UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 125 / LeClair Spring 2009

Exam I: Solution

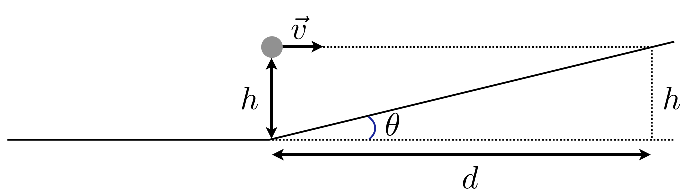
1. A pilot flies horizontally at 1300 km/h, at height $h=35$ m above initially level ground. However, at time $t=0$, the pilot begins to fly over ground sloping upward at angle $\theta=4.3°$. If the pilot does not change the airplane's heading, at what time t does the plane strike the ground?

This is problem 2.80 from your textbook.

Given: The initial velocity and height of a plane flying toward an upward slope of angle θ.

Find: How long before the plane hits the slope? At time $t=0$, the plane is at the beginning of the slope, a height h above level ground. Assuming the plane continues at the same horizontal speed, we wish to find the time at which the plane hits the slope. Given the plane's velocity and height and the slope's angle, we can relate the horizontal distance to intercept the ramp to the plane's height.

Sketch: Assume a spherical plane (it doesn't matter). If the plane is at altitude h, it will hit the ramp after covering a horizontal distance d, where $\tan\theta=h/d$.



Relevant equations: We can relate the horizontal distance to intersect the ramp to the plane's altitude using the known slope of ground:

$$\tan\theta = \frac{h}{d}$$

We can determine how long the horizontal distance d will be covered given the plane's constant horizontal speed v:

$$d = vt$$

Symbolic solution: Combining our equations above, the time t it takes for the plane to hit the slope is

$$t = \frac{d}{v} = \frac{h}{v\tan\theta}$$

Numeric solution: Using the numbers given, and converting units,

$$t = \frac{h}{v\tan\theta} = \frac{35\,\mathrm{m}}{1300\,\mathrm{km/h}\,(1000\,\mathrm{m/km})\,(1\,\mathrm{h}/3600\,\mathrm{s})\,(\tan 4.3°)} \approx 1.3\,\mathrm{s}$$

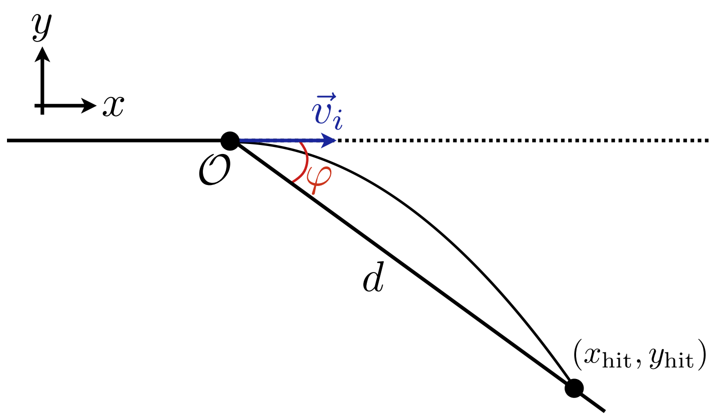
2. A ski jumper leaves the ski track moving in the horizontal direction with a speed of 35 m/s. The landing incline below falls off with a slope of $\varphi=25°$ relative to horizontal. **(a)** How far down the slope does the skier land? **(b)** At what angle does the skier land with respect to the slope? (Greater fall and greater angle can result in loss of control in the landing.)

Look familiar? This is basically the same as the projectile-ramp question from problem set 2 ...

Given: The initial velocity of a skier at the beginning of a downward slope of $\varphi = 25°$.

Find: The coordinates where the skier lands on the slope, and the skier's landing angle with respect to the slope.

Sketch: Let the y axis run vertically and the x axis horizontally, with the origin at the skier's launch position at the base of the slope. The skier's initial velocity then has only an x component. Denote the skier's landing position $(x_{\text{hit}}, y_{\text{hit}})$, and the lateral distance along the slope d.



