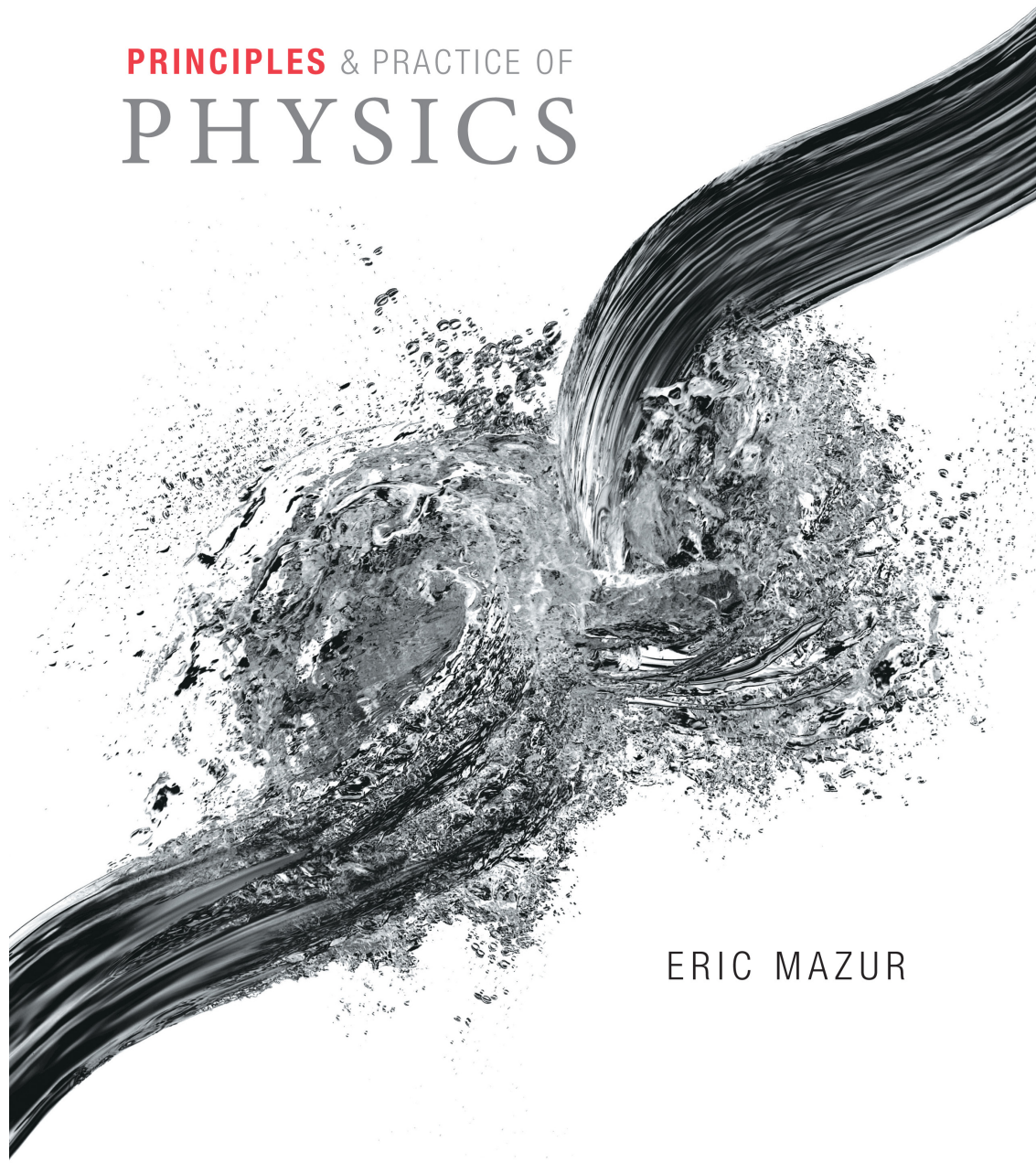


PRINCIPLES & PRACTICE OF  
PHYSICS

**Chapter 3**  
**Acceleration**



ERIC MAZUR

# Reading quiz Q1

## How does distance scale with time?

- On Mars, where air resistance is negligible, an astronaut drops a rock from a cliff and notes that the rock falls about  $d$  meters during the first  $t$  seconds of its fall. Assuming the rock does not hit the ground first, how far will it fall during the first  $4t$  seconds of its fall?
- $d \propto t^2$ , so if  $t$  doubles,  $d$  quadruples
  - If  $t$  increases 4 times,  $d$  increases 16 times

# Chapter 3: Acceleration

## Concepts

# Section 3.1: Changes in velocity

## Section Goals

- Define acceleration from velocity.
- Identify if an object is accelerating from several different graphical representations of motion.
- Understand the vector relationships between velocity and acceleration.

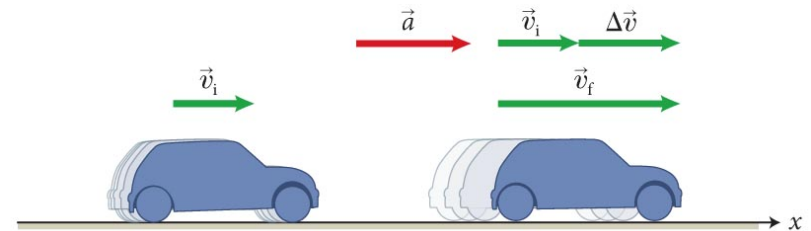
## Section 3.1: Changes in velocity

- If an object's velocity is changing, the object is accelerating.
- The  $x$  component of the average acceleration of an object is the change in the  $x$  component of the velocity divided by the time interval during which this change took place.
- The SI unit of acceleration is  $\text{m/s}^2$ .

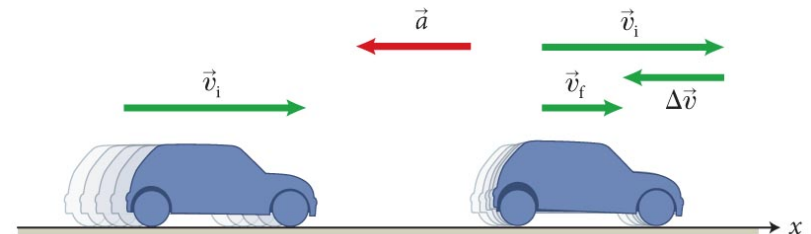
# Section 3.1: Changes in velocity

- Whenever an object's velocity vector  $\vec{v}$  and acceleration vector  $\vec{a}$  point in the same direction, the object speeds up.
- If  $\vec{v}$  and  $\vec{a}$  point in the opposite direction, the object slows down.

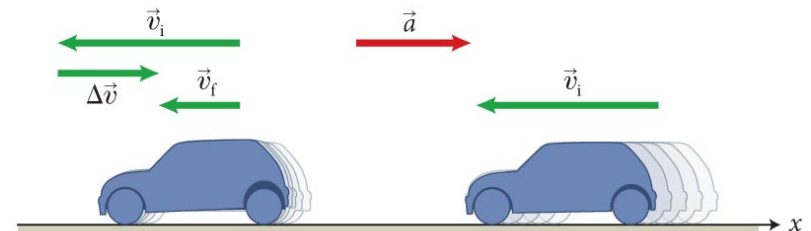
(a) Car speeds up in positive  $x$  direction



(b) Car slows down in positive  $x$  direction

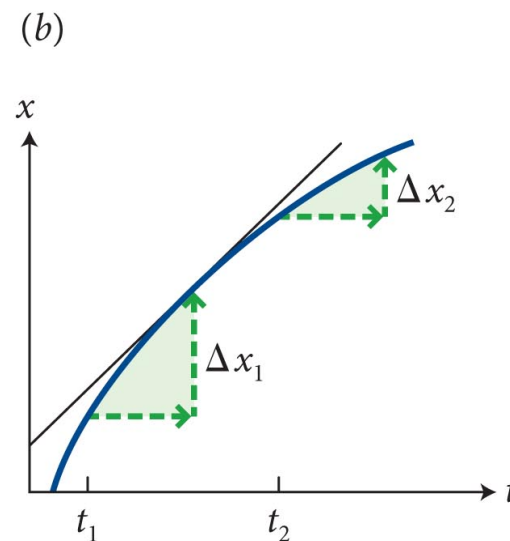
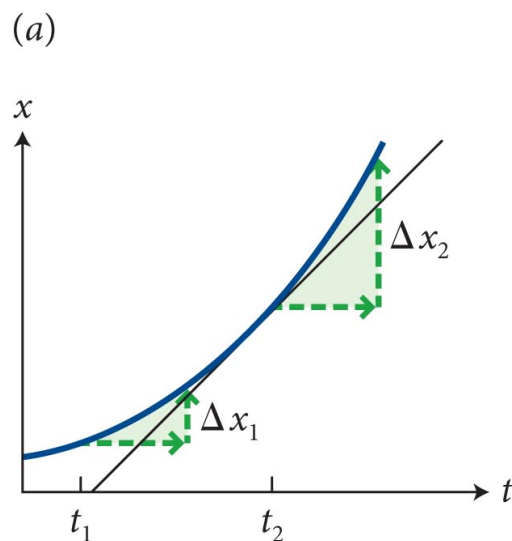


(c) Car slows down in negative  $x$  direction



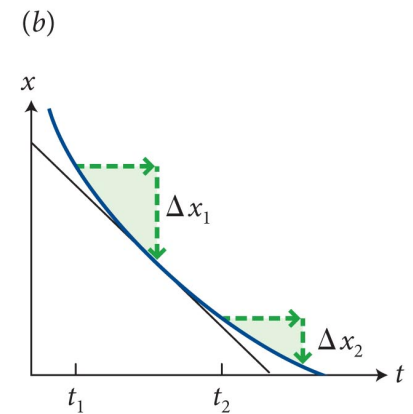
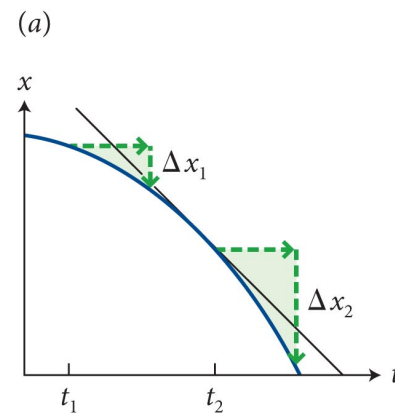
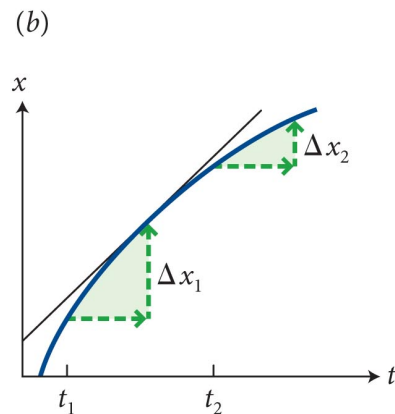
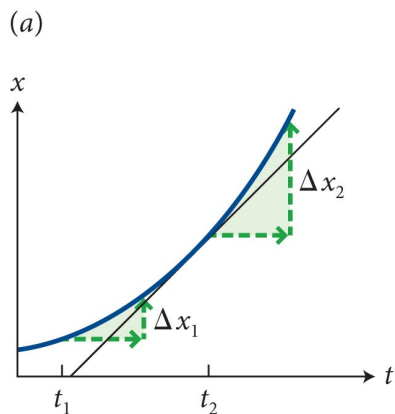
# Section 3.1: Changes in velocity

- For accelerating objects, the  $x(t)$  curve is a *not* a straight line.
- The figure shows the  $x(t)$  curve for two accelerating objects:
  - For each object, consider the displacements  $\Delta x_1$  and  $\Delta x_2$  during two equal time intervals ( $\Delta t$ ) at two different times.
  - If the displacement increases with time then the velocity is increasing (for example,  $\Delta x_2 > \Delta x_1$ ).
  - If the displacement decreases then velocity is decreasing.



