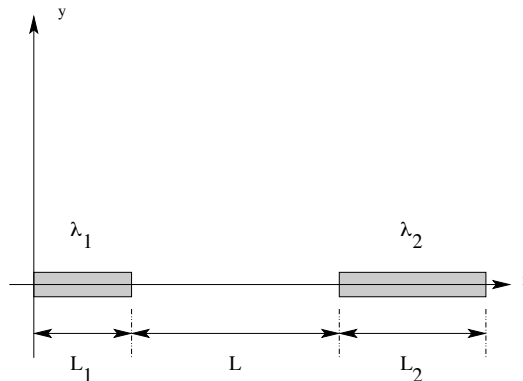


Problem Set 1: Solutions

1. *Purcell 1.4* At each corner of a square is a particle with charge q . Fixed at the center of the square is a point charge with opposite sign, of magnitude Q . What value must Q have to make the total force on each of the four particles zero? With Q set at that value, the system, in the absence of other forces, is in equilibrium. Do you think the equilibrium is stable?

Coming soon ...

2. Two thin rigid rods lie along the x axis, as shown below. Both rods are uniformly charged. Rod 1 has a length L_1 and a charge per unit length λ_1 . Rod 2 has a length L_2 and a charge per unit length λ_2 . The distance between the right end of rod 1 and the left end of rod 2 is L .



(a) Give an exact expression for the electrical force between the two rods, i.e. the force that one rod exerts on the other. If you get really stuck on the integral, you should always feel free to consult an integral table or try:

<http://integrals.wolfram.com>

(b) Use a first-order Taylor expansion to show that for $L_2 \gg L_1$ the electrical force on rod 1 is approximately

$$\vec{F}_1 = -\hat{x}\lambda_1\lambda_2 \ln\left(1 + \frac{L_1}{L}\right)$$

(c) Show that in the limit $L \gg L_1$ and $L \gg L_2$ your expression for the force between the rods reduces to the Coulomb force between two point charges. What are the magnitudes Q_1 and Q_2 of the point charges?

We first have to break each rod up into infinitesimal pieces, and calculate the sum of the forces from all the little bits of one rod on bits making up the other rod, and vice versa. This isn't as complex as it sounds. First, break the left rod up into tiny bits of length dx_1 , which will each then have a charge $dq_1 = \lambda_1 dx_1$. Similarly, we break the right rod up into bits of length dx_2 , which have charge $dq_2 = \lambda_2 dx_2$.

Each tiny bit of charge in the left rod dq_1 feels a force due to all the dq_2 in the right rod. Now consider the force between two given infinitesimal pieces of charge in each rod, dq_1 and dq_2 . Let dq_1 be at a position x_1 , and dq_2 at a position x_2 . The two tiny pieces of charge are then a distance $x_2 - x_1$ apart, and the force between them $d\vec{F}_{12}$ is easily calculated:

$$d\vec{F}_{12} = \frac{k_e dq_1 dq_2}{(x_2 - x_1)^2} \hat{\mathbf{x}} = \frac{k_e \lambda_1 \lambda_2 dx_1 dx_2}{(x_2 - x_1)^2} \hat{\mathbf{x}}$$

Now to find the total force a tiny section of the left rod dq_1 due to the *entire* right rod, we have to add up all of the infinitesimal $d\vec{F}_{12}$ due to all possible dq_2 in the right rod. This is done by integrating with respect to dx_2 over the length of the right rod - that is, the total force on a given section dq_1 due to all the bits dq_2 is found by integrating over all tiny slices of the right rod dx_2 . The right rod runs from $L_1 + L$ to $L_1 + L_2 + L$, thus:

$$d\vec{F}_{12,\text{tot}} = \int_{\text{right rod}} d\vec{F}_{12} = \int_{L_1+L}^{L_1+L_2+L} \frac{k_e \lambda_1 \lambda_2 dx_1}{(x_2 - x_1)^2} dx_2 \hat{\mathbf{x}}$$

We can readily carry out this integral:

$$d\vec{F}_{12,\text{tot}} = \int_{L_1+L}^{L_1+L_2+L} \frac{k_e \lambda_1 \lambda_2 dx_1}{(x_2 - x_1)^2} dx_2 \hat{\mathbf{x}} = k_e \lambda_1 \lambda_2 \left[\frac{-1}{x_2 - x_1} \right]_{L_1+L}^{L_1+L_2+L} dx_1 \hat{\mathbf{x}}$$

$$d\vec{F}_{12,\text{tot}} = k_e \lambda_1 \lambda_2 \left[\frac{1}{x_1 - (L_1 + L_2 + L)} - \frac{1}{x_1 - (L_1 + L)} \right] dx_1 \hat{\mathbf{x}}$$

