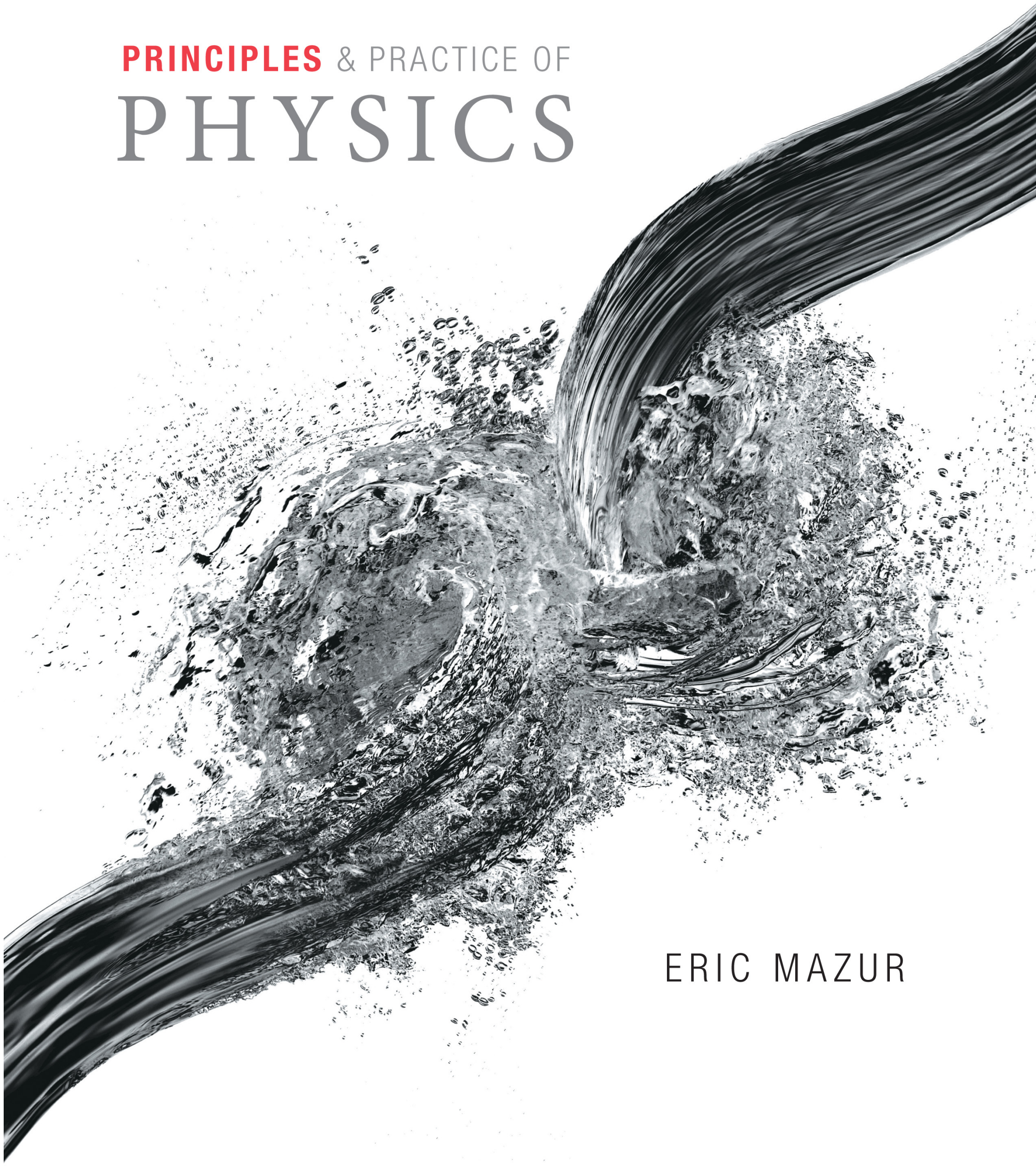
Chapter 5
Energy



Various & Sundry

- Exam 1
  - relax
  - ch. 2-4, 5.1-6 (heavy on 2-4)
  - will be in here at the usual lecture time
  - multiple choice
    - mix of quantitative and qualitative

  - *probably* about 20 questions
  - will put practice problems on MasteringPhysics
  - will get a formula sheet

Concepts

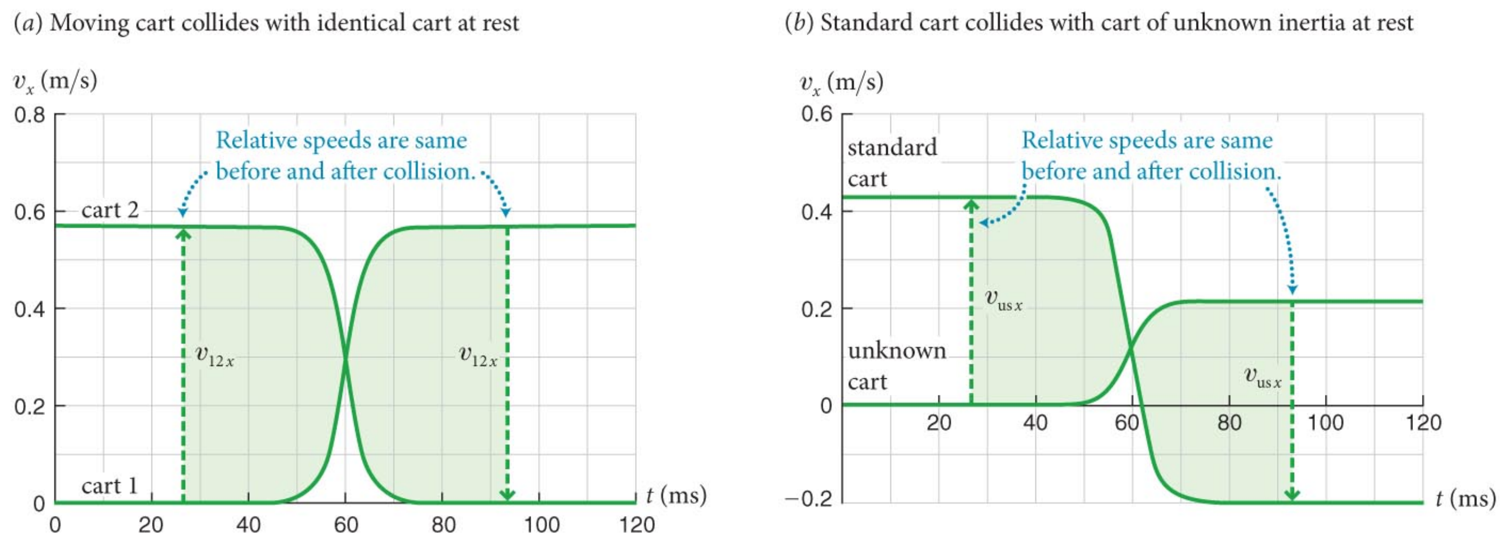
Section 5.1: Classification of collisions

- Below are the $v_x(t)$ curves for two carts colliding.
  - Notice that the velocity differences before and after collisions are highlighted.
- **Relative velocity** of the carts:

$$\vec{\upsilon}_{12} \equiv \vec{\upsilon}_2 - \vec{\upsilon}_1 \text{ is the velocity of cart 2 relative to cart 1.}$$

- **Relative speed** of the carts:

$$\vec{\upsilon}_{12} \equiv \left|\vec{\upsilon}_2 - \vec{\upsilon}_1\right| \text{ is the speed of cart 2 relative to cart 1.}$$



Section 5.1: Classification of collisions

- **Elastic collision**: A collision in which the relative speeds before and after the collision are the same.
- **Inelastic collision**: A collision in which the relative speed after the collision is lower than before the collision.
- **Totally inelastic collision**: A special type of inelastic collision in which the two objects stick together (i.e., relative speed is reduced to zero).

Checkpoint 5.2

5.2 (*a*) An outfielder catches a baseball. Is the collision between ball and glove elastic, inelastic, or totally inelastic?

totally inelastic, relative speed after is zero

(*b*) When a moving steel ball 1 collides head-on with a steel ball 2 at rest, ball 1 comes to rest and ball 2 moves away at the initial speed of ball 1. Which type of collision is this?

elastic, relative speed is unchanged

(*c*) Is the sum of the momenta of the two colliding objects constant in part *a*? In part *b*?

only depends on whether interactions are outside system

a – glove interacts with player, non-isolated, sum not constant

b – ignoring friction, isolated, sum constant

Section 5.1: Classification of collisions

Exercise 5.1 Classifying collisions

Are the following collisions elastic, inelastic, or totally inelastic?

(a) A red billiard ball moving at $v_{\mathrm{r}\,x,\mathrm{i}} = +2.2$ m/s hits a white billiard ball initially at rest.

After the collision, the red ball is at rest and the white ball moves at $v_{\mathrm{w}\,x,\mathrm{f}} = +1.9$ m/s.

Exercise 5.1 Classifying collisions (cont.)

SOLUTION

(a) The initial relative speed is

$$v_{\mathrm{wr,i}} = |v_{\mathrm{r}\,x,\mathrm{i}} - v_{\mathrm{w}\,x,\mathrm{i}}| = |+2.2\ \mathrm{m/s} - 0| = 2.2\ \mathrm{m/s}$$

the final relative speed is

$$v_{\mathrm{wr,f}} = |v_{\mathrm{r}\,x,\mathrm{f}} - v_{\mathrm{w}\,x,\mathrm{f}}| = |0 - 1.9\ \mathrm{m/s}| = 1.9\ \mathrm{m/s}$$

Final is lower than the initial, which means the collision is inelastic. ✔

Section 5.1: Classification of collisions

Exercise 5.1 Classifying collisions (cont.)

(*b*) Cart 1 moving along a track at $v_{1x,\mathrm{i}} = +1.2$ m/s hits cart 2 initially at rest.

After the collision, the two carts move at $v_{1x,\mathrm{f}} = +0.4$ m/s and $v_{2x,\mathrm{f}} = +1.6$ m/s.

Exercise 5.1 Classifying collisions (cont.)

SOLUTION

(*b*)

$$v_{12\mathrm{i}} = |v_{2x,\mathrm{i}} - v_{1x,\mathrm{i}}| = |0 - (+1.2\ \mathrm{m/s})| = 1.2\ \mathrm{m/s};$$

$$v_{12\mathrm{f}} = |v_{2x,\mathrm{f}} - v_{1x,\mathrm{f}}| = |+1.6\ \mathrm{m/s} - (+0.4\ \mathrm{m/s})| = 1.2\ \mathrm{m/s}.$$

Because the relative speeds are the same, the collision is elastic. ✔

Exercise 5.1 Classifying collisions (cont.)

(*c*) A piece of putty moving at $v_{\mathrm{p}\,x,\mathrm{i}} = +22$ m/s hits a wooden block moving at $v_{\mathrm{b}\,x,\mathrm{i}} = +1.0$ m/s. After the collision, the two move at $v_{x,\mathrm{f}} = +1.7$ m/s.

Exercise 5.1 Classifying collisions (cont.)

SOLUTION

(*c*) After the collision, both the putty and the block travel at the same velocity, making their relative speed zero.

It was obviously not zero before the collision.

