Problem Set 3 Solutions

Daily problems due 14 Feb 2014

1. A luge and its rider, with a total mass of 85 kg, emerge from a downhill track onto a horizontal straight track with an initial speed of 37 m/s. If a force slows them to a stop at a constant rate of $2.0\,\mathrm{m/s^2}$, **(a)** what magnitude F is required for the force, **(b)** what distance d do they travel while slowing, and **(c)** what work W is done on them by the force? What are **(d)** F, **(e)** d, and **(f)** W if they, instead, slow at $4.0\,\mathrm{m/s^2}$?

Solution: We know the acceleration a, so the force required must just be $F = ma$

$$F = ma = (85\,\mathrm{kg})\left(2\,\mathrm{m/s^2}\right) = 170\,\mathrm{N} \tag{1}$$

Knowledge of the starting and ending velocities and the acceleration lets us find the distance traveled d.

$$v_f^2 = 0 = v_i^2 + 2ad \tag{2}$$

$$d = \frac{v_i^2}{2a} = 342\,\mathrm{m} \tag{3}$$

Since the force applied is constant (it must be, since the acceleration is constant), the work done is just force times displacement. The work is negative, since energy is leaving the luge-rider system. Or, if you like, the force and displacement have opposite signs, so one of them must be negative. We arbitrarily pick the direction of displacement to be positive.

$$W = Fd = ma\left(\frac{v_i^2}{2a}\right) = \frac{1}{2}mv_i^2 = (-170\,\mathrm{N})\,(342\,\mathrm{m}) = -5.81\times10^4\,\mathrm{J} \tag{4}$$

We have also recovered the result that the work done must be equal to the change in kinetic energy of the luge and rider.

If the rider and luge instead slow at a new acceleration of $a' = 4.0\,\mathrm{m/s}$, one can see from the equations above that the force doubles (since it is proportional to a), the distance halves (since it is proportional to $1/a$), but the work done is exactly the same. Since the force doubled and the distance halved, their product is the same. Another way to put it: the change in kinetic energy is the same, so the work done must be the same - the details of *how* the luge stopped are irrelevant to figuring out how much energy it cost.

2. A 0.250 kg block of cheese lies on the floor of a 900 kg elevator cab that is being pulled upward by a cable through distance $d_1 = 2.40\,\mathrm{m}$ and then through a distance $d_2 = 10.5\,\mathrm{m}$. **(a)** Through d_1, if the normal force on the block from the floor has a constant magnitude $F_n = 3.00\,\mathrm{N}$, how much work is done on the cab by the force from the cable? **(b)** Through d_2, if the work done on the cab by the (constant) force from the cable is 92.61 kJ, what is the magnitude of F_n?

Solution: We should first just do force balances, one for the block of cheese and one for the elevator. From this we can get the net force pulling the elevator upward, and from that and the distance covered we can find the work.

For the cheese, it has a normal force upward and its weight downward, and this must equal its mass times acceleration. Its acceleration is *upward*, which we will choose as the positive direction. This makes the normal force positive and the weight negative.

$$\text{cheese alone:} \quad F_n - m_c g = m_c a \tag{5}$$

$$a = \frac{F_n}{m_c} - g \approx 2.19 \text{m/s}^2 \tag{6}$$

This gives us the acceleration of the cheese. Balancing forces for the elevator as a whole gets us the net force F. On the elevator, we have F acting upward and its weight pulling downward, giving an overall positive acceleration a which is the same as the cheese. Note that the weight here is that of the elevator plus the cheese, though the latter is utterly negligible in this force balance.

$$\text{elevator plus cheese:} \quad F - (m_e + m_c)\, g = (m_e + m_c)\, a \tag{7}$$

$$F = (m_e + m_c)(g + a) = (m_e + m_c)\left(g + \frac{F_n}{m_c} - g\right) = \left(\frac{m_e + m_c}{m_c}\right) F_n \tag{8}$$

The work done by the external force is the force F is this force times the displacement. For the first part of the motion, the displacement is d_1.

$$W = F d_1 = \left(\frac{m_e + m_c}{m_c}\right) F_n d_1 \approx 25.9\,\text{kJ} \tag{9}$$

For the second part, given the work done W, displacement d_2, and masses, we can solve for the normal force

$$F_n = \frac{W m_c}{(m_e + m_c)\, d_2} \approx 2.45\,\text{N} \tag{10}$$

Daily problems due 17 Feb 2014

3. A block of mass $m = 2.0\,\text{kg}$ is dropped from height 40 cm onto a spring of constant $k = 1960\,\text{N/m}$. Find the maximum distance the spring is compressed.

