Name & ID ____

University of Alabama
Department of Physics and Astronomy

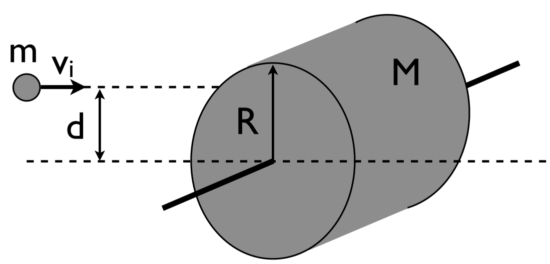
PH 125 / LeClair — Spring 2014

Exam III

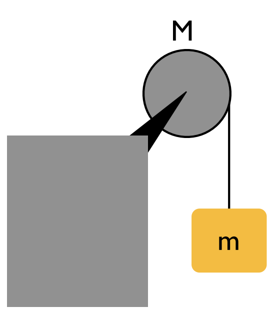
Instructions

1. **Solve 4 of 6 problems below.** All problems have equal weight.
2. You must answer all parts of multi-part questions for full credit.
3. Show your work for full credit. Significant partial credit will be given.
4. You are allowed a calculator and 2 sides of 8.5 x 11in paper with notes.
5. You have 1hr50min.

1. A wad of sticky clay with mass m and velocity v_i is fired at a solid cylinder of mass M and radius R as shown below. The cylinder is initially at rest and mounted on a fixed horizontal axle that runs through its center of mass. The line of motion of the projectile is perpendicular to the axis and at a distance $d<R$ from the center. Find the angular speed of the system just after the clay strikes and sticks to the surface of the cylinder. The moment of inertia of a solid cylinder is $I=\frac{1}{2}MR^2$, the moment of inertia of a point particle mass m a distance R from an axis of rotation is $I=mR^2$.



2. A uniform disk with mass $M=2.5\,\mathrm{kg}$ and radius $R=20\,\mathrm{cm}$ is mounted on a fixed horizontal axle, as shown below. A block of mass $m=1.2\,\mathrm{kg}$ hangs from a massless cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord. *Note: the moment of inertia of a disk about its center of mass is* $I=\frac{1}{2}MR^2$.



3. A hollow spherical iron shell floats almost completely submerged in water. The outer diameter is 0.60 m, and the density of iron is $7870\,\mathrm{kg/m^3}$. If the density of water is $1000\,\mathrm{kg/m^3}$, find the inner diameter of the sphere.

Name & ID

4. A (spherical) star of radius $R = 5 \times 10^5$ km has a rotational period of 60 days. Later in its life, its radius expands to $R = 5 \times 10^6$ km, though its mass M remains constant. What is the new rotational period after expansion? Presume the star's moment of inertial is kMR^2 at all times.

5. A small body of mass m hangs in equilibrium at one end of a light string of length l, the upper end of which is fixed. A small body of mass m moving horizontally with velocity $2\sqrt{gl}$ strikes the former body and adheres to it. Find:

(a) the velocity with which the combined bodies begin to move,
(b) the angle through which the string turns before coming to rest for an instant,

6. Ethanol of density $\rho = 791\,\text{kg/m}^3$ flows smoothly through a horizontal pipe that tapers in cross-sectional area from $A_1 = 1.20 \times 10^{-3}\,\text{m}^2$ to $A_2 = A_1/2$. The pressure difference between the wide and narrow sections of pipe is 4120 Pa. What is the volume flow rate (i.e., m^3/s) R_V of the ethanol?

