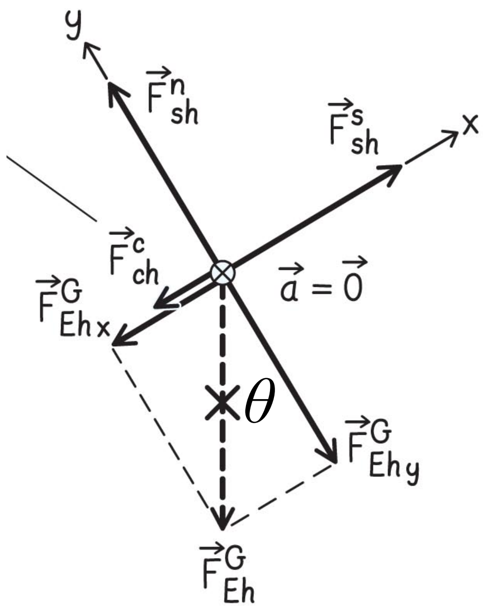
Example 10.8 Pulling a friend up a hill (cont.)

❸ EXECUTE PLAN Likewise, I have in the x direction

$$\sum F_x = F^G_{\mathrm{Eh}\,x} + F^s_{\mathrm{sh}} - F^{\mathrm{c}}_{\mathrm{ch}}$$

$$= -mg\sin\theta + F^s_{\mathrm{sh}} - F^{\mathrm{c}}_{\mathrm{ch}} = ma_x = 0$$

$$F^s_{\mathrm{sh}} = mg\sin\theta + F^{\mathrm{c}}_{\mathrm{ch}}$$



Example 10.8 Pulling a friend up a hill (cont.)

❸ EXECUTE PLAN If the hiker is not to slip, the force of static friction must not exceed its maximum value. Substituting and using Inequality 10.54, I get

$$mg\sin\theta + F^{c}_{ch} \le \mu_s (mg\cos\theta)$$

$$F^{c}_{ch} \le mg(\mu_s \cos\theta - \sin\theta).$$

Example 10.8 Pulling a friend up a hill (cont.)

❸ EXECUTE PLAN The problem asks me about the magnitude F^{c}_{hc} of the force exerted *by* the hiker *on* the cable, which is the same as the magnitude F^{c}_{ch} of the force exerted *by* the cable *on* the hiker, so

$$F^{c}_{hc} \leq mg(\mu_s \cos\theta - \sin\theta).$$

Example 10.8 Pulling a friend up a hill (cont.)

❸ EXECUTE PLAN Substituting the values given, I obtain

$$F_{\text{hc}}^{\text{c}} \leq (65 \text{ kg})(9.8 \text{ m/s}^2)(0.80 \cos 30° - \sin 30°)$$

$$= 1.2 \times 10^2 \text{ N.}$$ ✔

Example 10.8 Pulling a friend up a hill (cont.)