This is the force on an infinitesimal portion dq_1 of the left rod due to the entire right rod. The total force on the left rod is then found by summing up the forces from all the dx_1 in the left rod. In other words, we integrate with respect to dx_1 over length of the left rod, which runs from 0 to L_1 :

$$\begin{aligned}
\vec{\mathbf{F}}_{12,\text{tot}} &= \int_{\text{left rod}} d\vec{\mathbf{F}}_{12,\text{tot}} = \int_{\text{left}} \int_{\text{right}} d\vec{\mathbf{F}}_{12} = \int_0^{L_1} dx_1 \int_{(L_1+L)}^{(L_1+L_2+L)} \frac{k_e \lambda_1 \lambda_2}{(x_2 - x_1)^2} dx_2 \hat{\mathbf{x}} \\
&= \int_0^{L_1} k_e \lambda_1 \lambda_2 \left[\frac{1}{x_1 - (L_1 + L_2 + L)} - \frac{1}{x_2 - (L_1 + L)} \right] dx_1 \hat{\mathbf{x}} \\
&= k_e \lambda_1 \lambda_2 \left[\ln [x_1 - (L_1 + L_2 + L)] - \ln [x_1 - (L_1 + L)] \right]_0^{L_1} \hat{\mathbf{x}} \\
&= k_e \lambda_1 \lambda_2 \left[\ln [-L_2 - L] - \ln [-L] - \ln [-L_1 - L_2 - L] + \ln [-L_1 - L] \right] \hat{\mathbf{x}} \\
\Rightarrow \vec{\mathbf{F}}_{12,\text{tot}} &= k_e \lambda_1 \lambda_2 \ln \left[\frac{(L_2 + L)(L_1 + L)}{L(L + L_1 + L_2)} \right] \hat{\mathbf{x}}
\end{aligned}$$

(b) The integration to get the electric force between the two rods should look something like this after you finish part (a):

$$\vec{\mathbf{F}}_{12,\text{tot}} = \int_0^{L_1} dx_1 \int_{L_1+L}^{L_1+L_2+L} \frac{k_e \lambda_1 \lambda_2}{(x_2 - x_1)^2} dx_2 \hat{\mathbf{x}} \quad (1)$$

If you perform the integrals correctly, you should end up with something like this:

$$\begin{aligned}
\vec{\mathbf{F}}_{12,\text{tot}} &= -k_e \lambda_1 \lambda_2 \log \left[\frac{(L + L_2)(L + L_1)}{L(L_1 + L_2 + L)} \right] \hat{\mathbf{x}} \\
&= -k_e \lambda_1 \lambda_2 \left[\ln \left(\frac{L_1 + L}{L} \right) + \ln \left(\frac{L_2 + L}{L_2 + L_1 + L} \right) \right] \hat{\mathbf{x}}
\end{aligned}$$

In the second line, we regrouped the \ln terms into two more manageable fractions, which will be useful below. Now ... what happens when $L_2 \gg L_1$? This is equivalent to saying $\frac{L_1}{L_2} \ll 1$. That means that if we can rewrite the expression above to contain the small fraction $\frac{L_1}{L_2}$, we should be able to use some sort of approximation. For example, the Taylor expansion for $\ln(1+x)$:

$$\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} \quad (2)$$

To first order then, $\ln(1+x) \approx x$. Thus, if we can break the force expression into bits that look like $\ln\left(1 + \frac{L_1}{L_2}\right)$, we can approximate those terms by $\ln\left(1 + \frac{L_1}{L_2}\right) \approx \frac{L_1}{L_2}$. Remembering how to manipulate logarithms, we first need to ‘massage’ our previous result a bit:ⁱ

ⁱFor example, recalling that $\ln(ab) = \ln a + \ln b$ and $\ln a = -\ln(1/a)$.