Relevant equations: We first need the parametric equations for the skier's trajectory. The skier leaves the ground at the origin $(0, 0)$ and has a velocity purely along the x axis.

$$x(t) = v_i t$$
$$y(t) = -\frac{1}{2}gt^2$$

At the skier's landing point $(x_{\text{hit}}, y_{\text{hit}})$, the skier's distance along the slope will be $d = \sqrt{x_{\text{hit}}^2 + y_{\text{hit}}^2}$. Given the slope φ, we must also have $(x_{\text{hit}}, y_{\text{hit}}) = (d\cos\varphi, -d\sin\varphi)$. This relationship, along with the parametric equations above, is enough to find d.

The skier's landing angle θ *with respect to the x axis* will be given by the ratio of the x and y velocity components at the point of impact:

$$\tan\theta = \left.\frac{v_y}{v_x}\right|_{(x_{\text{hit}}, y_{\text{hit}})}$$

The x and y velocities can be found from the parametric equations for position above, $v_x = dx/dt$ and $v_y = dy/dt$. The angle relative to the slope must be the difference between this angle and that of the slope:

$$\text{landing angle} = |\theta| - \varphi$$

Here the absolute value signifies that already know the skier's angle has to be negative, since it is a downward slope. Since we refer to φ as being 'relative to the horizontal,' we are really ignoring its sign as well ...

Symbolic solution: We start with the skier's landing position and relate it to d and the slope's angle:

$$
\begin{aligned}
& x_{\text{hit}} = v_i t_{\text{hit}} = d\cos\varphi \\
\implies \quad & t_{\text{hit}} = \frac{d\cos\varphi}{v_i} \\
& y_{\text{hit}} = -\frac{1}{2}gt_{\text{hit}}^2 = -d\sin\varphi = \frac{-gd^2\cos^2\varphi}{2v_i^2}
\end{aligned}
$$

Rearranging the last line, we can find the distance d the skier lands down the slope

$$d = \frac{2v_i^2\sin\varphi}{g\cos^2\varphi}$$

This is the distance we are after. The angle the skier lands with respect to the x axis is easily found:

$$\tan\theta = \left.\frac{v_y}{v_x}\right|_{(x_{\text{hit}},y_{\text{hit}})} = -\frac{gt_{\text{hit}}}{v_i} = \frac{-gd\cos\varphi}{v_i^2}$$

The actual angle with respect to the slope is then $|\theta| - \varphi$, as noted above

Numeric solution: Given $v_i = 35\,\text{m/s}$ and $\varphi = 25°$,

$$d = \frac{2v_i^2\sin\varphi}{g\cos^2\varphi} \approx 130\,\text{m}$$

$$\theta = \tan^{-1}\left(\frac{-gd\cos\varphi}{v_i^2}\right) \approx -43°$$

The skier's velocity relative to the x axis is thus $43°$ below the horizontal. Since the slope is already $25°$ below the horizontal, the skier's angle relative to the slope is

$$\text{landing angle} = |\theta| - \varphi = 43° - 25° = 18°$$

3. A batter hits a pitched ball when the center of the ball is $1.22\,\text{m}$ above the ground. The ball leaves the bat at an angle of $45°$ with the ground. With that launch, the ball should have a horizontal range (returning to *launch* level) of $107\,\text{m}$. **(a)** Does the ball clear a $7.32\,\text{m}$-high fence that is $97.5\,\text{m}$ horizontally from the launch point? **(b)** At the fence, what is the distance between the fence top and the ball center?

This is problem 4.47 from your textbook.

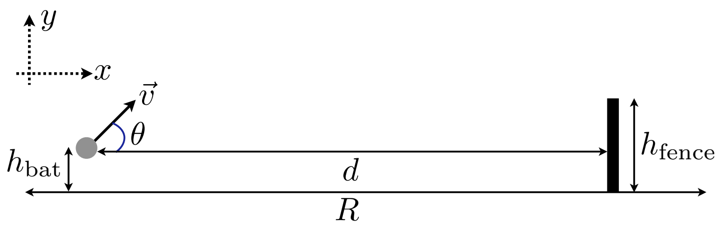
Find: Whether a batted baseball clears a fence, and by what amount it does or does not.

Given: The baseball's initial launch height and angle, the range the baseball would have without the fence, the distance to the fence and its height.