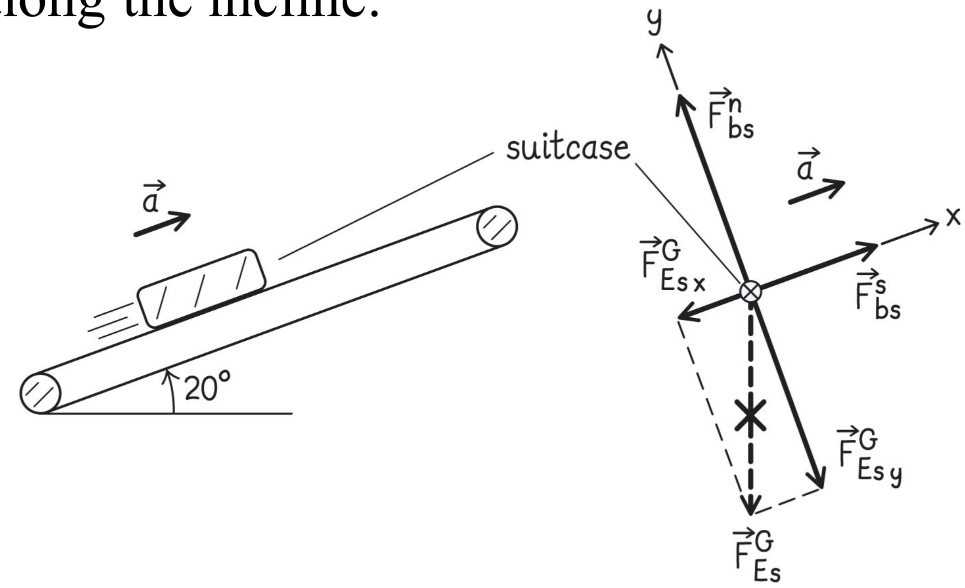
❹ EVALUATE RESULT A force of 1.2×10^2 N corresponds to the gravitational force exerted by Earth on an object that has an inertia of only 12 kg, and so the hiker cannot pull very hard before slipping. I know from experience, however, that unless I can lock a foot behind some solid object, it is impossible to pull hard on an inclined surface, and thus the answer I obtained makes sense.

Example 10.9 Object accelerating on a conveyor belt

While designing a conveyor belt system for a new airport, you determine that, on an incline of 20^{o}, the magnitude of the maximum acceleration a rubber belt can give a typical suitcase before the suitcase begins slipping is 4.0 m/s^2. What is the coefficient of static friction for a typical suitcase on rubber?

Example 10.9 Object accelerating on a conveyor belt (cont.)

❶ GETTING STARTED: I begin by making a sketch and drawing a free-body diagram for the suitcase, choosing the *x* axis along the conveyor belt in the direction of acceleration and the *y* axis upward and perpendicular to it (Figure 10.44). I also draw the upward acceleration of the suitcase along the incline.



Example 10.9 Object accelerating on a conveyor belt (cont.)

❷ DEVISE PLAN As in Example 10.8, I write out the equation of motion along both axes. The x component of the acceleration is $a_x = +a$, where the magnitude of the acceleration is $a = 4.0\ \text{m/s}^2$. I can then use Inequality 10.54 to determine the coefficient of static friction.

Example 10.9 Object accelerating on a conveyor belt (cont.)

❸ EXECUTE PLAN The equations of motion in the x and y directions are

$$\Sigma F_x = F^G_{\text{Es}\,x} + F^s_{\text{bs}} = -mg\sin\theta + F^s_{\text{bs}} = ma_x \qquad (1)$$

$$\Sigma F_y = F^G_{\text{Es}\,y} + F^n_{\text{bs}} = -mg\cos\theta + F^n_{\text{bs}} = ma_y = 0, \qquad (2)$$

with m the inertia of the suitcase.

Example 10.9 Object accelerating on a conveyor belt (cont.)

❸ EXECUTE PLAN If a_x represents the maximum acceleration at which the suitcase does not slip, the force of static friction must be maximum. Substituting Eq. 10.46 for $(F^s_{\text{bs}})_{\max}$ into Eq. 1, I get

$$-mg\sin\theta + \mu_s F^n_{\text{bs}} = m(+a).$$

Example 10.9 Object accelerating on a conveyor belt (cont.)

❸ EXECUTE PLAN Solving this equation for μ_s and substituting $F^n_{\text{bs}} = mg\cos\theta$ from Eq. 2, I obtain

$$\mu_s = \frac{ma + mg\sin\theta}{mg\cos\theta} = \frac{a + g\sin\theta}{g\cos\theta}$$

$$= \frac{(4.0\ \text{m/s}^2) + (9.8\ \text{m/s}^2)\sin 20^\circ}{(9.8\ \text{m/s}^2)(\cos 20^\circ)} = 0.80.$$ ✔

Example 10.9 Object accelerating on a conveyor belt (cont.)