Solution: Let the zero point for gravitational potential energy be the top of the spring as it is maximally compressed, as shown in the figure below. Once the block is dropped, it compresses the spring by a maximum distance x. At this point, the block is instantaneously at rest. The block starts off a distance $h + x$ above what we have defined as $U_{\text{grav}} = 0$. Given that the block starts at rest as well, kinetic energy is zero both in the initial and final states.

All we need to do is balance potential energy. Initially, the block has only gravitational potential energy, in amount $mg(h + x)$. At maximum compression, since we have defined that state to have zero gravitational potential energy we have only the potential energy of the spring to worry about.

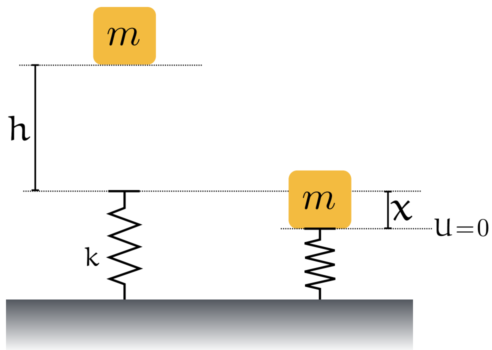


Figure 1: *A mass dropped onto a spring.*

$$
\begin{aligned}
K_i + U_i &= K_f + U_f && (11)\\
mg(h+x) &= \frac{1}{2}kx^2 && (12)\\
0 &= \frac{1}{2}kx^2 - mgx - mgh && (13)\\
x &= \frac{1}{k}\left(mg \pm \sqrt{m^2g^2 + 2kmgh}\right) \approx \{0.10, -0.08\}\,\mathrm{m} && (14)
\end{aligned}
$$

The negative root is unphysical because we know the spring is being compressed, not expanded, when the block strikes it. Thus, the maximum distance the spring is compressed is $x = 0.10\,\mathrm{m}$.

4. The block in the figure below lies horizontally on a frictionless surface, and the spring constant is $50\,\mathrm{N/m}$. Initially, the spring is at its relaxed length and the block is stationary at position $x = 0$. Then an applied force with a constant magnitude of $3.0\,\mathrm{N}$ pulls the block in the positive direction of the x axis, stretching the spring until the block stops. When that stopping point is reached, what are **(a)** the position of the block, **(b)** the work that has been done on the block by the applied force, and **(c)** the work that has been done on the block by the spring force? During the block's displacement, what are **(d)** the block's position when its kinetic energy is maximum, and **(e)** the value of that maximum kinetic energy.

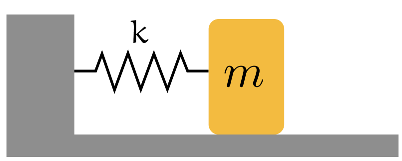


Figure 2: *A mass connected to a spring lies on a horizontal frictionless surface.*

Solution: When will the block stop? Since a constant force is pushing it and the spring, it will stop when the work done by the external force is equal to the potential energy change of the spring. One way to see this is conservation of energy - the work done by the force is being added to the system, and so long as it is greater than the energy required to compress the spring to a given distance, the net energy balance is positive and the block gains kinetic energy. At the point the block will stop, at its maximum compression, kinetic energy is zero and it must be the case that the work done by the force added to the system is used up entirely by the potential energy of the spring.

Given that, the stopping condition is

$$W = Fx = \frac{1}{2}kx^2 \tag{15}$$
$$x = \frac{2F}{k} \tag{16}$$

The work done by the force at that point is

$$W = Fx = \frac{2F^2}{k} \tag{17}$$

The work done by the block on the spring force must be exactly the same to conserve energy, but we can verify this:

$$W_{\text{on spring}} = \Delta U_{\text{spring}} = \frac{1}{2}kx^2 = \frac{2F^2}{k} \tag{18}$$

The kinetic energy change of the block will be the difference between the work done by the force and the spring's potential energy. In other words, the total work done by the external force is spent between the spring's potential energy and the block. Since the block starts at rest, $\Delta K = K_f - K_i = K_f$.

