Name & ID

PH125 Exam IV: A New Hope

Instructions

1. **Solve 3 of 5 problems below.** All problems have equal weight.
2. Show your work for full credit. Significant partial credit will be given.
3. You are allowed 2 sides of an 8.5 x 11 in piece of paper with notes and a calculator.

1. The space shuttle releases a 470 kg satellite while in an orbit 280 km above the surface of the earth. A rocket engine on the satellite boosts it to a geosynchronous orbit. How much energy is required for the orbit boost? (Note: the earth's radius is 6378 km, its mass is 5.98×10^{24} kg, and $G = 6.67 \times 10^{-11} N \cdot m^2 kg^{-2}$. Hint: "geosynchronous" means the satellite's period T is 24 hrs.)

2. Calculate the mass of the Sun given that the Earth's distance from the Sun is 1.496×10^{11}m. (Hint: you already know the period of the Earth's orbit.)

3. The free-fall acceleration on the surface of the Moon is about one sixth of that on the surface of the Earth. If the radius of the Moon is about $0.250\,R_E$, find the ratio of their average densities, $\rho_{\text{Moon}}/\rho_{\text{Earth}}$.

4. In the figure below, two masses are connected to each other and vertical walls by three identical springs. Presume $m_1 = m_2$ for simplicity. As it turns out, there are two stable frequencies of oscillation of the system. Find one of them. *Hint: there are two obvious ways the two masses can move relative to each other. One of them is really simple.*

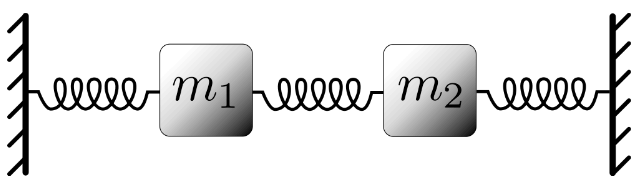

Figure 1: *From http://en.wikipedia.org/wiki/Normal_mode.*

5. *Energetics of diatomic systems.* An expression for the potential energy of two neutral atoms as a function of their separation x is given by the *Morse potential*,

$$U(x) = U_o \left[1 - e^{-a(x-x_o)}\right]^2 \tag{1}$$

where x_o is the equilibrium spacing. Calculate the force constant for small oscillations about $r = r_o$. *Hint: At equilibrium, the net force is zero. For small δ, one may approximate $e^\delta \approx 1 + \delta + \frac{1}{2}\delta^2 + \cdots$.*

Numbers & units:

$$g = 9.81\,\mathrm{m/s^2} \qquad M_e = 5.96\times 10^{24}\,\mathrm{kg} \quad \leftarrow \text{earth}$$

$$R_e = 6.37\times 10^{6}\,\mathrm{m} \quad \leftarrow \text{earth} \qquad G = 6.67\times 10^{11}\,\mathrm{N\,m^2/kg^2}$$

Math:

$$ax^2 + bx^2 + c = 0 \Longrightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha\pm\beta)\cos\frac{1}{2}(\alpha\mp\beta)$$

$$\cos\alpha \pm \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$$

$$c^2 = a^2 + b^2 - 2ab\cos\theta_{ab}$$

$$\frac{d}{dx}\sin ax = a\cos ax \qquad \frac{d}{dx}\cos ax = -a\sin ax$$

$$\int \cos ax\,\mathrm{dx} = \frac{1}{a}\sin ax \qquad \int \sin ax\,\mathrm{dx} = -\frac{1}{a}\cos ax$$

$$\sin\theta \approx \theta \qquad \text{small } \theta$$

$$\cos\theta \approx 1 - \frac{1}{2}\theta^2$$

$$\vec{\mathbf{a}}(t) = \frac{d^2s}{dt^2}\,\hat{\mathbf{T}} + \kappa|\vec{\mathbf{v}}|^2\,\hat{\mathbf{N}} = \frac{d^2s}{dt^2}\,\hat{\mathbf{T}} + \frac{|\vec{\mathbf{v}}|^2}{R}\,\hat{\mathbf{N}} \equiv a_N\,\hat{\mathbf{T}} + a_T\,\hat{\mathbf{N}}$$

