Name & ID

PH125 Final Exam

Instructions

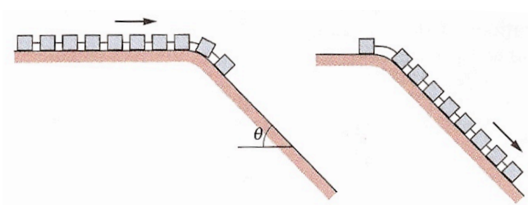
1. **Solve the required number of problems in each section below.**
2. Indicate which problems you have attempted by filling in the adjacent box.
3. Show your work for full credit. Significant partial credit will be given.
4. You are allowed 2 sides of an 8.5 x 11 in piece of paper with notes and a calculator.

Section I: Kinematics, Newton's laws. Solve 3 of 5

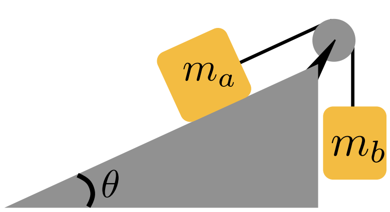
□ **1.** A rock is dropped from rest into a well, and it is heard to hit the surface of the water at bottom of the well 2.4 s later. How far below the top of the well is the surface of the water? The speed of sound in air (at ambient temperature) is 336 m/s.

□ **2.** A 3.00 kg object is moving in a plane, with its x and y coordinates in meters given by $x = 5t^2 - 1$ and $y = 3t^3 + 2$, where t is in seconds. What is the magnitude of the net force acting on this object at $t = 2.00$ s?

□ **3.** Consider a roller coaster having 10 identical cars with total mass M and massless interconnections. Figure a below shows the coaster just after the first car has begun its decent along a frictionless slope with an angle θ. Figure b shows the coaster just before the last car begins its descent. What is the acceleration of the coaster in these **two** situations?



□ **4.** A block of mass $m_a = 2.30$ kg on a frictionless plane is inclined at $\theta = 30°$ is connected by a cord over a massless, frictionless pulley to a second block of mass $m_b = 3.70$ kg. Find the acceleration (magnitude and direction) of both blocks and the tension in the cord.



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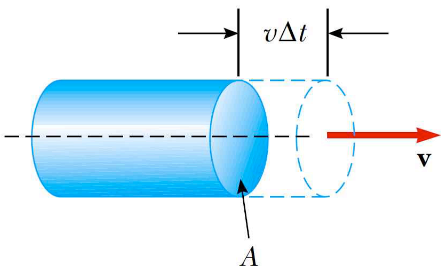
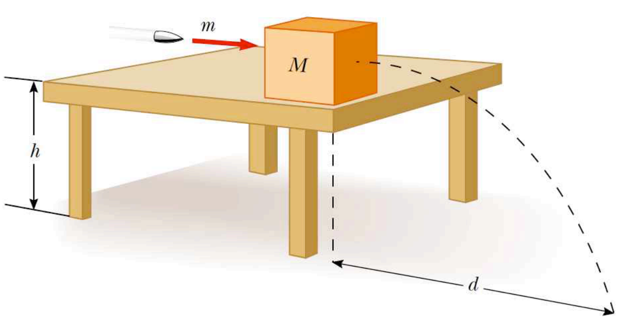
□ 5. What is the smallest radius of an unbanked (flat) track around which a bicyclist can travel if her speed is 29 km/h and the μ_s between tires and track is 0.32?

Section II: Work & Energy. Solve 2 of 4

□ 1. As it plows a parking lot, a snowplow pushes an ever-growing pile of snow in front of it. Suppose a car moving through the air is similarly modeled as a cylinder pushing a growing plug of air in front of it. The originally stationary air is set into motion at the constant speed v of the cylinder, as shown below. In a time interval δt, a new disk of air of mass δm must be moved a distance $v\delta t$ and hence must be given a kinetic energy $\frac{1}{2}(\delta m)^2 v^2$. Using this model, show that the resistive force due to the air is $F_{\text{drag}} = \frac{1}{2}\rho A v^2$, where ρ is the density of the air. Note that $F = dp/dt$, where p is momentum.



