

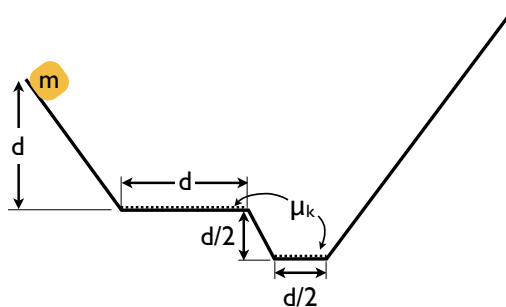
## Exam II

## Instructions

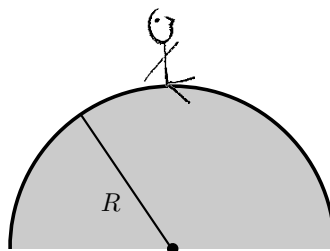
1. Solve 4 of 6 problems below.
2. Use the problem solving steps from the template. Attach other sheets as necessary.
3. All problems have equal weight.
4. You must answer all parts of multi-part questions for full credit.
5. Show your work for full credit. Significant partial credit will be given.
6. You are allowed 2 sides of a standard 8.5 x 11 in piece of paper with notes/formulas and a calculator.
7. You have 1h50m.

1. A block of mass  $m$  is released from rest at a height  $d = 40$  cm and slides down a frictionless ramp and onto a first plateau, which has length  $d$  and where the coefficient of kinetic friction is  $\mu_k = 0.5$ . If the block is still moving, it then slides down a second frictionless ramp through height  $d/2$  and onto a lower plateau, which has length  $d/2$  and where the coefficient of kinetic friction is again  $\mu_k = 0.5$ . If the block is still moving, it then slides up a frictionless ramp.

Where is the *final* stopping point of the block? If it is on a plateau, state which one and give the distance  $L$  from the *left* edge of that plateau.



2. A boy is initially seated on the top of a hemispherical ice mound of radius  $R$ . He begins to slide down the ice, with a negligible initial speed. Approximate the ice as being frictionless. At what height does the boy lose contact with the ice?



3. A uniformly dense rope of length  $b$  and mass per unit length  $\lambda$  is coiled on a smooth table. One end is lifted by hand with constant velocity  $v_0$ . Find the force of the rope held by the hand when the rope is a distance  $a$  above the table ( $b > a$ ).

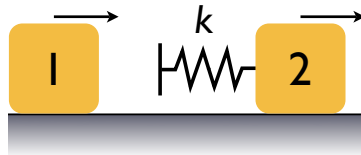
4. Block 1 of mass  $m_1$  is moving rightward at  $v_1$  while block 2 of mass  $m_2$  is moving rightward at  $v_2 < v_1$ . The surface is frictionless, and a spring of constant  $k$  is fixed to block 2. When the blocks collide, the compression of the spring is maximum the instant the blocks have the same velocity.

(a) Show that

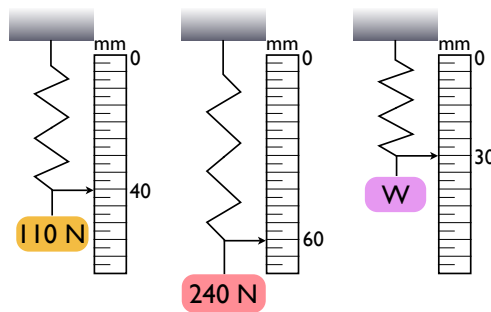
$$\Delta K = K_{1i} + K_{2i} - K_{12} = \frac{1}{2} \mu v_{\text{rel}}^2 \quad \text{with} \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

where  $K_{1i}$  and  $K_{2i}$  are the kinetic energies of blocks 1 and 2 before the collision, respectively,  $K_{12}$  is the kinetic energy of the system at the moment the spring compression is maximum, and  $v_{\text{rel}}$  is the relative velocity of the two blocks. The quantity  $\mu$  is known as the *reduced mass* of the system.

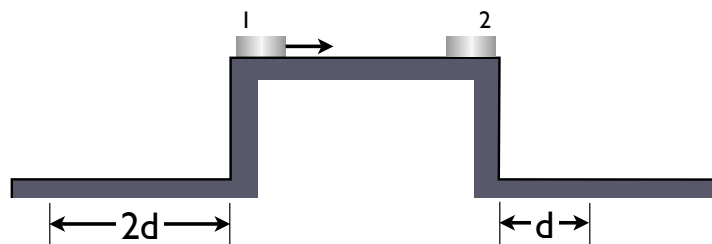
(b) Find the maximum compression of the spring.



5. A spring with a pointer attached is hanging next to a scale marked in millimeters. Three different packages are hung from the spring, in turn, as shown below. (a) Which mark on the scale will the pointer indicate when no package is hung from the spring? (b) What is the weight  $W$  of the third package?