The collision is totally inelastic.✔

Section 5.2: Kinetic energy

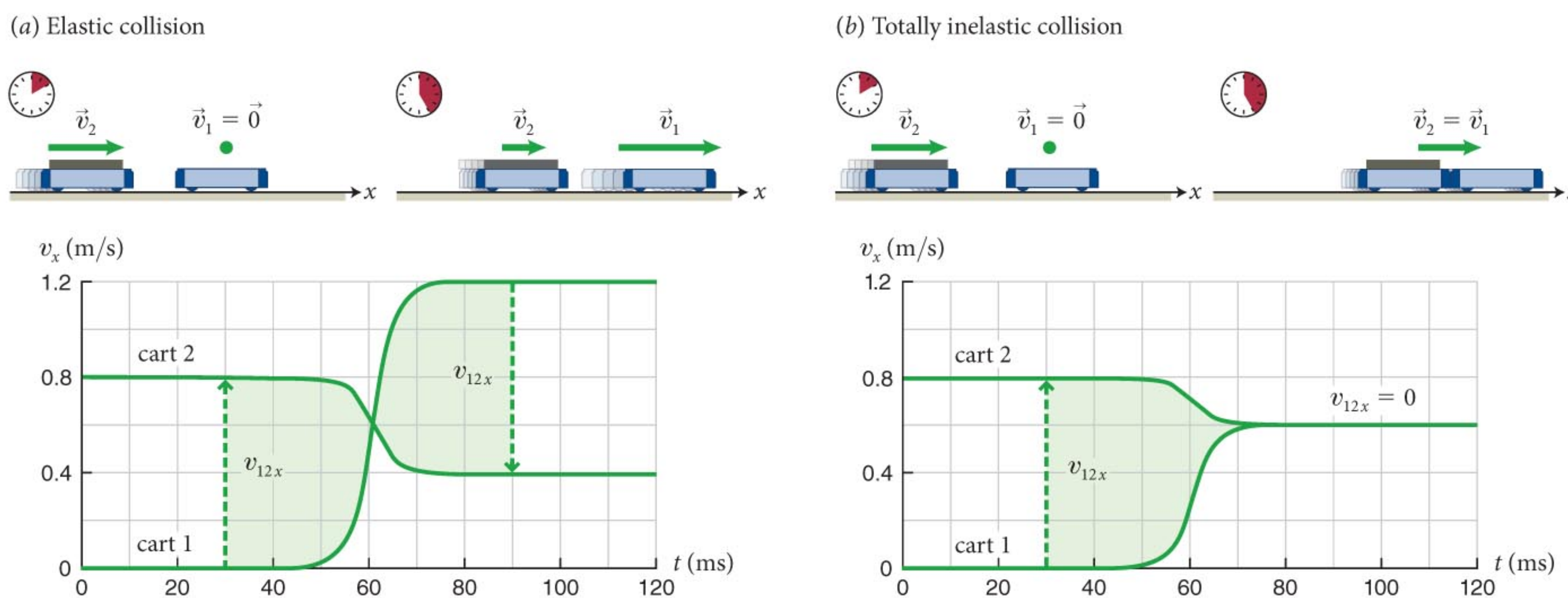
Section Goals

You will learn to

- Quantify the energy due to the motion of an object.
- Recognize that kinetic energy is a scalar quantity.
- Calculate the kinetic energy of an object from its inertia and speed.

Section 5.2: Kinetic energy

- The quantity, $K = \frac{1}{2}mv^2$ is called **kinetic energy** of the object, that is, "energy" associated with motion.
- Let us calculate the kinetic energy of the carts before and after the collisions (elastic collision and a totally inelastic collision) shown in the figure.



Section 5.2: Kinetic energy

Table 5.1 Kinetic energy in elastic and totally inelastic collisions

		ELASTIC				TOTALLY INELASTIC			
	Inertia m (kg)	Velocity v_x (m/s)		Kinetic energy $\frac{1}{2}mv^2$ ($\text{kg} \cdot \text{m}^2/\text{s}^2$)		Velocity v_x (m/s)		Kinetic energy $\frac{1}{2}mv^2$ ($\text{kg} \cdot \text{m}^2/\text{s}^2$)	
		before	*after*	*before*	*after*	*before*	*after*	*before*	*after*
Cart 1	0.12	0	+1.2	0	0.086	0	+0.60	0	0.022
Cart 2	0.36	+0.80	+0.40	0.12	0.029	+0.80	+0.60	0.12	0.065
Relative speed		0.80	0.80			0.80	0		
Kinetic energy of system				0.12	0.12			0.12	0.087

- Table 5.1 gives the initial and final kinetic energies.
- In general we observe:

  In an *elastic* collision, the sum of the kinetic energies of the object before is the same as the sum of kinetic energies after the collision.

Checkpoint 5.3

5.3 Is kinetic energy an extensive quantity?

Yes – it depends on the system size. Say an object has two parts, 1 and 2. The total kinetic energy is

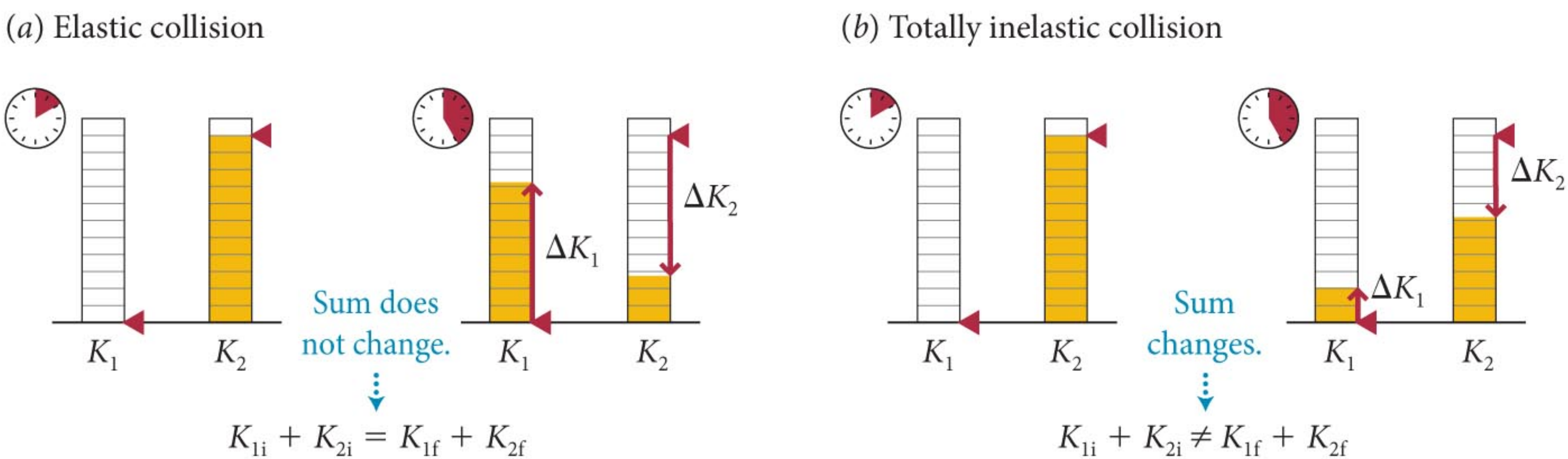
$$K = \tfrac{1}{2}(m_1+m_2)v^2$$

this is equal to the sum of the kinetic energies of the two parts

$$K = K_1 + K_2 = \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2$$

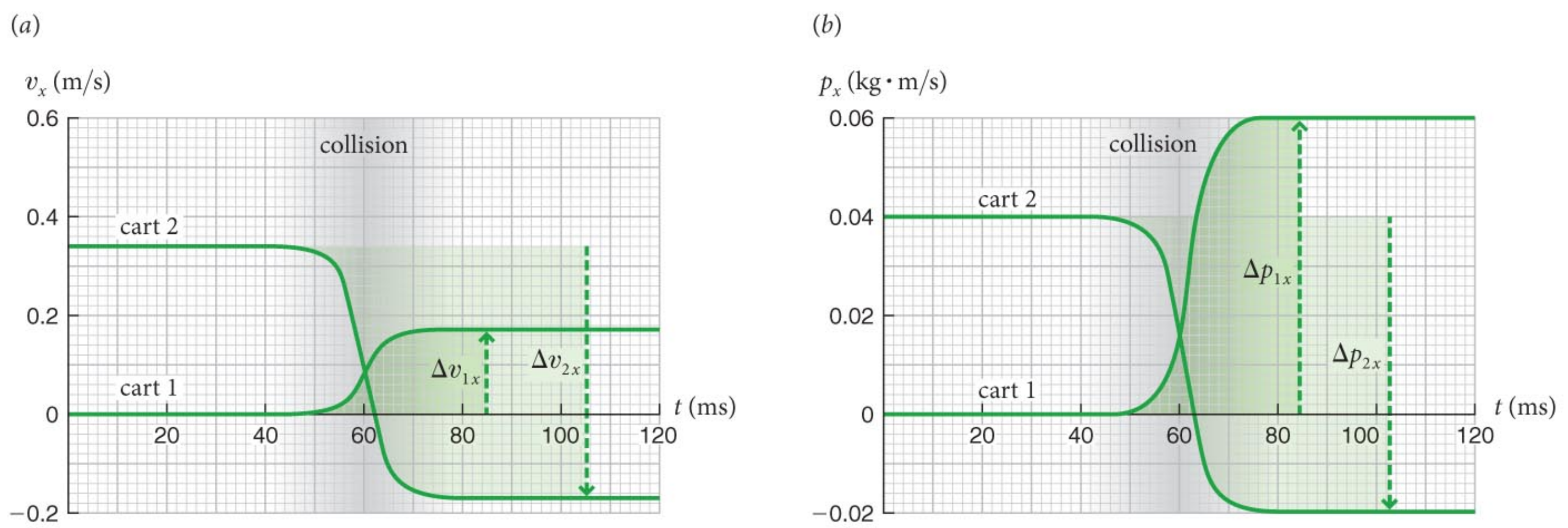
Section 5.2: Kinetic energy

- Because kinetic energy is a **scalar** extensive quantity, bar diagrams are a good way to visually represent changes in this quantity.
- Reinforces the main idea: bookkeeping
- Diagrams below from previous collisions



Example 5.2 Carts colliding

(*a*) Is the collision in figure below elastic, inelastic, or totally inelastic? How can you tell? (*b*) Verify your answer by comparing the initial kinetic energy of the two-cart system with the final kinetic energy.



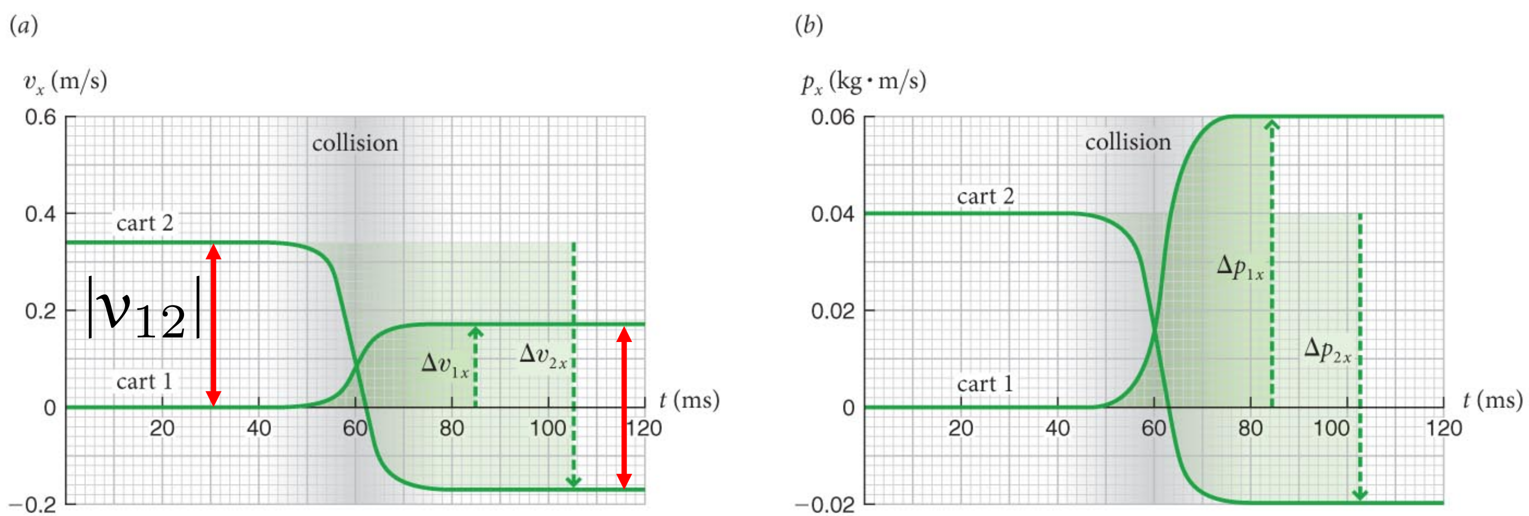
Example 5.2 Carts colliding (cont.)