$$\vec{\mathbf{F}}_{12,\text{tot}} = -k_e \lambda_1 \lambda_2 \left[\ln \left(\frac{L_2 + L}{L_2 + L_1 + L} \right) + \ln \left(\frac{L_1 + L}{L} \right) \right] \hat{\mathbf{x}} \quad (3)$$

$$= -k_e \lambda_1 \lambda_2 \left[-\ln \left(\frac{L_1 + L_2 + L}{L_2 + L} \right) + \ln \left(\frac{L_1 + L}{L} \right) \right] \hat{\mathbf{x}} \quad (4)$$

$$= -k_e \lambda_1 \lambda_2 \left[-\ln \left(1 + \frac{L_1}{L_2 + L} \right) + \ln \left(\frac{L_1}{L} + 1 \right) \right] \hat{\mathbf{x}} \quad (5)$$

$$(6)$$

Now we are close. The second term in square brackets is fine as it is - it does not involve $\frac{L_1}{L_2}$ at all, so it is not necessary to approximation (yet). The first term will be nearly zero so long as $L_2 \gg L_1$. More explicitly, we can use our Taylor expansion from above:

$$-\ln \left(1 + \frac{L_1}{L_2 + L} \right) \approx -\frac{L_1}{L_2 + L} = -\frac{1}{\frac{L_2}{L_1} + \frac{L}{L_1}} \approx 0 \quad (7)$$

So long as $L_2 \gg L_1$, the denominator of the right-most fraction above will be very large, which makes the whole term very small ... negligible in fact. Thus, to first order, we can approximate this term as zero. Putting it all together,

$$\vec{\mathbf{F}}_{12,\text{tot}} = -k_e \lambda_1 \lambda_2 \left[-\ln \left(1 + \frac{L_1}{L_2 + L} \right) + \ln \left(\frac{L_1}{L} + 1 \right) \right] \hat{\mathbf{x}} \quad (8)$$

$$\approx -k_e \lambda_1 \lambda_2 \left[0 + \ln \left(\frac{L_1}{L} + 1 \right) \right] \hat{\mathbf{x}} \quad (9)$$

$$\implies \vec{\mathbf{F}}_{12,\text{tot}} = -k_e \lambda_1 \lambda_2 \ln \left(\frac{L_1}{L} + 1 \right) \hat{\mathbf{x}} \quad (10)$$

This is the desired result for the second part of the question.

(c) Finally, we are asked to additionally consider $L \gg L_1$ and $L \gg L_2$. Under these conditions, $\frac{L_1}{L} \ll 1$, and $\frac{L_2}{L} \ll 1$. Starting from our last non-approximated expression,

$$\vec{\mathbf{F}}_{12,\text{tot}} = -k_e \lambda_1 \lambda_2 \left[\ln \left(\frac{L_1 + L}{L_2 + L_1 + L} \right) + \ln \left(\frac{L_2 + L}{L} \right) \right] \hat{\mathbf{x}} \quad (11)$$

$$= -k_e \lambda_1 \lambda_2 \left[-\ln \left(1 + \frac{L_1}{L_2 + L} \right) + \ln \left(\frac{L_1}{L} + 1 \right) \right] \hat{\mathbf{x}} \quad (12)$$

$$(13)$$

We first notice that the second term is now readily approximated:

$$\ln \left(\frac{L_1}{L} + 1 \right) \approx \frac{L_1}{L} \quad (14)$$

We don't need to further simplify this yet; we will first plug it back into our original expression and

simplify in the end. The first term we have already approximated before:

$$-\ln\left(\frac{L_1 + L_2 + L}{L_2 + L}\right) \approx -\frac{L_1}{L + L_2} \quad (15)$$

Now our full expression becomes:

$$\vec{\mathbf{F}}_{12,\text{tot}} = -k_e \lambda_1 \lambda_2 \left[-\ln\left(1 + \frac{L_1}{L_2 + L}\right) + \ln\left(\frac{L_1}{L} + 1\right) \right] \hat{\mathbf{x}} \quad (16)$$

$$\approx -k_e \lambda_1 \lambda_2 \left[-\frac{L_1}{L + L_2} + \frac{L_1}{L} \right] \hat{\mathbf{x}} \quad (17)$$

$$= -k_e \lambda_1 \lambda_2 \left[\frac{L_1 L_2}{L(L + L_2)} \right] \hat{\mathbf{x}} \quad (18)$$