□ 2. A bullet of mass m is fired into a block of mass M initially at rest at the edge of a frictionless table of height h as in the figure below. The bullet remains in the block, and after impact the block lands a distance d from the bottom of the table. Determine the initial speed of the bullet in terms of given quantities.



□ 3. The potential energy of an Argon dimer may be modeled by

$$U(r) = 4\epsilon\left(\frac{\sigma^{12}}{r^{12}} - \frac{\sigma^6}{r^6}\right)$$

(a) Find the equilibrium separation of the dimer (i.e., the value of r at equilibrium).
(b) Is the equilibrium stable? Justify your answer.

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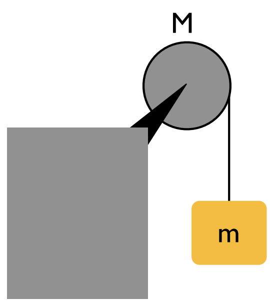
□ **4.** Force $\vec{\mathbf{F}} = (3x^2\,\mathrm{N})\ \hat{\imath} + (4\,\mathrm{N})\ \hat{\jmath}$, with x in meters, acts on a particle, changing only its kinetic energy.

(a) How much work is done on the particle as it moves from coordinates $(2\,\mathrm{m}, 3\,\mathrm{m})$ to $(7\,\mathrm{m}, -3\,\mathrm{m})$?
(b) Does the speed of the particle increase, decrease, or remain the same?

Section III: Rotation, Torque, and Angular Momentum. Solve 2 of 4

□ **1.** A uniform disk with mass $M = 2.5\,\mathrm{kg}$ and radius $R = 20\,\mathrm{cm}$ is mounted on a fixed horizontal axle, as shown below. A block of mass $m = 1.2\,\mathrm{kg}$ hangs from a massless cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord. *Note: the moment of inertia of a disk about its center of mass is* $I = \frac{1}{2}MR^2$.



□ **2.** A long uniform rod of length L and mass M is pivoted about a horizontal, frictionless pin through one end. The rod is released from rest in a vertical position. At the instant the rod is horizontal, find its angular speed. The moment of inertia of a solid rod about its center of mass is $I = \frac{1}{12}ML^2$.

□ **3.** A uniform ball of mass M and radius R rolls smoothly down a ramp at angle θ. The *center* of the ball starts at a vertical height h from the bottom of the ramp. How long does it take the ball to reach the bottom of the ramp?

□ **4.** A cockroach with mass m rides on a disk of mass $6.00m$ and radius R. The disk rotates like a merry-go-round around its central axis at angular speed $\omega_i = 1.50\,\mathrm{rad/s}$. The cockroach is initially at radius $r = 0.800R$, but then it crawls out to the rim of the disk. Treat the cockroach as a particle. What then is the angular speed?

Section IV: Gravitation. Solve 1 of 3

□ **1.** The free-fall acceleration on the surface of the Moon is about one sixth of that on the surface of the Earth. If the radius of the Moon is about $0.250\,R_E$, find the ratio of their average densities, $\rho_{\mathrm{Moon}}/\rho_{\mathrm{Earth}}$.

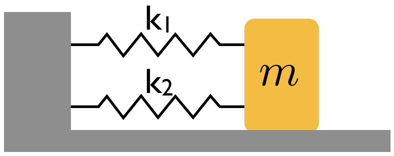
□ **2.** A playful astronaut releases a bowling ball of mass $m = 7.20\,\mathrm{kg}$ into circular orbit around Earth at an altitude h of $350\,\mathrm{km}$.
(a) What is the mechanical energy E of the ball in its orbit?

(b) What is the mechanical energy of the ball when it is on the launch pad at Cape Canaveral?
(c) From the launch pad to orbit, what is the change in the ball's mechanical energy?