❹ EVALUATE RESULT From Table 10.1, I see that the value I obtained for the coefficient for static friction is close to the coefficient for rubber against rubber and therefore reasonable. I also note that the inertia m of the suitcase, which is not given, drops out of the final result.

Section 10.10: Coefficients of Friction

- Once surfaces start to slip relative to each other, the force of kinetic friction is relative to the normal force

$$F_{12}^{k} = \mu_k F_{12}^{n}$$

where μ_k is called the **coefficient of kinetic friction**.

- The kinetic coefficient μ_k is always smaller then μ_s.
- Other than this, problem solving proceeds the same way

Chapter 10: Summary

Concepts: Vectors in two dimensions

- To add two vectors $\vec{A}$ and $\vec{B}$, place the tail of $\vec{B}$ at the head of $\vec{A}$. Their sum is the vector drawn from tail of $\vec{A}$ to the head of $\vec{B}$.
- To subtract $\vec{B}$ from $\vec{A}$, reverse the direction of $\vec{B}$ and then add the reversed $\vec{B}$ to $\vec{A}$.
- Any vector $\vec{A}$ can be written as

$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{i} + A_y\hat{j}$$

where A_x and A_y are the components of $\vec{A}$ along the x and y axes.

Quantitative Tools: Vectors in two dimensions

- The magnitude of any vector $\vec{A}$ is

$$A \equiv \left|\vec{A}\right| = +\sqrt{A_x^2 + A_y^2}$$

and the angle θ that $\vec{A}$ makes with the positive x axis is given by

$$\tan\theta = \frac{A_y}{A_x}.$$

- If $\vec{R} = \vec{A} + \vec{B}$ is the vector sum of $\vec{A}$ and $\vec{B}$, the component of $\vec{R}$ are

$$R_x = A_x + B_x$$

$$R_y = A_y + B_y.$$

- The **scalar product** of two vectors $\vec{A}$ and $\vec{B}$ that make an angle ϕ when placed tail to tail is

$$\vec{A} \cdot \vec{B} \equiv AB\cos\phi.$$

Concepts: Projectile motion in two dimensions

- For a projectile that moves near Earth's surface and experiences only the force of gravity, the acceleration has a magnitude *g* and is directed downward.
- The projectile's horizontal acceleration is zero, and the horizontal component of its velocity remains constant.
- At the highest point in the trajectory of a projectile, the vertical velocity component v_y is zero, but the vertical component of the acceleration is g and is directed downward.

Chapter 10: Summary

Quantitative Tools: Projectile motion in two dimensions

- If a projectile is at a position (x, y), its **position vector** $\vec{r}$ is

$$\vec{r} = x\hat{i} + y\hat{j}.$$

- If a projectile has position components x_i and y_i and velocity components $\upsilon_{x,i}$ and $\upsilon_{y,i}$ at one instant, then its acceleration, velocity, and position components a time interval Δt later are

$$a_x = 0$$

$$a_y = -g$$

$$\upsilon_{x,f} = \upsilon_{x,i}$$

$$\upsilon_{y,f} = \upsilon_{y,i} - g\Delta t$$

$$x_f = x_i + \upsilon_{x,i}\Delta t$$

$$y_f = y_i + \upsilon_{y,i}\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

Concepts: Collisions and momentum in two dimensions

- Momentum is a vector, so in two dimensions its changes in momentum must be accounted for by components.
- This means two equations, one for the *x* component and one for the *y* component of momentum.
- The coefficient of restitution is a scalar and is accounted for by a single equation.

