Problem Set 4

Instructions:

1. Answer all questions below. Show your work for full credit.
2. The first two problems are due at the start of class on 3 Mar 2014
3. The next two problems are due at the start of class on 5 Mar 2014
4. The remaining problems are due by the end of the day on 7 Mar 2014
5. You may collaborate, but everyone must turn in their own work.

Daily problems due 3 Mar 2014

1. A bullet of mass m is fired at velocity v_i. It strikes a wooden block of mass M, resulting in a completely inelastic collision in which the bullet ends up embedded in the block. After the collision, block plus bullet move smoothly along a surface with coefficient of kinetic friction μ_k. **(a)** What is the velocity of the block plus bullet immediately after the collision? **(b)** How far does the block slide before coming to a stop?

2. A long uniform rod of length L and mass M is pivoted about a horizontal, frictionless pin through one end. The rod is released from rest in a vertical position. At the instant the rod is horizontal, find its angular speed. The moment of inertia of a solid rod about its center of mass is $I = \frac{1}{12}ML^2$.

Daily problems due 5 Mar 2014

3. A uniformly dense rope of length b and mass per unit length λ is coiled on a smooth table. One end is lifted by hand with constant velocity v_o. Find the force of the rope on the hand when the rope is a distance a above the table ($b > a$).

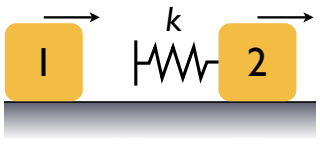
4. Block 1 of mass m_1 is moving rightward at v_1 while block 2 of mass m_2 is moving rightward at $v_2 < v_1$. The surface is frictionless, and a spring of constant k is fixed to block 2. When the blocks collide, the compression of the spring is maximum the instant the blocks have the same velocity.

(a) Show that

$$\Delta K = K_{1i} + K_{2i} - K_{12} = \frac{1}{2}\mu v_{\text{rel}}^2 \qquad \text{with} \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

where K_{1i} and K_{2i} are the kinetic energies of blocks 1 and 2 before the collision, respectively, K_{12} is the kinetic energy of the system at the moment the spring compression is maximum, and v_{rel} is the relative velocity of the two blocks. The quantity μ is known as the *reduced mass* of the system.

(b) Find the maximum compression of the spring.



The problems below are due by the end of the day on 7 Mar 2014.

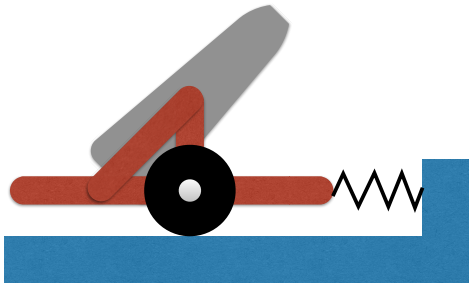
5. You are for some reason stuck on a rowboat on a calm lake, and you've lost all the oars. Inexplicably, you happen to have a large bucket filled with 500 baseballs in the boat. You decide to throw the baseballs, one by one, off of the back of the boat, figuring that conservation of momentum might get you to shore. You manage to throw the baseballs at a respectable 30.0 m/s, at a rate of 1 baseball per second, and estimate that each baseball has a mass of 0.145 kg, the rowboat has a mass of 80.0 kg, and your mass is 60.0 kg. Neglect friction, air resistance, and fluid drag forces.

(a) What is the speed that you and the boat achieve after tossing the n^{th} baseball? Don't forget the fact that the boat is getting lighter as you toss more and more baseballs out.

(b) How far has the boat gone after tossing the n^{th} baseball? You may neglect the amount of time it takes to actually toss the ball, and assume baseballs are emitted from the back of the boat instantaneously once every second.

6. A large howitzer is rigidly attached to a carriage, which can move along horizontal rails but is connected to a sturdy wall by a large spring, initially unstretched and with force constant $k = 1.90 \times 10^4$ N/m, as shown below. The cannon fires a 200 kg projectile at a velocity of 125 m/s directed 45.0° above the horizontal.

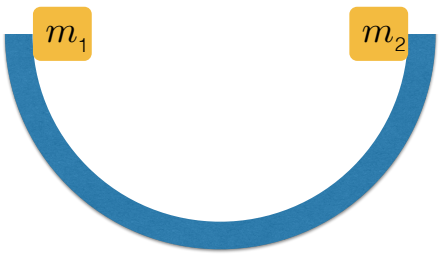
If the mass of the cannon and its carriage is 4780 kg, find the maximum extension of the spring.

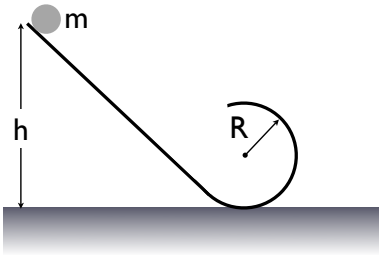


7. A cockroach with mass m rides on a disk of mass $6.00m$ and radius R. The disk rotates like a merry-go-round around its central axis at angular speed $\omega_i = 1.50$ rad/s. The cockroach is initially at radius $r = 0.800R$, but then it crawls out to the rim of the disk. Treat the cockroach as a particle. What then is the angular speed?

8. Two masses m_1 and m_2 are released opposite one another on the rim of a frictionless hemispherical bowl of radius R. **(a)** If the masses undergo a perfectly elastic collision, to what maximum height does each rise along the side of the bowl? **(b)** If the collision were completely inelastic (the masses stick together), to what height does the combined mass rise?

9. A solid brass ball of mass m will roll smoothly along a loop-the-loop track when released from rest along the straight section. The circular loop has radius R, and the ball has radius $r \ll R$. What is h if the ball is on the verge of leaving the track when it reaches the top of the loop? Assume the ball has a moment of inertia $I = kmr^2$, $k \in \mathbb{R} \,|\, 0 < k < 1$.





10. A uniform ball of mass M and radius R rolls smoothly down a ramp of angle θ. The *center* of the ball starts at a vertical height h from the bottom of the ramp. **(a)** How long does it take the ball to reach the bottom of the ramp? **(b)** How long would it take for a solid cylinder of mass M and radius R? **(c)** How about a pipe of mass M and radius R? Neglect the wall thickness of the pipe.