Sketch: Let the y axis run vertically and the x axis horizontally as shown below. Let the range the baseball would have without the fence be $R = 107\,\text{m}$, with the distance to the fence $d = 97.5\,\text{m}$ and its height $h_{\text{fence}} = 7.32\,\text{m}$. The baseball is batted at an angle $\theta = 45°$ at speed v_i a height of $h_{\text{bat}} = 1.22\,\text{m}$ above the ground.
Let the origin be at the position the ball leaves the bat. The height of the fence *relative to the height of the bat* is then

$$\delta h = h_{\text{fence}} - h_{\text{bat}}$$

What we really need to determine is the ball's y coordinate at $x = d$. If $y > \delta h$, the ball clears the fence. We can use the range the baseball would have without the fence and the launch angle to find the ball's speed, which will allow a complete calculation of the trajectory.



Relevant equations: We need only the equations for the range and trajectory of a projectile over level ground:

$$R = \frac{v_i^2 \sin 2\theta}{g}$$
$$y(x) = x \tan\theta - \frac{gx^2}{2v_i^2 \cos^2\theta}$$

Symbolic solution: From the range equation above, we can write the velocity in terms of known quantities:

$$v_i = \sqrt{\frac{Rg}{\sin 2\theta}}$$

The trajectory then becomes

$$y(x) = x \tan\theta - \frac{gx^2 \sin 2\theta}{2Rg \cos^2\theta} = x \tan\theta - \frac{x^2 \sin 2\theta}{2R \cos^2\theta}$$

The height difference between the ball and the fence is $y(d) - \delta h$. If it is positive, the ball clears the fence.

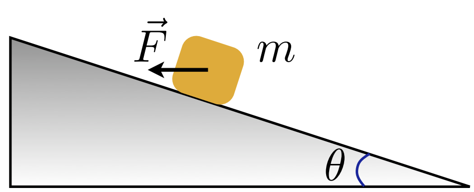
$$\text{clearance} = y(d) - \delta h = d \tan\theta - \frac{d^2 \sin 2\theta}{2R \cos^2\theta} - \delta h = d \tan\theta - \frac{d^2 \sin 2\theta}{2R \cos^2\theta} - h_{\text{fence}} + h_{\text{bat}}$$

Numeric solution: Using the numbers given, and noting $\tan 45° = 1$, $\sin 90° = 1$, and $\cos^2 45° = 1/2$

$$\text{clearance} = d - \frac{d^2}{R} - h_{\text{fence}} + h_{\text{bat}} = 97.5\,\text{m} - \frac{(97.5\,\text{m})^2}{107\,\text{m}} - 7.32\,\text{m} + 1.22\,\text{m} \approx 2.56\,\text{m}$$

The ball does clear the fence, by approximately 2.56 m.

4. A crate of mass $m = 100$ kg is pushed at constant speed up a horizontal ramp ($\theta = 30°$) by a horizontal force $\vec{\mathbf{F}}$. What are the magnitudes of **(a)** $\vec{\mathbf{F}}$ and **(b)** the force on the crate from the ramp?

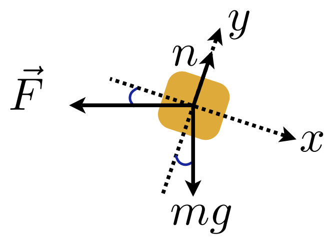


This is problem 5.32 from your textbook.

Find: The force required to push a crate up a 30° ramp at constant speed, meaning zero net acceleration, and the resulting force the crate from the ramp, meaning the normal force.

Given: The crate's mass, the ramp's angle, and a constraint of constant speed.

Sketch: We need a free-body diagram for the crate. Let the x axis be aligned with the ramp's plane, and the y axis normal to its surface. The relevant factors are the applied force F, the crate's weight mg, and the normal force n.



Relevant equations: We need only geometry and Newton's second law:

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

We add to this the constraints that both x and y components of acceleration are zero – the former due to the crate's constant speed, the latter because the crate must stay on the ramp.