$$\Delta K = K_f = W - U_{\text{spring}} = Fx - \frac{1}{2}kx^2 \tag{19}$$

The maximum kinetic energy is when $dK/dx = 0$.

$$\frac{dK}{dx} = F - kx = 0 \tag{20}$$
$$x_{\text{max K}} = \frac{F}{k} \tag{21}$$

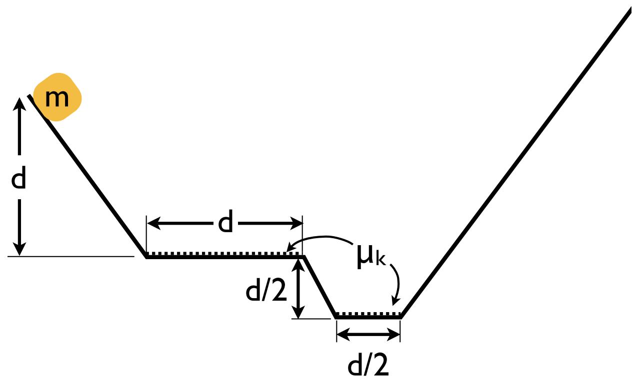
As expected, the maximum kinetic energy is when there is no net force on the block. When the spring force precisely balances the external force, the block has no net acceleration and maximum kinetic energy. The value of the kinetic energy at this point is

$$K_{\text{max}} = F_x - \frac{1}{2}kx^2 = \frac{F^2}{2k} \tag{22}$$

The problems below are due by the end of the day on 19 Feb 2014.

5. A block of mass m is released from rest at a height $d = 40\,\text{cm}$ and slides down a frictionless ramp and onto a first plateau, which has length d and where the coefficient of kinetic friction is $\mu_k = 0.5$. If the block is still moving, it then slides down a second frictionless ramp through height $d/2$ and onto a lower plateau, which has length $d/2$ and where the coefficient of kinetic friction is again $\mu_k = 0.5$. If the block is still moving, it then slides up a frictionless ramp.

Where is the *final* stopping point of the block? If it is on a plateau, state which one and give the distance L from the *left* edge of that plateau. If the block reaches the ramp, give the height H above the lower plateau where it momentarily stops.



Solution: The easiest thing to do is just keep track of all the kinetic energy gains and losses of the block. Let it start with energy zero. Adding up the gains and losses, when the balance reaches zero again the block stops. Every time it moves down through a height d, it gains kinetic energy mgd. Every time it goes upward through a height d, it loses $-mgd$. Over the flat sections with friction, the friction force will be $f_k = \mu_k n = \mu_k mg$. Over a distance d, the work done by the friction force takes away a kinetic energy $f_k d = \mu_k mgd$ and it is lost to the surroundings.

We should label the points of interest to keep things straight. Let A be the leftmost point of the flat section of length d, B the rightmost point of that section, C the leftmost portion of the flat section of length $d/2$ and D its rightmost point. For the block to get to A, we move down through a height d, gaining mgd in kinetic energy. Proceeding step by step to point d, our energy balance reads

$$\text{to A:} \quad +mgd > 0 \tag{23}$$

$$\text{to B:} \quad mgd - \mu_k mgd = mgd\,(1-\mu_k) = \frac{1}{2}mgd > 0 \tag{24}$$

$$\text{to C:} \quad mgd\,(1-\mu_k) + mg\frac{d}{2} = mgd\left(\frac{3}{2} - \mu_k\right) = mgd > 0 \tag{25}$$

$$\text{to D:} \quad mgd\left(\frac{3}{2} - \mu_k\right) - \mu_k mg\frac{d}{2} = \frac{3}{2}mgd\,(1-\mu_k) = \frac{3}{4}mgd > 0 \tag{26}$$

Given $\mu_k = 0.5$, our net kinetic energy change at point D is $\frac{3}{4}mgd$, which is still positive. That means the block proceeds up the ramp, and the height H it reaches will be when the block's net kinetic energy at point d is equal to the gravitational potential energy mgH:

$$mgH = \frac{3}{2}mgd\,(1-\mu_k) \tag{27}$$

$$H = \frac{3}{2}d\,(1-\mu_k) = \frac{3d}{4} \tag{28}$$

After reaching this height, since the slope has no friction the block returns to point D with the same kinetic energy it had when it reached there the first time. Now we run back from point D to point C and keep going until the kinetic energy balance is zero, that's when the block will stop.