❶ GETTING STARTED

Initial and final relative speeds are the same!

→ must be elastic

Does kinetic energy give the same conclusion?



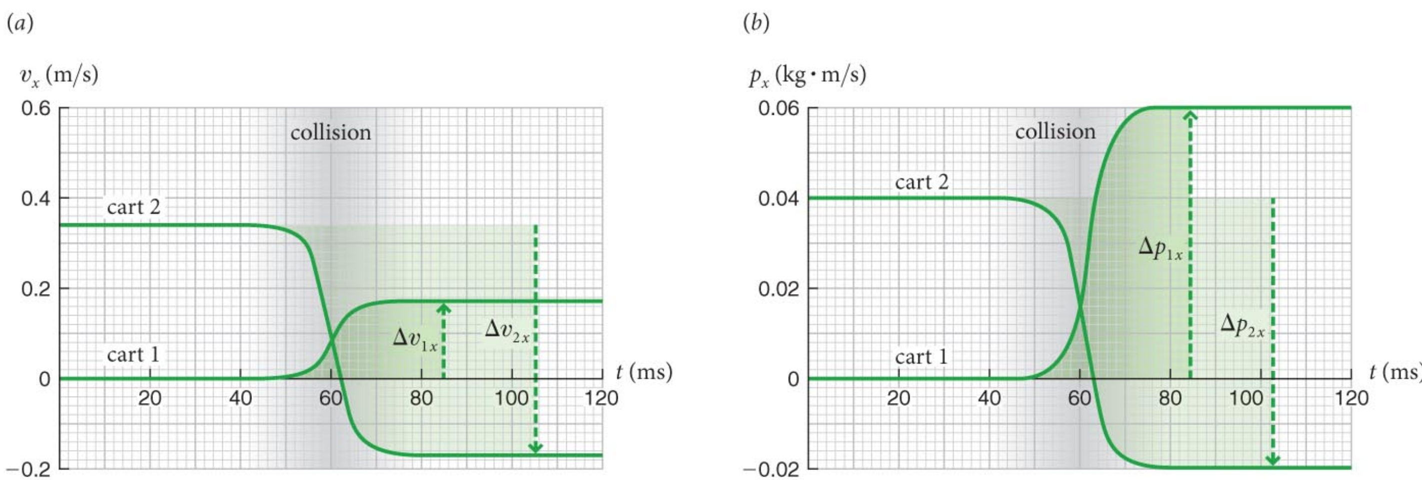
Example 5.2 Carts colliding (cont.)

❷ DEVISE PLAN momentum: need velocities of the carts to get relative speeds:

$$v_{1x,i} = 0;\ v_{2x,i} = +0.34\ \text{m/s};\ v_{1x,f} = +0.17\ \text{m/s};\ v_{2x,f} = -0.17\ \text{m/s}.$$

kinetic energy: use K=½mv². from the figure:

$$m_1 = 0.36\ \text{kg and } m_2 = 0.12\ \text{kg}.$$



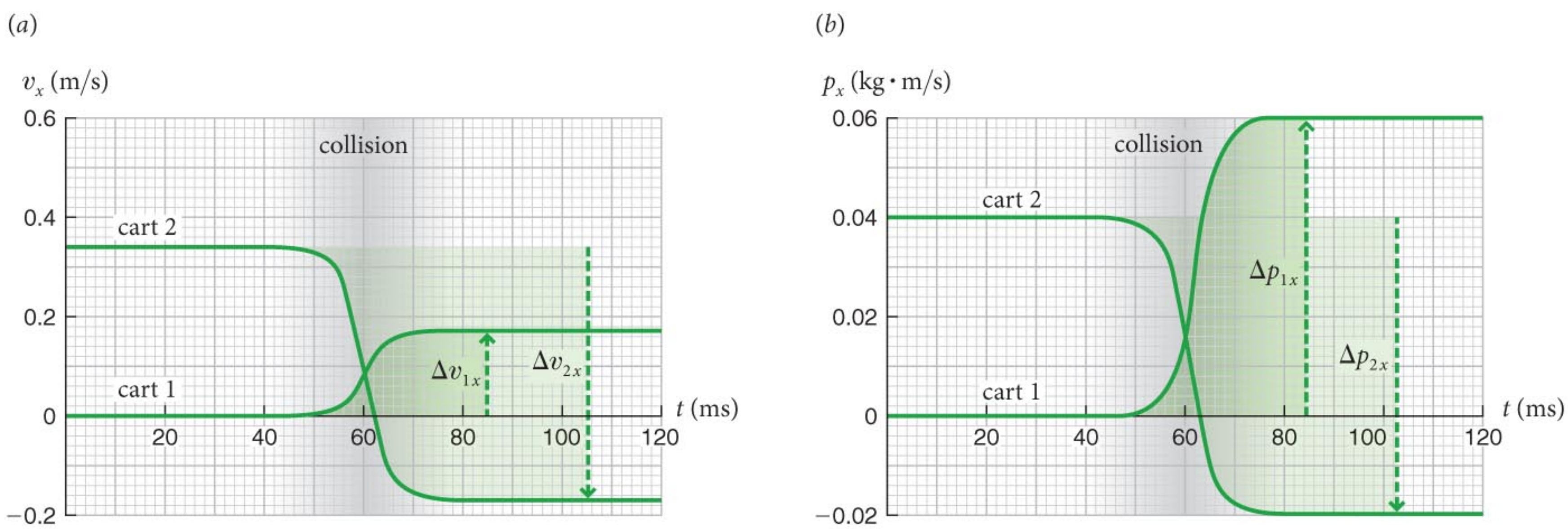
Example 5.2 Carts colliding (cont.)

❸ EXECUTE PLAN (*a*)

$$v_{12\mathrm{i}} = |v_{2x,\mathrm{I}} - v_{1x,\mathrm{i}}| = |(+0.34\ \mathrm{m/s}) - 0| = 0.34\ \mathrm{m/s};$$

$$v_{12\mathrm{f}} = |v_{2x,\mathrm{f}} - v_{1x,\mathrm{f}}| = |(-0.17\ \mathrm{m/s}) - (+0.17\ \mathrm{m/s})| = 0.34\ \mathrm{m/s}.$$

Relative speed is unchanged, collision is elastic.✔



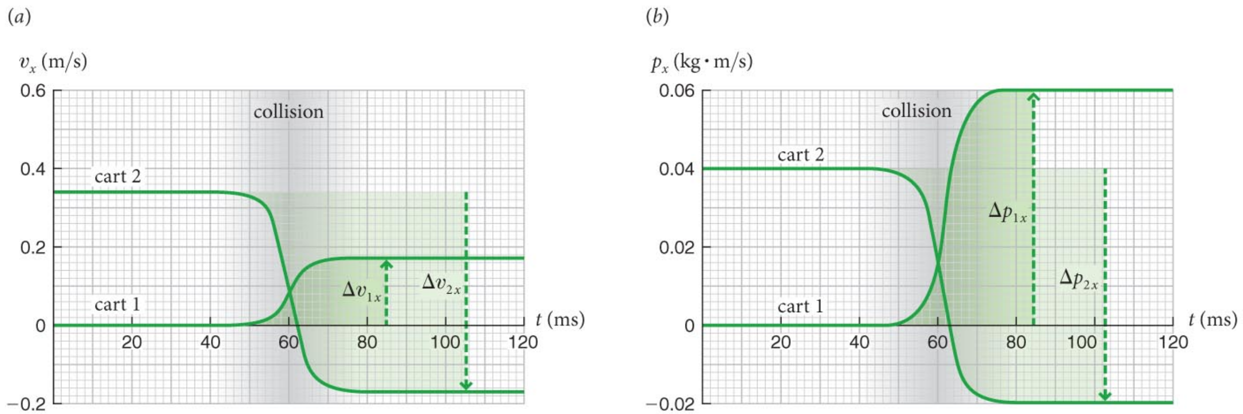
Example 5.2 Carts colliding (cont.)

❸ EXECUTE PLAN (*b*) The initial values are

$$K_{1i} = \tfrac{1}{2} m_1 \upsilon_{1i}^2 = \tfrac{1}{2}(0.36\ \text{kg})(0)^2 = 0$$

$$K_{2i} = \tfrac{1}{2} m_2 \upsilon_{2i}^2 = \tfrac{1}{2}(0.12\ \text{kg})(0.34\ \text{m/s})^2 = 0.0069\ \text{kg}\cdot\text{m}^2/\text{s}^2$$

so $$K_i = K_{1i} + K_{2i} = 0.0069\ \text{kg - m}^2/\text{s}^2.$$ ✔



Example 5.2 Carts colliding (cont.)

❸ EXECUTE PLAN The final values are

$$K_{1f} = \tfrac{1}{2} m_1 \upsilon_{1f}^2 = \tfrac{1}{2}(0.36\ \text{kg})(0.17\ \text{m/s})^2 = 0.0052\ \text{kg} \cdot \text{m}^2/\text{s}^2$$

$$K_{2f} = \tfrac{1}{2} m_2 \upsilon_{2f}^2 = \tfrac{1}{2}(0.12\ \text{kg})(-0.17\ \text{m/s})^2 = 0.0017\ \text{kg} \cdot \text{m}^2/\text{s}^2$$

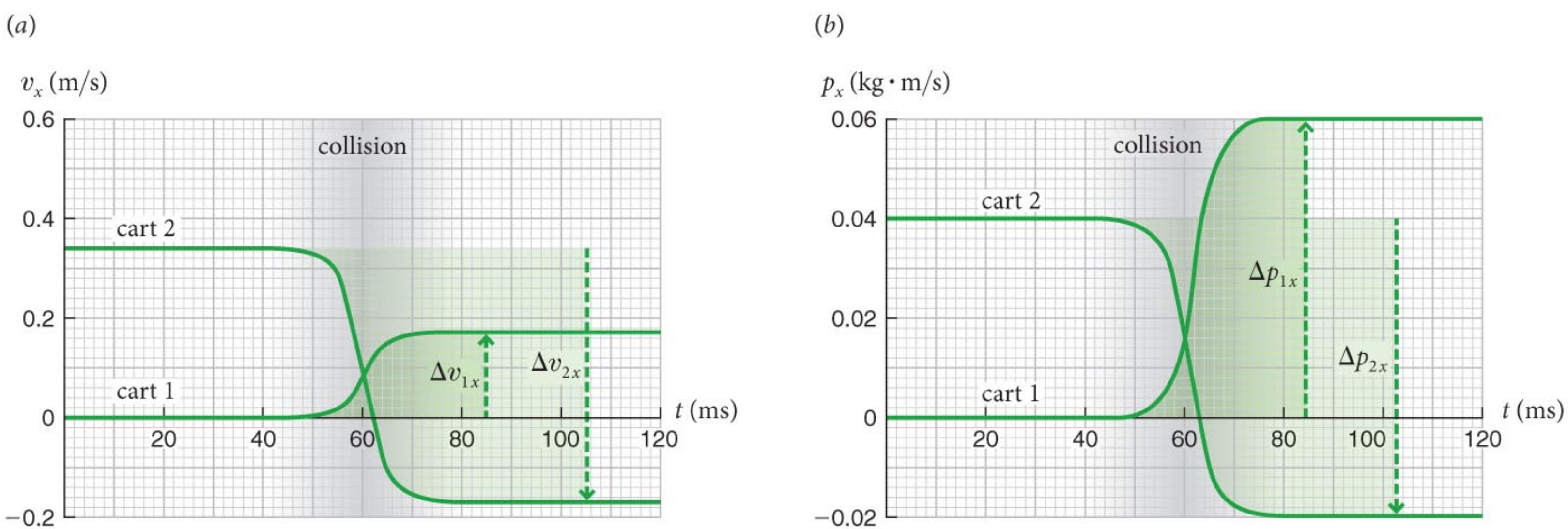
so

$$K_f = (0.0052\ \text{kg - m}^2/\text{s}^2) + (0.0017\ \text{kg - m}^2/\text{s}^2)$$
$$= 0.0069\ \text{kg - m}^2/\text{s}^2,$$

which is the same as before the collision, as it should be for an elastic collision.✔

Example 5.2 Carts colliding (cont.)

