

Exam I

Instructions

1. **Solve 4 of 7 problems below.** All problems have equal weight.
2. You must answer all parts of multi-part questions for full credit.
3. Show your work for full credit. Significant partial credit will be given.
4. Symbolic solutions give more partial credit than purely numerical ones.
5. You are allowed 2 sides of a standard 8.5 x 11 in piece of paper with notes/formulas and a calculator.

1. A stone is dropped into a river from a bridge 43.9 m above the water. Another stone is thrown vertically down 1.00 s after the first is dropped. The stones strike the water at the same time. What is the initial speed of the second stone? Neglect air resistance.
2. A football kicker can give the ball an initial speed of 25 m/s. What are the (a) least and (b) greatest elevation angles which he can kick the ball to score a field goal from a point 50.3 m (55 yd) in front of the goalposts whose horizontal bar is 3.35 m (10 ft) above the ground? Neglect air resistance.
3. A football player punts the football so that it will have a “hang time” (time of flight) of 4.5 s and land 46 m (~ 50 yd) away. If the ball leaves the player’s foot 1.50 m above the ground, what must be the (a) magnitude and (b) angle (relative to the horizontal) of the ball’s initial velocity? Neglect air resistance.
4. In the figure below, three ballot boxes are connected by cords, one of which wraps over a pulley having negligible friction on its axle and negligible mass. The three masses are $m_a = 30.0$ kg, $m_b = 40.0$ kg, and $m_c = 10.0$ kg. When the assembly is released from rest, (a) what is the tension in the cord connecting B and C , and (b) how far does A move in the first 0.250 s (assuming it does not reach the pulley)? The table may be assumed to be frictionless.

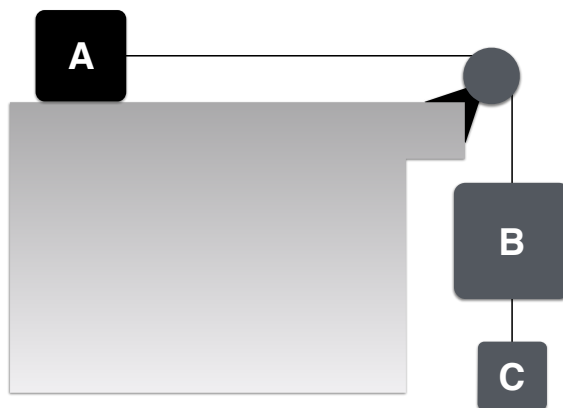


Figure 1: Three boxes connected by cords, one of which wraps over a pulley.

5. A puck of mass $m = 1.50$ kg slides in a circle of radius $r = 0.20$ m on a frictionless table while attached to a hanging cylinder of mass $M = 2.50$ kg by means of a cord that extends through a hole in the table (see figure

below). What speed keeps the hanging cylinder at rest? *Hint: think about the constraints of the forces acting on m given its path.*

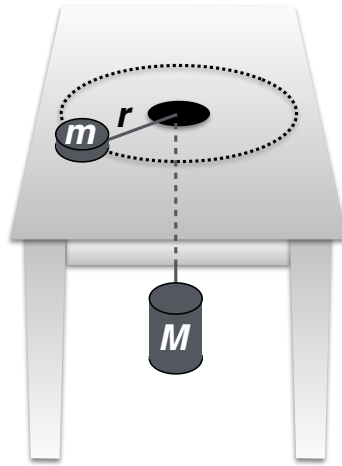
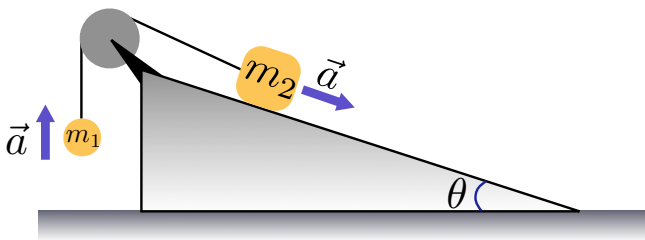


Figure 2: A puck slides in a circle on a table, holding up a cylindrical mass.