# Section 3.1: Changes in velocity

- The curvature of the  $x(t)$  curve is a measure of the  $x$  component of acceleration ( $a_x$ ).
- An upward curvature corresponds to a positive  $a_x$ :
  - The curve lies above the tangent; **faster than linear**
- A downward curvature corresponds to a negative  $a_x$ :
  - The curve lies below the tangent; **slower than linear**





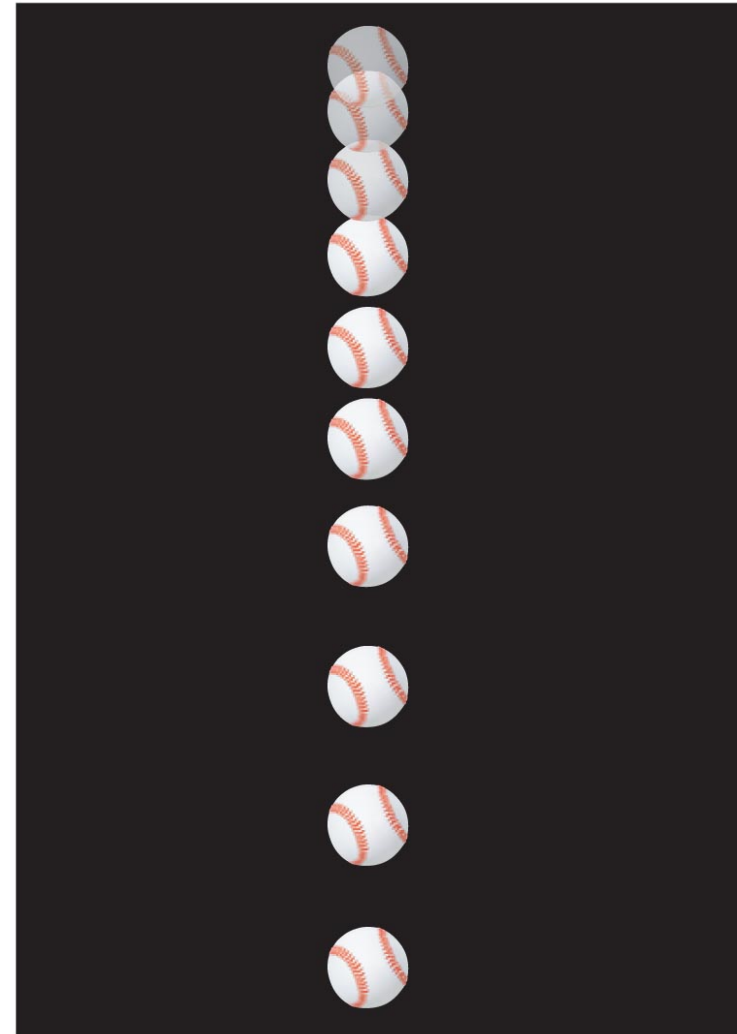
# Section 3.2: Acceleration due to gravity

## Section Goals

- Identify gravity as the cause of the vertical acceleration of an object moving near the surface of the Earth.
- Display the effects of gravity of an object using motion diagrams.
- Model the idealized case of vertical motion in the absence of other influences, such as air resistance, using the concept of free fall.

## Section 3.2: Acceleration due to gravity

- An object falling in a straight line toward the Earth's surface: **accelerated motion**.
- A falling ball recorded at equal time intervals of 0.05 s.
- The increasing displacements tells us that the speed increases as it falls: the ball accelerates.



## Checkpoint Question 3.5

- Does the acceleration of an object as it falls depend on the physical characteristics of the object?

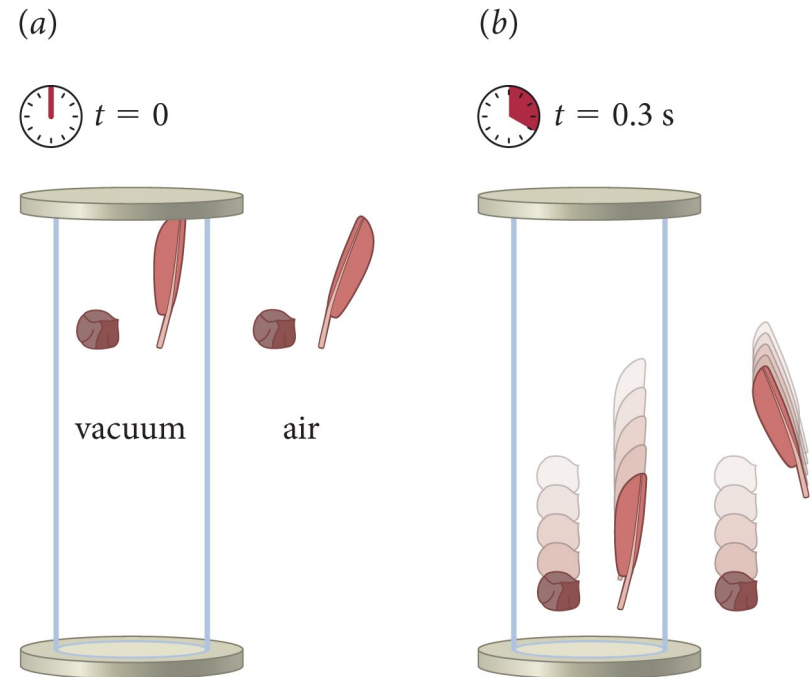


**3.5** Hold a book and a sheet of paper, cut to the same size as the book, side by side 1 m above the floor. Hold the paper parallel to the floor and the book with its covers parallel to the floor, and release them at the same instant. Which hits the floor first? Now put the paper on top of the book and repeat the experiment. What do you notice?

We need to separate *extrinsic* effects like air resistance from *intrinsic* effects like gravity. The latter is universal, the former depends on the details ...

# Section 3.2: Acceleration due to gravity

- What is the magnitude of acceleration due to gravity as an object falls?
  - feather vs stone
  - in a vacuum, no air resistance.
  - the acceleration **due to gravity** *does not* depend on the physical characteristics of object
    - The motion of an object under the influence of gravity only is called **free fall**.



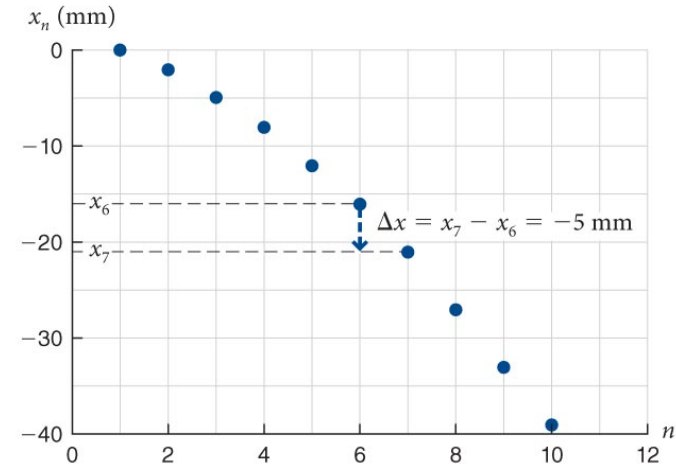
- what properties are most fundamental?
- which are circumstantial?

[https://youtu.be/frZ9dN\\_ATew](https://youtu.be/frZ9dN_ATew)

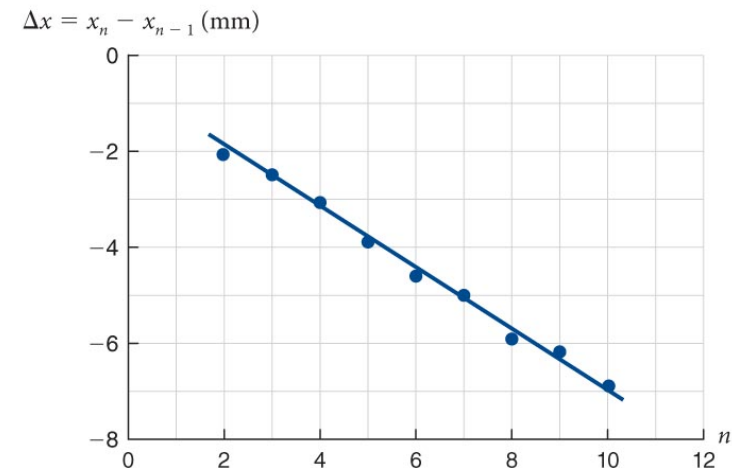
# Section 3.2: Acceleration due to gravity

- What is the magnitude of acceleration due to gravity as an object falls?
  - like last time, position vs time changes faster than linearly
  - *displacement* between adjacent times is linear
    - therefore, the velocity increases at a constant rate.

(a)



(b)



## Section 3.2: Acceleration due to gravity

- What is the magnitude of acceleration due to gravity as an object falls?
  - without air resistance, the magnitude of the acceleration of all falling objects is  $9.8 \text{ m/s}^2$ .
- Means? The amount of time it takes to fall from a certain height is the same for all falling objects.

# Section 3.3: Projectile motion

## Section Goals

- Define the motion of objects that are launched but not self-propelled as projectile motion.
- Model the vertical trajectory of projectiles as objects that are in free fall.
- Represent projectile motion graphically using motion diagrams and motion graphs.

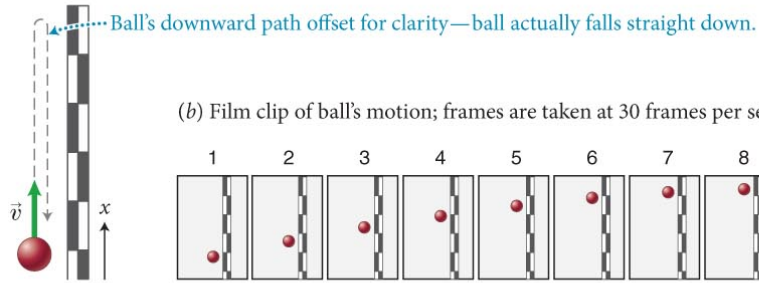


## Section 3.3: Projectile motion

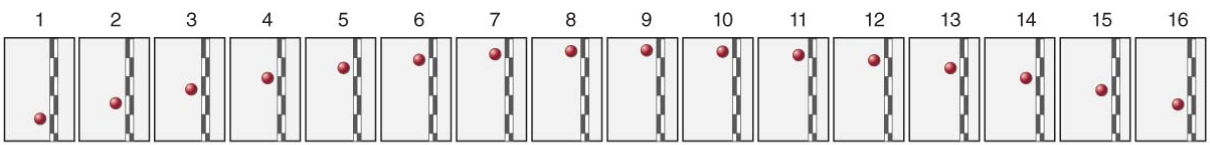
- An object that is launched but not self-propelled is called a **projectile**.
- Its motion is called **projectile motion**.
- The path the object follows is called its **trajectory**.