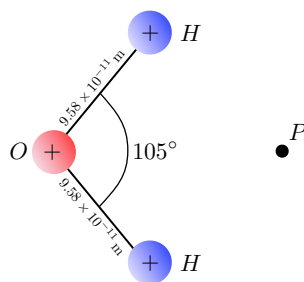
$$= -k_e \lambda_1 \lambda_2 \left[\frac{L_1 L_2}{L^2} \frac{1}{1 + \frac{L_2}{L}} \right] \hat{\mathbf{x}} \quad (19)$$

$$\approx -k_e \lambda_1 \lambda_2 \frac{L_1 L_2}{L^2} \quad (20)$$

In the second to last line, we found one more negligibly small factor of L_2/L to get rid of. Finally, we note that the total charge on rod 1 is just $Q_1 = \lambda_1 L_1$, and for rod 2 the total charge is $Q_2 = \lambda_2 L_2$. Thus,

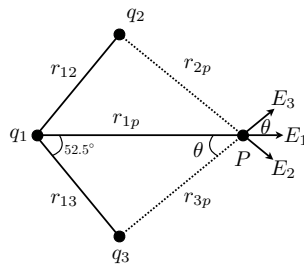
$$\vec{\mathbf{F}}_{12,\text{tot}} = -\frac{k_e Q_1 Q_2}{L^2} \hat{\mathbf{x}} \quad (L_1, L_2 \ll L) \quad (21)$$

3. The distance between the oxygen nucleus and each of the hydrogen nuclei in an H_2O molecule is 9.58×10^{-11} m, and the bond angle between hydrogen atoms is 105° . **(a)** Find the electric field produced by the nuclear charges (positive charges) at the point P a distance 1.2×10^{-10} m to the right of the oxygen nucleus. **(b)** Find the electric potential at P .



First, we need to define the geometry of the situation a bit more clearly, and label things properly. Have a look:

Rather than worry about which nucleus is which, we will simply label the charges q_1 , q_2 , and q_3 and be as general as possible. We will also label the distances in a generic but self-explanatory way: the distance from charge 1 to charge 2 is r_{12} , the distance from charge 3 to the point P is r_{3p} , and so on.



First, connect q_1 and P with a straight line. This is our x axis, and it nicely splits the problem into two symmetric halves. Since the bond angle was given as 105° , we know that the angle $\angle Pq_1q_3$ must be 52.5° , as must the angle $\angle Pq_1q_2$. The electric field due to charge 1 will clearly point directly along the x axis toward point P . The electric field due to charge 3 will make an angle θ with the x axis. Clearly, by symmetry, since $q_3 = q_2$ the electric fields from charges 2 and 3 will have the same x components, but equal and opposite y components - $E_{2x} = E_{3x}$, $E_{2y} = -E_{3y}$. Thus, the fields from charges 2 and 3 will in total have only an x component - so it is enough to compute only the x component of the field. And, since the x components are the same, we really only need to find one of them. In total, the field at P is then only composed of x components, and requires only two calculations:

$$\vec{\mathbf{E}}_P = [E_{2x} + E_{3x} + E_1] \hat{\mathbf{x}} = [2E_{3x} + E_1] \hat{\mathbf{x}}$$

First, we can easily find E_1 , since we are told $r_{1p} = 1.2 \times 10^{-10}$ m:

$$E_1 = \frac{k_e q_1}{r_{1p}^2}$$

In order to find E_{3x} , we need two things: the angle θ , and the distance of charge 3 to point P , viz., r_{3p} . We can find the latter in terms of known quantities using the law of cosinesⁱⁱ on the triangle $\triangle q_1 P q_3$ with the 52.5° angle

$$r_{3p}^2 = r_{1p}^2 + r_{13}^2 - 2r_{13}r_{1p} \cos 52.5^\circ \approx 9.79 \times 10^{-11} \text{ m}$$

Once we have r_{3p} , we can find the angle θ by using the law of cosines on the same triangle, this time about the angle θ :

$$\begin{aligned} r_{13}^2 &= r_{1p}^2 + r_{3p}^2 - 2r_{1p}r_{3p} \cos \theta \\ \implies \cos \theta &= \frac{r_{1p}^2 + r_{3p}^2 - r_{13}^2}{2r_{1p}r_{3p}} \approx 0.631 \\ \implies \theta &\approx 50.9^\circ \end{aligned}$$

Once we have the angle and distance, we can easily find E_3 , and then its x component:

ⁱⁱThis is a very useful trick, and remembering if you have forgotten. http://en.wikipedia.org/wiki/Law_of_cosines.