Chapter 10: Summary

Quantitative Tools: Collisions and momentum in two dimensions

$$\Delta p_x = \Delta p_{1x} + \Delta p_{2x} = m_1(v_{1x,\mathrm{f}} - v_{1x,\mathrm{i}}) + m_2(v_{2x,\mathrm{f}} - v_{2x,\mathrm{i}}) = 0$$

$$\Delta p_y = \Delta p_{1y} + \Delta p_{2y} = m_1(v_{1y,\mathrm{f}} - v_{1y,\mathrm{i}}) + m_2(v_{2y,\mathrm{f}} - v_{2y,\mathrm{i}}) = 0$$

Chapter 10: Summary

Concepts: Forces in two dimensions

- In two-dimensional motion, the component of the acceleration parallel to the instantaneous velocity changes the speed and the component of the acceleration perpendicular to the instantaneous velocity changes the direction of the instantaneous velocity.
- When choosing a coordinate system for a problem dealing with an accelerating object, if possible make one of the axes lie along the direction of the acceleration.

Chapter 10: Summary

Concepts: Friction

- When two surfaces touch each other, the component of the contact force perpendicular (normal) to the surfaces is called the **normal force** and the component parallel (tangential) to the surfaces is called the **force of friction.**

- The direction of the force of friction is such that the force opposes *relative* motion between the two surfaces. When the surfaces are not moving relative to each other, we have **static friction.**

Chapter 10: Summary

Concepts: Friction

- When the forces are moving relative to each other, we have **kinetic friction.**
- The magnitude of the force of kinetic friction is independent of the contact area and independent of the relative speeds of the two surfaces.

Quantitative Tools: Friction

- The maximum magnitude of the force of static friction between any two surfaces 1 and 2 is proportional to the normal face:

$$(F^s_{12})_{\max} = \mu_s F^n_{12},$$

where μ_s is the unitless **coefficient of static friction.** This upper limit means that the magnitude of the frictional force must obey the condition

$$F^s_{12} \leq \mu_s F^n_{12}.$$

- The magnitude of the force of kinetic friction is also proportional to the normal face:

$$F^k_{12} \leq \mu_k F^n_{12},$$

where $\mu_k \leq \mu_s$ is the unitless **coefficient of kinetic friction.**

Concepts: Work

- For a sliding object, the normal force does no work because this force is perpendicular to the direction of the object's displacement.
- The force of kinetic friction is a nonelastic force and thus causes energy dissipation.
- The force of static friction is an elastic force and so causes no energy dissipation.
- The work done by gravity is independent of path.

Quantitative Tools: Work

- The work done by a constant nondissipative force when the point of application of the force undergoes a displacement $\Delta\vec{r}_F$ is

$$W = \vec{F} \cdot \Delta\vec{r}_F.$$

- The work done by a variable nondissipative force when the point of application of the force undergoes a displacement is

$$W = \int_{\vec{r}_\text{i}}^{\vec{r}_\text{f}} \vec{F}(\vec{r}) \cdot d\vec{r}.$$

- This is the line integral of the force over the path traced out by the point of application of the force.

Chapter 10: Summary

Quantitative Tools: Work

- For a variable dissipative force, the change in the thermal energy is

$$\Delta E_{\text{th}} = -\int_{\vec{r}_\text{i}}^{\vec{r}_\text{f}} \vec{F}(\vec{r}_\text{cm}) \cdot d\vec{r}_\text{cm}.$$

- When an object descends a vertical distance h, no matter what path it follows, the work gravity does on it is

