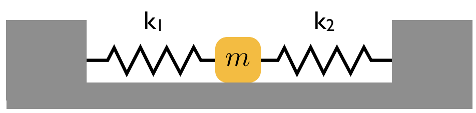
**Name & ID** ________________________________________________

# PH125 Exam III

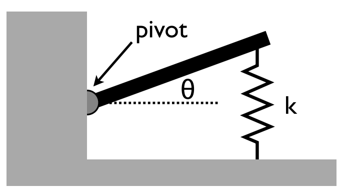
## Instructions

1. **Solve 5 of 8 problems below.** All problems have equal weight.
2. Show your work for full credit. Significant partial credit will be given.
3. You are allowed 2 sides of an 8.5 x 11 in piece of paper with notes and a calculator.

**1.** A block of mass $m$ is connected to two springs of force constants $k_1$ and $k_2$ as shown below. The block moves on a frictionless table after it is displaced from equilibrium and released. Determine the period of simple harmonic motion.




**2.** A horizontal plank of mass $m$ and length $L$ is pivoted at one end. The plank's other end is supported by a spring of force constant $k$. The moment of inertia of the plank about the pivot is $I = \frac{1}{3}mL^2$. The plank is displaced by an angle $\theta$ from horizontal equilibrium and released. Find the angular frequency $\omega$ of simple harmonic motion for small $\theta$. *Hint:* start with the torques at equilibrium, and the spring in equilibrium compression.




**3.** Assume the earth to be a solid sphere of uniform density. A hole is drilled through the earth, passing through its center, and a ball is dropped into the hole. Neglect friction.

**(a)** Calculate the time for the ball to return to the release point. *Hint:* what sort of motion results?
**(b)** Compare the result of part a to the time required for the ball to complete a circular orbit of radius $R_E$ about the earth

**4.** A wad of sticky clay with mass $m$ and velocity $v_i$ is fired at a solid cylinder of mass $M$ and radius $R$ as shown below. The cylinder is initially at rest and mounted on a fixed horizontal axle that runs through its center of mass. The line of motion of the projectile is perpendicular to the axis and at a distance $d < R$ from the center. Find the angular speed of the system just after the clay strikes and sticks to the surface of the cylinder. The moment of inertia of a solid cylinder is $I = \frac{1}{2}MR^2$, the moment of inertia of a point particle mass $m$ a distance $R$ from an axis of rotation is $I = mR^2$.

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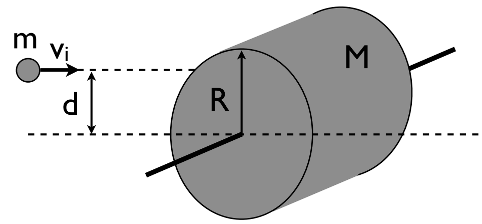



5. A satellite is in a circular Earth orbit of radius $r$. The area $A$ enclosed by the orbit depends on $r^2$ because $A=\pi r^2$. Determine how the following properties depend on $r$: **(a)** period, **(b)** kinetic energy, **(c)** angular momentum, and **(d)** speed.

6. Here are some functions:

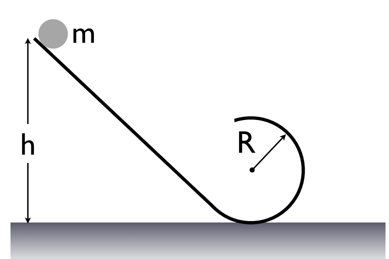
$$f_1(x,t) = Ae^{-b(x-vt)^2} \qquad f_2(x,t) = \frac{A}{b\,(x-vt)^2+1}$$

$$f_3(x,t) = Ae^{-b\left(bx^2+vt\right)} \qquad f_4(x,t) = A\sin\left(bx\right)\cos\left(bvt\right)^3$$

**(a)** Which ones satisfy the wave equation? Justify your answer with explicit calculations.
**(b)** For those functions that *satisfy* the wave equation, write down the corresponding functions $g(x,t)$ representing a wave of the same shape traveling in the opposite direction.