□ **3.** Assume the earth to be a solid sphere of uniform density. A hole is drilled through the earth, passing through its center, and a ball is dropped into the hole. Neglect friction. What is the speed of the ball when it reaches Earth's center?

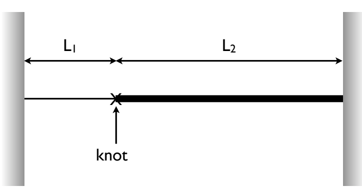
Section V: Waves & Oscillations. Solve 2 of 4

□ **1.** A mass is connected to two springs as shown below. Find the frequency of oscillation f.



□ **2.** In the figure below, two strings have been tied together with a knot and then stretched between two rigid supports. The strings have linear densities μ_1 and μ_2, with lengths L_1 and L_2, and tension T.

Simultaneously, on each string a pulse is sent from the rigid support end, toward the knot. The pulses from each end reach the knot at precisely the same moment. For that to be true, what is the relationship between L_1, L_2, μ_1, and μ_2?



□ **3.** A pendulum is formed by pivoting a long thin rod of mass M and length L about a point on the rod. If the pivot is a distance x from the rod's center, for what x is the period of the pendulum minimum? The moment of inertia for a thin rod about its center of mass is $I = \frac{1}{12}ML^2$.

□ **4.** A wave is governed by the equation

$$y = f(x,t) = e^{-(x-bt)^2 + ik_o(x-bt)}$$

Use the wave equation to find the speed of the wave. *Note:* here $i = \sqrt{-1}$.

Numbers & units:

$$g = 9.81\,\mathrm{m/s^2} \qquad M_e = 5.96 \times 10^{24}\,\mathrm{kg} \quad \leftarrow \text{earth}$$
$$R_e = 6.37 \times 10^6\,\mathrm{m} \quad \leftarrow \text{earth} \qquad G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$$

Math:

$$ax^2 + bx^2 + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$$
$$\cos\alpha \pm \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$$
$$c^2 = a^2 + b^2 - 2ab\cos\theta_{ab}$$
$$\frac{d}{dx}\sin ax = a\cos ax \qquad \frac{d}{dx}\cos ax = -a\sin ax$$
$$\int \cos ax\,\mathrm{dx} = \frac{1}{a}\sin ax \qquad \int \sin ax\,\mathrm{dx} = -\frac{1}{a}\cos ax$$
$$\sin\theta \approx \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2 \qquad \text{small } \theta$$
$$\vec{\mathbf{a}}(t) = \frac{d^2 s}{dt^2}\,\hat{\mathbf{T}} + \kappa|\vec{\mathbf{v}}|^2\,\hat{\mathbf{N}} = \frac{d^2 s}{dt^2}\,\hat{\mathbf{T}} + \frac{|\vec{\mathbf{v}}|^2}{R}\,\hat{\mathbf{N}} \equiv a_N\,\hat{\mathbf{T}} + a_T\,\hat{\mathbf{N}}$$

Vectors:

$$|\vec{\mathbf{F}}| = \sqrt{F_x^2 + F_y^2} \quad \text{magnitude}$$
$$\theta = \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{direction}$$
$$d\vec{\mathbf{l}} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$$
$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_x b_x + a_y b_y + a_z b_z = \sum_{i=1}^{n} a_i b_i = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$$
$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \quad |\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sin\theta$$

Waves:

$$y_{\rightarrow}(x,t) = y_m \sin(kx - \omega t) \qquad k = \frac{2\pi}{\lambda}$$
$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \quad \text{wave speed}$$
$$v = \sqrt{T/\mu} \quad \mu = M/L$$
$$P_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2 \quad \text{pwr}$$
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \text{wave equation}$$
$$y_t = 2y_m \cos\frac{\varphi}{2}\sin(kx - \omega t + \varphi/2) \quad \text{int.}$$
$$y_t = 2y_m \sin kx \cos\omega t \quad \text{standing}$$
$$f = \frac{v}{\lambda} = \frac{nv}{2L} \quad n \in \mathbb{N}$$