❹ EVALUATE RESULT Because I've reached the same conclusion—the collision is elastic—using two approaches, I can be pretty confident that my solution is correct.



Checkpoint 5.4

5.4 A moving cart collides with an identical cart initially at rest on a low-friction track, and the two lock together. What fraction of the initial kinetic energy of the system remains in this totally inelastic collision?

Could guess it is half …

Conservation of momentum:

$$mv_i = (2m)v_f \quad \rightarrow \quad v_f = \tfrac{1}{2}v_i$$

Initial K

$$K_i = \tfrac{1}{2}mv_i^2$$

Final K

$$K_f = \tfrac{1}{2}(2m)v_f^2 = \tfrac{1}{4}mv_i^2 = \tfrac{1}{2}K_i$$

Section 5.3: Internal energy

Section Goals

You will learn to

- Describe the **state** of an object by specifying physical parameters such as shape and temperature.
- Recognize that a **process** is a physical transformation in which an object or a set of objects changes from one state to another.
- Distinguish **reversible** from **irreversible** processes.
- Associate an **internal energy** with the physical state of an object.

Section 5.3: Internal energy

- In all **inelastic** collisions, the relative speed changes and therefore the total kinetic energy of the system changes.
- What happens to this energy?
  - Does it just appear from nowhere or simply vanish?
  - Let us answer this question by looking at inelastic collisions.
- The **state** of a system is the condition of an object completely specified by a set of parameters such as shape and temperature.
- The transformation of a system from an initial state to a final state is called a **process**.

Section 5.3: Internal energy

- Inelastic collisions are **irreversible processes**: The changes cannot spontaneously undo themselves.
- You cannot imagine watching it in reverse

Section 5.3: Internal energy

- Elastic collisions are **reversible**: no permanent changes
- You can easily imagine watching it in reverse

Section 5.3: Internal energy

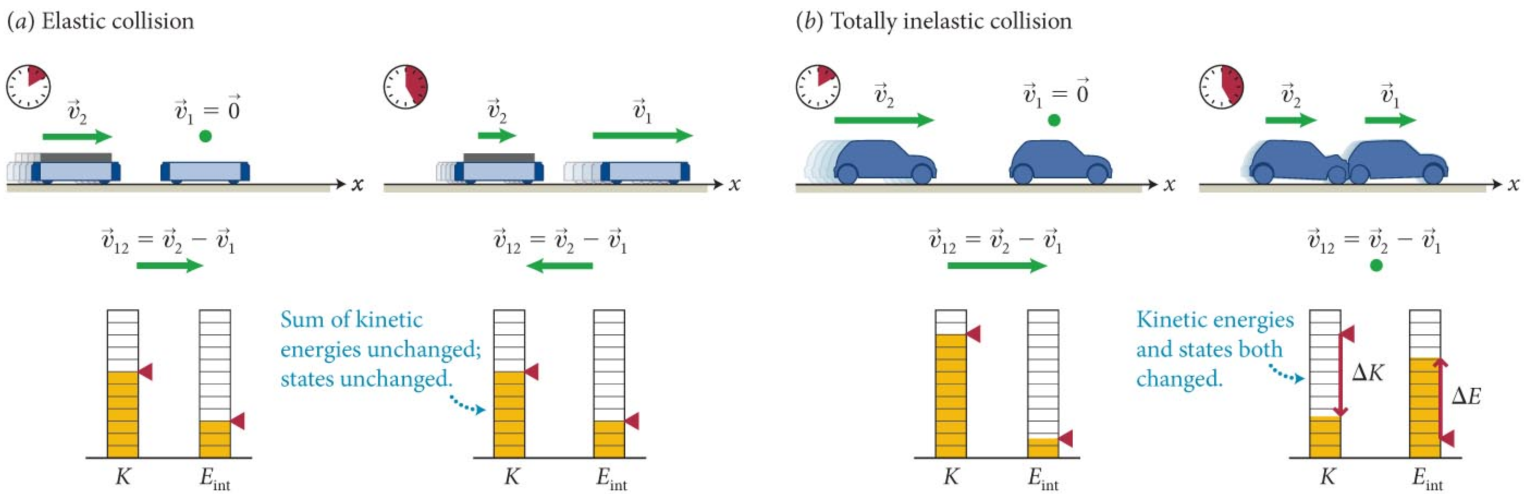
- Notice in the table below how the *change in total kinetic energy goes hand in hand with a change in the state.*
- To explore this connection further let us introduce a new quantity called **internal energy**:
  - In an inelastic collision one form of energy is converted to another form of energy (kinetic to internal).
  - The sum of kinetic and internal energy remains constant

Table 5.2 Elastic and inelastic collisions

Collision type	Relative speed	State
elastic	unchanged	unchanged
inelastic	changed	changed
totally inelastic	changed (becomes zero)	changed

Section 5.3: Internal energy

- Now we can make the following statements about collisions:
  - **Inelastic collision**: The states of the colliding objects change, the sum of their internal energies increases
    - Increase is equal to the decrease in the sum of kinetic energies.
  - Any collision: The ***total*** energy of a system of two colliding objects does not change during the collision.



Section 5.3: Internal energy

- A change in internal energy is associated with a change of state (inelastic collision)
- Internal energy must somehow relate to *how the object is made up*
  - a measure of the energy needed to build the object
  - due to the arrangement of (and motion of) its parts
  - (often intractable)

Example 5.3 Internal energy change

- A 0.2-kg cart 1 at rest is struck by an identical cart 2 traveling at $v_{2x,\mathrm{i}} = +0.5$ m/s. Ignore friction.
- After the collision, the velocity of cart 2 is reduced to $v_{2x,\mathrm{f}} = +0.2$ m/s.

(a) Is the collision elastic, inelastic, or totally inelastic

(b) By what amount does the internal energy of the two-cart system change?

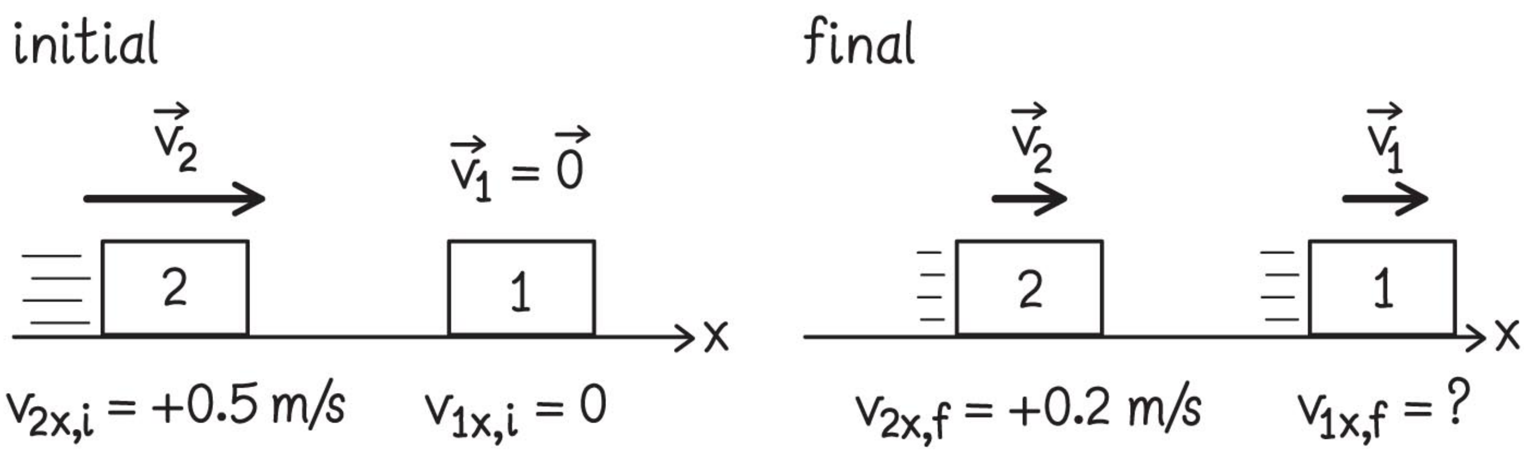
(c) Make a bar diagram showing the initial and final kinetic and internal energies of the two carts.

Example 5.3 Internal energy change (cont.)

❶ GETTING STARTED

Sketch!

To classify the collision, need the final relative speed, but the final velocity of cart 1 is not given.



Example 5.3 Internal energy change (cont.)

❷ DEVISE PLAN

- The two-cart system is isolated, and so the momentum of the system does not change
- I can use this to determine the final velocity of cart 1 and the final relative speed of the carts.
- By comparing the final and initial relative speeds, I can determine the type of collision.
- With initial and final velocities, I can calculate the kinetic energies determine what fraction of the initial kinetic energy has been converted to internal energy.

Example 5.3 Internal energy change (cont.)

❸ EXECUTE PLAN (*a*) The initial relative speed is

$$|v_{2x,\mathrm{i}} - v_{1x,\mathrm{i}}| = |(+0.5\ \mathrm{m/s}) - 0| = 0.5\ \mathrm{m/s}.$$

To determine $v_{1x,\mathrm{f}}$, I apply conservation of momentum to the system. The initial momentum of the system is

$$(0.2\ \mathrm{kg})(+0.5\ \mathrm{m/s}) + (0.2\ \mathrm{kg})(0) = (0.2\ \mathrm{kg})(+0.5\ \mathrm{m/s})$$

and its final momentum is

$$(0.2\ \mathrm{kg})(+0.2\ \mathrm{m/s}) + (0.2\ \mathrm{kg})(v_{1x,\mathrm{f}}).$$

Example 5.3 Internal energy change (cont.)

❸ EXECUTE PLAN Conservation of momentum requires these two momenta to be equal:

$$(0.2\ \text{kg})(+0.5\ \text{m/s}) = (0.2\ \text{kg})(+0.2\ \text{m/s}) + (0.2\ \text{kg})(v_{1x,\text{f}})$$

$$(+0.5\ \text{m/s}) = (+0.2\ \text{m/s}) + v_{1x,\text{f}}$$

$$v_{1x,\text{f}} = +0.3\ \text{m/s}.$$

Example 5.3 Internal energy change (cont.)

❸ EXECUTE PLAN The final relative speed is thus

$$|v_{2x,\mathrm{f}} - v_{1x,\mathrm{f}}| = |(+0.2\ \mathrm{m/s}) - (+0.3\ \mathrm{m/s})| = 0.1\ \mathrm{m/s},$$

which is different from the initial value. Thus the collision is inelastic.

(I know that the collision is not *totally* inelastic because the relative speed has not been reduced to zero.) ✔

Example 5.3 Internal energy change (cont.)