# Section 3.3: Projectile motion

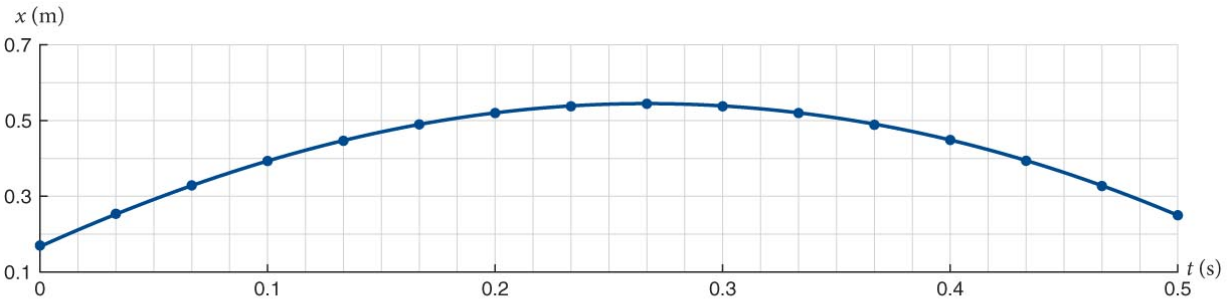
(a) Throw a ball straight up (with negligible air resistance)



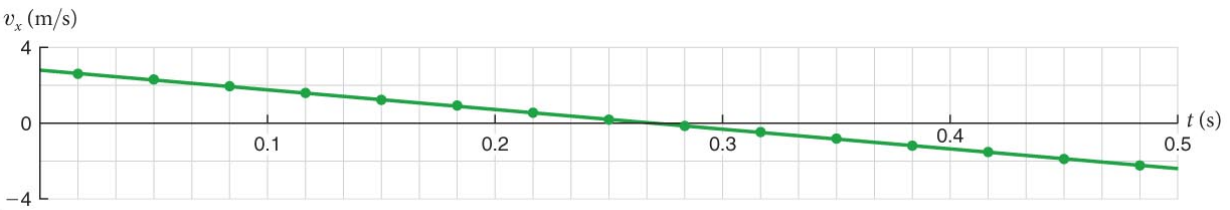
(b) Film clip of ball's motion; frames are taken at 30 frames per second



(c)  $x(t)$  curve for ball



(d)  $v_x(t)$  curve for ball



# Section 3.3: Projectile motion

- Consider  $x(t)$  and  $v(t)$  curves:
  - **As the ball moves upward** it slows down
    - $v$  and  $a$  are in opposite directions
    - since  $v$  is up,  $a$  must be down
  - **As the ball moves down** it speeds up
    - $v$  and  $a$  must be in the same direction
    - since  $v$  is down,  $a$  is down
  - the  $v(t)$  curve is a straight line *for the whole motion*
    - slope approximately the *acceleration due to gravity*.
- once the object is released, the rest of its motion is determined by gravity alone (free fall).

# Checkpoint 3.8



**3.8** Imagine throwing a ball downward so that it has an initial speed of 10 m/s.

What is its speed 1 s after you release it?  
2 s after?



Constant acceleration: gain/lose same speed each second

- launched downward, so it speeds up
- $\sim 10\text{m/s}^2$ , 1 second later: gain  $10\text{m/s}$   $\rightarrow$   $20\text{m/s}$
- 2 seconds later: gain another  $10\text{m/s}$   $\rightarrow$   $30\text{m/s}$

## Section 3.3: Projectile motion

- What happens at the very top of the trajectory of a ball launched upward?
  - At the top, velocity changes from up to down, which means that acceleration must be nonzero.
  - At the very top, the instantaneous velocity is zero.
  - Acceleration, however, is nonzero.
- *Acceleration is always  $\sim 9.8m/s^2$*

# Section 3.4: Motion diagrams

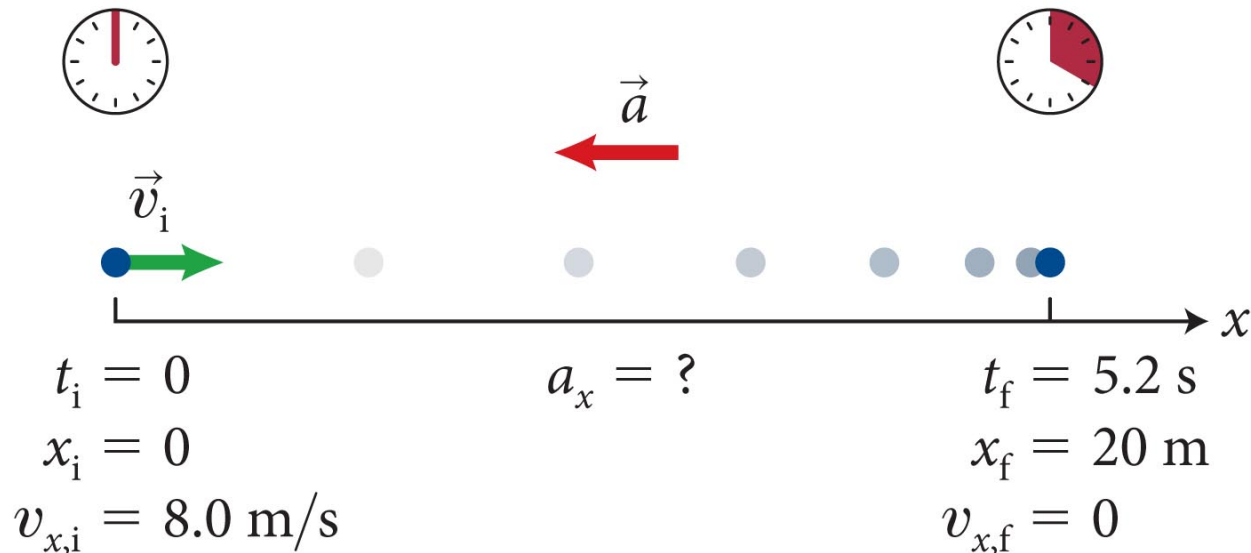
## Section Goals

You will learn to

- Generalize the “frame sequence” diagram introduced in Chapter 2 to a new visual representation called a **motion diagram**.
- Represent and correlate the kinematic quantities, position, displacement, velocity, and acceleration on motion diagrams.

# Section 3.4: Motion diagrams

- Motion diagrams are pictorial representations of objects in motion:
  - visualize the motion of an object described in a problem.
  - they show an object's  $x$ ,  $v$ , and  $a$  at several equally spaced instances (including at the start and end).
  - it is basically a cartoon
- Below: a motion diagram for a bicycle with an initial velocity of 8.0 m/s slowing down to a stop.





# Section 3.4: Motion diagrams

## Procedure: Analyzing motion using motion diagrams

Solving motion problems: a diagram summarizing what you have & what you want may all but solve the problem

1. Use dots to represent the moving object at equally spaced time intervals. If the object moves at constant speed, the dots are evenly spaced; if the object speeds up, the spacing between the dots increases; if the object slows down, the spacing decreases.
2. Choose an  $x$  (position) axis that is convenient for the problem. Most often this is an axis that ( $a$ ) has its origin at the initial or final position of the object and ( $b$ ) is oriented in the direction of motion or acceleration.

# Section 3.4: Motion diagrams

## Procedure: Analyzing motion using motion diagrams (cont.)

3. Specify  $x$  &  $v$  at all relevant instants. **Particularly, specify**
  - **the *initial conditions*** - position and velocity at the beginning of the time interval of interest
  - **the *final conditions*** - position and velocity at the end of that time interval.
  - also note where  $v$  reverses direction or a changes.
  - unknown parameters = question mark.
4. Indicate the acceleration of the object between all the instants specified

## Section 3.4: Motion diagrams

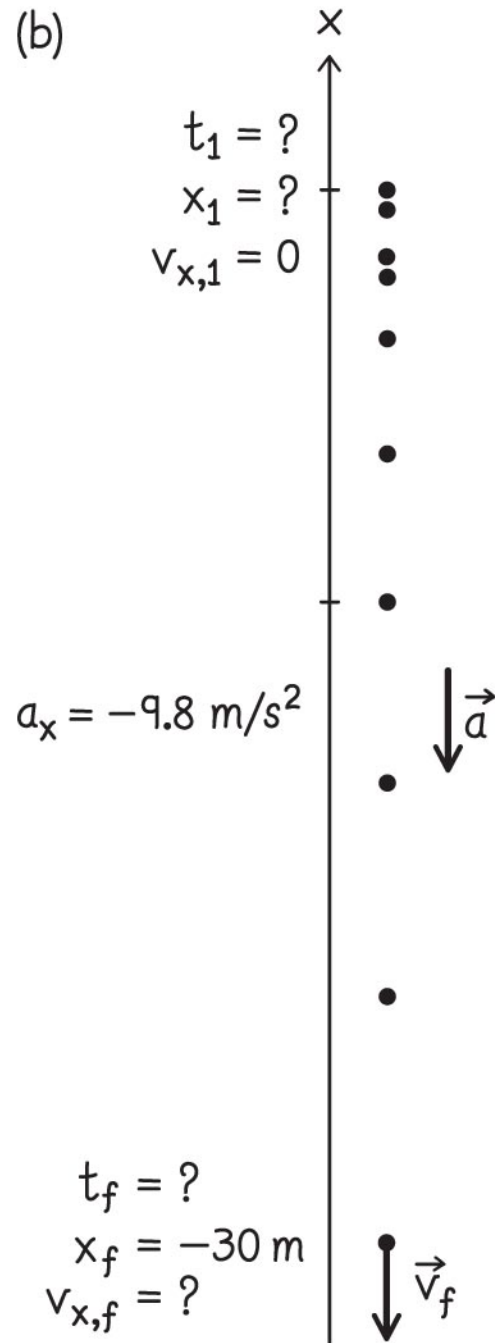
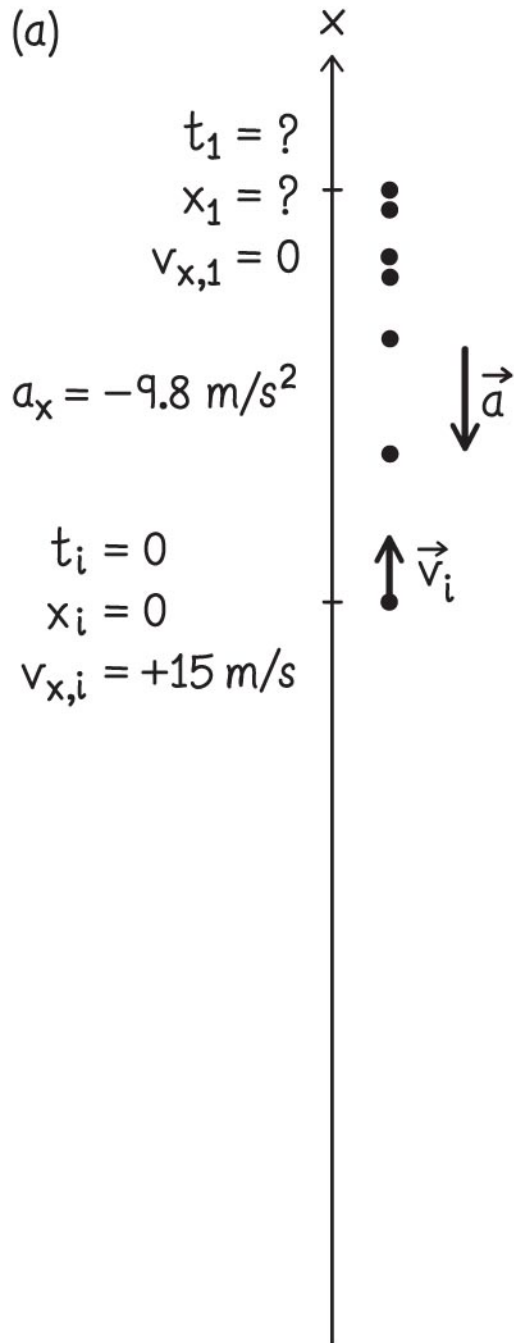
### Procedure: Analyzing motion using motion diagrams (cont.)