Name & ID

Formula sheet

$$g = 9.81\,\mathrm{m/s^2}$$

Derived unit	Symbol	equivalent to
newton	N	$\mathrm{kg{\cdot}m/s^2}$
joule	J	$\mathrm{kg{\cdot}m^2/s^2} = \mathrm{N{\cdot}m}$
watt	W	$\mathrm{J/s = m^2{\cdot}kg/s^3}$

Math:

$$ax^2 + bx^2 + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$$

$$\cos\alpha \pm \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$$

$$c^2 = a^2 + b^2 - 2ab\cos\theta_{ab}$$

$$\frac{d}{dx}\sin ax = a\cos ax \qquad \frac{d}{dx}\cos ax = -a\sin ax$$

$$\int \cos ax\,\mathrm{dx} = \frac{1}{a}\sin ax \qquad \int \sin ax\,\mathrm{dx} = -\frac{1}{a}\cos ax$$

$$\sin\theta \approx \theta \qquad \text{small } \theta \qquad \cos\theta \approx 1 - \frac{1}{2}\theta^2$$

Vectors:

$$|\vec{\mathbf{F}}| = \sqrt{F_x^2 + F_y^2} \quad \text{magnitude}$$

$$\theta = \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{direction}$$

$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_x b_x + a_y b_y + a_z b_z = \sum_{i=1}^{n} a_i b_i = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$$

$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \quad |\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sin\theta$$

Work-Energy:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\Delta K = K_f - K_i = W$$

$$W = \int F(x)\,dx = -\Delta U$$

$$U_g(y) = mgy$$

$$U_s(x) = \frac{1}{2}kx^2$$

$$F = -\frac{dU(x)}{dx}$$

$$K_i + U_i = K_f + U_f + W_{\text{ext}} = K_f + U_f + \int F_{\text{ext}}\,dx$$

Rotation: we use radians

$$s = \theta r \quad \leftarrow \text{arclength}$$

$$\omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d\omega}{dt}$$

$$a_t = \alpha r \quad \text{tangential} \qquad a_r = \frac{v^2}{r} = \omega^2 r \quad \text{radial}$$

$$I = \sum_i m_i r_i^2 \Rightarrow \int r^2\,dm = kmr^2$$

$$I_z = I_{com} + md^2 \quad \text{axis } z \text{ parallel, dist } d$$

$$\tau_{net} = \sum \vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}} = \frac{d\vec{\mathbf{L}}}{dt}$$

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} \qquad |\vec{\boldsymbol{\tau}}| = rF\sin\theta_{rF}$$

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = I\vec{\boldsymbol{\omega}}$$

$$K = \frac{1}{2}I\omega^2 = L^2/2I$$

$$\Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W = \int \tau\,d\theta$$

$$P = \frac{dW}{dt} = \tau\omega$$

Fluids:

$$\rho = m/V \quad \text{density}$$

$$p = F/A \quad \text{pressure}$$

$$p(h) = p(0) + \rho g h$$

$$F_b = m_{\text{fluid}}\, g \quad \text{buoyant force}$$

$$R_v = Av = \text{const vol flow rate}$$

$$R_m = \rho R_v = \text{mass flow rate}$$

$$\text{const} = p + \frac{1}{2}\rho v^2 + \rho g y \quad \text{Bernoulli}$$

Momentum, etc.:

$$x_{\text{com}} = \frac{1}{M_{\text{tot}}}\sum_{i=1}^{n} m_i x_i = \frac{m_1 x_1 + m_2 x_2 + \ldots m_n x_n}{m_1 + m_2 + \ldots m_n}$$

$$v_{\text{com}} = \frac{1}{M_{\text{tot}}}\sum_{i=1}^{n} m_i v_i = \frac{m_1 v_1 + m_2 v_2 + \ldots m_n v_n}{m_1 + m_2 + \ldots m_n}$$

$$F_{\text{net}} = M_{\text{tot}} a_{\text{com}} = \frac{dp}{dt}$$

$$p_{\text{tot}} = M_{\text{tot}} v_{\text{com}}$$

$$\text{elastic coll:} \begin{cases} v_{1f} = 2v_{\text{com}} - v_{1i} \\ v_{2f} = 2v_{\text{com}} - v_{2i} \end{cases}$$