

Numbers & units:

$$g = 9.81 \text{ m/s}^2 \quad M_e = 5.96 \times 10^{24} \text{ kg} \quad \leftarrow \text{earth}$$

$$R_e = 6.37 \times 10^6 \text{ m} \quad \leftarrow \text{earth} \quad G = 6.67 \times 10^{11} \text{ N m}^2/\text{kg}^2$$

Math:

$$ax^2 + bx^2 + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{1}{2}(\alpha \pm \beta) \cos \frac{1}{2}(\alpha \mp \beta)$$

$$\cos \alpha \pm \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

$$c^2 = a^2 + b^2 - 2ab \cos \theta_{ab}$$

$$\frac{d}{dx} \sin ax = a \cos x \quad \frac{d}{dx} \cos ax = -a \sin x$$

$$\int \cos ax \, dx = \frac{1}{a} \sin x \quad \int \sin ax \, dx = -\frac{1}{a} \cos x$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

$$\int \frac{x \, dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2 \sqrt{x^2 + a^2}}$$

$$\vec{a}(t) = \frac{d^2 s}{dt^2} \hat{T} + \kappa |\vec{v}|^2 \hat{N} = \frac{d^2 s}{dt^2} \hat{T} + \frac{|\vec{v}|^2}{R} \hat{N} \equiv a_N \hat{T} + a_T \hat{N}$$

Vectors:

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} \quad \text{magnitude}$$

$$\theta = \tan^{-1} \left[\frac{F_y}{F_x} \right] \quad \text{direction}$$

$$d\vec{l} = dx \hat{x} + dy \hat{y} + dz \hat{z}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = \sum_{i=1}^n a_i b_i = |\vec{a}| |\vec{b}| \cos \theta$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \quad |\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$$

Waves:

$$y_{\leftarrow}(x, t) = y_m \sin(kx - \omega t) \quad k = \frac{2\pi}{\lambda}$$

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \quad \text{wave speed}$$

$$v = \sqrt{T/\mu} \quad \mu = M/L$$

$$P_{avg} = \frac{1}{2} \mu v \omega^2 y_m^2 \quad \text{pwr}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \quad \text{wave}$$

$$y_t = 2y_m \cos \frac{\varphi}{2} \sin(kx - \omega t + \varphi/2) \quad \text{int.}$$

$$y_t = 2y_m \sin kx \cos \omega t \quad \text{standing}$$

$$f = \frac{v}{\lambda} = \frac{nv}{2L} \quad n \in \mathbb{N}$$

Rotation: we use radians

$$s = \theta r \quad \leftarrow \text{arclength}$$

$$\omega = \frac{d\theta}{dt} = \frac{v}{r} \quad \alpha = \frac{d\omega}{dt}$$

$$a_t = \alpha r \quad \text{tangential} \quad a_r = \frac{v^2}{r} = \omega^2 r \quad \text{radial}$$

$$I = \sum_i m_i r_i^2 \implies \int r^2 dm = k m r^2$$

$$I_z = I_{com} + m d^2 \quad \text{axis } z \text{ parallel, dist } d$$

$$\tau_{net} = \sum \vec{r} \times \vec{F} = I \vec{\alpha} = \frac{d\vec{L}}{dt}$$

$$\vec{\tau} = \vec{r} \times \vec{F} \quad |\vec{\tau}| = r F \sin \theta_{rF}$$

$$\vec{L} = \vec{r} \times \vec{p} = I \vec{\omega}$$

$$K = \frac{1}{2} I \omega^2$$

$$\Delta K = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2 = W = \int \tau \, d\theta$$

$$P = \frac{dW}{dt} = \tau \omega$$

Gravitation:

$$\vec{F}_{12} = \frac{G m_1 m_2}{r^2} \hat{r}_{12} = -\vec{\nabla} U_g$$

$$g = \frac{GM_e}{R_e^2}$$

$$U_g(r) = -\int F(r) \, dr = \frac{-GMm}{r}$$

$$K + U_g = 0 \quad \text{escape} \quad K + U_g < 0 \quad \text{bound}$$

$$\frac{dA}{dt} = \frac{1}{2} r^2 \omega = \frac{L}{2m}$$

$$T^2 = \left(\frac{4\pi^2}{GM} \right) r^3$$

$$E_{orbit} = \frac{-GMm}{2a} \quad \text{elliptical; } a \rightarrow r \text{ for circular}$$

Oscillations:

$$T = \frac{1}{f} \quad \omega = \frac{2\pi}{T} = 2\pi f$$

$$x(t) = x_m \cos(\omega t + \varphi)$$

$$a = -\omega^2 x \quad \frac{d^2 q}{dt^2} = -\omega^2 q$$

$$\omega = \sqrt{k/m} \quad \text{linear osc.}$$

$$T = \begin{cases} 2\pi \sqrt{I/\kappa} & \text{torsion pendulum} \\ 2\pi \sqrt{L/g} & \text{simple pendulum} \\ 2\pi \sqrt{I/mgh} & \text{physical pendulum} \end{cases}$$

$$E = -\frac{1}{2} k x^2 \quad E = -\frac{1}{2} \kappa \theta^2$$

$$x(t) = x_m e^{-bt/2m} \cos(\omega' t + \varphi) \quad \text{damped}$$

$$\omega' = \sqrt{k/m - b^2/4m}$$

Derived unit	Symbol	equivalent to
newton	N	kg·m/s ²
joule	J	kg·m ² /s ² = N·m
watt	W	J/s = m ² ·kg/s ³