❸ EXECUTE PLAN (*b*) The initial kinetic energies are

$$K_{1\mathrm{i}} = 0$$

$$K_{2\mathrm{i}} = \tfrac{1}{2}(0.2\ \mathrm{kg})(0.5\ \mathrm{m/s})^2 = 0.025\ \mathrm{kg \cdot m^2/s^2}$$

so $$K_{\mathrm{i}} = K_{1\mathrm{i}} + K_{2\mathrm{i}} = 0.025\ \mathrm{kg - m^2/s^2}.$$

Example 5.3 Internal energy change (cont.)

❸ EXECUTE PLAN The final kinetic energies are

$$K_{1f} = \tfrac{1}{2}(0.2\text{ kg})(0.3\text{ m/s})^2 = 0.009\text{ kg}\cdot\text{m}^2/\text{s}^2$$

$$K_{2f} = \tfrac{1}{2}(0.2\text{ kg})(0.2\text{ m/s})^2 = 0.004\text{ kg}\cdot\text{m}^2/\text{s}^2$$

so $K_f = K_{1f} + K_{2f} = 0.013\text{ kg - m}^2/\text{s}^2.$

Example 5.3 Internal energy change (cont.)

❸ EXECUTE PLAN The kinetic energy of the system has changed by an amount

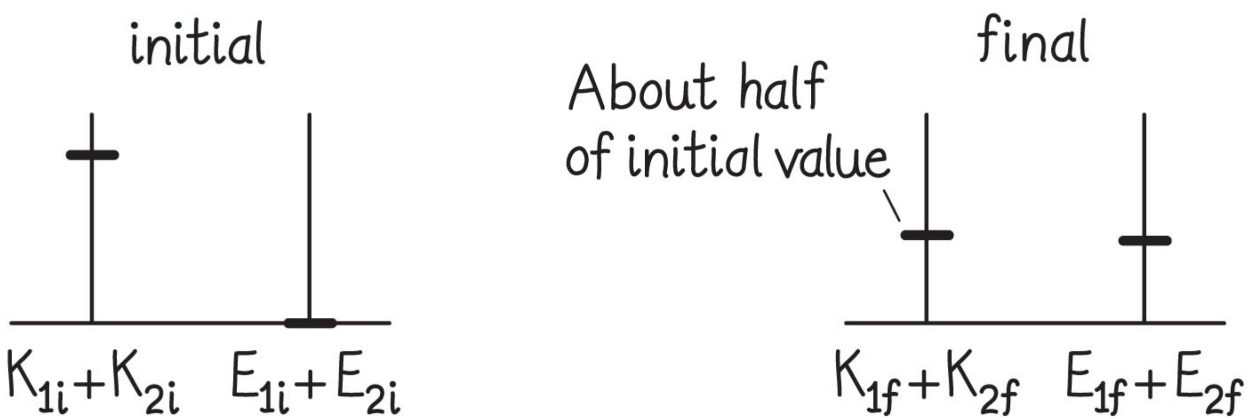
$$(0.013 \text{ kg - m}^2/\text{s}^2) - (0.025 \text{ kg - m}^2/\text{s}^2) = -0.012 \text{ kg - m}^2/\text{s}^2$$

To keep the energy of the system (the sum of its kinetic and internal energies) unchanged, the decrease in kinetic energy must be made up by an increase in internal energy.

This tells me that the internal energy of the system increases by $0.012 \text{ kg - m}^2/\text{s}^2$.✔

Example 5.3 Internal energy change (cont.)

❸ EXECUTE PLAN (*c*) Bar diagram. The final kinetic energy bar is about half of the initial kinetic energy bar. *Because I don't know the value of the initial internal energy, I set it to zero* and make the final internal energy bar equal in height to the difference in the kinetic energy bars.✔

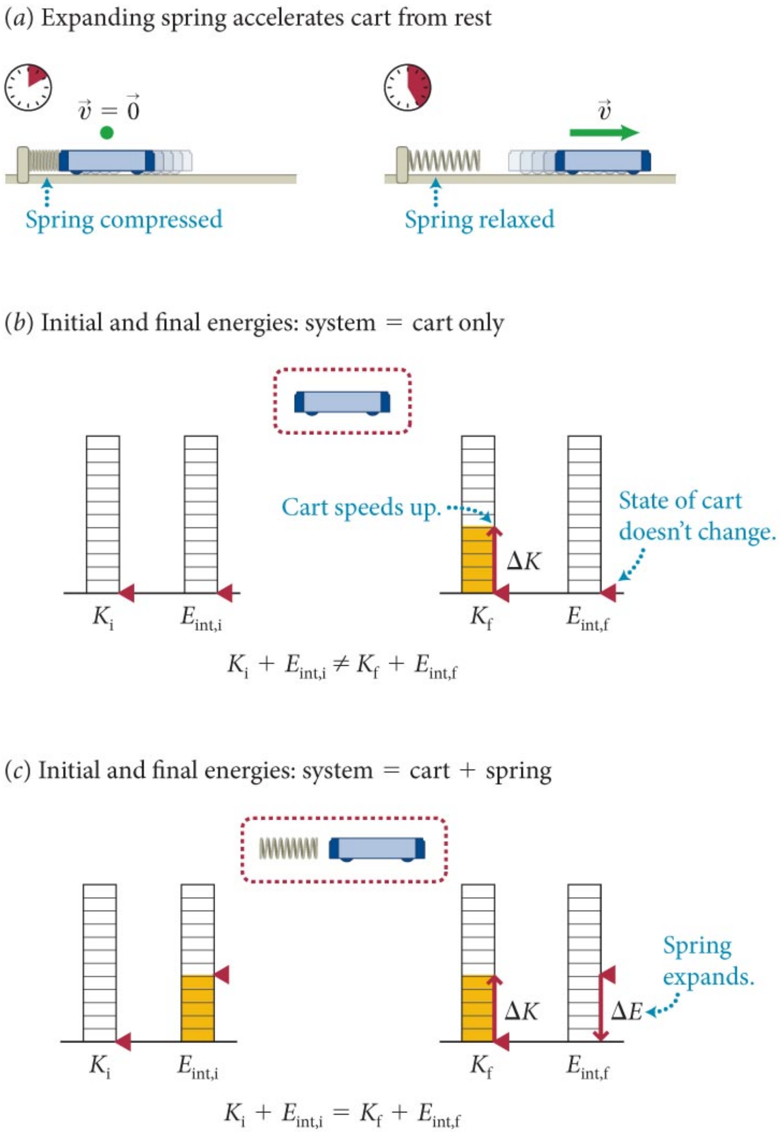


Section 5.3: Internal energy

- Can extend the idea of internal energy to other interactions. We assert:

  Energy can be transferred from one object to another or converted from one form to another, but energy cannot be destroyed or created.

- No observation has ever been found to violate this statement known as the law of **conservation of energy**.



Section 5.4: Closed systems

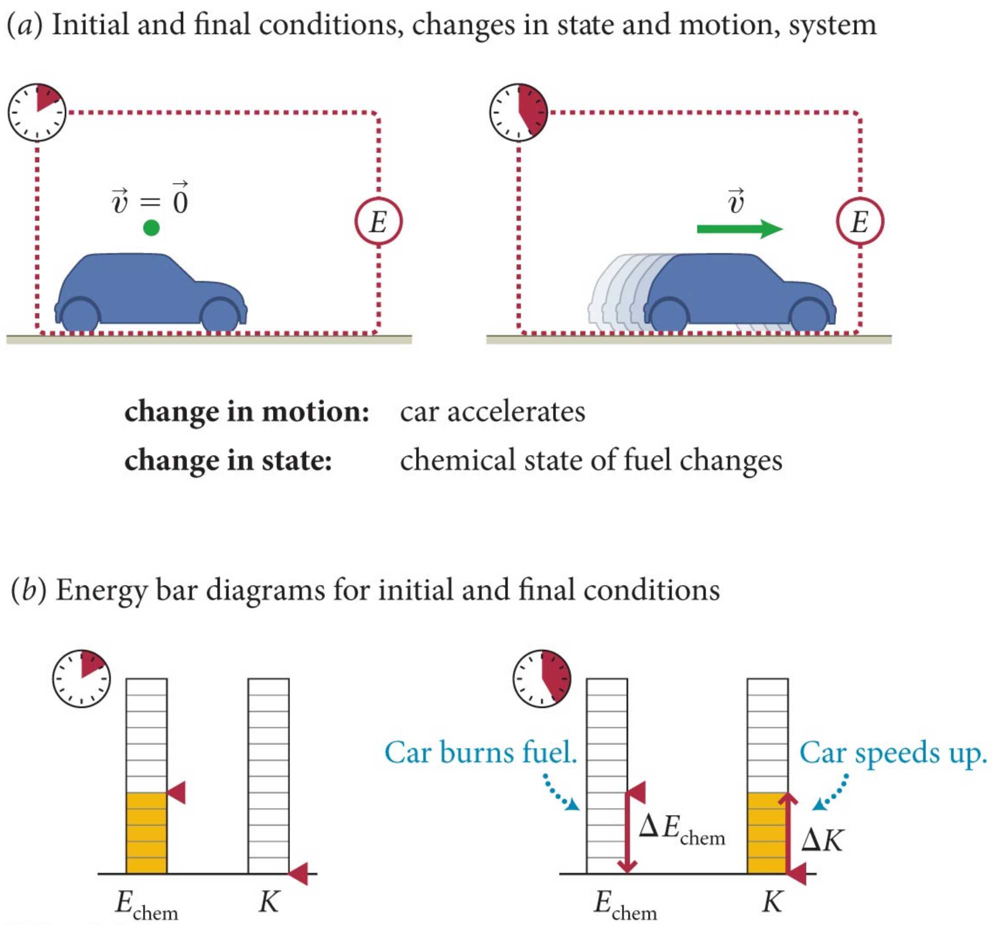
Section Goal

- A **closed system** is one in which no energy is transferred to or from it. (Choose like we did for **p**.)

- The only energy changes possible in a closed system are **transformations** from one type of energy to another.

Section 5.4: Closed systems

- Example:
  - Chemical energy stored in gasoline is converted to kinetic energy of a car.



Quantitative Tools

Section 5.5: Elastic collisions

Consider two objects colliding as shown in figure below.

