Name & ID ____

UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 125 / LeClair Spring 2014

Exam II

Instructions

1. **Solve 4 of 6 problems below.** All problems have equal weight.
2. You must answer all parts of multi-part questions for full credit.
3. Show your work for full credit. Significant partial credit will be given.
4. You are allowed a calculator and 2 sides of 8.5 x 11in paper with notes.
5. You have 50 min.

1. For an isolated system, conservation of energy holds, and the total energy of the system may be written as the sum of kinetic and potential energies, $E_{\text{tot}} = K + U$. Presume the potential energy U is a function of position alone, $U(x)$. Show that to move between positions x_i and x_f given $U(x)$ the time required is

$$t = \sqrt{\frac{m}{2}} \int\limits_{x_i}^{x_f} \frac{dx}{\sqrt{E_{\text{tot}} - U(x)}} \tag{1}$$

To begin, note that $K = E_{\text{tot}} - U(x) = \frac{1}{2}mv^2$ and $v = \frac{dx}{dt}$.

2. To stretch a spring a distance d from equilibrium takes an amount W_o of work. **(a)** How much work does it take to stretch the spring from d to $2d$ from equilibrium? **(b)** From Nd to $(N+1)d$ from equilibrium?

3. A stone is tied to a string of length R. A person whirls this stone in a vertical circle. Assume that the energy of the stone remains constant as it moves around the circle. Show that if the string is to remain taut at the top of the circle, the speed of the stone at the bottom of the circle must be at least $\sqrt{5gR}$.

4. A 2.5 g Ping-Pong ball is dropped from a window and strikes the ground 20 m below with a speed of 9.0 m/s. What fraction of its initial potential energy was lost to air friction?

5. A package is dropped onto a horizontal conveyor belt. The mass of the package is m, the speed of the conveyor belt is v, and the coefficient of kinetic friction for the package on the belt is μ_k. **(a)** For what length of time will the package slide on the belt? **(b)** How far does it move in this time? **(c)** How much energy is dissipated by friction? *Hint: would it make any difference if the belt were stationary and the box were moving at velocity v?*

6. An alternative to using Newton's laws and forces to analyze mechanical problems is to use an energy-based approach known as the Lagrangian method. The Lagrangian function of a system is defined as the *difference* between kinetic and potential energy:

$$L = K - U = \frac{1}{2}mv^2 - U \tag{2}$$

Here v is the particle's velocity and U its (in general position-dependent) potential energy. The path a particle takes is the one that extremizes this Lagrangian function, integrated over the entire path. That condition can be summarized by the Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial L}{\partial v} - \frac{\partial L}{\partial x} = 0 \tag{3}$$

(The ∂ symbol means 'take the derivative with respect to the given variable leaving all other variables constant.' If this is unfamiliar, just treat it like a normal derivative.)

Show that the above two equations reproduce Newton's 2nd law ($\sum F = ma$) for **(a)** a particle under the influence of gravity alone, $U(x) = mgx$, and **(b)** a particle interacting with a spring, $U(x) = \frac{1}{2}kx^2$. **(c)** Now consider an arbitrary $U(x)$. What relationship between U and the net force can be derived from the above equations? Note: writing $U(x)$ means U is a function of x **only**.

Formula sheet

$$g = 9.81\,\mathrm{m/s^2}$$

$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$1\,\mathrm{N} = 1\,\mathrm{kg}\cdot\mathrm{m/s^2}$$

$$1\,\mathrm{J} = 1\,\mathrm{kg\,m^2/s^2} = 1\,\mathrm{N\,m}$$

1-D motion:

$$v(t) = \frac{d}{dt}x(t)$$

$$a(t) = \frac{d}{dt}v(t) = \frac{d^2}{dt^2}x(t)$$

const. acc. $\downarrow$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$v_f^2 = v_i^2 + 2a_x\Delta x$$

$$v_f = v_i + at$$

Projectile motion:

$$v_x(t) = v_{ix} = |\vec{\mathbf{v}}_i|\cos\theta$$

$$v_y(t) = |\vec{\mathbf{v}}_i|\sin\theta - gt = v_{iy}\sin\theta - gt$$

$$x(t) = x_i + v_{ix}t$$

$$y(t) = y_i + v_{iy}t - \frac{1}{2}gt^2$$

over level ground:

$$\text{max height} = H = \frac{v_i^2\sin^2\theta_i}{2g}$$

$$\text{Range} = R = \frac{v_i^2\sin 2\theta_i}{g}$$

Force:

$$\sum\vec{\mathbf{F}} = \vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = \frac{d\vec{\mathbf{p}}}{dt}$$

$$\sum F_i = ma_i \quad \text{by component}$$

$$\vec{\mathbf{F}}_c = \sum F_{\mathrm{r}} = -\frac{mv^2}{r}\,\hat{\mathbf{r}}$$

$$f_k = \mu_k n$$

$$F_s = -kx$$

$$F_g = -mg$$

2-D motion:

$$\vec{\mathbf{r}} = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath}$$

$$x(t) = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$

$$y(t) = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$

$$\vec{\mathbf{a}}(t) = \frac{d^2s}{dt^2}\,\hat{\mathbf{T}} + \kappa|\vec{\mathbf{v}}|^2\,\hat{\mathbf{N}}$$

$$= \frac{d^2s}{dt^2}\,\hat{\mathbf{T}} + \frac{|\vec{\mathbf{v}}|^2}{R}\,\hat{\mathbf{N}} \equiv a_N\,\hat{\mathbf{T}} + a_T\,\hat{\mathbf{N}}$$

$$\vec{\mathbf{a}}_c = -\frac{v^2}{r}\,\hat{\mathbf{r}} \quad \text{circ.}$$

$$T = \frac{2\pi r}{v} \quad \text{circ.}$$

Work-Energy:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\Delta K = K_f - K_i = W$$

$$W = \int F(x)\,dx = -\Delta U$$

$$U_g(y) = mgy$$

$$U_s(x) = \frac{1}{2}kx^2$$

$$F = -\frac{dU(x)}{dx}$$

$$K_i + U_i = K_f + U_f + W_{\text{ext}} = K_f + U_f + \int F_{\text{ext}}\,dx$$

Power	Prefix	Abbreviation
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	µ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T