

### Exercise 4: Solution

1. A “free” electron and a “free” proton are placed in an identical electric field. Which of the following statements are true? *Check all that apply.* Note that the electron mass is  $9.11 \times 10^{-31}$  kg, and the proton mass is  $1.67 \times 10^{-27}$  kg.

- Each particle is acted on by the same electric force and has the same acceleration.
- The electric force on the proton is greater in magnitude than the force on the electron, but in the opposite direction.
- The electric force on the proton is equal in magnitude to the force on the electron, but in the opposite direction.
- The magnitude of the acceleration of the electron is greater than that of the proton.
- Both particles have the same acceleration.

See the solutions to quiz 1.

2. Two isolated identical conducting spheres have a charge of  $q$  and  $-3q$ , respectively. They are connected by a conducting wire, and after equilibrium is reached, the wire is removed (such that both spheres are again isolated). What is the charge on each sphere?

Once we connect the two spheres with a conducting wire, we have basically made one large conductor, over which the charges can move freely. If the wire is negligibly small compared to the spheres, the total charge originally on the two spheres will now spread out uniformly over both spheres. The total charge originally contained on the two spheres is  $q + (-3q) = -2q$ , and spread out over two equal spheres, this gives  $-q$  per sphere.

3. When we power a light bulb, are we using up charges and converting them to light?

- Yes, moving charges produce “friction” which heats up the filament and produces light
- Yes, charges are emitted and observed as light
- No, charge is conserved. It is simply converted to another form such as heat and light.
- No, charge is conserved. Moving charges produce “friction” which heats up the filament and produces light.

No, charge is conserved. Charges moving through the filament produce “friction” which heats up the filament and produces light.

Charges are not used up, and charge cannot be converted to heat or light. The “friction” charges experience is resistance, which leads to a conversion of the charges’ electrical potential energy into vibrational energy in the wire (heat) through collisions between the charges and atoms in the wire. The filament heats up due to the collisions between the charges and its atoms, and glows as it gets hotter.

4. In semiconductors such as Si, the number of carriers is not fixed, it depends on *e.g.*, temperature. For a certain sample of Si, the number of carriers doubles but their drift velocity decreases by 10 times. By how much does the sample’s resistance change?

This is easily answered with some algebra. First, we recall the relation between *current* and drift velocity:

$$I = nqAv_d$$

What we are really after is the resistance, however, which we can find with Ohm’s law:

$$R = \frac{\Delta V}{I} = \frac{\Delta V}{nqAv_d} \propto \frac{1}{nv_d}$$

So the resistance is inversely proportional to the carrier density and drift velocity. Let's say the initial resistance is  $R_0$ , and the resistance after changing  $n$  and  $v_d$  is just  $R$ . If we double the number of carriers, the resistance goes *down* by a factor of two. If we decrease the drift velocity by 10 times, the resistance goes *up* by 10 times.

$$\begin{aligned} R_0 &\propto \frac{1}{nv_d} \\ R &\propto \frac{1}{(2n)\left(\frac{v_d}{10}\right)} = \frac{10}{2nv_d} = \frac{5}{nv_d} \\ \implies R &= 5R_0 \end{aligned}$$

Even though we don't know what the actual resistance  $R_0$  is, we can say that  $R$  is five times more after the carrier concentration and velocity change. The one tricky step here is to write down the proper relationship between *resistance* and the given quantities, not just the relationship between *current* and the given quantities.

5. An electric current of 1 mA flows through a conductor, which results in a 150 mV potential difference. The resistance of the conductor is:

This is just plugging the numbers straight in to Ohm's law:

$$R = \frac{\Delta V}{I} = \frac{150 \text{ mV}}{1 \text{ mA}} = \frac{150 \times 10^{-3} \text{ V}}{1 \times 10^{-3} \text{ A}} = 150 \text{ V/A} = 150 \Omega$$

6. The resistance of a 150 W, 115 V light bulb is  $88 \Omega$  when the light bulb is at its operating temperature. What current passes through the light bulb when in operation?

Here we need to recall the definition of electrical power and Ohm's law. Power is voltage times current. Since we know the power and voltage, we can solve for current. Remember that a Watt is a Volt times an Amp ...

$$\begin{aligned} \mathcal{P} &= \Delta V I \\ 150 \text{ W} &= (115 \text{ V}) I \\ \implies I &= \frac{150 \text{ W}}{115 \text{ V}} = 1.30 \text{ W/V} = 1.30 \text{ A} \end{aligned}$$

7. How many electrons per second does the current above correspond to?

The definition of current is the amount of charge flowing through a surface in one second. Each electron carries a single unit charge of  $1.6 \times 10^{-19} \text{ C}$ , so the total charge is just how many electrons there are times how much charge each one carries.

$$\begin{aligned} 1.3 \text{ [A]} &= 1.3 \text{ [C/s]} = \left[ 1.6 \times 10^{-19} \text{ [C/electron]} \right] \left[ \text{[electrons/s]} \right] \\ \implies \text{[electrons/s]} &= \frac{1.3 \text{ [A]}}{1.6 \times 10^{-19} \text{ [C/s]}} = 8.2 \times 10^{18} \end{aligned}$$

For that last line, we note that one Amp is just one Coulomb per second, so the units cancel - the number of electrons is just a number, and has no units.