5. With more than one object, draw separate diagrams side by side, using one common  $x$  axis.
6. If the object reverses direction, separate the motion diagram into two parts, one for each direction

# Checkpoint 3.9



**3.9** Make a motion diagram for the following situation: A seaside cliff rises 30 m above the ocean surface, and a person standing at the edge of the cliff launches a rock vertically upward at a speed of 15 m/s. After reaching the top of its trajectory, the rock falls into the water.



# Chapter 3: Self-Quiz #1

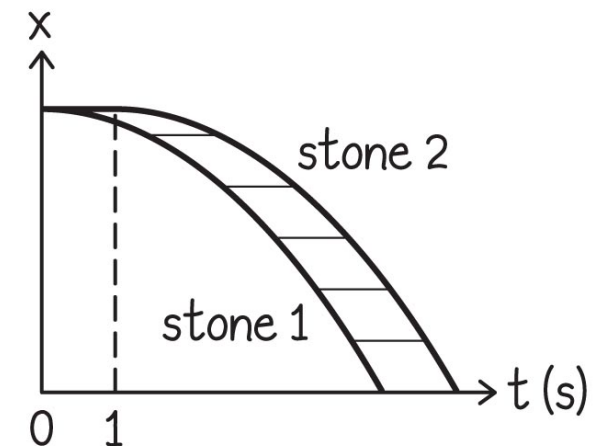
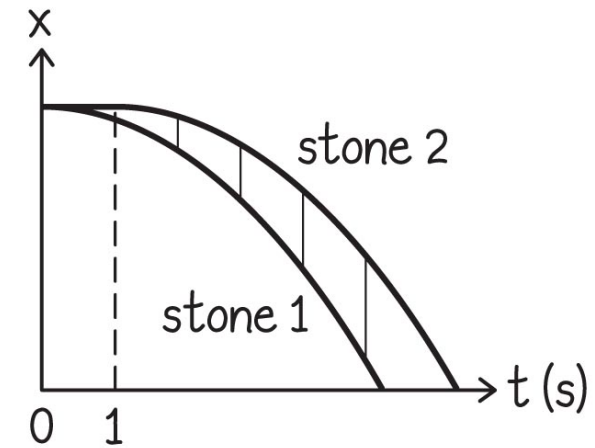
Two stones are released from rest at a certain height, one 1 s after the other.

- (a) Once the second stone is released, does the difference in their speeds increase, decrease, or stay the same?
- (b) Does their separation increase, decrease, or stay the same?
- (c) Is the time interval between the instants at which they hit the ground less than, equal to, or greater than 1 s? (Use  $x(t)$  curves to help you visualize this problem.)

# Chapter 3: Self-Quiz #1

## Answer

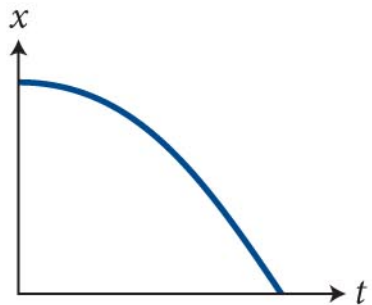
- (a) Both stones accelerate at about  $10 \text{ m/s}^2$ , so the speeds increase at the same rate, the **difference in the speeds remains the same**.
- (b) The separation increases because the speed of the first stone is always greater. As a result, for a given time interval the first stone always goes farther. (Position goes as  $v$  times  $t$ )
- (c) the second stone always remains 1 s behind, this is how time works



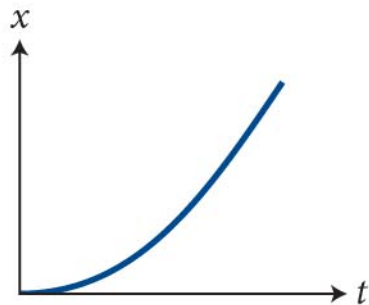
# Chapter 3: Self-Quiz #2

Which of the graphs in Figure 3.12 depict(s) an object that starts from rest at the origin and then speeds up in the positive  $x$  direction?

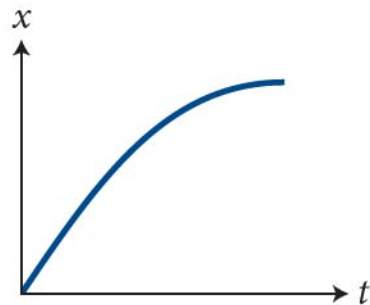
(a)



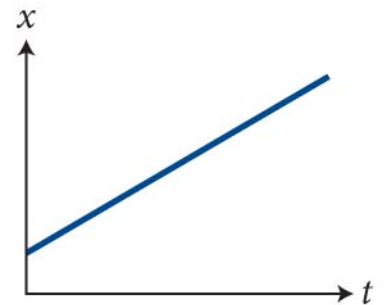
(b)



(c)



(d)

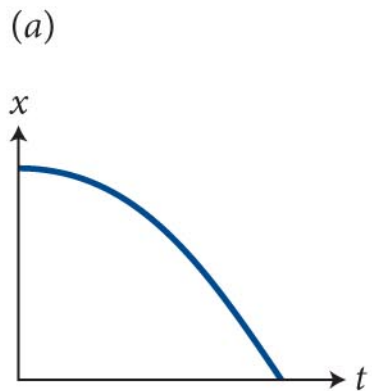




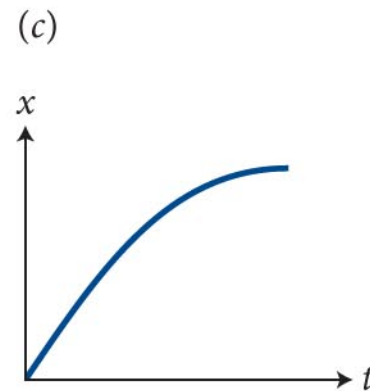
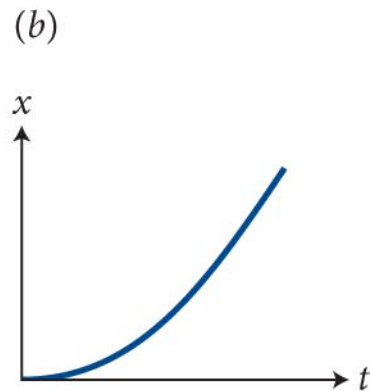
# Chapter 3: Self-Quiz #2

## Answer

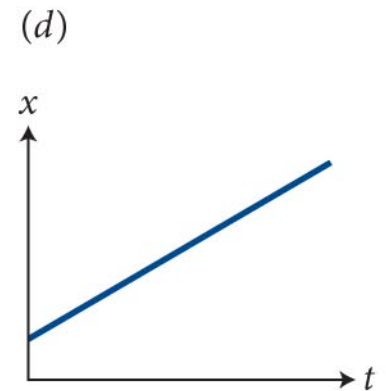
Choice *b* is the correct answer because its initial position is zero and the slope is initially zero but then increasing, indicating that the object speeds up.



speeds in  $-x$   
no start at origin



slows

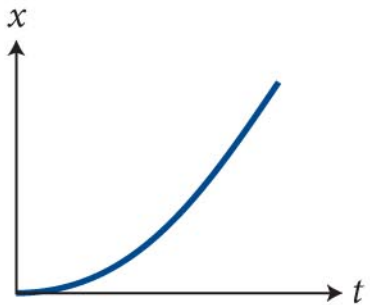


no accel  
no start at origin

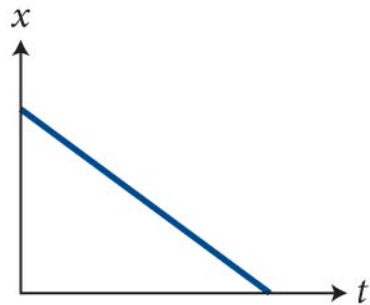
# Chapter 3: Self-Quiz #3

Which of the graphs in Figure 3.13 depict(s) an object that starts from a positive position with a positive  $x$  component of velocity and accelerates in the negative  $x$  direction?

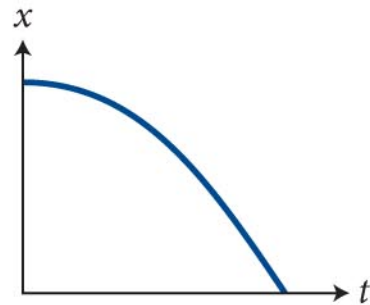
(a)



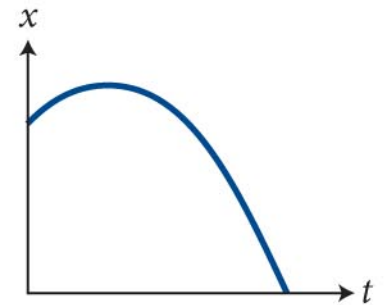
(b)



(c)



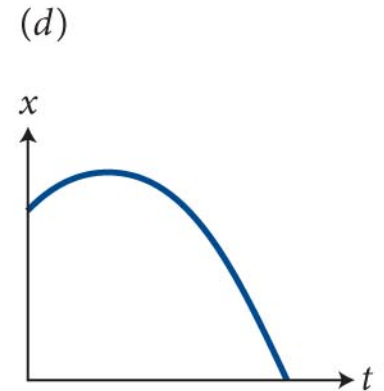
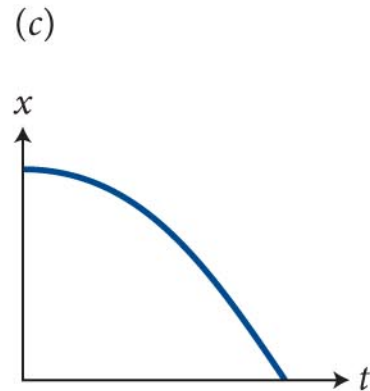
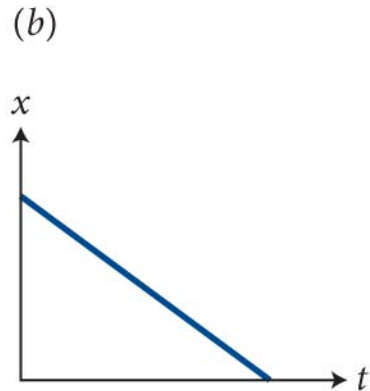
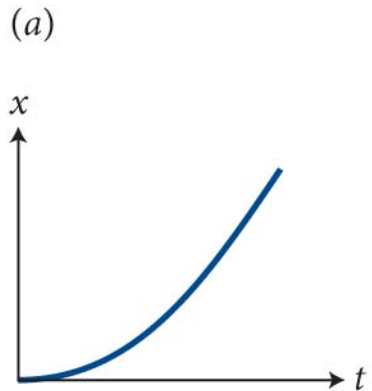
(d)



# Chapter 3: Self-Quiz #3

## Answer

Choice *d*. Choice *a* does not have a positive initial position. Choice *b* represents zero acceleration. Choice *c* represents zero initial velocity.