- Relative velocity of cart 2 relative to cart 1 is

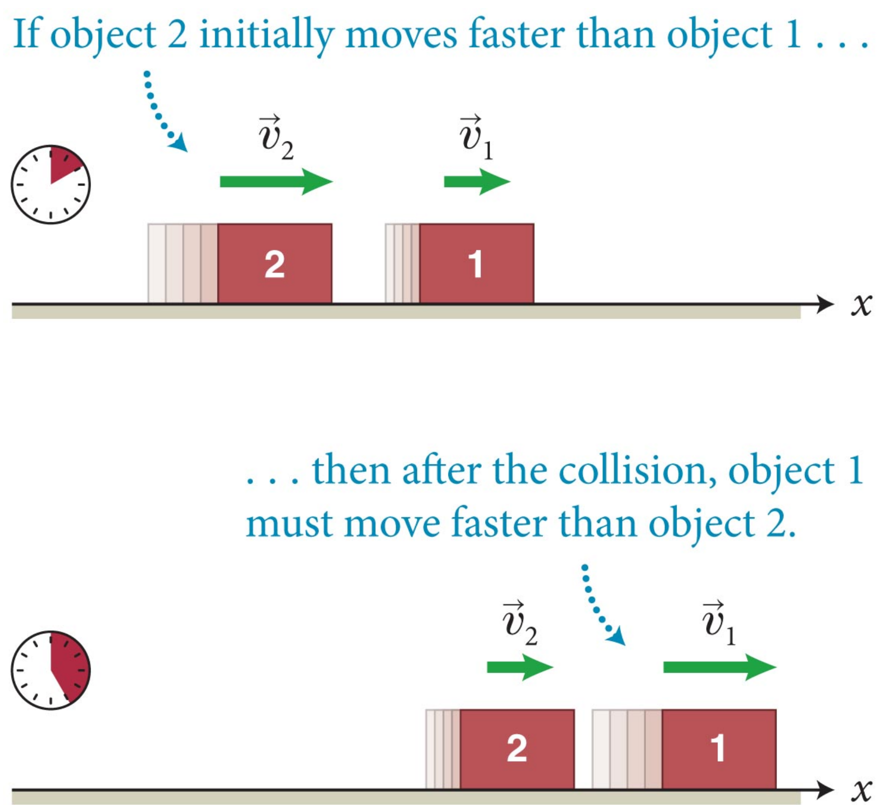
$$\vec{v}_{12} \equiv \vec{v}_2 - \vec{v}_1$$

- Relative velocity of cart 2 relative to cart 1 is

$$\vec{v}_{21} \equiv \vec{v}_1 - \vec{v}_2 = -\vec{v}_{12}$$

- For elastic collisions, **relative speeds** before and after the collision are the same:

$$v_{12\mathrm{i}} = v_{12\mathrm{f}} \text{ (elastic collision)}$$



Section 5.5: Elastic collisions

- For two objects moving along the x axis, we can write the previous equation as

$$v_{2x,\mathrm{i}} - v_{1x,\mathrm{i}} = -(v_{2x,\mathrm{f}} - v_{1x,\mathrm{f}}) \quad \text{(elastic collision)}$$

- Considering the two colliding carts to be an isolated system, conservation of momentum gives

$$m_1 v_{1x,\mathrm{i}} + m_2 v_{2x,\mathrm{i}} = m_1 v_{1x,\mathrm{f}} + m_2 v_{2x,\mathrm{f}} \quad \text{(isolated system)}$$

- Algebraic manipulation of the above equations will yield a new *constant of motion*

$$\tfrac{1}{2} m_1 v_{1\mathrm{i}}^2 + \tfrac{1}{2} m_2 v_{2\mathrm{i}}^2 = \tfrac{1}{2} m_1 v_{1\mathrm{f}}^2 + \tfrac{1}{2} m_2 v_{2\mathrm{f}}^2$$

Section 5.5: Elastic collisions

- This is why we define

$$K \equiv \tfrac{1}{2} m\upsilon^2$$

- Rewrite:

$$\tfrac{1}{2} m_1 \upsilon_{1i}^2 + \tfrac{1}{2} m_2 \upsilon_{2i}^2 = \tfrac{1}{2} m_1 \upsilon_{1f}^2 + \tfrac{1}{2} m_2 \upsilon_{2f}^2$$

$$K_{1i} + K_{2i} = K_{1f} + K_{2f} \quad \text{(elastic collision)}$$

- Thus, elastic collision, $K_i = K_f$
- SI unit kg · m²/s² – also ***joule***: 1 kg – m²/s² = 1 J

Section 5.5 Elastic collisions

- Example 5.5: if you use conservation of both energy & momentum for an elastic collision?
- Additional constraint, less unknowns, relate final velocities to initial velocities

$$v_{1x,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1x,i} + \frac{2m_2}{m_1 + m_2} v_{2x,i}$$

$$v_{2x,f} = \frac{2m_1}{m_1 + m_2} v_{1x,i} + \frac{m_1 - m_2}{m_1 + m_2} v_{2x,i}$$

- If you know masses and initial velocities, can predict final state!
- "Elastic collision equations"

Section 5.5 Elastic collisions

- Do the equations make sense? Limiting cases

$$v_{1x,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1x,i} + \frac{2m_2}{m_1 + m_2} v_{2x,i}$$

$$v_{2x,f} = \frac{2m_1}{m_1 + m_2} v_{1x,i} + \frac{m_1 - m_2}{m_1 + m_2} v_{2x,i}$$

- $v_{2x,i} = 0$ (strike an object at rest)

$$v_{1x,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1x,i}$$

$$v_{2x,f} = \frac{2m_1}{m_1 + m_2} v_{1x,i}$$

- Now consider huge m_2 (like a wall) – object 1 rebounds

Example 5.6 Collision and kinetic energy

A rubber ball of inertia $m_b = 0.050$ kg is fired along a track toward a stationary cart of inertia $m_c = 0.25$ kg.

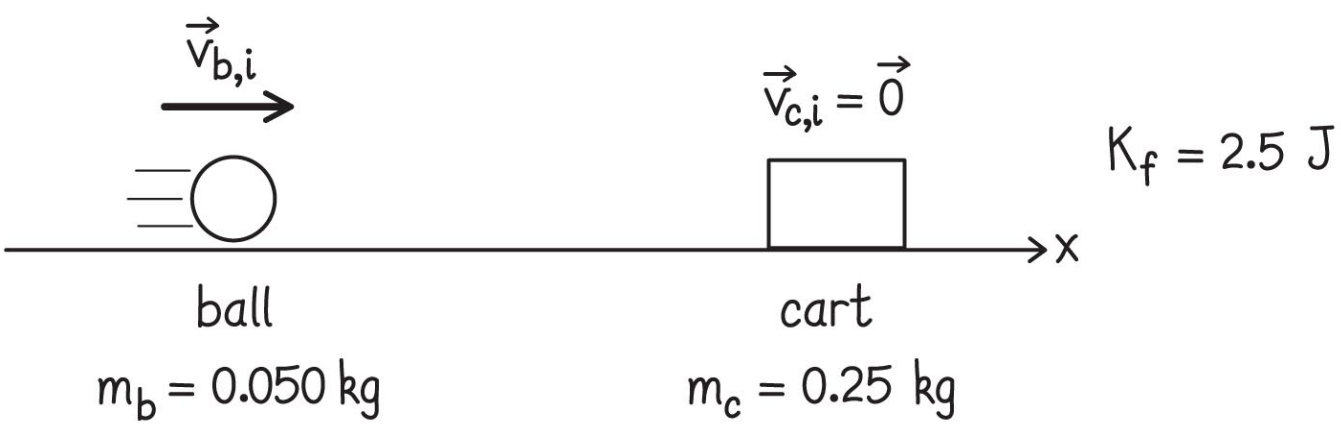
The kinetic energy of the system after the two collide elastically is 2.5 J.

(a) What is the initial velocity of the ball?

(b) What are the final velocities of the ball and the cart?

Example 5.6 Collision and kinetic energy (cont.)

➊ GETTING STARTED Organize the problem graphically. Choose the *x* axis in the direction of the incoming rubber ball. Only one initial velocity is given. I need to determine the other initial velocity and both final velocities.



Example 5.6 Collision and kinetic energy (cont.)

❷ DEVISE PLAN Because the collision is elastic, I know that the kinetic energy of the system doesn't change, which means that the final value (2.5 J) is the same as the initial value.

Because the cart is initially at rest, all of this kinetic energy belongs initially to the ball.

Once I have this information, I know the initial velocities of both colliding objects and I can calculate the final velocities.

Example 5.6 Collision and kinetic energy (cont.)

❸ EXECUTE PLAN (*a*) $K = \frac{1}{2}mv^2$, we know K and m, so solve for v:

$$v_{\mathrm{b,i}} = \sqrt{\frac{2K_{\mathrm{b,i}}}{m_{\mathrm{b}}}} = \sqrt{\frac{2(2.5\ \mathrm{J})}{0.05\ \mathrm{kg}}} = 10\ \mathrm{m/s}.$$

Because the ball is initially moving in the positive x direction, its initial velocity is given by $v_{\mathrm{b}\,x,\mathrm{i}} = +10$ m/s. ✔

Example 5.6 Collision and kinetic energy (cont.)

❸ EXECUTE PLAN (*b*) I can now substitute the two initial velocities and the inertias into the elastic collision equations. With object 2 initial at rest, they are simpler:

$$v_{1x,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1x,i}$$

$$v_{2x,f} = \frac{2m_1}{m_1 + m_2} v_{1x,i}$$

This gives

$$v_{\mathrm{b}\,x,\mathrm{f}} = -6.7 \text{ m/s}$$

$$v_{\mathrm{c}\,x,\mathrm{f}} = +3.3 \text{ m/s}$$ ✔

relative v still 10 m/s

Example 5.6 Collision and kinetic energy (cont.)

❹ EVALUATE RESULT It makes sense that the velocity of the ball is reversed by the collision because the inertia of the cart is so much greater than that of the ball.

Now that I know both the initial and final velocities, I can also check to make sure that the relative speed remains the same, which it does, as required for an elastic collision.

Section 5.6: Inelastic collisions

Section Goals

You will learn to

- Identify **inelastic collisions** from the relative velocity of the colliding objects.
- Analyze inelastic collisions mathematically using the law of conservation of momentum (because that’s all you’ve got)

Section 5.6: Inelastic collisions

- In totally inelastic collisions, the objects move together after the collision. Therefore,

$$v_{12f} = 0 \text{ (totally inelastic collision)}$$

- Most collisions fall between the two extremes of elastic and totally inelastic.

Table 5.4 Coefficient of restitution for various processes

Process	Relative speed	Coefficient of restitution
totally inelastic collision	$v_{12f} = 0$	$e = 0$
inelastic collision	$0 < v_{12f} < v_{12i}$	$0 < e < 1$
elastic collision	$v_{12f} = v_{12i}$	$e = 1$
explosive separation*	$v_{12f} > v_{12i}$	$e > 1$

*See Section 5.8.

Section 5.6: Inelastic collisions

- For these cases, it is convenient to define the quantity called the **coefficient of restitution**

$$e \equiv \frac{v_{12\mathrm{f}}}{v_{12\mathrm{i}}}$$

- In component form,

$$e = -\frac{v_{2x,\mathrm{f}} - v_{1x,\mathrm{f}}}{v_{2x,\mathrm{i}} - v_{1x,\mathrm{i}}} = -\frac{v_{12x,\mathrm{f}}}{v_{12x,\mathrm{i}}}$$

Table 5.4 Coefficient of restitution for various processes

Process	Relative speed	Coefficient of restitution
totally inelastic collision	$v_{12\mathrm{f}} = 0$	$e = 0$
inelastic collision	$0 < v_{12\mathrm{f}} < v_{12\mathrm{i}}$	$0 < e < 1$
elastic collision	$v_{12\mathrm{f}} = v_{12\mathrm{i}}$	$e = 1$
explosive separation*	$v_{12\mathrm{f}} > v_{12\mathrm{i}}$	$e > 1$

*See Section 5.8.

Section 5.7: Conservation of energy

Section Goals

You will learn to

- Understand the law of **conservation of energy** for a closed system.
- Identify some types of internal energy changes that occur in physical systems.