Symbolic solution: Along the x direction, we have components of the applied force and weight:

$$\sum F_x = mg\sin\theta - F\cos\theta = (mg - F)\sin\theta = 0$$
$$\implies \quad F = mg\tan\theta$$

Along the y direction we have we have components of the applied force and weight as well as the normal force:

$$\sum F_y = n - mg\cos\theta - F\sin\theta = 0$$
$$\implies \quad n = mg\cos\theta + F\sin\theta = mg\left(\cos\theta + \tan\theta\sin\theta\right)$$

Numeric solution: Using the numbers given,

$$F = mg\tan\theta = (100\,\mathrm{kg})\left(9.81\,\mathrm{m/s^2}\right)\tan 30^\circ \approx 566\,\mathrm{N}$$

$$n = mg\left(\cos\theta + \tan\theta\sin\theta\right) \approx 1132\,\mathrm{N}$$

5. An elevator cab that weights $27.8\,\mathrm{kN}$ moves upward. What is the tension in the cable if the cab's speed is **(a)** increasing at a rate of $1.22\,\mathrm{m/s^2}$ and **(b)** decreasing at a rate of $1.22\,\mathrm{m/s^2}$?

This is problem 5.41 from your textbook.

Given: The weight of an elevator accelerating upward or downward.

Find: The tension in the elevator's cable for either sign of acceleration.

Sketch: We have only two forces: the downward force of the elevator cab's weight, and the upward force of the cable's tension. The sum of the two must equal the cab's mass times the resulting acceleration. For concreteness, let the upward direction be $+y$ and the downward direction be $-y$.

Relevant equations: The force balance in the vertical direction gives us the acceleration of the elevator.

$$T - mg = ma$$

Here the acceleration can be either positive or negative, depending on whether the cab is increasing or decreasing its speed. Additionally, since we are given the cab's *weight* w, we need to calculate its mass:

$$w = mg \qquad \Longrightarrow \qquad m = \frac{w}{g}$$

Symbolic solution: When the elevator is moving upward and *increasing* its speed, this corresponds to positive acceleration. Thus,

$$T_{\text{incr}} = m\left(g + a\right)$$

When the elevator is moving upward and *decreasing* its speed, the acceleration is negative:

$$T_{\text{decr}} = m\left(g - a\right)$$

Numeric solution: With a weight of 27.8 kN, the elevator cab's mass is

$$m = \frac{27.8 \times 10^3\,\text{N}}{9.81\,\text{m/s}^2} = 2830\,\text{kg}$$

The tension in either case is then easily found:

$$T_{\text{incr}} = (2830\,\text{kg})\left(9.81\,\text{m/s}^2 + 1.22\,\text{m/s}^2\right) = 31.3\,\text{kN}$$
$$T_{\text{decr}} = (2830\,\text{kg})\left(9.81\,\text{m/s}^2 - 1.22\,\text{m/s}^2\right) = 24.3\,\text{kN}$$

Qualitatively, it makes sense that the tension is greater when the elevator cab's speed is increasing.

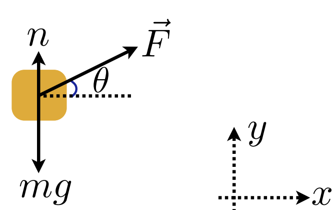
6. A block of mass $m = 5.00\,\text{kg}$ is pulled along a horizontal frictionless floor by a cord that exerts a force of magnitude $F = 12.0\,\text{N}$ at an *upward* angle of $25°$. **(a)** What is the magnitude of the block's acceleration? **(b)** The force magnitude F is slowly increased. What is its value just before the block is lifted (completely) off the floor? **(c)** What is the magnitude of the block's acceleration just before it is lifted (completely) off the floor?

This is problem 5.47 from your textbook.

Given: A block pulled along a frictionless floor by a force making an angle θ with the horizontal.

Find: The block's acceleration, the maximum force before the block leaves the floor, and the block's acceleration at that point.

Sketch: Let the x and y axes be horizontal and vertical, respectively. We have only the block's weight, the normal force, and the applied force.



Relevant equations: We need only Newton's second law and geometry.

Symbolic solution: Along the vertical direction, a force balance must give zero for the block to remain on the floor. This immediately yields the normal force.

$$\sum F_y = n - mg + F\sin\theta = 0 \qquad \Longrightarrow \qquad n = mg - F\sin\theta$$

A horizontal force balance gives us the acceleration:

$$\sum F_x = F\cos\theta = ma_x \qquad \Longrightarrow \qquad a_x = \frac{F}{m}\cos\theta$$

If the magnitude of the force is increased, the block will leave the floor as the normal force becomes zero:

$$n = mg - F\sin\theta = 0 \qquad \Longrightarrow \qquad F = \frac{mg}{\sin\theta}$$