# Chapter 3: Acceleration

## Quantitative Tools

# Section 3.5: Motion with constant acceleration

## Section Goals

You will learn to

- Represent motion with constant acceleration using motion graphs and mathematics.
- Construct self-consistent position-versus-time, velocity-versus-time, and acceleration-versus-time graphs for specific motion situations.

## Section 3.5: Motion with constant acceleration

- We can write down the definition for the  $x$  component of *average acceleration*:

$$a_{x,av} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{x,f} - v_{x,i}}{t_f - t_i}$$

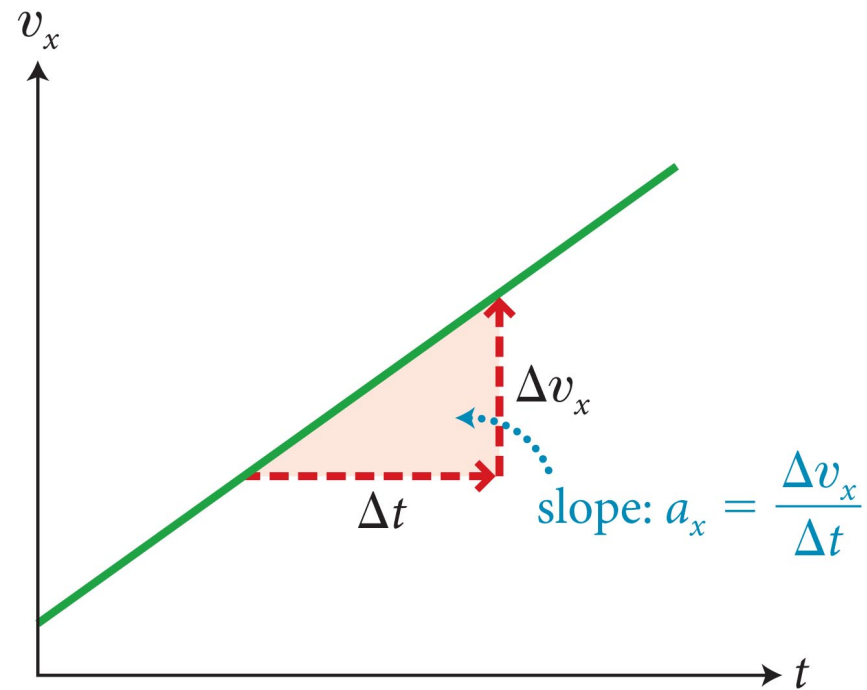
- Notice the similarity between this definition and the definition of average velocity in chapter 2:

$$v_{x,av} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

# Section 3.5: Motion with constant acceleration

- Now let us consider the motion of an object with **constant acceleration**:
  - For motion with constant acceleration,  $a_{x,av} = a_x$  and  $v_x(t)$  curve is a straight line.
  - Rewriting Equation we can get the  $x$ -component of final velocity:

$$v_{x,f} = v_{x,i} + a_x \Delta t \quad (\text{constant acceleration})$$



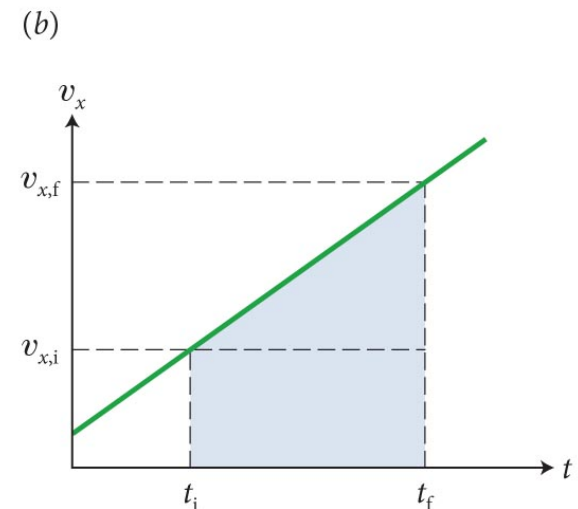
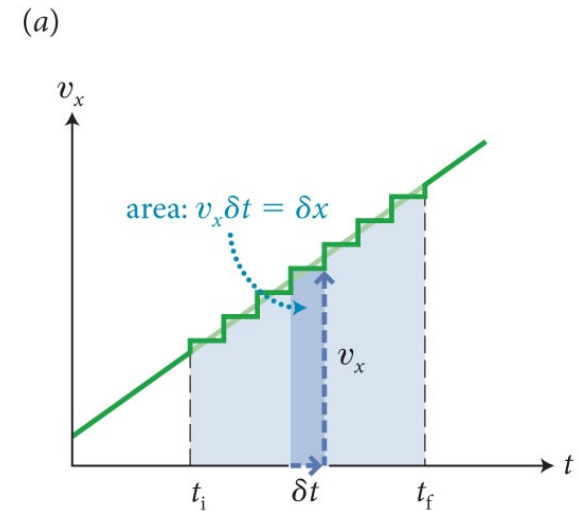
# Section 3.5: Motion with constant acceleration

- displacement is the area under the  $v_x(t)$  curve.
- for an object in motion with constant acceleration, the displacement ( $\Delta x = x_f - x_i$ ) in time interval ( $\Delta t = t_f - t_i$ ) is given by the area of the shaded trapezoid
- Setting  $t_i = 0$ , the object's final position can be written as

$$x_f = x_i + v_{x,i} t_f + \frac{1}{2} a_x t_f^2 \quad (\text{constant acceleration})$$

- we can determine the object's final velocity

$$v_{x,f} = v_{x,i} + a_x t_f \quad (\text{constant acceleration})$$





## Section 3.5: Motion with constant acceleration

- Since  $t_f$  is an arbitrary instant in time in the object's motion, we can drop the subscript  $f$  and rewrite as

$$x(t) = x_i + v_{x,i}t + \frac{1}{2}a_x t^2 \quad (\text{constant acceleration})$$

$$v_x(t) = v_{x,i} + a_x t \quad (\text{constant acceleration})$$

- This is easier with calculus, assuming constant  $a$

$$a = \frac{dv}{dt} \implies v = \int a dt = at + C$$

- Noting  $v(t=0) = v_i$ :  $v(t) = v_i + at$

- Once more:

$$x = \frac{dv}{dt} \implies x = \int v dt = v_i t + \frac{1}{2}at^2 + C'$$

- $C'$  is  $x(t=0)$  or  $x_i$


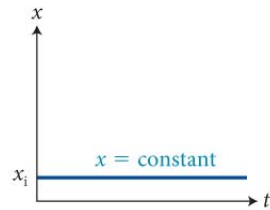
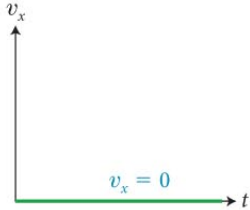
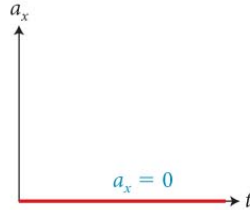

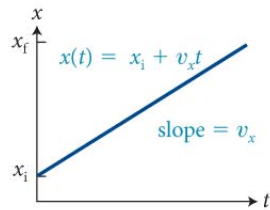
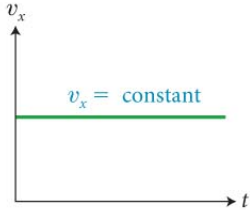
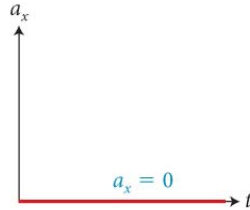
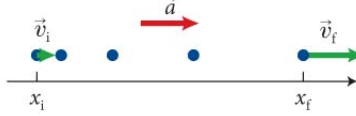
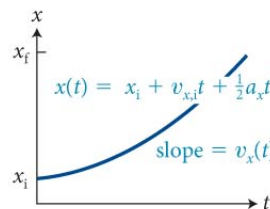
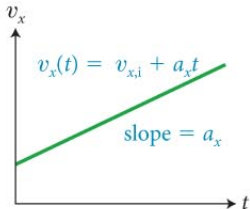
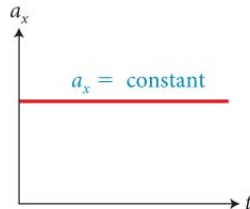
$$x(t) = x_i + v_i t + \frac{1}{2}at^2$$

# In terms of displacement

$$\Delta x = x_f - x_i = v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$

# Section 3.5: Motion with constant acceleration

**Table 3.1** Kinematics graphs for three basic types of motion

Motion diagram	Position versus time	Velocity versus time	Acceleration versus time
<p>At rest</p> 	 <p><math>x = \text{constant}</math></p>	 <p><math>v_x = 0</math></p>	 <p><math>a_x = 0</math></p>
<p>Constant velocity</p> 	 <p><math>x(t) = x_i + v_x t</math> slope = <math>v_x</math></p>	 <p><math>v_x = \text{constant}</math></p>	 <p><math>a_x = 0</math></p>
<p>Constant acceleration</p> 	 <p><math>x(t) = x_i + v_{x,i}t + \frac{1}{2}a_x t^2</math> slope = <math>v_x(t)</math></p>	 <p><math>v_x(t) = v_{x,i} + a_x t</math> slope = <math>a_x</math></p>	 <p><math>a_x = \text{constant}</math></p>

## Section 3.5: Motion with constant acceleration

### Example 3.4 Collision or not?

You are bicycling at a steady  $6.0 \text{ m/s}$  when someone suddenly walks into your path  $2.5 \text{ m}$  ahead. You immediately apply the brakes, which slow you down at  $6.0 \text{ m/s}^2$ . Do you stop in time to avoid a collision?

# Section 3.5: Motion with constant acceleration

## Example 3.4 Collision or not? (cont.)

### ① GETTING STARTED

In order to avoid a collision, you must come to a stop in less than 2.5 m.

Need to calculate the distance traveled under the given conditions. Is it more or less than 2.5 m?