6. A rifle that shoots a bullet at 460 m/s is to be aimed at a target 45.7 m away. If the center of the target is level with the rifle, how high above the target must the rifle barrel be pointed so that the bullet hits dead center?
7. A ball of mass m_1 and a block of mass m_2 are connected by a lightweight cord that passes over a frictionless pulley of negligible mass, as shown below. The block lies on a frictionless incline of angle θ . Find the magnitude of the acceleration of the two objects. *Note: the direction for \vec{a} shown is just an arbitrary choice, it implies nothing about the problem.*



Formula sheet

$$g = 9.81 \text{ m/s}^2$$

$$0 = ax^2 + bx^2 + c \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$$

Vectors:

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

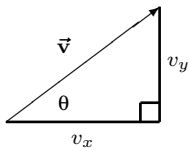
$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

$$\tan \theta = \frac{a_y}{a_x}$$

$$\vec{a} + \vec{b} = (a_x + b_x) \hat{i} + (a_y + b_y) \hat{j} + (a_z + b_z) \hat{k}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$|\vec{a} \cdot \vec{b}| = |\vec{a}| |\vec{b}| \cos \varphi$$



$$v_y = |\vec{v}| \sin \theta$$

$$v_x = |\vec{v}| \cos \theta$$

$$\tan \theta = \frac{v_y}{v_x}$$

1-D motion:

$$v(t) = \frac{d}{dt} x(t)$$

$$a(t) = \frac{d}{dt} v(t) = \frac{d^2}{dt^2} x(t)$$

const. acc. \downarrow

$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

$$v_f^2 = v_i^2 + 2a_x \Delta x$$

$$v_f = v_i + at$$

Projectile motion:

$$v_x(t) = v_{ix} = |\vec{v}_i| \cos \theta$$

$$v_y(t) = |\vec{v}_i| \sin \theta - gt = v_{iy} \sin \theta - gt$$

$$x(t) = x_i + v_{ix} t$$

$$y(t) = y_i + v_{iy} t - \frac{1}{2} gt^2$$

$$y(x) = x \tan \theta - \frac{gx^2}{2|\vec{v}_i|^2 \cos^2 \theta}$$

over level ground:

$$\text{max height} = H = \frac{v_i^2 \sin^2 \theta_i}{2g}$$

$$\text{Range} = R = \frac{v_i^2 \sin 2\theta_i}{g}$$

Force:

$$\sum \vec{F} = \vec{F}_{\text{net}} = m\vec{a}$$

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$F_{\text{grav}} = mg = \text{weight}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$f_s \leq \mu_s n$$

$$f_{s, \text{max}} = \mu_s n$$

$$f_k = \mu_k n$$

$$\vec{F}_{\text{centr.}} = -\frac{mv^2}{r} \hat{r} \quad \text{circular}$$

2-D motion:

$$\vec{r} = x(t) \hat{i} + y(t) \hat{j}$$

$$x(t) = x_i + v_{ix} t + \frac{1}{2} a_x t^2$$

$$y(t) = y_i + v_{iy} t + \frac{1}{2} a_y t^2$$

$$\vec{v} = v_x(t) \hat{i} + v_y(t) \hat{j}$$

$$v_x(t) = \frac{dx}{dt} = v_{xi} + a_x t$$

$$v_y(t) = \frac{dy}{dt} = v_{yi} + a_y t$$

$$\vec{a} = a_x(t) \hat{i} + a_y(t) \hat{j}$$

$$a_x(t) = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$$

$$\vec{a}_c = -\frac{v^2}{r} \hat{r} \quad \text{circ.}$$

$$T = \frac{2\pi r}{v} \quad \text{circ.}$$

Math:

$$ax^2 + bx^2 + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{1}{2} (\alpha \pm \beta) \cos \frac{1}{2} (\alpha \mp \beta)$$

$$\cos \alpha \pm \cos \beta = 2 \cos \frac{1}{2} (\alpha + \beta) \cos \frac{1}{2} (\alpha - \beta)$$

$$c^2 = a^2 + b^2 - 2ab \cos \theta_{ab}$$

$$\frac{d}{dx} \sin ax = a \cos ax \quad \frac{d}{dx} \cos ax = -a \sin ax$$

Power	Prefix	Abbreviation
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^3	kilo	k
10^6	mega	M
10^9	giga	G
10^{12}	tera	T