$$\text{back to C:} \quad \frac{3}{2}mgd\,(1-\mu_k) - \mu_k mg\frac{d}{2} = mgd\left(\frac{3}{2} - 2\mu_k\right) - \frac{1}{2}mgd > 0 \tag{29}$$

At this point, the balance is still positive, $\frac{1}{2}mgd$. Going back to point B requires moving upward through a height $d/2$, so we lose kinetic energy.

$$\text{back to B:} \quad mgd\left(\frac{3}{2} - 2\mu_k\right) - mg\frac{d}{2} = mgd\,(1-2\mu_k) = 0 \tag{30}$$

Reaching point B takes all the energy we have left, so the block stops exactly at point B while moving to the left, on the edge of the lower ramp. One could argue that it might just fall back of the edge and end up at point C, but we will assume that if the block can reach B it stays there.

More generally, one could have just argued that one will stop when the net change in height is equal to μ_k times the net lateral distance. In this case, ending at B, the net change in height is d, and laterally the block covered a total of $2d$, and $d = 2d\mu_k$. That doesn't really ensure a unique solution or the *first* point in the block's journey when that happens, so the rigorous thing to do is just go step by step as we have done.

6. The figure below shows a cord attached to a cart that can slide across a frictionless horizontal rail aligned along an x axis. The left end of the cord is pulled over a pulley, of negligible mass and friction and at cord height $h = 1.20\,\text{m}$, so the cart slides from $x_1 = 3.00\,\text{m}$ to $x_2 = 1.00\,\text{m}$. During the move, the tension in the cord is a constant $25.0\,\text{N}$. What is the change in the kinetic energy of the cart during the move? *Hint: One method is just to find $\int \vec{\mathbf{T}} \cdot dx$ over the cart's motion. A second and shorter method is to recognize how much work is done in creating the initial and final situations.*

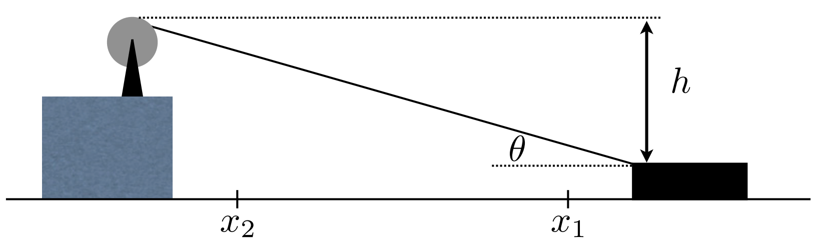


Figure 3: *A cart is pulled along a surface by a cord connected to a pulley.*

Solution: **Find:** The change in kinetic energy of a block pulled on a frictionless surface from x_1 to x_2 by a string with tension T a height h above the block.

Given: The string's tension, the geometry of the system.

Sketch: Already given.

Symbolic solution: There are two straightforward ways to do this problem. First, the more formal. The work done by the tension force of the rope must be equal to the block's change in kinetic energy. At any point during the block's motion, the angle at which the rope is pulling changes, which means that the horizontal force on the block changes. Let the x axis be horizontal, with the $+x$ direction running to the right, and the y axis be vertical with the $+y$ direction being upward. Let the origin be at the base of the platform holding the pulley.

At any horizontal distance x, we can resolve the rope's tension along the x and y axes using the geometry of the system:

$$\vec{\mathbf{T}} = -T\cos\theta\,\hat{\boldsymbol{\imath}} + T\sin\theta\,\hat{\boldsymbol{\jmath}} = \frac{-Tx}{\sqrt{x^2+h^2}}\,\hat{\boldsymbol{\imath}} + \frac{Th}{\sqrt{x^2+h^2}}\,\hat{\boldsymbol{\jmath}}$$

The block moves purely along the x axis, so an incremental displacement of the block can be written $d\vec{\mathbf{x}}=dx\,\hat{\boldsymbol{\imath}}$. The work done by the rope's tension is the integral of $\vec{\mathbf{T}}\cdot d\vec{\mathbf{x}}$ along the block's path:

$$W = \int_{x_1}^{x_2} \vec{\mathbf{T}}\cdot d\vec{\mathbf{x}} = \int_{x_1}^{x_2} \frac{-Tx}{\sqrt{x^2+h^2}}\,dx = -T\sqrt{x^2+h^2}\Big|_{x_1}^{x_2} = T\sqrt{h^2+x_1^2} - T\sqrt{h^2+x_2^2}$$