# Section 3.5: Motion with constant acceleration

## Example 3.4 Collision or not? (cont.)

② DEVISE PLAN I have equations for displacement, but I don't know the time interval  $\Delta t$ .

From the definition of acceleration:

$$\Delta t = (v_{x,f} - v_{x,i})/a_x$$

which contains no unknowns on the right side.

# Section 3.5: Motion with constant acceleration

## Example 3.4 Collision or not? (cont.)

**3** EXECUTE PLAN Substituting the expression for the time interval gives the  $x$  component of the displacement necessary to stop:

$$\Delta x = v_{x,i} \frac{v_{x,f} - v_{x,i}}{a_x} + \frac{1}{2} a_x \left( \frac{v_{x,f} - v_{x,i}}{a_x} \right)^2 = \frac{v_{x,f}^2 - v_{x,i}^2}{2a_x} \quad (1)$$



# Section 3.5: Motion with constant acceleration

## Example 3.4 Collision or not? (cont.)

### ③ EXECUTE PLAN

With  $+x$  along the direction of the motion

$$v_{x,i} = +6.0 \text{ m/s}$$

$$v_{x,f} = 0$$

$$a_x = -6.0 \text{ m/s}^2.$$

$$\Delta x = \frac{0 - (+6.0 \text{ m/s})^2}{2(-6.0 \text{ m/s}^2)} = +3.0 \text{ m}$$

more than the 2.5 m required. You will totally collide. ✓

## Section 3.5: Motion with constant acceleration

- Notice: rearranging we can find the final velocity of an object under constant acceleration over a certain displacement ( $\Delta x$ ):

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x \Delta x \quad (\text{constant acceleration})$$

- advantage: don't need to know time!
- In general: motion is *overdetermined* by  $x, v, a, t$  – can eliminate 1

# Checkpoint 3.10



**3.10** Determine the velocity of the stone dropped from the top of the Empire State Building in Example 3.2 just before the stone hits the ground.

(The empire state building is  $\sim 300\text{m}$  tall)



$$v_{x,f}^2 = v_{x,i}^2 + 2a_x \Delta x \quad (\text{constant acceleration})$$

initial velocity is zero

displacement  $\sim 300$  m

acceleration  $\sim 10$  m/s<sup>2</sup>

$$v_{x,f} \sim 80 \text{ m/s}$$

# Section 3.6: Free-fall equations

## Section Goals

- Model free-fall motion using the concept of gravity and the definitions of velocity and acceleration.
- Manipulate the equations for free-fall into a form that allows the prediction of the future motion of an object from its present state of motion.

## Section 3.6: Free-fall equations

- The magnitude of the **acceleration due to gravity** is designated by the letter  $g$ :

$$g \equiv \left| \vec{a}_{\text{free fall}} \right|$$

- Near Earth's surface  $g = 9.8 \text{ m/s}^2$ .
- The direction of the acceleration is downward, and if we chose a positive axis pointing upward,  $a_x = -g$ .
- If an object is dropped from a certain height with zero velocity along an **upward-pointing x-axis**, then

$$x_f = x_i - \frac{1}{2} g t_f^2$$

$$v_{x,f} = -g t_f$$

## Section 3.6: Free-fall equations

### Example 3.5 Dropping the ball

Suppose a ball is dropped from height  $h = 20$  m above the ground. How long does it take to hit the ground, and what is its velocity just before it hits?

# Section 3.6: Free-fall equations

## Example 3.5 Dropping a ball (cont.)

### 1 GETTING STARTED

$x$  axis that points upward

origin at the initial position of the ball

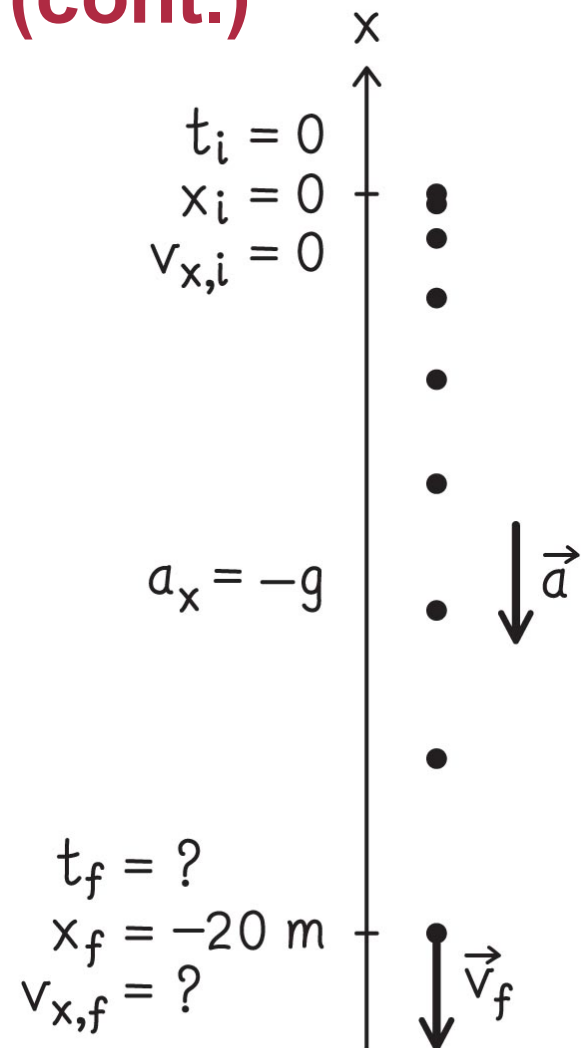
#### assumptions:

released from rest ( $v_{x,i} = 0$  at  $t_i = 0$ )

ignore air resistance

initial conditions are

$$t_i = 0, x_i = 0, v_{x,i} = 0$$





# Section 3.6: Free-fall equations

## Example 3.5 Dropping a ball (cont.)

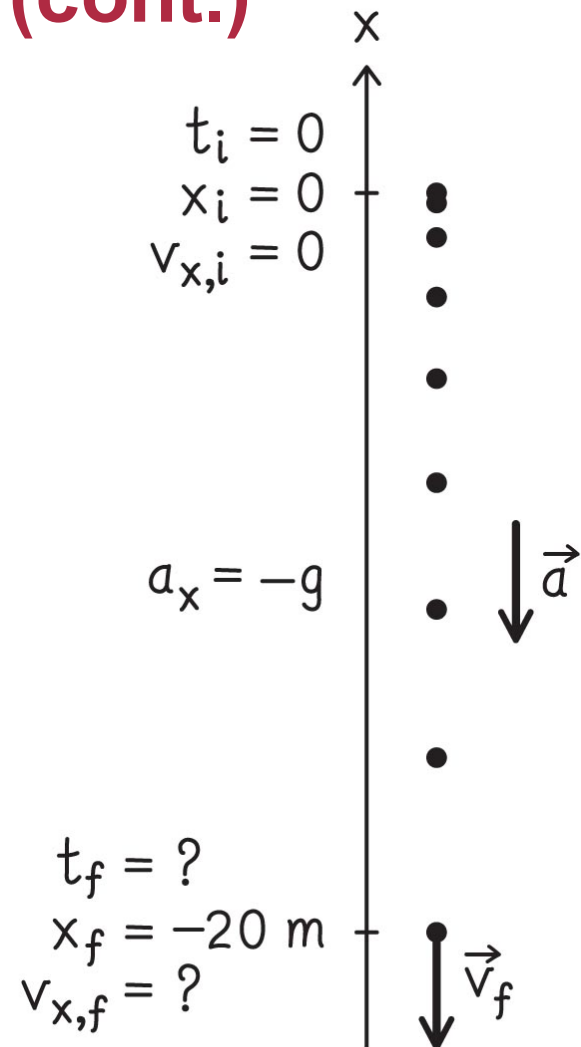
### 1 GETTING STARTED

final position  $x_f$  at instant  $t_f$  is a distance  $h$  below the initial position

just before impact at instant  $t_f$ , the final conditions are

$$t_f = ?, x_f = -h, v_{x,f} = ?$$

acceleration is negative,  $a_x = -g$ .



# Section 3.6: Free-fall equations

## Example 3.5 Dropping a ball (cont.)

### ② DEVISE PLAN

Acceleration is constant, so our equations work.

Gives me two equations & two unknowns:  $t_f$  and  $v_{x,f}$

$$x(t) = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_f = v_i + a t$$

# Section 3.6: Free-fall equations

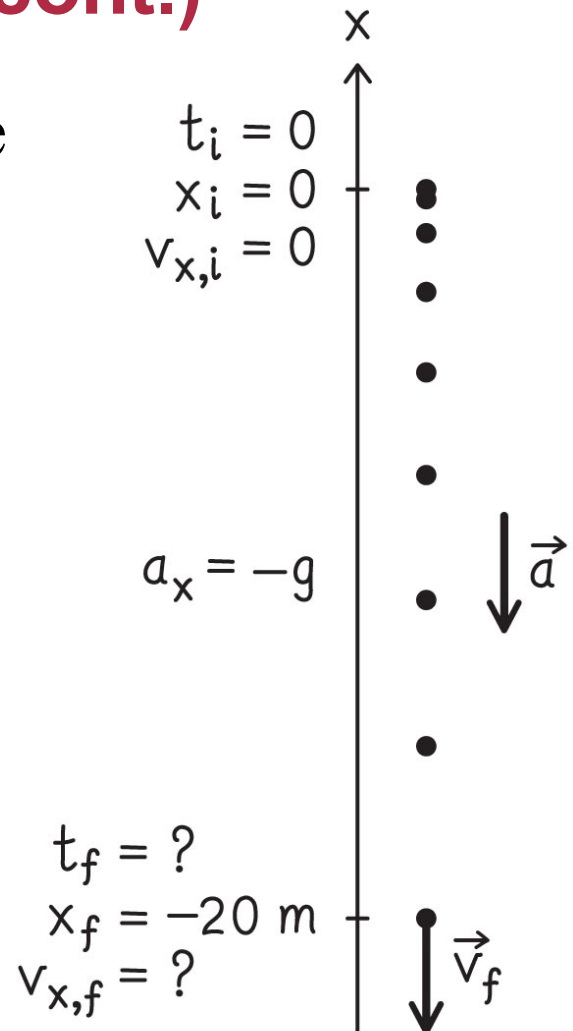
## Example 3.5 Dropping a ball (cont.)