$$E_3 = \frac{k_3 q_3}{r_{3p}^2}$$

$$E_{3x} = E_3 \cos \theta = \frac{k_3 q_3}{r_{3p}^2} \cos \theta$$

Since the x component of the field from charge 2 is the same (and the y components of E_2 and E_3 cancel), we are ready to find the total field at point P :

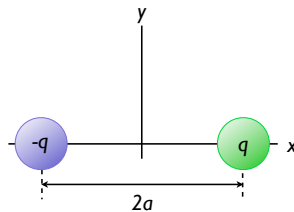
$$\vec{E}_P = [2E_{3x} + E_1] \hat{x} = \left[2 \left(\frac{k_3 q_3}{r_{3p}^2} \cos \theta \right) + \frac{k_e q_1}{r_{1p}^2} \right] \hat{x} \approx [9.9 \times 10^{11} \text{ V/m}] \hat{x}$$

Now, when you get to the point of actually plugging in numbers, remember: the charge on a hydrogen nucleus, with a single proton, is $+e$, while that on an oxygen nucleus is $+8e$.

What about the potential at point P ? Far easier, no vectors! We have two charges a distance r_{3p} away, and one a distance r_{1p} away (again, we know that the contributions from charges 2 and 3 will be the same):

$$V_P = \frac{k_e q_1}{r_{1p}} + \frac{k_e q_2}{r_{2p}} + \frac{k_e q_3}{r_{3p}} = \frac{k_e q_1}{r_{1p}} + 2 \frac{k_e q_3}{r_{3p}} \approx 125 \text{ V}$$

4. Two point charges q and $-q$ are situated along the x axis a distance $2a$ apart as shown below. Show that the electric field at a distant point along $|x| > a$ along the x axis is $E_x = 4k_e q a / x^3$.



Starting this one is not complicated: write down the electric field at a point along the x axis for each charge. The superposition principle says that the total electric field at that point is the sum of the fields from each charge alone. If we are at a point $(x, 0)$, then the $-q$ charge is a distance $x+a$ away, and the $+q$ charge is $x-a$ away. Thus:

$$\begin{aligned} E_{\text{tot}} &= E_q + E_{-q} \\ &= \frac{k_e q}{(x-a)^2} + \frac{k_e (-q)}{(x+a)^2} = \frac{k_e q (x+a)^2}{(x-a)^2 (x+a)^2} - \frac{k_e q (x-a)^2}{(x-a)^2 (x+a)^2} \\ &= \frac{k_e q (x^2 + 2ax + a^2) - k_e q (x^2 - 2ax + a^2)}{(x^2 - a^2)^2} = \frac{4_e k q a x}{(x^2 - a^2)^2} \end{aligned}$$

Now what? The key is that when we specify that we want the field at a "distant" point, we mean

the distance x is much, much larger than the spacing a , i.e., $x \gg a$. Large enough that we can use mathematical approximations, basically. First, some rearranging:

$$E_{\text{tot}} = \frac{4k_e q a x}{(x^2 - a^2)^2} = \frac{4k_e q a x}{x^4 (1 - a^2/x^2)^2}$$

If we specify that $x \gg a$, then the larger x gets, the smaller a^2/x^2 gets, and for large distances $1 - a^2/x^2 \approx 1$. More directly, before rearranging anything we might have just claimed that since when $x \gg a$, $x^2 - a^2 \approx x^2$ - ignore the a^2 since it is much smaller anyway. Formally, this is considered Not OK, even though it works here. Typically, to make an approximation like this you want to get an expression such that in the limit x tends toward infinity, some term goes to zero and can be ignored - in this case, a^2/x^2 goes to zero, so we drop it. In some sense this is just being pedantic, but this more general trick is very useful for more complicated equations.

In any case, the effect here is the same: the denominator can be approximated as x^4 . Using this approximation,

$$E_{\text{tot}} \approx \frac{4k_e q a x}{x^4} = \frac{4k_e q a}{x^3}$$

A positive and a negative charge like this is a *dipole*, something that comes up a lot - for instance, it is a reasonable approximation of a diatomic molecule (e.g., HCl).