At that point, its acceleration will be

$$a_x = \frac{F}{m}\cos\theta = \frac{g}{\tan\theta}$$

Numeric solution: Using the numbers given, the initial acceleration is

$$a_x = \frac{F}{m}\cos\theta = \frac{12.0\,\mathrm{N}}{5.00\,\mathrm{kg}}\cos 25^\circ \approx 2.18\,\mathrm{m/s^2}$$

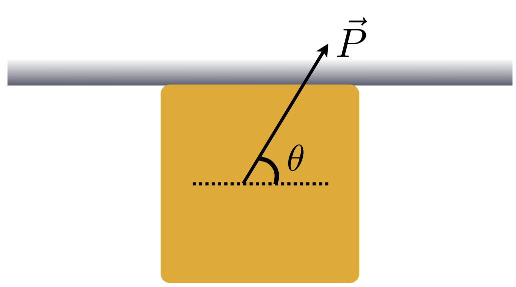
At the point the block is about to leave the floor, the required force is

$$F = \frac{mg}{\sin\theta} = \frac{(5.00\,\mathrm{kg})\left(9.81\,\mathrm{m/s^2}\right)}{\sin 25^\circ} \approx 116\,\mathrm{N}$$

At that point, the block's acceleration is

$$a_x = \frac{g}{\tan\theta} = \frac{9.81\,\mathrm{m/s^2}}{\tan 25^\circ} \approx 21.0\,\mathrm{m/s^2}$$

7. A student, crazed by exams, uses a force $\vec{\mathbf{P}}$ of magnitude $80\,\mathrm{N}$ and angle 70° to push a $5.0\,\mathrm{kg}$ block across the ceiling of his room. If the coefficient of kinetic friction between the block and the ceiling is 0.40, what is the magnitude of the block's acceleration?



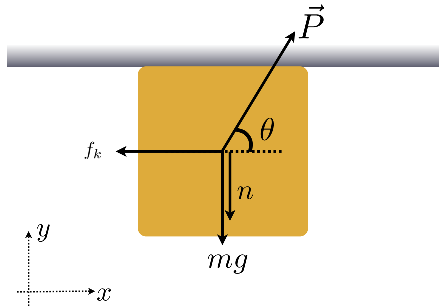
This is problem 6.87 from your textbook.

Given: The force on a block of mass $5.0\,\mathrm{kg}$ pushed against the ceiling and the coefficient of kinetic friction between the block and ceiling.

Find: The acceleration of the block. Clearly, the acceleration will be purely horizontal.

Sketch: Let the x and y axes be horizontal and vertical, respectively. A free-body diagram for the block requires the normal force n pointing *away* from the ceiling, the block's weight, the applied force $\vec{\mathbf{P}}$, and the kinetic friction force f_k.

Relevant equations: The friction force is found from the normal force:



$$f_k = \mu_k n$$

We can find the normal force by a force balance along the y axis, which must yield zero, and we can find the block's horizontal acceleration from horizontal force balance:

$$\sum F_y = 0$$
$$\sum F_x = m a_x$$

Symbolic solution: Along the vertical direction,

$$\begin{aligned} &\sum F_y = P\sin\theta - mg - n = 0 \\ \implies \quad & n = P\sin\theta - mg \end{aligned}$$

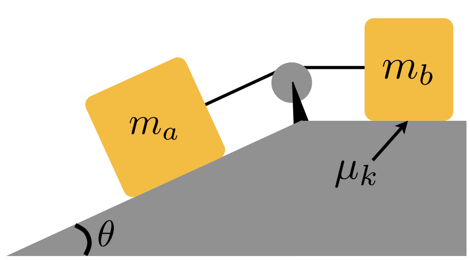
Along the horizontal direction,

$$\begin{aligned} &\sum F_x = P\cos\theta - f_k = P\cos\theta - \mu_k n = P\cos\theta + \mu_k mg - \mu_k P\sin\theta = m a_x \\ \implies \quad & a_x = \frac{P}{m}\left(\cos\theta - \mu_k \sin\theta\right) + \mu_k g \end{aligned}$$

Numeric solution: Using the numbers given,

$$a_x = \frac{P}{m}\left(\cos\theta - \mu_k\sin\theta\right) + \mu_k g = \frac{80\,\mathrm{N}}{5.0\,\mathrm{kg}}\left(\cos 70^\circ - 0.4\sin 70^\circ\right) + 0.4\left(9.81\,\mathrm{m/s^2}\right) \approx 3.4\,\mathrm{m/s^2}$$

