

Problem Set 8: Induction

Instructions:

1. Answer all questions below. Show your work for full credit.
2. Due at the end of **Friday** 10 Nov. 2008
3. You may collaborate, but everyone must turn in their own work

1. **10 points.** A thin ring of radius a carries a static charge q . This ring is in a magnetic field of strength B_0 , parallel to the ring's axis, and is supported so that it is free to rotate about that axis. If the field is switched off, how much angular momentum will be added to the ring? If the ring has mass m , show that it will acquire an angular velocity $\omega = qB_0/2mc$.

2. **10 points.** There is evidence that a magnetic field exists in most of the interstellar space with a strength between 10^{-9} and 10^{-10} T. Adopting 3×10^{-10} T as a typical value, find the total energy stored in the magnetic field of the galaxy. Assume the galaxy is a disk roughly 10^{21} m in diameter and 10^{19} m thick. Assuming stars radiate about 10^{37} W, how many years of starlight is the magnetic energy worth?

3. **10 points.** Find the magnetic field at a point P midway between the plates of capacitor a distance r from the axis of symmetry. A current I is flowing through the capacitor.

4. **10 points.** At $t = 0$, the open switch in the figure below is closed. By using Kirchoff's rules for the instantaneous currents and voltages in this two-loop circuit, show that the current in the inductor at time $t > 0$ is

$$I(t) = \frac{\Delta V}{R_1} \left[1 - e^{-(R'/L)t} \right]$$

where $R' = R_1 R_2 / (R_1 + R_2)$

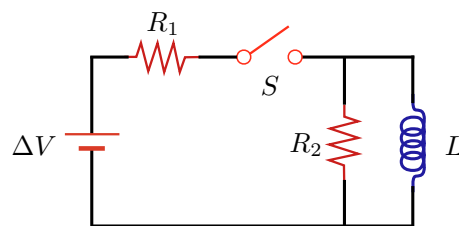


Figure 1: Problem 4: an RL circuit.

5. **10 points.** A cell membrane typically has a capacitance around $1 \mu\text{F}/\text{cm}^2$. It is believed the membrane consists of material having a dielectric constant of $\kappa \sim 3$. Find the thickness this implies. Other electrical measurements have indicated that the resistance of 1 cm^2 of cell membrane is around 1000Ω . Show that the time constant of such a leaky capacitor is independent of the area of the capacitor. How large is it in this case? What is the resistivity?

6. **10 points.** Two inductors having self-inductance L_1 and L_2 are connected in parallel to a time-varying source of current $I(t)$. The mutual inductance between the two inductors is M_{12} . Determine the equivalent self-inductance L_{eq} for the system.