Section 5.7: Conservation of energy

- For a closed system, conservation of energy requires that

$$K_i + E_{int,i} = K_f + E_{int,f} \text{ (closed system)}$$

- The total energy of the system is given by

$$E \equiv K + E_{int}$$

- Now we can rewrite the first equation as

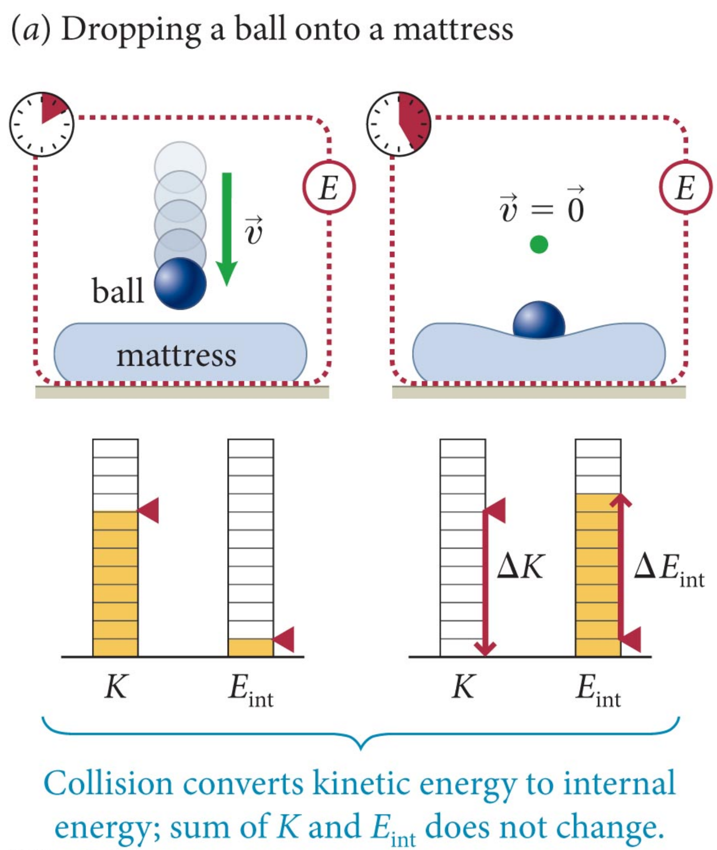
$$E_i = E_f \quad \text{(closed system)}$$

- Even though we cannot yet calculate E_{int}, the previous equation allows us to compute ΔE_{int}

$$\Delta E_{int} = -\Delta K \quad \text{(closed system)}$$

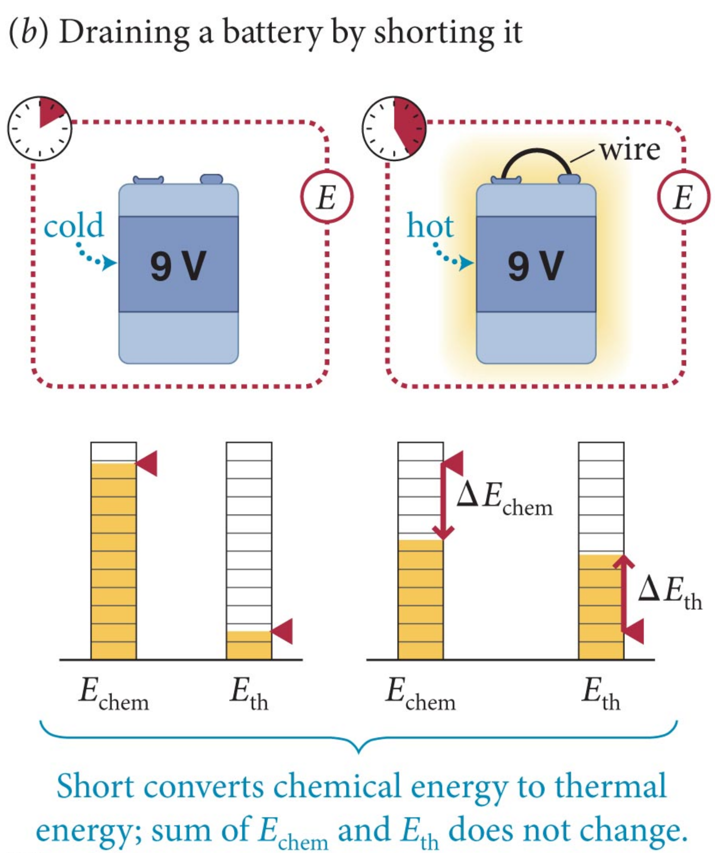
Section 5.7: Conservation of energy

- As an example, consider the situation in the figure below, where a ball is dropped onto a mattress:
  - Energy conservation requires the loss of kinetic energy to be equal to the gain in internal energy.



Section 5.7: Conservation of energy

- Another example is shown below, that is, when a battery is drained rapidly, it becomes hot:
  - Energy conservation requires the loss of chemical energy to be equal to the gain in thermal energy.



Example 5.8 Making a light

A 0.20-kg steel ball is dropped into a ball of dough, striking the dough at a speed of 2.3 m/s and coming to rest inside the dough. If it were possible to turn all of the energy converted in this totally inelastic collision into light, how long could you light a desk lamp? It takes 25 J to light a desk lamp for 1.0 s.

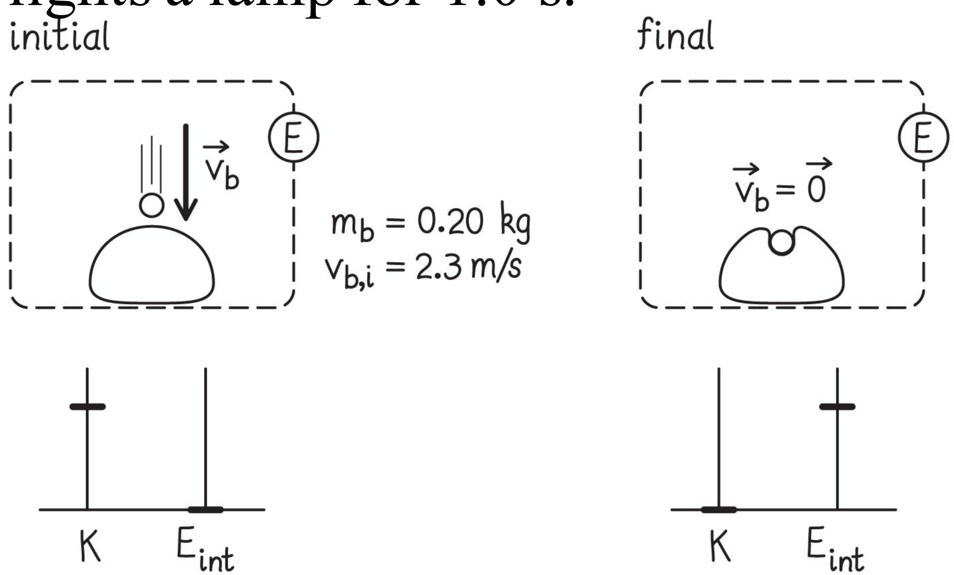
Example 5.8 Making a light (cont.)

➊ GETTING STARTED I begin by applying the procedure for choosing a closed system. Although the problem doesn’t specify it explicitly, I’m assuming the dough is at rest both before and after the steel ball is dropped in it; it could, for example, be at rest on a countertop.

Example 5.8 Making a light (cont.)

❶ GETTING STARTED Only the steel ball has kinetic energy initially, and all of this energy is converted to internal energy as the ball comes to rest in the dough (Figure 5.20).

I have to calculate the initial kinetic energy of the ball and determine how long that amount of energy could light a lamp, given that 25 J lights a lamp for 1.0 s.



Example 5.8 Making a light (cont.)

❷ DEVISE PLAN To determine the initial kinetic energy of the ball, I use Eq. 5.12. Then I divide this result by 25 J to determine how many seconds I can light a lamp.

Example 5.8 Making a light (cont.)

❸ EXECUTE PLAN The initial kinetic energy of the ball is

$$K_{\mathrm{b,i}} = \tfrac{1}{2} m_{\mathrm{b}} \upsilon_{\mathrm{b,i}}^2 = \tfrac{1}{2}(0.20\ \mathrm{kg})(2.3\ \mathrm{m/s})^2 = 0.53\ \mathrm{J}.$$

Given that a desk lamp requires 25 J per second, this 0.53 J lights a lamp for

$$\frac{\text{energy available}}{\text{energy needed per second}} = \frac{0.53\ \mathrm{J}}{25\ \mathrm{J/s}} = 0.021\ \mathrm{s}.$$

Great, just have to do this 50 times per second …

Example 5.8 Making a light (cont.)

❹ EVALUATE RESULT The length of time I obtained, two hundreths of a second, is not very much!

However, a 0.20-kg steel ball moving at 2.3 m/s does not have much kinetic energy: I know from experience that a small steel ball's ability to induce state changes—to crumple or deform objects, for example—is very limited.

It makes sense that one can't light a desk lamp very long.

Checkpoint 5.12

5.12 A gallon of gasoline contains approximately 1.2×10^8 J of energy. If all of this energy were converted to kinetic energy in a 1200-kg car, how fast would the car go?

$$v = \sqrt{2K/m}$$

With the given K and m, $v \sim 4.5 \times 10^2$ m/s

Section 5.8: Explosive separations

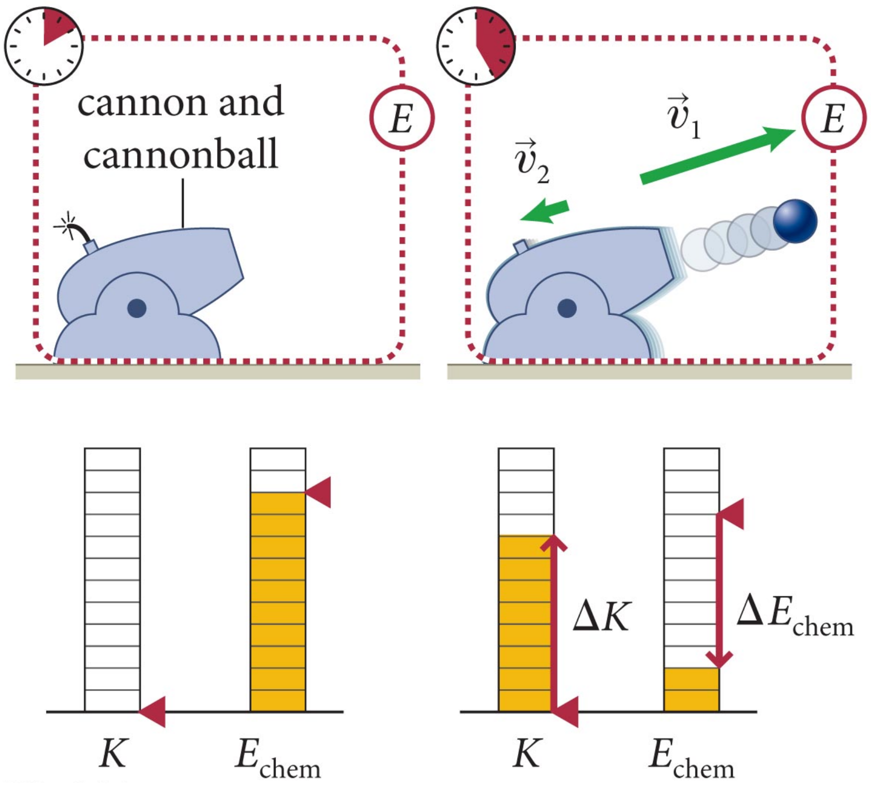
Section Goals

You will learn to

- Recognize that **explosive separations** involve a process in which internal energy is converted into kinetic energy.
- Use the law of conservation of momentum to calculate the relative final velocity of the explosion fragments.

Section 5.8: Explosive separations

- Is it possible to have a process in which kinetic energy is gained at the expense of internal energy?
  - Yes, in any type of explosive separation, where the object breaks apart.
  - Firing a cannon is one such example, as seen in the figure.