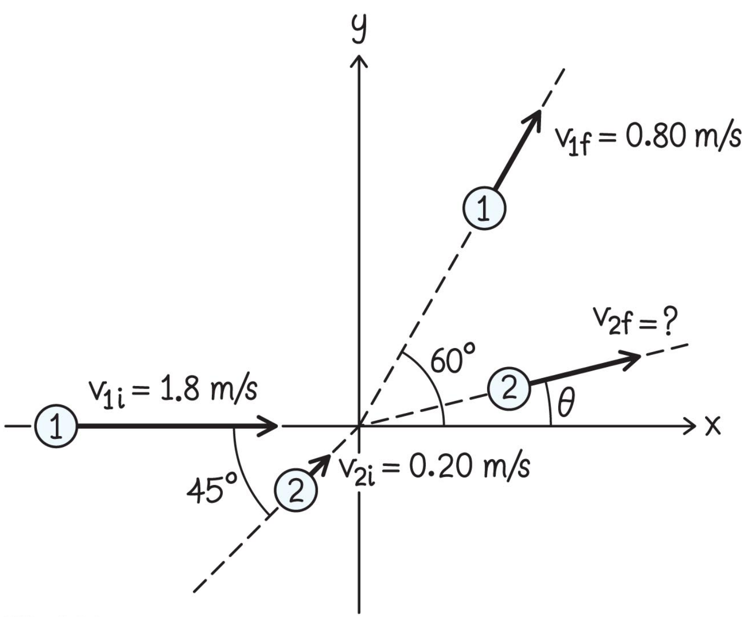
$$W = mgh.$$

Example 10.7 Pucks colliding

Pucks 1 and 2 slide on ice and collide. The inertia of puck 2 is twice that of puck 1. Puck 1 initially moves at 1.8 m/s; puck 2 initially moves at 0.20 m/s in a direction that makes an angle of 45^{o} with the direction of puck 1. After the collision, puck 1 moves at 0.80 m/s in a direction that makes an angle of 60^{o} with its original direction. What are the speed and direction of puck 2 after the collision?

Example 10.7 Pucks colliding (cont.)

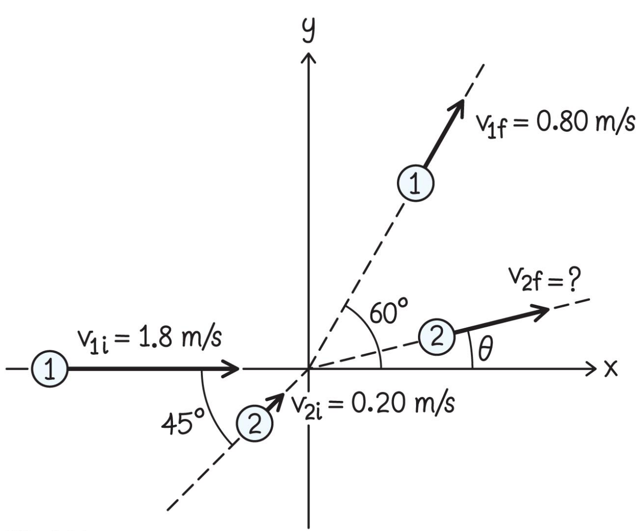
❶ GETTING STARTED Because it takes place on a surface, the collision is two-dimensional. I begin by organizing the given information in a sketch (Figure 10.33). I let puck 1 move along the *x* axis before the collision. Then I add puck 2, which initially moves at an angle of 45° to the *x* axis.



Section 10.8: Collisions and momentum in two dimensions

Example 10.7 Pucks colliding (cont.)

❶ GETTING STARTED I draw puck 1 after the collision moving upward and to the right along a line that makes an angle of 60° with the x axis. I don't know anything about the motion of puck 2 after the collision, and so I arbitrarily draw it moving to the right and upward. I label the pucks and indicate the speeds I know. I need to determine the final speed v_{2f} of puck 2 and the angle θ between its direction of motion after the collision and the x axis.



Example 10.7 Pucks colliding (cont.)

❷ DEVISE PLAN Equations 10.21 and 10.22 give the relationship between the initial and final velocities of the two pucks. I know the x and y components of both initial velocities and, after decomposing $\vec{v}_{1f}$, the x and y components of the final velocity of puck 1. Thus I can use Eqs. 10.21 and 10.22 to obtain values for the two components of $\vec{v}_{2f}$. Using these component values, I can calculate v_{2f} and θ.

Example 10.7 Pucks colliding (cont.)