6. In the figure below, puck 1 of mass  $m_1 = 0.20 \text{ kg}$  is sent sliding across a frictionless lab bench, to undergo a one-dimensional elastic collision with stationary puck 2. Puck 2 then slides off the bench and lands a distance  $d$  from the base of the bench. Puck 1 rebounds from the collision and slides off the opposite edge of the bench, landing a distance  $2d$  from the base of the bench. What is the mass of puck 2?



# Formula sheet

$$g = 9.81 \text{ m/s}^2$$

$$0 = ax^2 + bx^2 + c \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$$

$$1 \text{ J} = 1 \text{ kg m}^2/\text{s}^2 = 1 \text{ N m}$$

## 1-D motion:

$$v(t) = \frac{d}{dt}x(t)$$

$$a(t) = \frac{d}{dt}v(t) = \frac{d^2}{dt^2}x(t)$$

const. acc. ↓

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$v_f^2 = v_i^2 + 2a_x \Delta x$$

$$v_f = v_i + at$$

## Projectile motion:

$$v_x(t) = v_{ix} = |\vec{v}_i| \cos \theta$$

$$v_y(t) = |\vec{v}_i| \sin \theta - gt = v_{iy} \sin \theta - gt$$

$$x(t) = x_i + v_{ix}t$$

$$y(t) = y_i + v_{iy}t - \frac{1}{2}gt^2$$

over level ground:

$$\text{max height} = H = \frac{v_i^2 \sin^2 \theta_i}{2g}$$

$$\text{Range} = R = \frac{v_i^2 \sin 2\theta_i}{g}$$

## Force:

$$\sum \vec{F} = \vec{F}_{\text{net}} = m\vec{a} = \frac{d\vec{p}}{dt}$$

$$\sum F_i = ma_i \quad \text{by component}$$

$$\vec{F}_c = \sum F_r = -\frac{mv^2}{r} \hat{r}$$

$$f_k = \mu_k n$$

$$F_s = -kx$$

$$F_g = -mg$$

## 2-D motion:

$$\vec{r} = x(t) \hat{i} + y(t) \hat{j}$$

$$x(t) = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$

$$y(t) = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$

$$\vec{a}(t) = \frac{d^2 s}{dt^2} \hat{T} + \kappa |\vec{v}|^2 \hat{N}$$

$$= \frac{d^2 s}{dt^2} \hat{T} + \frac{|\vec{v}|^2}{R} \hat{N} \equiv a_N \hat{T} + a_T \hat{N}$$

$$\vec{a}_c = -\frac{v^2}{r} \hat{r} \quad \text{circ.}$$

$$T = \frac{2\pi r}{v} \quad \text{circ.}$$

## Work-Energy:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\Delta K = K_f - K_i = W$$

$$W = \int F(x) dx = -\Delta U$$

$$U_g(y) = mgy$$

$$U_s(x) = \frac{1}{2}kx^2$$

$$F = -\frac{dU(x)}{dx}$$

$$K_i + U_i = K_f + U_f + W_{\text{ext}} = K_f + U_f + \int F_{\text{ext}} dx$$

## Momentum, etc.:

$$x_{\text{com}} = \frac{1}{M_{\text{tot}}} \sum_{i=1}^n m_i x_i = \frac{m_1 x_1 + m_2 x_2 + \dots + m_n x_n}{m_1 + m_2 + \dots + m_n}$$

$$v_{\text{com}} = \frac{1}{M_{\text{tot}}} \sum_{i=1}^n m_i v_i = \frac{m_1 v_1 + m_2 v_2 + \dots + m_n v_n}{m_1 + m_2 + \dots + m_n}$$

$$F_{\text{net}} = M_{\text{tot}} a_{\text{com}} = \frac{dp}{dt}$$

$$p_{\text{tot}} = M_{\text{tot}} v_{\text{com}}$$

$$\Delta p = p_f - p_i = J = \int_{t_i}^{t_f} F(t) dt = F_{\text{avg}} \Delta t$$

$$\Delta p = 0 \quad \text{closed}$$

$$\text{elastic coll: } \begin{cases} v_{1f} = 2v_{\text{com}} - v_{1i} \\ v_{2f} = 2v_{\text{com}} - v_{2i} \end{cases}$$

Power	Prefix	Abbreviation
10 <sup>-12</sup>	pico	p
10 <sup>-9</sup>	nano	n
10 <sup>-6</sup>	micro	μ
10 <sup>-3</sup>	milli	m
10 <sup>-2</sup>	centi	c
10 <sup>3</sup>	kilo	k
10 <sup>6</sup>	mega	M
10 <sup>9</sup>	giga	G
10 <sup>12</sup>	tera	T