**3** EXECUTE PLAN Substituting the initial and final conditions into  $x(t)$ :

$$-h = 0 + 0 - \frac{1}{2}gt_f^2 = -\frac{1}{2}gt_f^2$$

and so

$$t_f = \sqrt{\frac{2h}{g}}$$

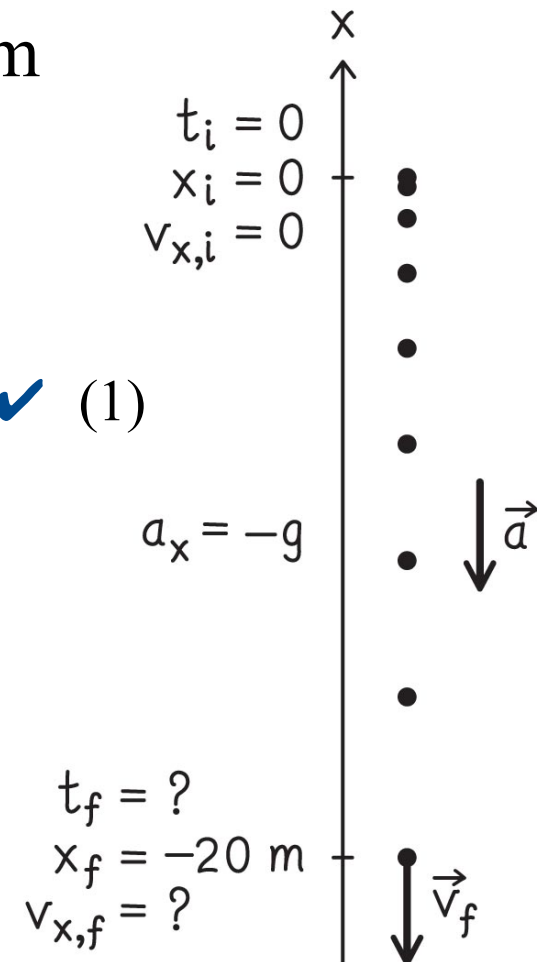


# Section 3.6: Free-fall equations

## Example 3.5 Dropping a ball (cont.)

**3** EXECUTE PLAN Substituting  $h = 20$  m and  $g = 9.8$  m/s<sup>2</sup> into  $v(t)$ :

$$\Delta t = t_f - t_i = \sqrt{\frac{2h}{g}} - 0 = \sqrt{\frac{2(20 \text{ m})}{9.8 \text{ m/s}^2}} = \sqrt{4.0 \text{ s}^2} = 2.0 \text{ s} \quad \checkmark \quad (1)$$



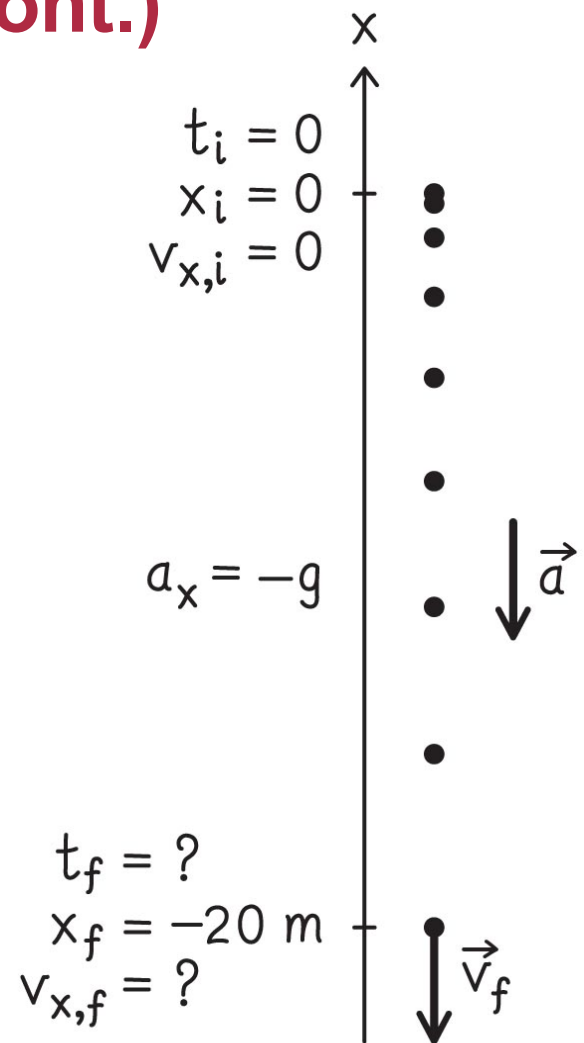
# Section 3.6: Free-fall equations

## Example 3.5 Dropping a ball (cont.)

### 3 EXECUTE PLAN

Because the ball starts from rest:

$$\begin{aligned}v_{x,f} &= 0 - gt_f = -gt_f = -(9.8 \text{ m/s}^2)(2.0 \text{ s}) \\ &= -20 \text{ m/s. } \checkmark\end{aligned}$$



# Section 3.6: Free-fall equations

## Example 3.5 Dropping a ball (cont.)

### ④ EVALUATE RESULT

Time is reasonable based on everyday experience

Final velocity  $\Delta x_f = -20$  m/s also makes sense:

- negative because it points in the negative  $x$  direction
- if the ball was at a constant speed of 20 m/s, it would cover the 20-m distance in 1 s. It moves at that speed only at the end of the drop, so it takes longer to fall.

# Section 3.7: Inclined planes

## Section Goals

You will learn to

- Identify that one-dimensional motion along an incline plane can be related to free-fall motion along a non-vertical direction.
- Establish that purely horizontal and purely vertical motion are the special cases of motion along an incline plane.

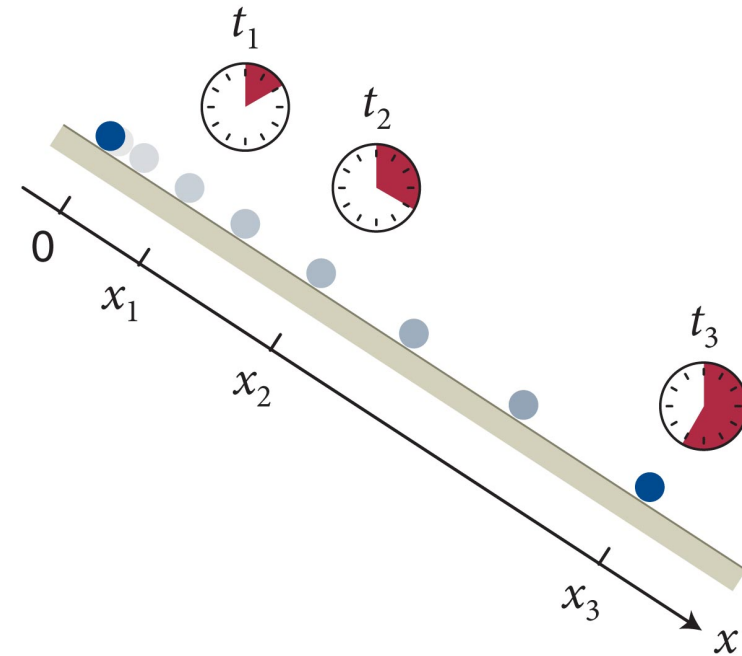
# Section 3.7: Inclined planes

- Galileo used inclined planes to study motion of objects that are accelerated due to gravity:
  - He found that when a ball rolls down an incline starting at rest, the ratio of the distance traveled to the square of the amount of time needed to travel that distance is constant:

$$\frac{x_1}{t_1^2} = \frac{x_2}{t_2^2} = \frac{x_3}{t_3^2}$$

- Using this and setting  $x_i = 0$  and  $t_i = 0$  we can show that this ratio is proportional to  $a_x$ :

$$\frac{x_f}{t_f^2} = \frac{1}{2} a_x$$

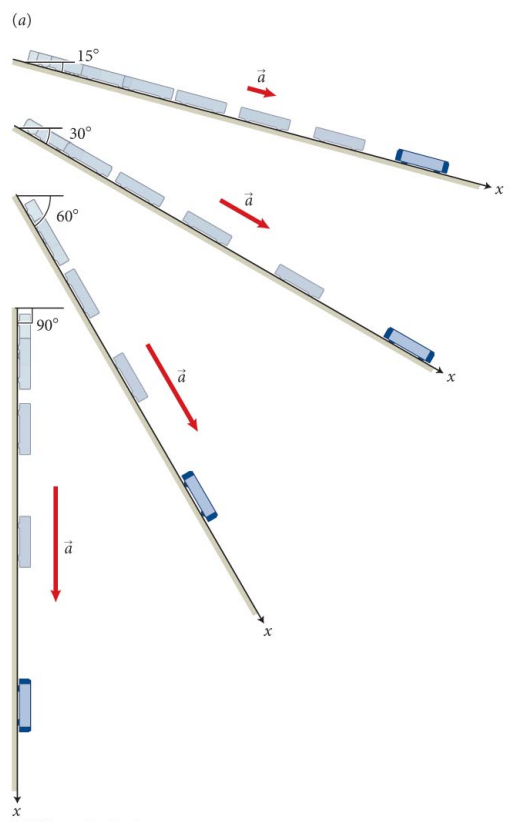




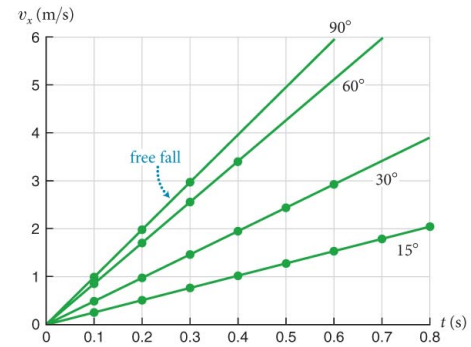
# Section 3.7: Inclined planes

- Galileo observed that
  - For each value of the angle  $\theta$ ,  $a_x$  along the incline is a constant.
  - $a_x$  along the incline increases as  $\theta$  increases.
  - Experimentally we can determine that the  $x$  component of the acceleration along the incline obey the relationship

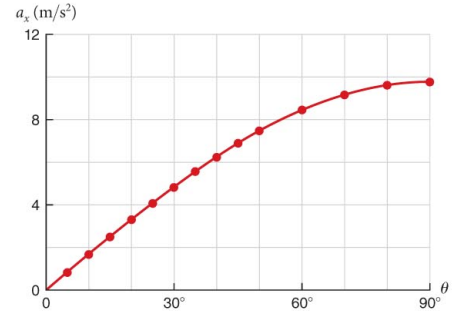
$$a_x = +g \sin \theta$$



(b) Component of cart's velocity along incline as function of time



(c) Component of acceleration along incline as function of angle of incline



# Inclined planes

- establishes gravity is vertical, constant acceleration
- it is a vector, and only the *vertical component* matters
- for inclined plane, the component along the plane is  
 $a_x = +g \sin \theta$

## 3.68 (HW2)

- A block has an initial speed of  $8.0 \text{ m/s}$  up an inclined plane that makes an angle of  $35^\circ$  with the horizontal. Ignoring friction, what is the block's speed after it has traveled  $2.0\text{m}$ ?

# Section 3.8: Instantaneous acceleration

## Section Goals

You will learn to

- Generalize the mathematical definition of the average acceleration of a moving object to instantaneous acceleration by use of a limiting process.
- Represent motion with continuous changes in velocity using motion graphs and mathematics.
- Relate the concept of a tangent line on a velocity-versus-time graph with the instantaneous acceleration.