❸ EXECUTE PLAN The two unknown components are $\upsilon_{2x,\text{f}}$ and $\upsilon_{2y,\text{f}}$. Solving Eq. 10.21, for $\upsilon_{2x,\text{f}}$, I get

$$\upsilon_{2x,\text{f}} = -\frac{m_1}{m_2}(\upsilon_{1x,\text{f}} - \upsilon_{1x,\text{i}}) + \upsilon_{2x,\text{i}}$$

$$= -\frac{1}{2}\ [(0.80\ \text{m/s})\cos 65^\circ - (1.8\ \text{m/s})]$$

$$+ (0.20\ \text{m/s})\cos 45^\circ$$

$$= 0.84\ \text{m/s}.$$

Example 10.7 Pucks colliding (cont.)

❸ EXECUTE PLAN Solving Eq. 10.22 for $\upsilon_{2y,\mathrm{f}}$, I get

$$\upsilon_{2y,\mathrm{f}} = -\frac{m_1}{m_2}(\upsilon_{1y,\mathrm{f}} - \upsilon_{1y,\mathrm{i}}) + \upsilon_{2y,\mathrm{i}} = -\frac{m_1}{m_2}\upsilon_{1y,\mathrm{f}} + \upsilon_{2y,\mathrm{i}}$$

$$= -\frac{1}{2}(0.80\ \mathrm{m/s})\sin 60^{\circ} + (0.20\ \mathrm{m/s})\sin 45^{\circ}$$

$$= -0.21\ \mathrm{m/s}.$$

Example 10.7 Pucks colliding (cont.)

❸ EXECUTE PLAN Using these values, I obtain the final speed of puck 2 and its direction of motion:

$$\upsilon_{2\text{f}} = \sqrt{\upsilon_{2x,\text{f}}^2 + \upsilon_{2y,\text{f}}^2} = \sqrt{(0.84\text{ m/s})^2 + (-0.21\text{ m/s})^2}$$

$$= 0.87\text{ m/s} \checkmark$$

$$\tan\ \theta = \frac{\upsilon_{2y,\text{f}}}{\upsilon_{2x,\text{f}}} = \frac{-0.21\text{ m/s}}{0.84\text{ m/s}} = -0.24$$

or $\theta = -14^{\circ}.\ \checkmark$

Example 10.7 Pucks colliding (cont.)

❹ EVALUATE RESULT My positive result for $v_{2x,\mathrm{f}}$ and negative result for $v_{2y,\mathrm{f}}$ tell me that after the collision, puck 2 moves in the positive x direction and the negative y direction. Both of these directions make sense: I can see from my sketch that the x component of the velocity of puck 1 decreases in the collision. The corresponding decrease in the x component of its momentum must be made up by an increase in the x component of the momentum of puck 2.

Section 10.8: Collisions and momentum in two dimensions

Example 10.7 Pucks colliding (cont.)

❹ EVALUATE RESULT Given that puck 2 initially moves in the positive x direction, it must continue to do so after the collision. In the y direction, I note that because $v_{1y,\mathrm{i}}$ is zero, the y component of the momentum of puck 1 increases in the positive y direction by an amount $m_1 v_{1y,\mathrm{f}} = m_1[(0.80 \text{ m/s}) \sin 60^{\circ}] = m_1(0.69 \text{ m/s})$. Therefore the y component of the momentum of puck 2 must undergo a change of equal amount in the negative y direction.

Section 10.8: Collisions and momentum in two dimensions

Example 10.7 Pucks colliding (cont.)

❹ EVALUATE RESULT Because the initial y component of the momentum of puck 2 is smaller than this value, $m_2 v_{2y,i} = 2m_1 v_{2y,i} = 2m_1[(0.20 \text{ m/s}) \sin 45^o] = m_1(0.28 \text{ m/s})$, puck 2 must reverse its direction of motion in the y direction, yielding a negative value for $v_{2y,f}$, in agreement with what I found.

Checkpoint 10.10

10.10 Is the collision in Example 10.7 elastic?