The work done by the tension must be equal to the change in the block's kinetic energy.

$$W = K_f - K_i = T\sqrt{h^2+x_1^2} - T\sqrt{h^2+x_2^2}$$

The second method is to recognize how much work is done in creating the initial and final situations. At the beginning, we have a certain length of rope l_i with a certain tension T applied. It takes $W_i=Tl_i$ worth of work to apply tension T to a length l_i of rope. At the end of the block's motion, we have a shorter length of rope l_f with the same tension applied. The difference in work required to tension the rope from start to finish must be the work applied to the block, and this difference must be gained by the block as kinetic energy. Simple geometry gives us the starting and ending lengths of rope:

$$K_f - K_i = Tl_f - Tl_i = T\sqrt{h^2+x_1^2} - T\sqrt{h^2+x_2^2}$$

Numeric solution: Given $T=25.0\,\mathrm{N}$, $h=1.20\,\mathrm{m}$, $x_1=3.00$ and $x_2=1.00$

$$W = \Delta K = T\left[\sqrt{h^2+x_1^2} - \sqrt{h^2+x_2^2}\right] = (25.0\,\mathrm{N})\left[\sqrt{1.20^2+3.00^2}\,\mathrm{m} - \sqrt{1.20^2+1.00^2}\,\mathrm{m}\right] \approx 41.7\,\mathrm{J}$$

7. A funny car accelerates from rest through a measured track distance in time T with the engine operating at a constant power P. If the track crew can increase the engine power by a differential amount dP, what is the change in time required for the run?

Solution: Since the power is fixed, the amount of work done by the car is just power times time. This work done must be the car's change in kinetic energy. Since the car starts at rest, the change in kinetic energy is just the final kinetic energy $\frac{1}{2}mv^2$, where m is the mass of the car.

$$W = PT = \Delta K = K_f = \frac{1}{2}mv^2 \tag{31}$$

We can solve this to get the final speed:

$$v = \sqrt{2PT/m} \tag{32}$$

The speed isn't of much use. The one other thing we know is that the length of the track is a fixed distance x. We would like to relate the power and time to this distance, since it is the only other thing we know. To do that, we can note that $v = dx/dt$ and separate and integrate the equation above.

$$\frac{dx}{dT} = \sqrt{2PT/m} \tag{33}$$

$$dx = \sqrt{2PT/m}\, dT \tag{34}$$

$$x = \int_0^x \sqrt{2PT/m}\, dT = \sqrt{\frac{4}{3m}} P^{1/2} T^{3/2} \tag{35}$$

How to proceed? What we want to figure out is how much T changes (δT) for some small change in power δP. First let's solve for P.

$$P = \frac{3mx}{4} \frac{1}{T^3} = P(x, T) \tag{36}$$

We know have P explicitly as a function of the distance x and time T. For small changes in x and T (δx and δT, respectively), the change in power δP for small δP would be given by a the first term in a Taylor series:[i]

$$P(x + \delta x, T + \delta T) \approx P(x, T) + \frac{dP}{dx} \delta x + \frac{dP}{dT} \delta T \tag{37}$$

$$\delta P = P(x + \delta x, T + \delta T) - P(x, T) = \frac{dP}{dx} \delta x + \frac{dP}{dT} \delta T \tag{38}$$

Since the track length is fixed, $\delta x = 0$, and

$$\delta P = \frac{dP}{dT} \delta T = -3 \frac{3mx}{4} \frac{1}{T^4} \delta T = -\frac{3}{T} \frac{3mx}{4} \frac{1}{T^3} \delta T = -\frac{3\delta T}{T} P \tag{39}$$

$$\frac{\delta T}{T} = -\frac{1}{3} \frac{\delta P}{P} \tag{40}$$

Thus, if we change the power by some fraction $\delta P/P$, the fractional change in time is only a third as much. The minus sign means that increasing power decreases time, which makes sense. Figuring out the changes in some function as a result of its variables changing a little bit is more generally known as propagation of uncertainty, and it is an important tool for assessing overall experimental uncertainty.