Rotation: we use radians

$$s = \theta r \quad \leftarrow \text{arclength}$$
$$\omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d\omega}{dt}$$
$$a_t = \alpha r \quad \text{tangential} \qquad a_r = \frac{v^2}{r} = \omega^2 r \quad \text{radial}$$
$$I = \sum_i m_i r_i^2 \Rightarrow \int r^2\,dm = kmr^2$$
$$I_z = I_{com} + md^2 \quad \text{axis } z \text{ parallel, dist } d$$
$$\tau_{net} = \sum \vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}} = \frac{d\vec{\mathbf{L}}}{dt}$$
$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} \qquad |\vec{\boldsymbol{\tau}}| = rF\sin\theta_{rF}$$
$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = I\vec{\boldsymbol{\omega}}$$
$$K = \frac{1}{2}I\omega^2 = L^2/2I$$
$$\Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W = \int \tau\,d\theta$$
$$P = \frac{dW}{dt} = \tau\omega$$

Gravitation:

$$\vec{\mathbf{F}}_{12} = \frac{Gm_1 m_2}{r^2}\,\hat{\mathbf{r}}_{12} = -\vec{\boldsymbol{\nabla}}U_g$$
$$g = \frac{GM_e}{R_e^2}$$
$$U_g(r) = -\int F(r)\,dr = \frac{-GMm}{r}$$
$$K + U_g = 0 \quad \text{escape} \qquad K + U_g < 0 \quad \text{bound}$$
$$\frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} \qquad T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$
$$E_{\text{orbit}} = \frac{-GMm}{2a} \quad \text{elliptical; } a \rightarrow r \text{ for circular}$$

Oscillations:

$$T = \frac{1}{f} \quad \omega = \frac{2\pi}{T} = 2\pi f$$
$$x(t) = x_m \cos(\omega t + \varphi)$$
$$a = -\omega^2 x \qquad \frac{d^2 q}{dt^2} = -\omega^2 q$$
$$\omega = \sqrt{k/m} \quad \text{linear osc.}$$
$$T = \begin{cases} 2\pi\sqrt{I/\kappa} & \text{torsion pendulum} \\ 2\pi\sqrt{L/g} & \text{simple pendulum} \\ 2\pi\sqrt{I/mgh} & \text{physical pendulum} \end{cases}$$
$$U = -\frac{1}{2}kx^2 \quad U = -\frac{1}{2}\kappa\theta^2 \quad F = -\frac{dU}{dx} = ma \quad \text{SHM}$$
$$x(t) = x_m e^{-bt/2m}\cos(\omega' t + \varphi) \quad \text{damped}$$
$$\omega' = \sqrt{k/m - b^2/4m}$$

1-D motion:

$$\begin{aligned} v(t) &= \frac{\mathrm{d}}{\mathrm{d}t}x(t) \\ a(t) &= \frac{\mathrm{d}}{\mathrm{d}t}v(t) = \frac{\mathrm{d}^2}{\mathrm{d}t^2}x(t) \end{aligned}$$

const. acc.

$$\begin{aligned} x_f &= x_i + v_{xi}t + \frac{1}{2}a_x t^2 \\ v_{xf}^2 &= v_{xi}^2 + 2a_x\Delta x \\ v_f &= v_i + at \end{aligned}$$

Projectile motion:

$$\begin{aligned} v_x(t) &= v_i\cos\theta \\ v_y(t) &= v_i\sin\theta - gt \\ x(t) &= x_i - v_x t \\ y(t) &= y_i + v_{yi}t + \frac{1}{2}a_y t^2 \end{aligned}$$

over level ground:

$$\begin{aligned} \text{max height } &= H = \frac{v_i^2\sin^2\theta_i}{2g} \\ \text{Range } &= R = \frac{v_i^2\sin 2\theta_i}{g} \end{aligned}$$