7. A string under tension $T_i$ oscillates in the third harmonic ($n=3$) at frequency $f_3$, and the waves on the string have wavelength $\lambda_3$. If the tension is increased to $T_f = 4T_i$, and the string is again made to oscillate in the third harmonic, what are then the **(a)** frequency of oscillation in terms of $f_3$ and **(b)** the wavelength of the waves in terms of $\lambda_3$?

8. A solid brass ball of mass $m$ will roll smoothly along a loop-the-loop track when released from rest along the straight section. The circular loop has radius $R$, and the ball has radius $r \ll R$. What is $h$ if the ball is on the verge of leaving the track when it reaches the top of the loop? Assume the ball has a moment of inertia $I=kmr^2$, $k \in \mathbb{R}\,|\,0<k<1$.



**Numbers & units:**

$$g = 9.81\,\mathrm{m/s^2} \qquad M_e = 5.96\times10^{24}\,\mathrm{kg} \quad \leftarrow \text{earth}$$

$$R_e = 6.37\times10^6\,\mathrm{m} \quad \leftarrow \text{earth} \qquad G = 6.67\times10^{11}\,\mathrm{N\,m^2/kg^2}$$

**Math:**

$$ax^2+bx^2+c=0 \Longrightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\sin\alpha\pm\sin\beta = 2\sin\frac{1}{2}(\alpha\pm\beta)\cos\frac{1}{2}(\alpha\mp\beta)$$

$$\cos\alpha\pm\cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$$

$$c^2=a^2+b^2-2ab\cos\theta_{ab}$$

$$\frac{d}{dx}\sin ax = a\cos ax \qquad \frac{d}{dx}\cos ax = -a\sin ax$$

$$\int\cos ax\,\mathrm{dx}=\frac{1}{a}\sin ax \qquad \int\sin ax\,\mathrm{dx}=-\frac{1}{a}\cos ax$$

$$\sin\theta\approx\theta \qquad \text{small }\theta$$

$$\cos\theta\approx 1-\frac{1}{2}\theta^2$$

$$\vec{\mathbf{a}}(t)=\frac{d^2s}{dt^2}\,\hat{\mathbf{T}}+\kappa|\vec{\mathbf{v}}|^2\,\hat{\mathbf{N}}=\frac{d^2s}{dt^2}\,\hat{\mathbf{T}}+\frac{|\vec{\mathbf{v}}|^2}{R}\,\hat{\mathbf{N}}\equiv a_N\,\hat{\mathbf{T}}+a_T\,\hat{\mathbf{N}}$$

**Vectors:**

$$|\vec{\mathbf{F}}|=\sqrt{F_x^2+F_y^2} \quad \text{magnitude}$$

$$\theta=\tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{direction}$$

$$d\vec{\mathbf{l}}=dx\,\hat{\mathbf{x}}+dy\,\hat{\mathbf{y}}+dz\,\hat{\mathbf{z}}$$

$$\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=a_xb_x+a_yb_y+a_zb_z=\sum_{i=1}^{n}a_ib_i=|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$$

$$\vec{\mathbf{a}}\times\vec{\mathbf{b}}=\begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix} \qquad |\vec{\mathbf{a}}\times\vec{\mathbf{b}}|=|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sin\theta$$