`http://en.wikipedia.org/wiki/Propagation_of_uncertainty`

8. A phenomenological expression for the potential energy of a bond as a function of spacing is given by

$$U(r) = \frac{A}{r^n} - \frac{B}{r^m} \tag{41}$$

For a stable bond, $m < n$. Show that the molecule will break up when the atoms are pulled apart to a

[i] Strictly speaking, we should use partial derivatives like $\partial P/\partial x$, but that is beyond the scope of our math prerequisites.

distance

$$r_b = \left(\frac{n+1}{m+1}\right)^{1/(n-m)} r_o \tag{42}$$

where r_o is the equilibrium spacing between the atoms. Be sure to note your criteria for breaking used to derive the above result.

Solution: The condition defining equilibrium is that the force vanishes, or equivalently that the potential energy is a minimum. If the equilibrium spacing is r_o, then

$$F(r_o) = -\left.\frac{dU}{dr}\right|_{r_o} = \frac{nA}{r_o^{n+1}} - \frac{mB}{r_o^{m+1}} \tag{43}$$

$$\frac{nA}{mB} = \frac{r_o^{n+1}}{r_o^{m+1}} = \frac{r \cdot r_o^n}{r \cdot r_o^m} = r_o^{n-m} \tag{44}$$

$$\Longrightarrow \qquad r_o = \left(\frac{nA}{mB}\right)^{\frac{1}{n-m}} \tag{45}$$

Is this really a minimum? We can check with the second derivative test: if $d^2U/dr^2 = -dF/dr > 0$ at r_o, have a maximum. We will need dF/dr shortly anyway. You didn't really need to do this on the test, since you were given that the stability condition $m<n$, but it is instructive:

$$-\frac{dF}{dr} = \frac{d^2U}{dr^2} = \frac{n\,(n+1)\,A}{r^{n+2}} - \frac{m\,(m+1)\,B}{r^{m+2}} \tag{46}$$

$$\left.\frac{d^2U}{dr^2}\right|_{r_o} = n\,(n+1)\,A\left(\frac{mB}{nA}\right)^{\frac{n+2}{n-m}} - m\,(m+1)\,B\left(\frac{mB}{nA}\right)^{\frac{m+2}{n-m}} \tag{47}$$

$$= \left(\frac{mB}{nA}\right)^2 \left[n\,(n+1)\,A\left(\frac{mB}{nA}\right)^{\frac{n}{n-m}} - m\,(m+1)\,B\left(\frac{mB}{nA}\right)^{\frac{m}{n-m}}\right] \tag{48}$$

$$= \left(\frac{mB}{nA}\right)^2 \left(\frac{mB}{nA}\right)^{\frac{n}{n-m}} \left[n\,(n+1)\,A - m\,(m+1)\,B\left(\frac{mB}{nA}\right)^{\frac{m-n}{n-m}}\right] \tag{49}$$

$$= \left(\frac{mB}{nA}\right)^2 \left(\frac{mB}{nA}\right)^{\frac{n}{n-m}} \left[n\,(n+1)\,A - m\,(m+1)\,B\left(\frac{nA}{mB}\right)\right] \tag{50}$$

$$= \left(\frac{mB}{nA}\right)^{\frac{n+2}{n-m}} \left[n\,(n+1)\,A - n\,(m+1)\,A\right] \tag{51}$$

$$= nA\left(\frac{mB}{nA}\right)^{\frac{n+2}{n-m}} \left[n-m\right] > 0 \tag{52}$$

Clearly, the only way this expression will be positive is if $n>m$, as previously stated. This means that the repulsive force has a higher index than the attractive force, and it is of shorter range.

What about breaking the molecule? For distances smaller than r_o, the force is repulsive, while for distances greater than r_o it is attractive – in either case, it serves to try and restore the equilibrium position. However, the competition between the shorter-range repulsive force and longer-range attractive force means that there is a critical distortion of the molecule for $r>r_o$ at which the force is maximum, and any stronger force (or larger displacement) will separate the constituents to an arbitrarily large distance – the molecule will be broken.