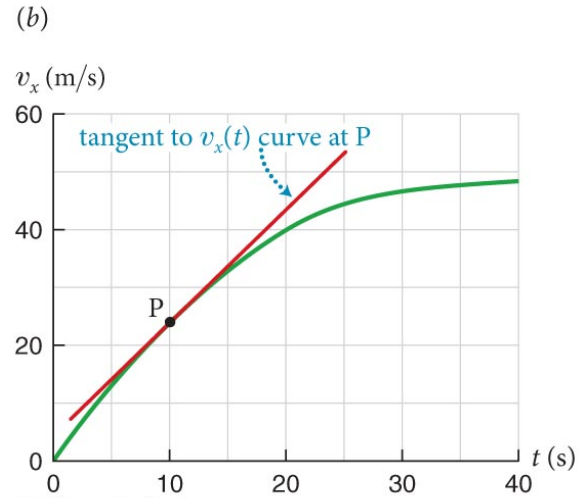
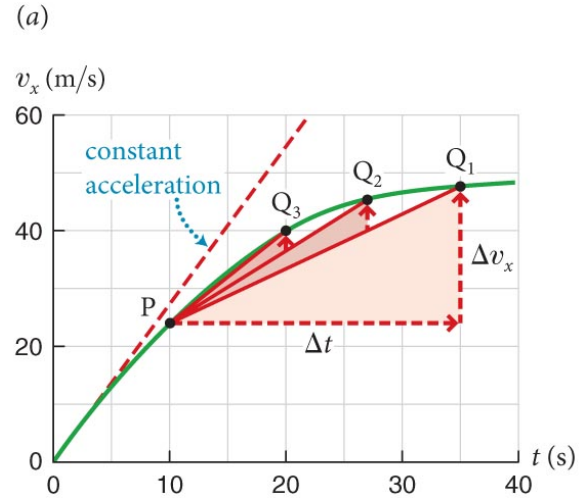
# Section 3.8: Instantaneous acceleration

- What if acceleration is not constant? Use the calculus we did.
  - The figure shows the  $v_x(t)$  curve for a motion where the acceleration is not constant.
  - The instantaneous acceleration  $a_x$  is the slope of the tangent of the  $v_x(t)$  curve at time  $t$ :

$$a_x = \frac{dv_x}{dt}$$

• Or

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt} \left( \frac{dx}{dt} \right) \equiv \frac{d^2x}{dt^2}$$

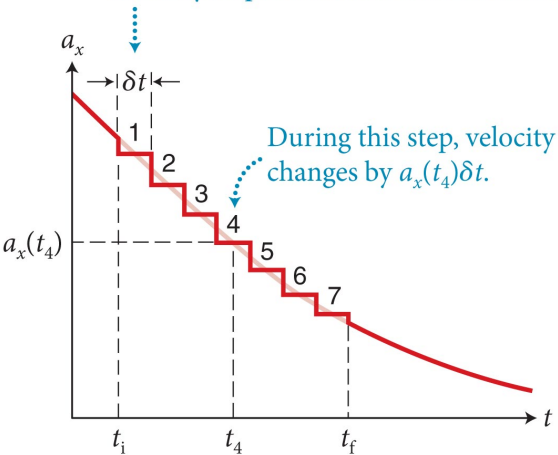


# Section 3.8: Instantaneous acceleration

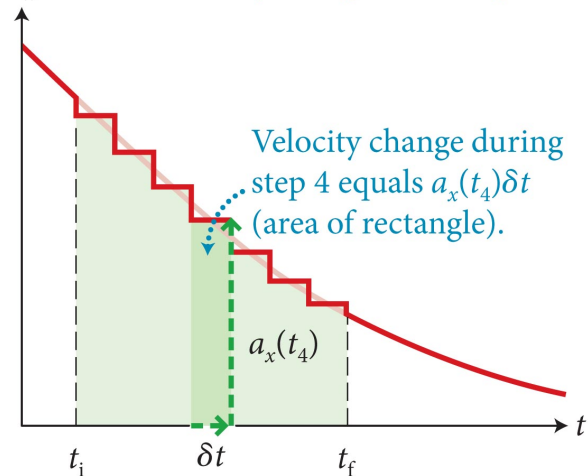
- To find the change in velocity during the time interval ( $\Delta t$ ), we can use the area under the  $a_x(t)$  curve in the figure.
- Although, acceleration is not constant, we can divide motion into small intervals of  $\Delta t$  in which it is constant.
- In the limit  $\Delta t \rightarrow 0$ , we can find

$$\Delta v_x = \int_{t_i}^{t_f} a_x(t) dt$$

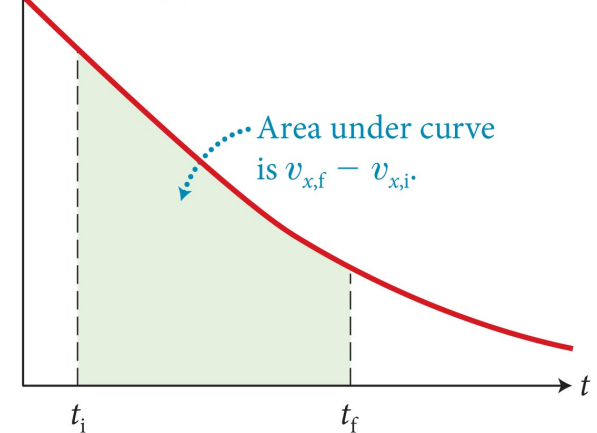
(a) To find change in velocity over time interval, we approximate acceleration curve by steps of constant acceleration.



(b) Velocity change over entire interval is sum of velocity changes over steps.



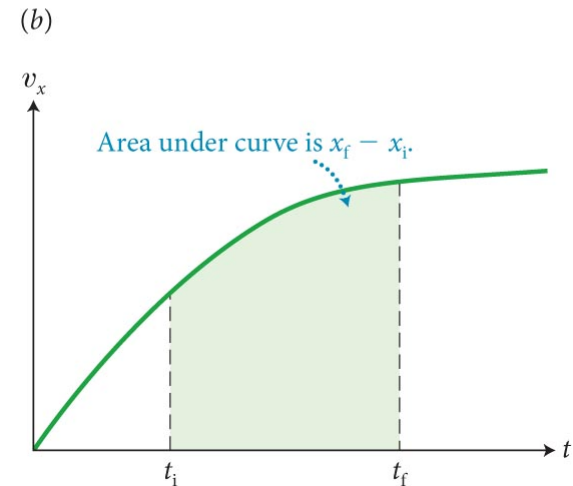
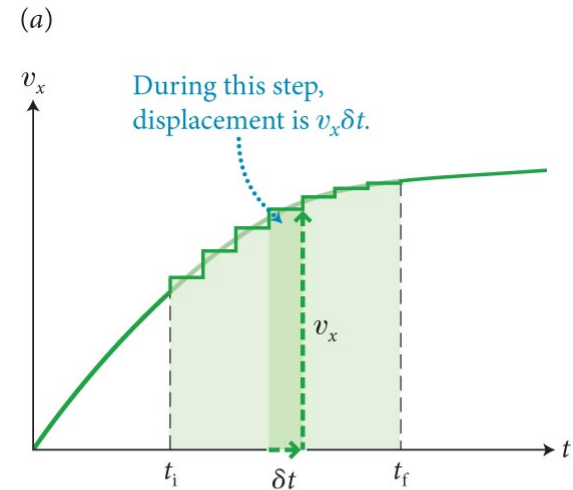
(c) As we make  $\delta t$  smaller, approximation improves and sum of rectangular areas approaches area under curve.



# Section 3.8: Instantaneous acceleration

- Once we know the velocity, we can use the same approach to obtain displacement:

$$\Delta x = \int_{t_i}^{t_f} v_x(t) dt$$



# Checkpoint 3.14



**3.14** Take the first and second time derivatives of  $x_f$  in Eq. 3.9. What do you notice?

$$x_f = x_i + v_{x,i} t_f + \frac{1}{2} a_x t_f^2 \quad (\text{constant acceleration})$$



## 3.80 (HW2)

- The acceleration of a particular car during braking has magnitude  $bt$ , where  $t$  is the time in seconds from the instant the car begins braking, and  $b = 1.6 \text{ m/s}^3$ . If the car has an initial speed of  $45 \text{ m/s}$ , how far does it travel before it stops?
- If the forward direction is positive, this implies
  - $a = -bt$                       and                       $v_0 = +45 \text{ m/s}$
- Get  $v(t)$  by integrating,  $x(t)$  by integrating again

# Chapter 3: Summary

## Concepts: Accelerated motion

- If the velocity of an object is changing, the object is **accelerating**. The  $x$  component of an object's **average acceleration** is the change in the  $x$  component of its velocity divided by the time interval during which this change takes place.
- The  $x$  component of the object's **instantaneous acceleration** is the  $x$  component of its acceleration at any given instant.
- A **motion diagram** shows the positions of a moving object at equally spaced time intervals.

# Chapter 3: Summary

## Quantitative Tools: Accelerated motion

- The  $x$  component of the **average acceleration** is

$$a_{x,\text{av}} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{x,f} - v_{x,i}}{t_f - t_i}$$

- The  $x$  component of the **instantaneous acceleration** is

$$a_x \equiv \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

- The  $x$  component of the change in velocity over a time interval is given by

$$\Delta v_x = \int_{t_i}^{t_f} a_x(t) dt$$

- The  $x$  component of the displacement over a time interval is given by

$$\Delta x = \int_{t_i}^{t_f} v_x(t) dt$$

# Chapter 3: Summary

## Concepts: Motion with constant acceleration

- If an object has constant acceleration, the  $v_x(t)$  curve is a straight line that has a nonzero slope and the  $a_x(t)$  curve is a horizontal line.

# Chapter 3: Summary

## Quantitative Tools: Motion with constant acceleration

- If an object moves in the  $x$  direction with constant acceleration  $a_x$  starting at  $t = 0$ , with initial velocity  $v_{x,i}$  at initial position  $x_i$ , its  $x$  coordinate at any instant  $t$  is given by

$$x(t) = x_i + v_{x,i}t + \frac{1}{2}a_x t^2$$

- The  $x$  component of its instantaneous velocity is given by

$$v_x(t) = v_{x,i} + a_x t$$

- And the  $x$  component of its final velocity is given by

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x \Delta x$$

# Chapter 3: Summary

## Concepts: Free fall and projectile motion

- An object subject only to gravity is in **free fall**. All objects in free fall near the surface of Earth have the same acceleration, which is directed downward. We call this acceleration the **acceleration due to gravity** and denote its magnitude by the letter  $g$ .
- An object that is launched but not self-propelled is in **projectile motion**. Once it is launched, it is in free fall. The it follows is called its **trajectory**.

# Chapter 3: Summary

## Quantitative Tools: Free fall and projectile motion

- The magnitude  $g$  of the downward acceleration due to gravity is

$$g = \left| \vec{a}_{\text{free fall}} \right| = 9.8 \text{ m/s}^2 \text{ (near Earth's surface)}$$

# Chapter 3: Summary

## Concepts: Motion along an inclined plane

- An object moving up or down an inclined plane on which friction is negligible has a constant acceleration that is directed parallel to the surface of the plane and points downward along the surface.



# Chapter 3: Summary

## Quantitative Tools: Free fall and projectile motion

- When friction is negligible, the  $x$  component of acceleration  $a_x$  for an object moving on an inclined plane that rises at an angle  $\theta$  above the horizontal is

$$a_x = +g \sin \theta$$

when the  $x$  axis is directed downward along the plane.