**Waves:**

$$y_{\rightarrow}(x,t)=y_m\sin(kx-\omega t) \qquad k=\frac{2\pi}{\lambda}$$

$$v=\frac{\omega}{k}=\frac{\lambda}{T}=\lambda f \quad \text{wave speed}$$

$$v=\sqrt{T/\mu} \quad \mu=M/L$$

$$P_{avg}=\frac{1}{2}\mu v\omega^2y_m^2 \quad \text{pwr}$$

$$\frac{\partial^2y}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2y}{\partial t^2} \quad \text{wave}$$

$$y_t=2y_m\cos\frac{\varphi}{2}\sin(kx-\omega t+\varphi/2) \quad \text{int.}$$

$$y_t=2y_m\sin kx\cos\omega t \quad \text{standing}$$

$$f=\frac{v}{\lambda}=\frac{nv}{2L} \quad n\in\mathbb{N}$$

**Rotation: we use radians**

$$s=\theta r \quad \leftarrow \text{arclength}$$

$$\omega=\frac{d\theta}{dt}=\frac{v}{r} \qquad \alpha=\frac{d\omega}{dt}$$

$$a_t=\alpha r \quad \text{tangential} \qquad a_r=\frac{v^2}{r}=\omega^2r \quad \text{radial}$$

$$I=\sum_i m_ir_i^2 \Rightarrow \int r^2\,dm=kmr^2$$

$$I_z=I_{com}+md^2 \quad \text{axis } z \text{ parallel, dist } d$$

$$\tau_{net}=\sum\vec{\boldsymbol{\tau}}=I\vec{\boldsymbol{\alpha}}=\frac{d\vec{\mathbf{L}}}{dt}$$

$$\vec{\boldsymbol{\tau}}=\vec{\mathbf{r}}\times\vec{\mathbf{F}} \qquad |\vec{\boldsymbol{\tau}}|=rF\sin\theta_{rF}$$

$$\vec{\mathbf{L}}=\vec{\mathbf{r}}\times\vec{\mathbf{p}}=I\vec{\boldsymbol{\omega}}$$

$$K=\frac{1}{2}I\omega^2=L^2/2I$$

$$\Delta K=\frac{1}{2}I\omega_f^2-\frac{1}{2}I\omega_i^2=W=\int\tau\,d\theta$$

$$P=\frac{dW}{dt}=\tau\omega$$

**Gravitation:**

$$\vec{\mathbf{F}}_{12}=\frac{Gm_1m_2}{r^2}\,\hat{\mathbf{r}}_{12}=-\vec{\boldsymbol{\nabla}}U_g$$

$$g=\frac{GM_e}{R_e^2}$$

$$U_g(r)=-\int F(r)\,dr=\frac{-GMm}{r}$$

$$K+U_g=0 \quad \text{escape} \qquad K+U_g<0 \quad \text{bound}$$

$$\frac{dA}{dt}=\frac{1}{2}r^2\omega=\frac{L}{2m} \qquad T^2=\left(\frac{4\pi^2}{GM}\right)r^3$$

$$E_{\text{orbit}}=\frac{-GMm}{2a} \quad \text{elliptical; } a\to r \text{ for circular}$$

**Oscillations:**

$$T=\frac{1}{f} \quad \omega=\frac{2\pi}{T}=2\pi f$$

$$x(t)=x_m\cos(\omega t+\varphi)$$

$$a=-\omega^2x \qquad \frac{d^2q}{dt^2}=-\omega^2q$$

$$\omega=\sqrt{k/m} \quad \text{linear osc.}$$

$$T=\begin{cases}2\pi\sqrt{I/\kappa} & \text{torsion pendulum}\\ 2\pi\sqrt{L/g} & \text{simple pendulum}\\ 2\pi\sqrt{I/mgh} & \text{physical pendulum}\end{cases}$$

$$U=-\frac{1}{2}kx^2 \quad U=-\frac{1}{2}\kappa\theta^2 \quad F=-\frac{dU}{dx}=ma \quad \text{SHM}$$

$$x(t)=x_me^{-bt/2m}\cos(\omega't+\varphi) \quad \text{damped}$$

$$\omega'=\sqrt{k/m-b^2/4m}$$

| Derived unit | Symbol | equivalent to |
|---|---|---|
| newton | N | kg·m/s$^2$ |
| joule | J | kg·m$^2$/s$^2$ = N·m |
| watt | W | J/s=m$^2$·kg/s$^3$ |