We have the force between the molecular constituents above:

$$F(r) = \frac{nA}{r^{n+1}} - \frac{mB}{r^{m+1}} \tag{53}$$

so we can readily calculate the maximum force with which the bond may try to restore its equilibrium. The force above is the force with which the molecule will respond if we push or pull on it.[ii] The maximum force will occur when $dF/dr = 0$, at a radius r_b

$$\frac{dF}{dr} = -\frac{n(n+1)A}{r_b^{n+2}} + \frac{m(m+1)B}{r_b^{m+2}} = 0 \tag{54}$$

$$\frac{n(n+1)A}{m(m+1)B} = \frac{r_b^{n+2}}{r_b^{m+2}} = r_b^{n-m} \tag{55}$$

$$\implies \qquad r_b = \left(\frac{nA}{mB}\right)^{\frac{1}{n-m}} \left(\frac{n+1}{m+1}\right)^{\frac{1}{n-m}} = r_o \left(\frac{n+1}{m+1}\right)^{\frac{1}{n-m}} \tag{56}$$

Now, how do we know this is the *maximum* force, and not a minimum force? We grind through another derivative ... we must have $d^2F/dr^2 > 0$ for a maximum:

$$\frac{d^2F}{dr^2} = \frac{n(n+1)(n+2)A}{r^{n+3}} - \frac{m(m+1)(m+2)B}{r^{m+3}} = r^{n+3}\left[n(n+1)(n+2)A - \frac{m(m+1)(m+2)B}{r^{m-n}}\right]$$

$$\left.\frac{d^2F}{dr^2}\right|_{r_b} = r_o^{n+3}\left(\frac{n+1}{m+1}\right)^{\frac{n+3}{n-m}} \left[n(n+1)(n+2)A - m(m+1)(m+2)Br_o^{n-m}\left(\frac{n+1}{m+1}\right)^{\frac{n-m}{n-m}}\right]$$

$$= r_o^{n+3}\left(\frac{n+1}{m+1}\right)^{\frac{n+3}{n-m}} \left[n(n+1)(n+2)A - m(n+1)(m+2)Br_o^{n-m}\right] \tag{57}$$

$$= r_o^{n+3}\left(\frac{n+1}{m+1}\right)^{\frac{n+3}{n-m}} \left[n(n+1)(n+2)A - m(n+1)(m+2)B\left(\frac{nA}{mB}\right)\right] \tag{58}$$

$$= r_o^{n+3}\left(\frac{n+1}{m+1}\right)^{\frac{n+3}{n-m}} \left[n(n+1)(n+2)A - n(n+1)(m+2)A\right] \tag{59}$$

$$= An(n+1)r_o^{n+3}\left(\frac{n+1}{m+1}\right)^{\frac{n+3}{n-m}} \left[n-m\right] > 0 \tag{60}$$

For the second to last line, we noted that $r_o^{n-m} = nA/mB$. Once again, if $n > m$, the second derivative is positive, and thus the force is maximum at r_b. Applying a force sufficiently strong to stretch the bond to a separation r_b will serve to break it. Incidentally, the maximum force required is

[ii] See HW9, problem 5.

$$F(r_b) = \frac{nA}{r_o^{n+1}} \left(\frac{n+1}{m+1}\right)^{\frac{n+1}{m-n}} - \frac{mB}{r_o^{m+1}} \left(\frac{n+1}{m+1}\right)^{\frac{m+1}{m-n}} = \left(\frac{n+1}{m+1}\right)^{\frac{n+1}{m-n}} \left[\frac{nA}{r_o^{n+1}} - \frac{mB}{r_o^{m+1}} \left(\frac{n+1}{m+1}\right)\right]$$

$$= \left(\frac{n+1}{m+1}\right)^{\frac{n+1}{m-n}} \left[nA \left(\frac{nA}{mB}\right)^{\frac{n+1}{m-n}} - mB \left(\frac{nA}{mB}\right)^{\frac{m+1}{m-n}} \left(\frac{n+1}{m+1}\right)\right] \tag{61}$$

$$= \left(\frac{n+1}{m+1}\right)^{\frac{n+1}{m-n}} \left(\frac{nA}{mB}\right)^{\frac{n+1}{m-n}} \left[nA - nA \left(\frac{n+1}{m+1}\right)\right] \tag{62}$$

$$= nA \left(\frac{n+1}{m+1}\right)^{\frac{n+1}{m-n}} \left(\frac{nA}{mB}\right)^{\frac{n+1}{m-n}} \left(\frac{m-n}{m+1}\right) = \frac{nA}{r_b^{n+1}} \left(\frac{m-n}{m+1}\right) \tag{63}$$