2-D motion:

$$\begin{aligned} \vec{\mathbf{r}} &= x(t)\,\hat{\imath} + y(t)\,\hat{\jmath} \\ x(t) &= x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y(t) &= y_i + v_{iy}t + \frac{1}{2}a_y t^2 \\ \vec{\mathbf{v}} &= v_x(t)\,\hat{\imath} + v_y(t)\,\hat{\jmath} \\ v_x(t) &= \frac{dx}{dt} = v_{xi} + a_x t \\ v_y(t) &= \frac{dy}{dt} = v_{yi} + a_y t \\ \vec{\mathbf{a}} &= a_x(t)\,\hat{\imath} + a_y(t)\,\hat{\jmath} \\ a_x(t) &= \frac{dv_x}{dt} \\ \vec{\mathbf{a}}(t) &= \frac{d^2s}{dt^2}\,\hat{\mathbf{T}} + \kappa|\vec{\mathbf{v}}|^2\,\hat{\mathbf{N}} = \frac{d^2s}{dt^2}\,\hat{\mathbf{T}} + \frac{|\vec{\mathbf{v}}|^2}{R}\,\hat{\mathbf{N}} \equiv a_N\,\hat{\mathbf{T}} + a_T\,\hat{\mathbf{N}} \\ a_c &= \frac{v^2}{r} \quad \text{circ.} \\ T &= \frac{2\pi r}{v} \quad \text{circ.} \end{aligned}$$

Force:

$$\begin{aligned} \Sigma\vec{\mathbf{F}} &= \vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} \\ \Sigma F_x &= ma_x \\ \Sigma F_y &= ma_y \\ F_{\text{grav}} &= mg = \text{weight} \\ \vec{\mathbf{F}}_{12} &= -\vec{\mathbf{F}}_{21} \\ f_s &\le \mu_s n \\ f_{s,\text{max}} &= \mu_s n \\ f_k &= \mu_k n \\ \vec{\mathbf{F}}_{\text{drag}} &= -\frac{1}{2}C\rho A|\vec{\mathbf{v}}|\vec{\mathbf{v}} \\ \vec{\mathbf{F}}_c &= -\frac{mv^2}{r}\,\hat{\mathbf{r}} \end{aligned}$$

Work-Energy:

$$\begin{aligned} K &= \frac{1}{2}mv^2 = \frac{p^2}{2m} \\ \Delta K &= K_f - K_i = W \\ W &= \int F(x)\,dx = -\Delta U \\ U_g(y) &= mgy \\ U_s(x) &= \frac{1}{2}kx^2 \\ F &= -\frac{dU(x)}{dx} \\ K_i + U_i &= K_f + U_f + W_{\text{ext}} = K_f + U_f + \int F_{\text{ext}}\,dx \end{aligned}$$

Momentum, etc.:

$$\begin{aligned} x_{\text{com}} &= \frac{1}{M_{\text{tot}}}\sum_{i=1}^{n} m_i x_i = \frac{m_1x_1 + m_2x_2 + \ldots m_nx_n}{m_1 + m_2 + \ldots m_n} \\ v_{\text{com}} &= \frac{1}{M_{\text{tot}}}\sum_{i=1}^{n} m_i v_i = \frac{m_1v_1 + m_2v_2 + \ldots m_nv_n}{m_1 + m_2 + \ldots m_n} \\ F_{\text{net}} &= M_{\text{tot}}a_{\text{com}} = \frac{dp}{dt} \\ p_{\text{tot}} &= M_{\text{tot}}v_{\text{com}} \\ \Delta p &= p_f - p_i = J = \int_{t_i}^{t_f} F(t)\,dt = F_{\text{avg}}\Delta t \\ \Delta p &= 0 \quad \text{closed} \end{aligned}$$

$$\text{elastic coll:} \begin{cases} v_{1f} = 2v_{\text{com}} - v_{1i} \\ v_{2f} = 2v_{\text{com}} - v_{2i} \end{cases}$$

Derived unit	Symbol	equivalent to
newton	N	kg·m/s^2
joule	J	kg·m^2/s^2 = N·m
watt	W	J/s=m^2·kg/s^3