8. Block A has mass $m_a = 4.0\,\mathrm{kg}$, and block B has mass $m_b = 2.0\,\mathrm{kg}$. The coefficient of kinetic friction between block B and the horizontal plane is $\mu_k = 0.50$. The inclined plane is frictionless, and at angle $\theta = 30^\circ$. The pulley serves only to change the direction of the cord connecting the two blocks. The cord has negligible mass. Find **(a)** the tension in the cord and **(b)** the magnitude of the acceleration of the two blocks.

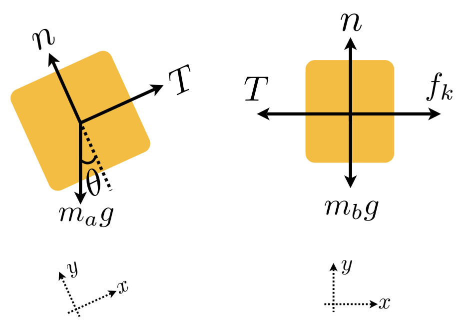


This is problem 6.81 from your textbook.

Find: The tension in a cord connecting two blocks and the system's acceleration, with one block on a frictionless incline and the second on a flat surface with coefficient of kinetic friction μ_k.

Given: The mass of both blocks, the coefficient of friction for the block on the flat surface, and the angle of incline for the ramp.

Sketch: We need free-body diagrams for each mass. Note the axis definitions for each mass. The system will obviously accelerate to the left, as mass A slides down the ramp and pulls mass B with it, meaning the acceleration is in the $-x$ direction according to the sketch below.



Only for mass B do we need to consider the friction force. Since the rope is presumably taut the entire time of interest, the acceleration is the same for both blocks. For the same reason, the tension applied to both blocks is the same.

Relevant equations: Newton's second law and geometry will suffice. Along the y direction for either mass, the forces must sum to zero, while along the x direction, the forces must give the acceleration for each mass.

$$\sum F_y = 0$$
$$\sum F_x = ma_x$$

Symbolic solution: First consider mass A. The free body diagram above yields the following, noting that the acceleration will be purely along the $-x$ direction:

$$\sum F_y = n - m_a g \cos\theta = 0$$
$$\sum F_x = T - m_a g \sin\theta = -m_a a_x \qquad \Longrightarrow \qquad a_x = g \sin\theta - \frac{T}{m_a}$$

For mass B, we must include the frictional force, $f_k = \mu_k n$.

$$\sum F_y = n - m_b g = 0 \qquad \Longrightarrow \qquad n = m_b g$$
$$\sum F_x = f_k - T = \mu_k m_b g - T = -m_b a_x$$

Using the latter equation with our result for the acceleration above allows us to solve for the tension in terms of known quantities:

$$T = \mu_k m_b g + m_b a_x = \mu_k m_b g + m_b g \sin\theta - \frac{m_b}{m_a} T$$
$$T\left(1 + \frac{m_b}{m_a}\right) = \mu_k m_b g + m_b g \sin\theta$$
$$T = \frac{\mu_k m_b g + m_b g \sin\theta}{1 + \frac{m_b}{m_a}} = g\left(\mu_k + \sin\theta\right)\left[\frac{m_a m_b}{m_b + m_a}\right]$$

Finally, this gives an expression for the acceleration in terms of known quantities:

$$a_x = g\sin\theta - \frac{T}{m_a} = g\sin\theta - g\left(\mu_k + \sin\theta\right)\left[\frac{m_b}{m_b + m_a}\right] = g\left[\frac{m_a\sin\theta - \mu_k m_b}{m_b + m_a}\right]$$

Numeric solution: Given $m_a = 4.0\,\mathrm{kg}$, $m_b = 2.0\,\mathrm{kg}$, $\mu_k = 0.5$, and $\theta = 30^\circ$,

$$T = g\left(\mu_k + \sin\theta\right)\left[\frac{m_a m_b}{m_b + m_a}\right] \approx 13\,\mathrm{N}$$

and

$$a_x = g\left[\frac{m_a\sin\theta - \mu_k m_b}{m_b + m_a}\right] \approx 1.64\,\mathrm{m/s^2}$$