9. In the figure below, a small block of mass $m = 0.032\,\text{kg}$ can slide along the frictionless loop-the-loop, with loop radius $R = 12\,\text{cm}$. The block is released from rest at a point P, at height $h = 5.0R$ above the bottom of the loop. How much work does the gravitational force do on the block as the block travels from point P to **(a)** point Q and **(b)** the top of the loop? If the gravitational potential energy of the block-Earth system is taken to be zero at the bottom of the loop, what is that potential energy when the block is **(c)** at point P, **(d)** at point Q, and **(e)** at the top of the loop? **(f)** If, instead of merely being released, the block is given some initial speed downward along the track, do the answers to (a) through (e) increase, decrease, or remain the same?

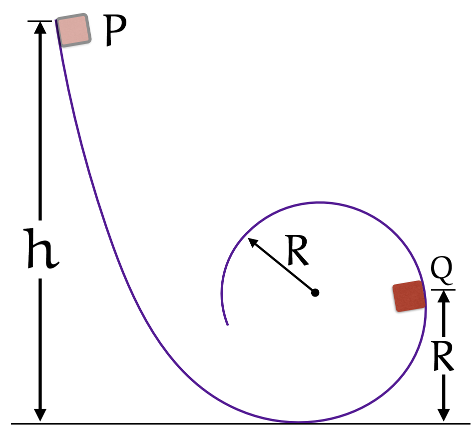


Figure 4: *A block slides along a loop-the-loop.*

Solution: Since there is no friction present, the work done by the gravitational force is just weight times change in height. Thus, for a and b we just need the change in height from the top of the loop to the points of interest.

$$\text{P to Q}: \quad W = mgh - mgR - mg(h-R) = mg(5.0R - R) = 4mgR = 0.15\,\text{J} \tag{64}$$

$$\text{P to top of loop}: \quad W = mgh - mr(2R) = mg(h - 2R) = 3mgR = 0.11\,\text{J} \tag{65}$$

If we call the ground level $U = 0$, the potential energy is just weight times the height above ground level.

$$\text{at P}: \quad U = mgh = 5mgR = 0.19\,\text{J} \tag{66}$$
$$\text{at Q}: \quad U = mgR = 0.038\,\text{J} \tag{67}$$
$$\text{at top of loop}: \quad U = mg(2R) = 0.075\,\text{J} \tag{68}$$

If the block had some starting kinetic energy, nothing changes above - the work done still just depends on the change in height. At each point the kinetic energy would be higher by the same amount (by the starting kinetic energy), but the *changes* would remain the same, since they are due only to the changes in height.

10. An 8.00 kg stone sits on top of a spring which is resting on the ground. The spring is compressed 10.0 cm by the stone sitting on it. **(a)** What is the spring constant k? **(b)** The stone is pushed down an additional 30.0 cm and released. What is the elastic potential energy of the compressed spring just before the release? **(c)** What is the change in the gravitational potential energy of the stone-Earth system when the stone moves from the release point to its maximum height? **(d)** What is that maximum height, measured from the release point?

Solution: The resting point of the stone sitting on the spring must occur when the stone's weight pulling down is equal to the restoring force of the spring pushing back up:

$$\sum F = kx - mg = 0 \tag{69}$$
$$k = \frac{mg}{x} \approx 785\,\text{N/m} \tag{70}$$

If the stone is pushed down an additional 30.0 cm, for a total of $x_{\text{tot}} = 40.0\,\text{cm}$ (or 0.40 m), the spring's potential energy is

$$U_{\text{spring}} = \frac{1}{2}kx_{\text{tot}}^2 \approx 62.8\,\text{J} \tag{71}$$

When the stone is released, all of the spring's potential energy is converted to kinetic energy. As the stone rises, kinetic energy is converted to gravitational potential energy. At the highest point, the kinetic energy is zero and all of the spring's original potential energy is converted to gravitational potential energy. Thus,

$$U_{\text{spring}} = U_{\text{grav}} = mgh = \frac{1}{2}kx_{\text{tot}}^2 \approx 62.8\,\text{J} \tag{72}$$

Given this, the maximum height is readily found in terms of given quantities.

$$h = \frac{kx_{\text{tot}}^2}{2mg} \approx 0.8\,\text{m} \tag{73}$$