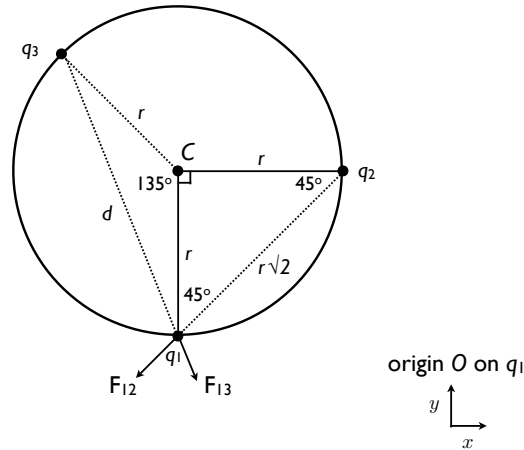
5. *Purcell 1.32* Suppose three positively charged particles are constrained to move on a fixed circular track. If all the charges were equal, an equilibrium arrangement would obviously be a symmetrical one with the particles spaced 120° apart around the circle. Suppose two of the charges have equal charge q , and the equilibrium arrangement is such that these two charges are 90° apart rather than 120° . What must be the relative magnitude of the third charge?

The first thing we need to do is figure out the geometry and draw a picture. First, all three charges are confined to a circular track, which we will say has radius r . Two of the charges are the same, which we will call q_1 and q_2 , and they sit 90° apart on the circle. Where will the third, unequal charge (q_3) sit? In order for the forces on it due to charges 1 and 2 to be balanced, it must be equidistant from both on the circle. If charges 1 and 2 are 90° apart, then there are 270° left in the circle, and the third charge must sit halfway around that - the third charge must be 135° from both of the other charges.

Next, we should pick a coordinate system and origin. For reasons I hope will be clear soon, we will choose the origin to be on charge q_1 , with the $+y$ direction pointing toward the center of the circle and the x axis tangential to the circle, as shown below. We could have equally chosen q_2 as the origin, since it is identical to q_1 , it makes no difference.ⁱⁱⁱ For convenience, we label the center of the circle as point C so we can easily refer to it later.

Since charges q_1 and q_2 are 90° apart on the circle, we can form a 45-45-90 triangle with point C . Based on this, we can find the distance between q_1 and q_2 in terms of the radius r : $r_{12} = r\sqrt{2}$. Charges q_1 and q_2 are identical, and therefore experience a repulsive force of magnitude F_{12} directed along the line connecting them. This force must be at a 45° angle to the x and y axes, based on the geometry above. Charge q_3 has a different magnitude, but the same sign as q_1 , and thus the force between them

ⁱⁱⁱOne could choose any point as the origin and get the same result, but in my opinion the geometry is more transparent in the present case.



F_{13} is also repulsive.

In order for the charges to stay in the positions above, what must be true? For charge q_1 , the forces in the y direction are irrelevant, since q_1 is constrained to stay on the circle anyway. Only net forces along the x direction will force it to move around the circle one way or the other. Thus, in order for this situation to be the equilibrium configuration, the forces in the x direction on q_1 must cancel. Since q_1 and q_2 are identical, the forces along the direction of the circle will also vanish for q_2 automatically. Finally, since the system is symmetric, q_3 must also have no net force along the direction of the circle if neither of the other charges do. Thus, it is sufficient to find the forces in the x direction for q_1 and equate them. This means we need to find the x components of F_{12} and F_{13} , set them equal to one another, and solve for q_3 .

First, we focus on F_{12} , whose x component we will label $F_{12,x}$. We now know the distance between q_1 and q_2 , so the magnitude of the *total* force is easily written down with Coulomb's law:

$$F_{12} = \frac{k_e q_1 q_2}{r_{12}^2} = \frac{k_e q_1 q_2}{(r\sqrt{2})^2} = \frac{k_e q_1 q_2}{2r^2} \quad (22)$$

In order to find the x component, we just need to know the angle that \vec{F}_{12} makes with the x axis - 45° . You should be able to convince yourself this is true based on the geometry above (the inset to the second figure below may help). The x component is then just $F_{12,x} = F_{12} \sin 45^\circ$. Noting that $\sin 45^\circ = \frac{\sqrt{2}}{2}$:

$$F_{12,x} = F_{12} \sin 45^\circ = F_{12} \frac{\sqrt{2}}{2} = \frac{\sqrt{2} k_e q_1 q_2}{4r^2} \quad (23)$$

Now, what about the force between charges 1 and 3, F_{31} ? We can write down the force between them easily:

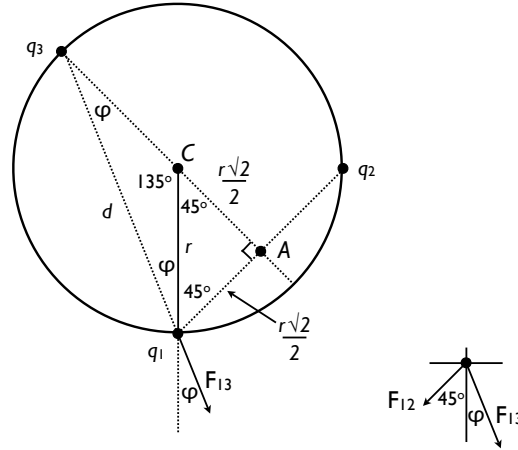
$$F_{13} = \frac{k_e q_1 q_3}{r_{13}^2} = \frac{k_e q_1 q_3}{d^2} \quad (24)$$

What is the distance d between q_1 and q_3 ? For this, we will need the law of cosines (and the fact that

$\cos 135^\circ = -\sqrt{2}/2$):

$$d^2 = r^2 + r^2 - 2 \cdot r \cdot r \cdot \cos 135^\circ = 2r^2 - 2r^2 \left(-\frac{\sqrt{2}}{2} \right) = 2r^2 \left(1 + \frac{\sqrt{2}}{2} \right) \quad (25)$$

Before we combine that with our expression for F_{13} , let us find the x component, for which we need the angle that \vec{F}_{13} makes with our axes. The figure below will help us:



The triangle defined by q_1 , q_3 , and C gives us two equal angles φ . Since the angles of a triangle must add up to 180° , we must have $\varphi = (180^\circ - 135^\circ) / 2 = 22.5^\circ$. This is the angle that \vec{F}_{13} makes with the y axis, and thus $F_{13,x} = F_{13} \sin \varphi$. The inset in the lower right of the figure should help you see this. If we look at the triangle formed by q_1 , q_3 , and point A , we can find $\sin \varphi$ analytically. Look at the φ nearest q_3 : $\sin \varphi = \frac{r\sqrt{2}/2}{d} = \frac{\sqrt{2}r}{2d}$. now we have everything to find $F_{13,x}$:

$$F_{13,x} = F_{13} \sin \varphi = \frac{k_e q_1 q_3}{d^2} \frac{r\sqrt{2}}{2d} = \frac{\sqrt{2} r k_e q_1 q_3}{2d^3} \quad (26)$$

Finally, we have the x components of both forces acting on q_1 . All we need to do now is equate them, and solve for q_3 :

$$F_{13,x} = F_{12,x} \quad (27)$$

$$\frac{\sqrt{2} r k_e q_1 q_3}{2d^3} = \frac{\sqrt{2} k_e q_1 q_2}{4r^2} \quad (28)$$

$$\frac{\sqrt{2} r k_e q_1 q_3}{2d^3} = \frac{\sqrt{2} k_e q_1 q_2}{4r^2} \quad (29)$$

$$\frac{r q_3}{d^3} = \frac{q_2}{2r^2} \quad (30)$$

$$\implies q_3 = \frac{q_2 d^3}{2r^3} \quad (31)$$

Plugging in our expression for d^2 we can find q_3 in terms of only q_2 and numerical factors:

$$q_3 = \frac{1}{2} \left(\frac{d}{r} \right)^3 q_2 \quad (32)$$

$$= \frac{1}{2} \left(\frac{d^2}{r^2} \right)^{\frac{3}{2}} q_2 \quad (33)$$

$$= \frac{1}{2} \left(\frac{2r^2 \left(1 + \frac{\sqrt{2}}{2} \right)}{r^2} \right)^{\frac{3}{2}} q_2 \quad (34)$$

$$= \frac{1}{2} \left(2 + \sqrt{2} \right)^{\frac{3}{2}} q_2 \approx 3.15q_2 \quad (35)$$

Thus, the charge q_3 must be approximately 3.15 times as big as q_1 and q_2 in order for the latter two charges to be 90° apart. Physically, it makes sense that q_3 is bigger - q_1 and q_2 are closer together than they would be if all three charges are equal, so they must be feeling more repulsion from q_3 than from each other, which means q_3 must be bigger.

6. A charge of $100 \mu\text{C}$ is at the center of a cube of side 0.8 m. (a) Find the total flux through each face of the cube. (b) Find the flux through the whole surface of the cube. (c) Would your answers to the first two parts change if the charge were not at the center of the cube?

Coming in lecture ...