Vectors:

$$|\vec{\mathbf{F}}| = \sqrt{F_x^2 + F_y^2} \quad \text{magnitude}$$

$$\theta = \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{direction}$$

$$d\vec{\mathbf{l}} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$$

$$\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = a_xb_x + a_yb_y + a_zb_z = \sum_{i=1}^{n} a_ib_i = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$$

$$\vec{\mathbf{a}}\times\vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \qquad |\vec{\mathbf{a}}\times\vec{\mathbf{b}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sin\theta$$

Waves:

$$y_{\rightarrow}(x,t) = y_m \sin(kx - \omega t) \qquad k = \frac{2\pi}{\lambda}$$

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \quad \text{wave speed}$$

$$v = \sqrt{T/\mu} \quad \mu = M/L$$

$$P_{avg} = \frac{1}{2}\mu v\omega^2 y_m^2 \quad \text{pwr}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \text{wave}$$

$$y_t = 2y_m\cos\frac{\varphi}{2}\sin(kx - \omega t + \varphi/2) \quad \text{int.}$$

$$y_t = 2y_m \sin kx\cos\omega t \quad \text{standing}$$

$$f = \frac{v}{\lambda} = \frac{nv}{2L} \quad n\in\mathbb{N}$$

Rotation: we use radians

$$s = \theta r \quad \leftarrow \text{arclength}$$

$$\omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d\omega}{dt}$$

$$a_t = \alpha r \quad \text{tangential} \qquad a_r = \frac{v^2}{r} = \omega^2 r \quad \text{radial}$$

$$I = \sum_i m_i r_i^2 \Rightarrow \int r^2\,dm = kmr^2$$

$$I_z = I_{com} + md^2 \quad \text{axis } z \text{ parallel, dist } d$$

$$\tau_{net} = \sum \vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}} = \frac{d\vec{\mathbf{L}}}{dt}$$

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}\times\vec{\mathbf{F}} \qquad |\vec{\boldsymbol{\tau}}| = rF\sin\theta_{rF}$$

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}\times\vec{\mathbf{p}} = I\vec{\boldsymbol{\omega}}$$

$$K = \frac{1}{2}I\omega^2 = L^2/2I$$

$$\Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W = \int \tau\,d\theta$$

$$P = \frac{dW}{dt} = \tau\omega$$

Gravitation:

$$\vec{\mathbf{F}}_{12} = \frac{Gm_1m_2}{r^2}\,\hat{\mathbf{r}}_{12} = -\vec{\boldsymbol{\nabla}}U_g$$

$$g = \frac{GM_e}{R_e^2}$$

$$U_g(r) = -\int F(r)\,dr = \frac{-GMm}{r}$$

$$K + U_g = 0 \quad \text{escape} \qquad K + U_g < 0 \quad \text{bound}$$

$$\frac{dA}{dt} = \frac{1}{2}r^2\omega = \frac{L}{2m} \qquad T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

$$E_{\text{orbit}} = \frac{-GMm}{2a} \quad \text{elliptical; } a\to r \text{ for circular}$$

Oscillations:

$$T = \frac{1}{f} \quad \omega = \frac{2\pi}{T} = 2\pi f$$

$$x(t) = x_m\cos(\omega t + \varphi)$$

$$a = -\omega^2 x \qquad \frac{d^2q}{dt^2} = -\omega^2 q$$

$$\omega = \sqrt{k/m} \quad \text{linear osc.}$$

$$T = \begin{cases} 2\pi\sqrt{I/\kappa} & \text{torsion pendulum} \\ 2\pi\sqrt{L/g} & \text{simple pendulum} \\ 2\pi\sqrt{I/mgh} & \text{physical pendulum} \end{cases}$$

$$U = -\frac{1}{2}kx^2 \quad U = -\frac{1}{2}\kappa\theta^2 \quad F = -\frac{dU}{dx} = ma \quad \text{SHM}$$

$$x(t) = x_m e^{-bt/2m}\cos\left(\omega' t + \varphi\right) \quad \text{damped}$$

$$\omega' = \sqrt{k/m - b^2/4m}$$