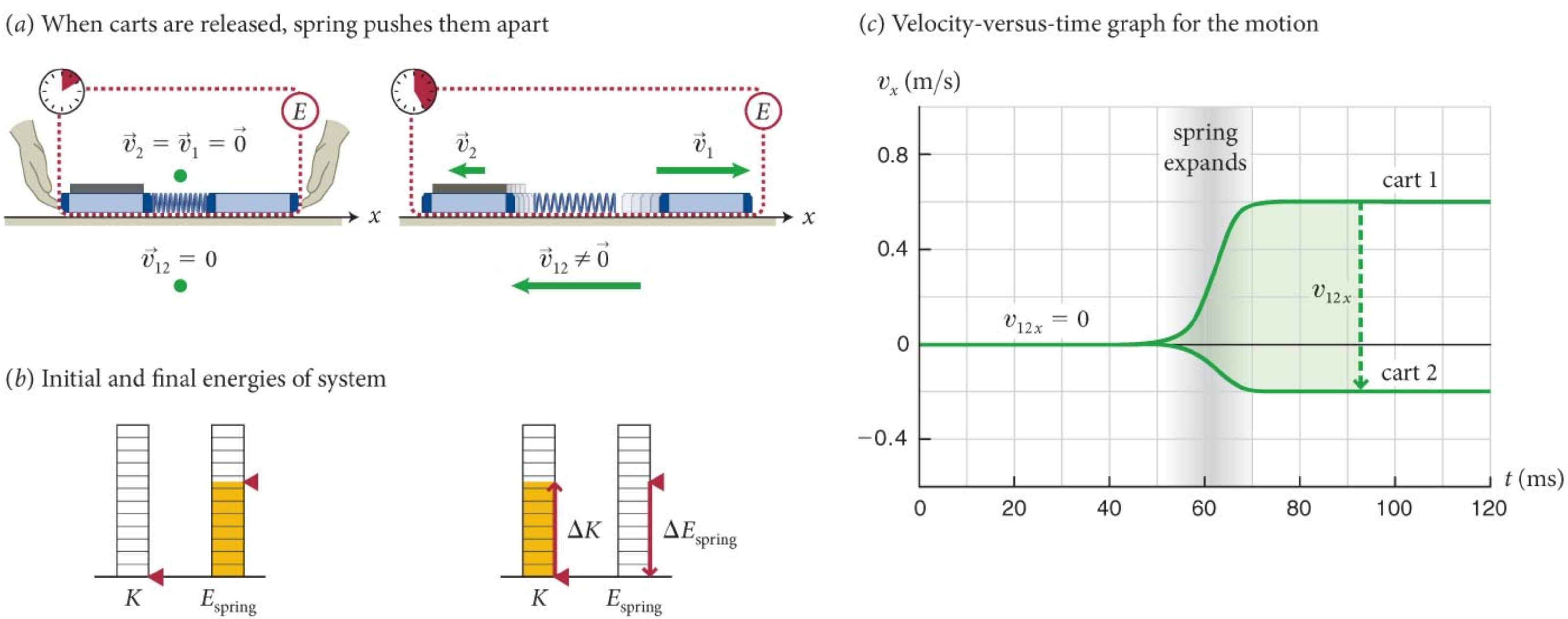


- The figure below shows an explosive separation involving two carts.
  - Because $v_{1x,i} = v_{2x,i} = 0$, using conservation of momentum we can write

$$0 = m_1 v_{1x,f} + m_2 v_{2x,f}$$

  - Applying energy conservation we get

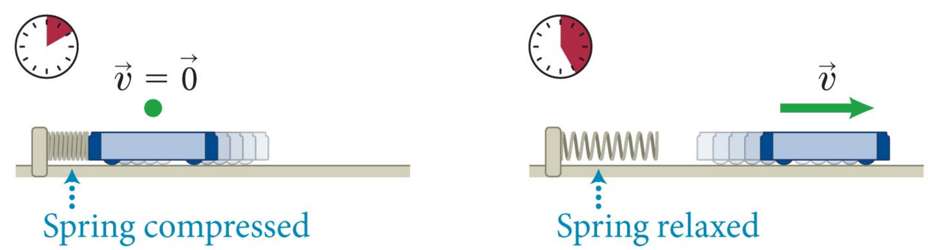
$$\Delta K + \Delta E_{int} = \tfrac{1}{2} m_1 \upsilon_{1f}^2 + \tfrac{1}{2} m_2 \upsilon_{2f}^2 + \Delta E_{int} = 0$$



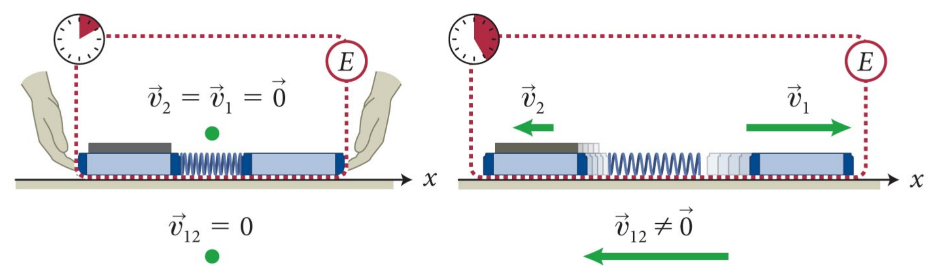
Example 5.9 Spring energy

A 0.25-kg cart is held at rest against a compressed spring as in Figure 5.8*a* and then released. The cart's speed after it separates from the spring is 2.5 m/s. The spring is then compressed by the same amount between a 0.25-kg cart and a 0.50-kg cart, as shown in Figure 5.22*a*, and the carts are released from rest. What are the carts' speeds after separating from the spring?

(*a*) Expanding spring accelerates cart from rest



(*a*) When carts are released, spring pushes them apart



Example 5.9 Spring energy (cont.)

❶ GETTING STARTED The key point in this problem is the identical compression of the spring in the two cases: The initial state of the spring is therefore the same before both releases. Because the spring ends in the same uncompressed state in both cases, the change in its internal energy must be the same in both cases. In the first case, all of this energy is transferred to the 0.25-kg cart. In the second case, the same amount of energy is distributed between the two carts.

Example 5.9 Spring energy (cont.)

❷ DEVISE PLAN To calculate the kinetic energy of the single cart in the first release, I use Eq. 5.12. This gives me the amount of energy stored in the compressed spring. The final velocities of the two carts in the second case are then given by Eqs. 5.28 and 5.29.

Example 5.9 Spring energy (cont.)

❸ EXECUTE PLAN From Eq. 5.12, I get

$$K = \tfrac{1}{2}m\upsilon^2 = \tfrac{1}{2}(0.25\,\text{kg})(2.5\,\text{m/s})^2 = 0.78\,\text{J}$$

and so the change in the spring's internal energy is $\Delta E_{\text{int}} = -0.78$ J. Next I rewrite Eq. 5.28 as $v_{1x,\text{f}} = -(m_2/m_1)\,v_{2x,\text{f}}$. Substituting this result in Eq. 5.29, I get

$$\tfrac{1}{2}m_1\left(\frac{m_2}{m_1}\right)^2 \upsilon^2_{2x,\text{f}} + \tfrac{1}{2}m_2\upsilon^2_{2x,\text{f}} = -\Delta E_{\text{int}}.$$

Example 5.9 Spring energy (cont.)

❸ EXECUTE PLAN Solving for the final velocity of cart 2 gives

$$v_{2x\mathrm{f}} = \sqrt{\frac{-2m_1 \Delta E_{\mathrm{int}}}{m_2(m_1 + m_2)}}$$

$$v_{2x\mathrm{f}} = \sqrt{\frac{-2(0.25\ \mathrm{kg})(-0.78\ \mathrm{J})}{(0.50\ \mathrm{kg})(0.25\ \mathrm{kg} + 0.50\ \mathrm{kg})}} = 1.0\ \mathrm{m/s}. ✔$$

Substituting this result into my rewritten Eq. 5.28, $v_{1x,\mathrm{f}} = -(m_2/m_1)v_{2x,\mathrm{f}}$, I get $v_{1x,\mathrm{f}} = -2.0$ m/s. ✔

Example 5.9 Spring energy (cont.)

❹ EVALUATE RESULT The carts move in opposite directions, as expected. I also note that cart 1 moves at twice the speed of cart 2, as it should to keep the final momentum of the system zero. Finally, because my assignment of m_1 and m_2 is arbitrary, I verify that I get the same result when I substitute $m_1 = 0.50$ kg and $m_2 = 0.25$ kg. (You may want to check this yourself. When you reverse the inertias, why does the velocity of cart 1 reverse to positive and the velocity of cart 2 reverse to negative?)

Checkpoint 5.13

5.13 Does each cart in Example 5.9 get half of the spring's energy? Why or why not?

Concepts: Kinetic energy

- The **kinetic energy** of an object is the energy associated with its motion.
- Kinetic energy is a positive scalar quantity and is independent of the direction of motion.

Chapter 5: Summary

Quantitative Tools: Kinetic energy

- The **kinetic energy** K of an object of inertia m moving at speed v is

$$K = \tfrac{1}{2}mv^2.$$

- The SI unit of kinetic energy is the **joule** (J):

$$1\ \text{J} = 1\ \text{kg} \cdot \text{m}^2/\text{s}^2.$$

Concepts: Relative velocity, states, and internal energy

- In a collision between two objects, the velocity of one object relative to the velocity of the other object is the **relative velocity** $\vec{v}_{12}$. The magnitude of the relative velocity is the **relative speed** v_{12}.
- The **state** of an object is its condition as specified by some complete set of physical parameters. Energy associated with the object's state but not with its motion is called the **internal energy** of the object.
- We can consider a system of two colliding objects to be isolated during the collision. Therefore the momentum of the system remains constant during all the collisions we study.

Chapter 5: Summary

Quantitative Tools: Relative velocity, states, and internal energy

- The **relative velocity** $\vec{v}_{12}$ of object 2 relative to object 1 is

$$\vec{v}_{12} \equiv \vec{v}_2 - \vec{v}_1.$$

- The **relative speed** v_{12} of object 2 relative to object 1 is the magnitude of $\vec{v}_{12}$:

$$v_{12} = \left|\vec{v}_2 - \vec{v}_1\right|.$$

- Because momentum is a conserved quantity, the momentum of a system remains constant during a collision:

$$p_{x,\mathrm{i}} = p_{x,\mathrm{f}}.$$

Concepts: Types of collisions

- The **coefficient of restitution** *e* for a collision is a positive, unitless quantity that tells how much of the initial relative speed is restored after the collision.

- For an **elastic collision**, the relative speed is the same before and after the collision, and the coefficient of restitution is equal to 1. The collision is **reversible**, and the kinetic energy of the system made up of the colliding objects is constant.

Concepts: Types of collisions

- For an **inelastic collision**, the relative speed after the collision is less than it was before the collision. The coefficient of restitution is between 0 and 1, and the collision is **irreversible**. The kinetic energy of the objects changes during the collision, but the energy of the system does not change. If the objects stick together, the final relative speed is zero; the collision is **totally inelastic**, and the coefficient of restitution is 0.
- For an **explosive separation**, kinetic energy is gained during the collision and the coefficient of restitution is greater than 1.

Quantitative Tools: Types of collisions

- The **coefficient of restitution** e is

$$e = \frac{v_{12\text{f}}}{v_{12\text{i}}} = -\frac{v_{2x,\text{f}} - v_{1x,\text{f}}}{v_{2x,\text{i}} - v_{1x,\text{i}}}.$$

- For an **elastic collision,**

$$v_{12\text{i}} = v_{12\text{f}}$$

$$K_\text{i} = K_\text{f}$$

$$e = 1.$$

- For an **inelastic collision,**

$$v_{12\text{f}} < v_{12\text{i}}$$

$$K_\text{f} < K_\text{i}$$

$$0 < e < 1.$$

Quantitative Tools: Types of collisions

- For a **totally inelastic collision,**

$$v_{12f} = 0$$
$$e = 0.$$

- For an **explosive separation,**

$$v_{12f} > v_{12i}$$
$$K_f > K_i$$
$$e > 1.$$

Chapter 5: Summary

Concepts: Conservation of energy

- The energy of any system is the sum of the kinetic energies and internal energies of all the objects that make up the system.
- The law of **conservation of energy** states that energy can be transferred from one object to another or converted from one form to another, but it cannot be destroyed or created.
- A **closed system** is one in which no energy is transferred in or out. The energy of such a system remains constant.

Chapter 5: Summary

Quantitative Tools: Conservation of energy

- The energy of a system is

$$E = K + E_{\text{int}}.$$

- The law of conversation of energy requires the energy of a **closed system** to be constant:

$$E_{\text{i}} = E_{\text{f}}.$$